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A Simple Chaotic Model of Economic Growth  
and Unemployment

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## Abstract

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*The study of natural fluctuations has been of great interest for economists for decades. Although the earliest formal models of business cycles, around the 1950s, were endogenous (Hicks, Kaldor, Goodwin), the dominant strategy during the most of the last century has been to assume model specifications for which the equilibrium is determinate and intrinsically stable, so that in the absence of continuing exogenous shocks the economy would tend toward a steady state growth path. By the early 1980's, however, there was a revival of interest in the hypothesis that aggregate fluctuations might represent an endogenous phenomenon that would persist even in the absence of stochastic shocks to the economy. Some authors began to elaborate models with the objective of showing how the nonlinear internal dynamics of the economy generate complex behaviour that appears to be random, providing excellent fits to the behaviour of economic series. In these models, fluctuations result mainly from high level of impatience, imperfect markets or erroneous foresights of the individuals. The purpose of this paper is to show how complex behaviour can emerge from quite simple economic structures. In particular, when sufficient nonlinearities are present, the interaction of some essential elements of developed economies generates irregular fluctuations endogenously. The elements we will consider as representative of a developed economy are economic growth, sustained by knowledge accumulation, and unemployment.*

## Resumen

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*Desde hace décadas los economistas han mostrado un enorme interés por el estudio de las fluctuaciones. El objetivo de este trabajo es mostrar como modelos económicos sencillos pueden generar fluctuaciones muy complejas. Para ello elaboramos un modelo de crecimiento endógeno con acumulación de capital humano y desempleo. Este último es introducido mediante una dinámica de salarios, en particular una curva de Phillips no lineal. Mostramos cómo la introducción de desequilibrios genera dinámicas más complejas de la que habitualmente obtienen los modelos neoclásicos de crecimiento. La tasa de desempleo y la renta per capita fluctúan a lo largo de ciclos de diferentes periodos, e incluso trayectorias caóticas. En particular, cuanto mayor es el grado de desarrollo de la economía mayor es la posibilidad de obtener dinámicas complejas. Además, en la misma línea que los modelos de negociación de salario, obtenemos que cuanto mayor es el poder de negociación de los trabajadores, menores son la tasa de desempleo y la producción per capita.*

# 1 Introduction

The study of natural fluctuations has been of great interest for economists for decades. Although the earliest formal models of business cycles, around the 1950s, were endogenous (Hicks, Kaldor, Goodwin), the dominant strategy during the most of the last century has been to assume model specifications for which the equilibrium is determinate and intrinsically stable, so that in the absence of continuing exogenous shocks the economy would tend toward a steady state growth path. By the early 1980's, however, there was a revival of interest in the hypothesis that aggregate fluctuations might represent an endogenous phenomenon that would persist even in the absence of stochastic shocks to the economy. Some authors began to elaborate models with the objective of showing how the nonlinear internal dynamics of the economy generate complex behaviour that appears to be random, providing excellent fits to the behaviour of economic series. In these models, fluctuations result mainly from high level of impatience, imperfect markets or erroneous foresights of the individuals. The purpose of this paper is to show how complex behaviour can emerge from quite simple economic structures. In particular, when sufficient nonlinearities are present, the interaction of some essential elements of developed economies generates irregular fluctuations endogenously. The elements we will consider as representative of a developed economy are economic growth, sustained by knowledge accumulation, and unemployment.

The study of long run growth has given rise to one of the most fruitful areas in economic theory, the *Economic Growth* literature. The purpose of the economic growth literature has been to explain both long run growth in per capita income and the huge differences in the growth pattern over economies. One habitual assumption of the neoclassical literature of economic growth is full employment of labour. This assumption and the basic

hypotheses of neoclassical microeconomics give rise to a very simple dynamic behaviour: either an equilibrium point, or regular cycles. In our research we show how this simple behaviour changes when we consider labour market disequilibrium and some nonlinearity. The interaction of economic growth and unemployment (which is generally strongly fluctuating) is a dynamic process, under which each one affects the other, which is able to generate the irregular fluctuations that distinguish the economic series.

In this paper we elaborate a model of endogenous growth with human capital accumulation in the tradition of Lucas (1988). It is assumed that because of historical, cultural or sociological reasons, the economy decides the allocation of resources between the production of final goods and knowledge. The savings and investment decisions of the model is also found in Goodwin (1967). Finally, in order to introduce the unemployment effect, we consider non market real wage clearing, which is modelled by a non linear Phillips Curve.

The analysis of the dynamic behaviour is one of the most important points of our research. We will show how the introduction of disequilibrium will give rise to more complex dynamics than what is usually obtained in the neoclassical growth model. This analysis shows that both unemployment rate and per capita income dynamics fluctuate along cycles of different periods, and they may even have chaotic paths. In particular, as the level of knowledge development and the rigidity of the labour market increases, the possibility of complex dynamics increases as well. Besides cyclical behaviour, the dynamic analysis points to a positive income growth trend- sustained by knowledge accumulation- but there are periods where per capita income decreases. Moreover, in the same line as wage bargaining models (Layard and Nickell, 1985, 1986), we get the result that the higher is workers' bargaining power, the lower are both the employment rate and per capita production.

The rest of the paper is organized as follows. In section 2 we explain the model and its main hypothesis. In section 3 we analyze the dynamic behaviour of the model's variables. In Section 4, we extend the wage dynamics foundation with a more realistic assumption regarding wage behaviour. The conclusions of the paper are summarized in section 5.

## 2 The model

### 2.1 Final Good and Knowledge Production

Consider a single economy that uses labour  $L_t$  and knowledge  $h_t$  to produce a final good  $Y_t$ . The market for the final good is perfectly competitive. The aggregate production function is:

$$Y_t = F(L_t, h_t) = \mu (\gamma h_t L_t)^\alpha \hat{h}_t^\beta, \quad 0 < \alpha < 1, 0 < \gamma < 1, \beta > 0, \mu > 0$$

where  $\mu$  is the sector productivity.  $\gamma$  is the constant exogenous fraction of time that people devote to the production of final goods. Here, the individual's knowledge  $h_t$  multiplies labor inputs, so the labour input is measured in efficient units.

In addition to the effects of an individual's knowledge on his own productivity, we also consider an external effect (Lucas, 1988). Specifically, the average level of knowledge  $\hat{h}_t$  also contributes to the productivity of all factors of production. If we assume that all workers are identical, the average level of knowledge  $\hat{h}_t$  is just  $h_t$ , in which case output production  $Y$  can be rewritten as

$$Y_t = F(L_t, h_t) = \mu (\gamma L_t)^\alpha h_t^{\alpha+\beta} \tag{1}$$

For simplicity, we consider constant physical capital<sup>1</sup> which could be included in the

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<sup>1</sup> This hypothesis is habitual in some traditional models of endogenous growth (Aghion-Howitt, 1998).

parameter  $\mu$ . We can assume a economy where physical accumulation is stable and only by accumulating human capital can the economy obtain production improvements.

Besides producing final goods, the economy also produces knowledge. In the literature on economic growth, some authors separate the rival component of knowledge, that is, knowledge embodied in some kind of tangible capital such as conventional physical capital or human capital, from the non rival or intangible component (Romer, 1990). But, in the tradition of Lucas, we consider and formalize a more general concept of human capital. According to Lucas, human capital accumulation is a “*social* activity” which survives individuals, and so it can be accumulated without limit and it guarantees long run economic growth. Consequently, we are not going to distinguish between private human capital of individuals and knowledge of the society as a whole, therefore  $h$  can be interpreted as a composite good made up of knowledge and human capital.

The knowledge technology is<sup>2</sup>

$$h_{t+1} = e^{\delta(1-\gamma)} h_t, \quad 0 < \delta < 1 \quad (2)$$

where  $(1 - \gamma)$  is the fraction of time devoted to knowledge accumulation.  $\delta$  is the productivity of the sector. Equation (2) is an approximation to Lucas’s knowledge technology:

$$h_{t+1} = h_t + h_t \delta (1 - \gamma) \quad 0 < \delta < 1$$

We consider equation (2) because it simplifies the later analysis considerably.

## 2.2 Consumers-Workers

Each individual is endowed with one unit of time that can be used either in producing the final good and or in producing knowledge. Labour is supplied inelastically. At the end

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<sup>2</sup> The model can be easily rewritten in terms of a two sector economy, in which there exists a constant proportionality between the populations devoted to the productions of final goods and knowledge, and workers are paid at the same wage.

of each period, individuals receive all the income from labour supplied in these activities, and all labour is paid at the same wage denoted by  $w_t$ . We assume that all income is spent on consumption. So, in each period the total demand for the final good is equal to the wage income paid at the end of the previous period:

$$D_{t+1} = C_{t+1} = w_t L_t \quad (3)$$

where  $D_{t+1}$  is total demand in period  $t + 1$ ,  $C_{t+1}$  is total consumption in period  $t + 1$ ,  $L_t$  is total employment and  $w_t L_t$  is the total wage income paid at the end of period  $t$ .

On the other hand, we assume that the population  $N$  grows at the fixed exogenous rate  $n$ . Moreover, we assume, consistent with the empirical evidence<sup>3</sup>, that the total population is a constant fraction of the labour force  $A$ ,  $N = \lambda A$  ( $\lambda > 1$ ). Thus

$$\frac{A_{t+1}}{A_t} = \frac{N_{t+1}}{N_t} = 1 + n, \quad n > 0 \quad (4)$$

### 2.3 Production of Final Good and Labour Demand

We assume the productive sector produces whatever individuals demand. The production process for the final good takes one period. In each period, final production is equal to the total demand in the next period

$$Y_t = D_{t+1}$$

From this last condition, we get the total labour demand. At the beginning of each period, the productive sector demands labour in order that supply equals the demand for goods in the next period. Substituting the demand and supply expressions

$$\mu (\gamma L_t)^\alpha h_t^{\alpha+\beta} = w_t L_t \quad (5)$$

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<sup>3</sup> For example, in Spain this relation is rather stable, although less than in other European countries, except during particular periods such as when women were incorporated into the labour market.

and solving for  $L_t$  we get the total labour demand of the economy

$$L_t = \left( \frac{\mu\gamma^\alpha h_t^{\alpha+\beta}}{w_t} \right)^{\frac{1}{1-\alpha}}$$

Given  $h$  and  $w$ , if the productive sector demands less than  $L_t$ , we have excess supply of the final good  $Y_t > w_t L_t$ . Labor is very productive and demand is insufficient to absorb supply. To provide an incentive for demand, the productive sector needs to employ more workers, which generates more income and, consequently, greater demand. This result is shown in figure 1, where we have drawn the production and cost functions for particular values of  $h_t$  and  $w_t$ . There are no incentives to employ more than  $L_t$  because firms would suffer losses. In this case, there is a continuum of excess demand,  $Y_t < w_t L_t$ , and the level of employment (and therefore production) does not increase because the demand that is generated would be unsatisfied. Very strong and rigid diminishing returns to labor in productive capacity emerge. It is necessary to introduce new investments while firms fire workers in order not to suffer losses. We are considering the profits that firms get by only producing final goods,  $B^o = Y_t - w_t L_t$ . As will be explained in the next section, total profits are received both from final goods and knowledge production. Increasing productive efficiency -which would shift the production function in figure 1 upwards- would allow the excess demand to be satisfied and employment and production would also be increased<sup>4</sup>. Therefore, the  $L^*$  which satisfies (5) is the equilibrium level of employment.

(Figure 1 see below)

In the dynamic analysis we will show that  $L_t$  may not equal labour supply, so we could get unemployment  $L_t < A_t$ . Similarly, we could get  $L_t > A_t$ , that is, the production

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<sup>4</sup> Later on, we will show that this is one of the reasons why employment depends positively on technological change.

sector's demand for labour could be rationed. Because our objective is to explain the dynamic relationship between growth and unemployment, this last case will not be analysed. Therefore, we make the necessary assumptions in order that  $L_t \leq A_t$ , and the market for the final good is always in equilibrium<sup>5</sup>.

## 2.4 Savings and Investment Decisions

The savings and investment decisions are in line with Goodwin (1967): all the wages are consumed, all savings automatically invested. If  $L_t$  is used in the production of final goods, that is, if the economy does not invest in knowledge, the final production (potential product) would be  $\tilde{Y}_t$ :

$$\tilde{Y}_t = \mu L_t^\alpha h_t^{\alpha+\beta}$$

So, if all the labour is used in the production of final goods the economy will get more product than if some labour is allocated to the production of knowledge. The difference between the two production levels is:

$$\mu (L_t)^\alpha h_t^{\alpha+\beta} - \mu (\gamma L_t)^\alpha h_t^{\alpha+\beta} = (1 - \gamma^\alpha) \tilde{Y}_t$$

This equation shows the opportunity cost of the use of labour inputs in the production of knowledge instead of final goods. This difference is the final good production that the economy renounce, and can be interpreted as the total savings of the economy. These savings are invested ex-ante in the production of knowledge. Therefore  $\gamma^\alpha$  is the fraction of income that individuals devote to consumption and  $1 - \gamma^\alpha$  is the fraction they devote to saving or investing in knowledge.

Although the production sector is that which produces knowledge, it is the economy itself that decides to invest in knowledge. On the one hand, workers invest in knowledge

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<sup>5</sup> This is a usual assumption in the related literature (see Pohjola, 1981).

in order to improve their skills and get a better job in the future. On the other hand, firms invest in knowledge in order to either improve the productivity of the labour input or to get process and product innovations<sup>6</sup>. Therefore, the fractions of time devoted to each activity, although exogenous, are a decision of the entire economy. These decisions can be interpreted as cultural or social characteristic. For simplicity in the following analysis we assume it is exogenous. However, we should point out that the choice of  $\gamma$  determines the time evolution of the economy. Two similar economies with different  $\gamma$  will follow different time paths. In a specific way, this idea is similar to Lucas (1988, pag. 19), where he states that “human capital is a social activity, involving groups of people”<sup>7</sup>.

## 2.5 Wage Dynamics

The labour market is assumed to be in disequilibrium and the wage dynamics is determined by this disequilibrium. In particular, we assume a Phillips equation, where the wage is increasing with the rate of employment

$$\frac{w_{t+1}}{w_t} = h(l_t), \quad h'(l) > 0$$

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<sup>6</sup> As was explained earlier, the Lucas concept of knowledge is very general, and it does not exclude the possibility of knowledge embodied in the physical capital of the firm. In this case the economy decides to invest in knowledge because by doing so it increases the tangible part of its capital.

<sup>7</sup> In the same line, Jones (2001) relates the exogenous fraction of time devoted to knowledge accumulation with the role of institutional development, which promotes the production of knowledge with the creation, definition and enforcement of property rights. Through numerical simulations, he shows that the improvement in property rights in the 20th century played a critical role in the timing of Industrial Revolution.

where  $l_t = \frac{L_t}{A_t}$  is the rate of employment. In particular, we consider the following nonlinear curve<sup>8</sup> :

$$\frac{w_{t+1}}{w_t} = \exp(-a + bl_t), \quad b > a > 0 \quad (6)$$

Although the function that is normally known as the “Phillips curve” is  $\varphi(l) = \frac{w_{t+1} - w_t}{w_t}$ , abusing language somewhat, we also use the term “Phillips curve” to refer to the function  $h(l)$ . Both have the same properties because  $h(l)$  is a translation of  $\varphi(l)$ :  $h(l) = 1 + \varphi(l)$ . The functional relationship between the wage dynamics and the rate of employment is illustrated in figure 2.

(Figure 2 see below)

This modeling<sup>9</sup> is based on the ideas of Phillips (1958) and Lipsey (1960) regarding the non linearity of the Phillips curve<sup>10</sup> (“excess demand conditions are much more inflationary than excess supply conditions are disinflationary” as stated by Clark et al. (1996)) and the keynesian ideas about downward wage rigidity due to market imperfections analyzed in the *New Keynesian Economics* literature (Mankiw and Romer, 1991). As far as the dynamics is concerned, we try to introduce, in a simple way, the labour market

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<sup>8</sup> Introducing imperfections in the labour market through wage dynamics is usual in disequilibrium models with money (Chiarella and Flaschel, 1999 and Chiarella et al., 2000) and regime switching models (Ito, 1978, 1980; Picard, 1983). Generally, the theoretical elements of bargaining models are modelled in terms of a “wage curve” (Blanchard and Oswald, 1995) which captures the inverse relation between employment and wages instead of their growth rate. However, Chiarella et al. (2000) assert: “the theory based level form formulations of such wage and price equations should be reducible to rates of growth, possibly considering demand as well as cost pressure terms”. So, in their work (chapt. 5) they reformulate the wage curve in terms of growth rates, introducing theoretical elements of wage bargaining models, insiders-outsiders and hysteresis in a Phillips Curve. We think that the formulation in growth rates, as in the model of Chiarella et al., provides a better fit to the formal structure of our model.

<sup>9</sup> For simplicity, Goodwin (1967) introduces a linear approximation to  $h(l_t)$ , but, as he said, the linear approximation is “quite satisfactorily for moderate movements of  $l$  near to the point 1”. Also, the Goodwin model presents an economic inconsistency:  $l$  could be greater than one. Desai et al. (2003) prove that by introducing a nonlinear Phillips curve or assuming that a fraction of capitalist profits are not invested  $l < 1$  is guaranteed.

<sup>10</sup> There are a number of empirical papers from the 1990s, concerning the nonlinearities or *asymmetries* between wage changes and unemployment. See Chada, Masson and Meredith (1992); Laxton, Meredith and Rose (1995); Clark, Laxton and Rose (1996).

disequilibrium caused by different imperfections which explain wage and price rigidity from the optimizing behavior of individuals (efficiency wage, insiders-outsiders and the wage bargain between employers and unions). In particular, and in the same line as models of wage bargaining, the labour market situation is interpreted as a measure of the workers' bargaining power. Near to full employment firms raise wage rates in order to attract the most suitable workers. However, when unemployment is high, workers are unwilling to accept wage reductions.

Finally, regarding  $a$  and  $b$  we make the following assumptions. On the one hand, in order to guarantee that wages do not rise when  $l = 0$ , it is necessary that  $a > 0$ , that is,  $e^{-a} < 1$ . On the other hand, under full employment, the growth of wages must be high, so that  $b - a$  is large, in particular we require  $e^{b-a} > 1$ . Therefore we assume  $b > a$  and  $a > 0$ . Usually these parameters are characteristic of labour market (Pohjola, 1981). It will be discussed later.

### **3 Dynamic Analysis**

#### **3.1 The dynamics of $l_t$**

We begin with the analysis of the dynamics of the employment rate  $l$ . The analysis shows how complex behaviour can emerge from the rather simple economic structures described above. When sufficient nonlinearities are present and the labour market is in disequilibrium, the interaction between unemployment and the accumulation of knowledge alone can lead to growth cycles that exhibit behaviour that is more complex than what is obtained in neoclassical growth models. However this is an appropriate qualitative description of the observed oscillations in the real world.

Beginning with the decision regarding the production of final goods we get:

$$\frac{Y_{t+1}}{Y_t} = \frac{D_{t+2}}{D_{t+1}}$$

and substituting the expressions for production and demand:

$$\left(\frac{h_{t+1}}{h_t}\right)^{\alpha+\beta} \left(\frac{L_{t+1}}{L_t}\right)^\alpha = \frac{w_{t+1}}{w_t} \frac{L_{t+1}}{L_t}$$

Using equations (2) and (6), taking logarithms and then exponentials, we obtain the following nonlinear dynamic equation for the employment rate:

$$\frac{l_{t+1}}{l_t} = \frac{1}{1+n} \exp\left(\frac{(\alpha+\beta)\delta(1-\gamma)+a}{(1-\alpha)}\right) \cdot \exp\left(\frac{-bl_t}{(1-\alpha)}\right)$$

Letting  $r = \frac{1}{1+n} \exp\left(\frac{(\alpha+\beta)\delta(1-\gamma)+a}{(1-\alpha)}\right)$  and  $s = \frac{b}{(1-\alpha)}$  the equation is reduced to a one-dimensional, nonlinear discrete time dynamical system:

$$l_{t+1} = f(l_t) = r \exp(-sl_t) l_t \tag{7}$$

(where we assume that  $b + \alpha > 1$  and  $r < es$  in order to guarantee that  $L_t < A_t$  is always satisfied, that is  $l < 1$ ). Figure 3 shows  $f(l_t)$  for a selection of values of  $r$ .

(Figure 3 see below)

Equation (7) is known as the Ricker-Moran equation (Ricker, 1954; Moran, 1950; Cook, 1965; Macfadyen, 1963), and it has been employed in ecology, especially in the study of fish populations. The dynamics of this equation is well documented in the ecological and mathematical literature (May 1976). Because its dynamical behaviour has been well studied, we will briefly summarize the main results (see May, 1975; May and Oster, 1976).

Table 1 summarizes the dynamics for each value of  $r$  (the parameter  $s$  affects the value of equilibrium points, but does not affect the behaviour of the dynamical system).

(Table 1 see below)

The transition to chaotic behaviour can be assessed in the bifurcation diagram (figure 4). This shows how increasing  $r$  gives rise to a sequence of bifurcating stable points of period  $2^n$ . Note that although this bifurcation process produces an infinite sequence of cycles of period  $2^n$  ( $n \rightarrow \infty$ ) it is bounded by some parameter value, in particular,  $r_c \approx 14.767075$ . Beyond the limit point  $r_c$ , we enter in the “chaotic” regime characterized<sup>11</sup> by a finite number of attracting fixed points, an infinite number of repelling fixed points, and an uncountable number of points (initial conditions) whose trajectories are totally aperiodic. Periodic doubling is an example of a “route to chaos.”- the way in which the dynamics of a system change as a parameter is changed, leading ultimately to the appearance of chaos.

(Figure 4 see below)

Figure 5 shows the erratic behaviour which is a characteristic of a regime of chaos that is generated by the Ricker-Moran equation for a value of  $r$  in the chaotic regime,  $r = 18$ . Taking standard values for the parameters,  $\alpha = 0.7, \beta = 0.1, \gamma = 0.7, \delta = 0.1$  and  $n = 0.02$  (Lucas, 1988; Barro and Sala-i-Martin, 1995), and  $a = 0.8$  y  $b = 2.7$  the possibility of chaos is “reasonable” from an economic point of view.

(Figure 5 see below)

Let us interpret the dynamics of  $l$  in terms of the original parameters. The parameter  $b$  affects the employment rate but it does not affect the dynamic behaviour. The greater the value of  $b$ , that, the greater is  $s = \frac{b}{1-\alpha}$ , the lower is the employment rate. Because  $s$  can be interpreted as reflecting workers’ bargaining power, since, given  $a$ , an increase in

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<sup>11</sup> A sufficient condition for the existence of topological chaos, may be established by the Li-Yorke theorem “Period Three Implies Chaos” (1975).

$b$  (that is in  $s$ ) implies a greater growth of the wage (Pohjola, 1981), the model implies (in the same way as bargaining wage models (Layard and Nickell, 1985, 1986)) that a lower unemployment rate goes along with greater bargaining power. Taking into account the parameters that determine  $r = \frac{1}{1+n} \exp\left(\frac{(\alpha+\beta)\delta(1-\gamma)+a}{(1-\alpha)}\right)$ , the employment rate and the possibility of chaos increase with the effectiveness of investment in human capital  $\delta$ , the fraction of time devoted to knowledge accumulation  $(1-\gamma)$ , the external effect  $\beta$ , the production function returns  $\alpha$  and the parameter  $a$ . As was discussed earlier, in economies with excess demand, increasing knowledge raises employment because of the increase in demand, and thus in production, more than the increase in productivity. For this reason, the employment rate depends positively on the knowledge parameters<sup>12</sup>.

The parameter  $r$  is formed by the knowledge production parameters and the Phillips curve's parameter  $a$ ; weighted by the rate of population growth. For one hand, given  $a$ , the parameter  $r$  can be interpreted as reflecting the medium level of development of the knowledge sector. Hence, the model predicts that  $l$  and the probability of chaos increase with the level of development of the knowledge sector<sup>13</sup>. Because most developed societies have a high level of development of the knowledge sector, the model also predicts that higher levels of economic development are to be associated with higher probabilities of complex dynamics. On the other hand, the parameter  $r$  could be also interpreted to reflect labour market rigidity since  $a$  determines the elasticity of the wage's growth to

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<sup>12</sup> Most of the economic literature has shown the negative relation between growth and unemployment (Pissarides, 1990; Bean and Pissarides, 1993), except for the positive relation between unemployment and growth (technological unemployment) in "creative destruction" models (Aghion and Howitt, 1992) based on the ideas of Schumpeter (1934). Bean et al. assessing the empirical evidence, suggest that the relationship between both phenomenon is negative in the short run, and it could be either positive, negative or null in the long run. The sign is ambiguous because both variables can be endogenously and jointly determined, which implies that the relationship between them depends on exactly which economic structure we are considering.

<sup>13</sup> However, although empirical evidence shows that the rate of employment and the irregularity of its dynamic behaviour, usually depend positively on the level of development, some developed economies have shown high rates of unemployment (even though not as high as developing countries).

the employment rate. As is known, the more sensitive growth wage to changes of the employment rates the higher flexibility in the labour market. Since an increase in  $a$  (that is in  $r$ ) implies a lower elasticity, the model predicts that the more imperfect the labour market<sup>14</sup> the greater the possibility of complex dynamics<sup>15</sup>. These results agrees with the ideas of Keynes in the *General Theory* (1936, chap.22), and Schumpeter, in *The Theory of Economic Development* (1934, chap.VI), regarding the economic development of capitalist societies. As the level of development increases rigidities and market imperfections begin to interfere with this development, oscillations become more irregular.

As far as population growth is concerned, we find that the employment rate and irregular behaviour decrease with population growth. Given  $L$ , the greater is the population size  $A$ , the lower is the employment rate,  $l = \frac{L}{A}$ . The historical evolution of population growth (Kremer, 1993; Galor and Weil, 2000) shows a consistent fall in fertility rates in developed economies. So, in line with the previous paragraph, a higher level of development is associated with lower population growth and irregular dynamic behaviour.

### 3.2 The dynamics of $y_t$

The results obtained in the last section concern the employment rate. They can, however, be referred to the rate of economic growth:

$$\frac{y_{t+1}}{y_t} = \left(\frac{h_{t+1}}{h_t}\right)^{\alpha+\beta} \left(\frac{l_{t+1}}{l_t}\right)^{\alpha} (1+n)^{\alpha-1} = z \cdot \left(\frac{l_{t+1}}{l_t}\right)^{\alpha} \quad (8)$$

where  $z = e^{\delta(1-\gamma)(\alpha+\beta)} (1+n)^{\alpha-1}$ . So, the rate of economic growth depends of the evolution of the employment rate and the parameter  $z$ . The qualitative conclusions obtained concerning the dynamics of the employment rate apply directly to the growth rate.

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<sup>14</sup> The rigidity of the labour market is explained from different imperfections such as cost of mobility, informational imperfections, mismatch between the workers looking for jobs and the vacancies available, minimum wages and union wage setting, etc.

<sup>15</sup> The parameter  $b$  also determines the elasticity of the wage, but it does not affect the dynamic behaviour of the unemployment rate.

The numerical simulations show that, whenever  $z < 1$ , the growth of knowledge productivity does not absorb population growth, and the per capita production of the economy  $y_{t+1}$  tends to zero. This is because we have assumed that  $h$  depends on the fraction of time that individuals devote to knowledge production instead of the total number of workers employed in this sector<sup>16, 17</sup>.

However, in order for the economy to disappear, the values which must be taken by the rate of population growth are unrealistic<sup>18</sup>. Considering realistic values of  $n$ , the economy would disappear only if it does not invest in knowledge ( $\gamma = 1$ ) or if it does at an infinitesimal level<sup>19</sup>. So, in the same way as traditional models of endogenous growth, sustained growth is due to knowledge production, in particular, to the productivity  $\delta$  and the fraction of time devoted to knowledge accumulation ( $1 - \gamma$ ).

Equation (8) also shows that the rate of economic growth increases with the knowledge external effect,  $\beta$ . This result is in agreement with Lucas (1988) for high levels of risk aversion. As far as worker's bargaining power  $b$  goes, lower levels of the rate of economic growth are associated with higher bargaining power; this is because greater bargaining

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<sup>16</sup> This assumption eliminates the so-called "scale effect" in the literature of economic growth: the population size affects human capital accumulation positively and, therefore, it also has a positive effect on economic growth. In our model, if we consider that human capital accumulation depends on the total number of workers employed in this sector (Romer, 1990; Grossman and Helpman, 1991; Aghion and Howitt, 1992) the scale effect would cause a overflowing on human accumulation, economic growth and employment when the population's size is large. In particular, economies with large populations would not have unemployment. Moreover, we must choose the initial population size, which is too arbitrary. So we have chosen modeling (2).

<sup>17</sup> The inverse relation between population and economic growth rates reflects the Malthusian idea that the greater is population growth the lower is per capita income.

<sup>18</sup> See Kremer (1993) for the evolution of world population growth rate since one million years C.B until 1990. The greatest rate of growth was 2.01% in 1960. So, for example, for the parameters values that we have considered and  $n = 2\%$ ,  $z = 1.01822$ .

<sup>19</sup> If we had considered endogenous population growth, so that  $n$  depends on income growth (as is assumed in endogenous population growth models) this result would coincide with the Malthusian idea concerning the stagnation of the economy. As the population rises, in a non industrialized economy and with limited resources, at some moment the amount of food falls below the subsistence level. This leads to a situation in which the economy is stagnant, and where the population no longer rises.

power implies greater unemployment, and so, less resources are used<sup>20</sup> .

If  $z > 1$ , per capita income shows a long term positive growth trend that is sustained by knowledge production, and its dynamics behaviour depends on the dynamics of the employment rate<sup>21</sup> . The qualitative conclusions obtained about the dynamics of the employment rate apply directly to per capita income  $y_t$ . Again, both the flexibility of the labour market and the level of development of knowledge sector determine the complex dynamics of per capita income. For example, for a value of  $r$  in the chaotic regime  $r = 18$  (figure 6) the trajectory of  $y_t$  is quite irregular. This behaviour provides an excellent match to the behaviour of the series of per capita income<sup>22</sup> . Again, workers' bargaining power does not affect the dynamics of per capita income.

(Figure 6 see below)

Finally, although for all numerical simulations the model generates a positive income growth trend, in figure 6 we can see periods in which per capita income decreases  $\frac{y_{t+1}}{y_t} < 1$ , which is consistent with what we see in the real world. Most of the neoclassical theory of growth focuses on explaining the positive trend of income growth, and it does not normally take into account the existence of the periods in which income decreases, and in those cases when it does the decreases are generated exogenously. The singularity of our model is that the nonlinear nature of the model generates both the growth trend and the possibility of decreases in income without requiring exogenous shocks.

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<sup>20</sup> This result coincides with other growth models that introduce the unemployment assumption (Davieri and Tabellini, 2000; Alonso, Echevarría and Tran, 2002)

<sup>21</sup> For  $z = 1$ , the positive growth trend disappears. However, the parameters values for which  $z = 1$ , in particular the value of  $n$ , are not realistic for the same reason that we have already explained for  $z < 1$ .

<sup>22</sup> In figure 6,  $a = 0.62$ . The rest of parameters values are the standard values.

## 4 Wage Dynamics Extension

Looking at the trajectories of employment and per capita income, the sizes of the oscillations of the time series are too large when compared with their true (real world) behaviour. The reasons for this result are the simple nonlinear modelings in some of the economic relationships that we have assumed, for example the wage dynamics. In this section we will now show how introducing more realistic assumptions about the non linearity of the Phillips curve, in particular for  $l$  values with neither full employment nor total unemployment, the size of the oscillations decreases and are close to full employment. In particular, we consider the following alternative modeling of the Phillips Curve:

$$\frac{w_{t+1}}{w_t} = h(l_t) = \exp(-a + bl_t^\sigma + c(1 - l_t)^{-\varepsilon}) \quad (9)$$

with  $a, b, c > 0$ ,  $0 < \sigma < 1$ ,  $0 < \varepsilon < 1$ , the resulting behaviour is illustrated in figure 7.

(Figure 7 see below)

We have made the following assumptions. First, we assume there is a large area, corresponding to an interval  $(\varepsilon_1, 1 - \varepsilon_2)$  with  $\varepsilon_1$  and  $\varepsilon_2$  small, where wages are very insensitive to employment changes. This modeling considers, in a more precise way, wage behaviour far from either full employment or total unemployment, and it is based on Rose<sup>23</sup> (1967): “In some neighbourhood of the  $l$  at which unemployment is balanced by unfilled vacancies, frictions and imperfections weaken the responsiveness of wage inflation to changes in  $l$ ”.

Secondly, we assume a vertical tangency  $l = 0$  in order to capture the rapid fall of wages<sup>24</sup> when the rate of unemployment is very close to 100% ( $l$  close to 0). Finally, we

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<sup>23</sup> Rose introduces a kind of nonlinearity in Phillips Curve with the purpose of preventing local explosive dynamics. The qualitative behaviour which he obtained is similar to the dynamic behaviour that we get with the curve assumed here.

<sup>24</sup> Some macroeconomic growth models with money and disequilibrium (Chiarella et al., 2000; Chiarella and Flaschel, 2000) assume the so-called “kinked” Phillips Curve. This curve is an extreme case of the

assume a vertical asymptote at  $l = 1$ . This asymptote is designed to reflect the fact that near to full employment it is more and more difficult to hire workers<sup>25</sup> and the labour market pressure accelerates the growth of wages; which near to full employment would be huge.

Taking into account the Phillips curve (9), the dynamics of the employment rate are

$$l_{t+1} = f(l_t) = \frac{1}{1+n} \exp\left(\frac{(\alpha + \beta)\delta(1 - \gamma) - q(l_t)}{(1 - \alpha)}\right) \quad (10)$$

where now  $q(l_t)$  is

$$q(l_t) = -a + bl_t^\sigma + c(1 - l_t)^{-\varepsilon}$$

In figure 8 we have drawn the intertemporal evolution of  $l_t$  for normal parameter values and setting the parameters of the curve<sup>26</sup> (9) at  $a = 1.716, b = 1.455, \sigma = 0.01, \varepsilon = 0.1$ . The resulting  $l_t$  dynamics are complex and irregular, but now since we are close to full employment values, they provide an excellent fit to empirical evidence<sup>27</sup>.

(Figure 8 see below)

In the same way as in the previous section, complexity is transferred to per capita production dynamics. As is shown in figure 9, the intertemporal evolution of  $y_t$  follows a cyclical growth with a positive trend, but with the possibility of per capita production net

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nonlinear Phillips Curve because it introduces downward rigidity in nominal wages; wages are constant if the growth rate is negative. They affirm that this curve is more realistic than assuming the possibility of wage decreases. The objective of this work is to extend the habitual nonlinearity of the Phillips Curve, a “kinked curve”, and to use numerical simulations to show how the dynamical behaviour of the model changes drastically and would be able to generate chaotic paths.

<sup>25</sup> This assumption is very usual both in theoretical work which introduces the Phillips Curve (Phillips, 1958; Lipsey, 1960; Samuelson and Solow, 1960); Rose, 1967; Chiarella and Flaschel, 1996) and in empirical work (Chada, Masson y Meredith, 1992; Laxton, Meredith and Rose, 1995; Clark, Laxton and Rose, 1996).

<sup>26</sup> As in the previous section, the parameter values of the Phillips curve have been chosen so that the case  $l > 1$  is excluded.

<sup>27</sup> The dynamics behaviour of (10) is qualitatively similar to the function (7). Both are unimodal functions which generate cyclical temporal evolution of the rate of employment. The difference is that now the oscillations are much more close to full employment values.

decreases at some periods. The only difference is that now the oscillations have smaller amplitude.

(Figure 9 see below)

In conclusion, as we postulated at the outset, the simple adaptation of the Phillips Curve to theoretical patterns that fit better its assumed behaviour is enough to generate more realistic intertemporal evolutions of the employment rate and per capita production.

## 5 Conclusions

The objective of this research has been to elaborate an endogenous growth model where the labour market is in disequilibrium and its internal dynamics are able to generate both the positive growth trend and the irregular behaviour which characterizes the intertemporal evolution of per capita income. The introduction of unemployment has made it possible to study the effect of this variable on both phenomena and their interaction over time, thereby enriching the theoretical results. Also, we have shown that the application of a chaotic system to business cycle theory can enrich and complete the literature which tries to explain the origin and nature of fluctuations.

The dynamic analysis shows how per capita production and employment rate oscillate from stable cycles of different periods to a chaotic path. The dynamic behaviour of these variables depends on both the knowledge sector's level of development and the flexibility of the labour market. In particular, the higher the level of knowledge development and the rigidity of the labor market the greater the possibility of complex dynamics. Besides cyclical behaviour, per capita production shows a positive long run growth trend but there are periods with net decreases of per capita income without considering exogenous shocks. As far as the growth determinants go, our model coincides with traditional endogenous

growth results. Technical progress, interpreted endogenously as knowledge, ensures economic growth in the long term.

Although the model has not been elaborated to policy analysis, some conclusions can be extracted. Policies focused on providing an incentive for investment in knowledge will promote economic growth and employment (because there is excess demand, technological unemployment does not exist), but will increase instability. However, policies which improve the flexibility of the labour market will decrease instability. On the other hand, in the same way as in models of wage bargaining, we find that greater levels of workers' bargaining power are associated with lower levels of both the employment rate and per capita production. Thus, policies which reduce workers' bargaining power will have a positive effect on economic growth, increasing the employment rate.

Additionally, the possibility of chaos has some interesting implications in terms of economic policy. In our model, persistent oscillations emerge solely because of the internal structure of the economy. Therefore the undesired fluctuations could be eliminated or reduced through appropriate structural interventions rather than the traditional business cycle theory policies that are focused on attenuating exogenous shocks to economic conditions (temporary changes in technologies and preferences). However, this does not mean that oscillations are purely deterministic, but the endogenous shocks are not the only source of economic irregularity. We find these preliminary results encouraging enough to perform further work, both at the theoretical and empirical level. These will be possible extensions of our research.

## **6 References**

Aghion, P., Howitt, P., 1992. A Model of Growth through Creative Destruction. *Econometrica* 60, 323-351.

- Aghion, P., Howitt, P., 1998. *Endogenous Growth Theory*. MIT Press, Cambridge Mass.
- Alonso, A., Echevarría, C., Tran, K., 2002. Long Run Economic Performance and the Labour Market. *Southern Economic Journal*, 70, 905-919.
- Barro, R.J., Sala-i-Martin, X., 1995. *Economic Growth*. McGraw-Hill, New York.
- Bean, C., Pissarides, C., 1993. Unemployment, consumption and growth. *European Economic Review* 37, 837-859.
- Blanchard, O.J., Oswald, A.J., 1995. An Introduction to the Wage Curve. *The Journal of Economics Perspectives* 9, 153-167.
- Clark, P., Laxton, D., Rose, D., 1996. Asymmetry in the U.S. Output-Inflation Nexus. *IMF Staff Papers* 43, 216-251.
- Cook, L.M., 1965. Oscillation in the Simple Logistic Growth Model. *Nature* 207, 316.
- Chadha, B., Masson, P., Meredith, G., 1992. Models of Inflation and the Costs of Disinflation. *IMF Staff Papers* 39, 395-431.
- Chiarella, C.P., Flaschel, G., Groh, G., Semmler, W., 2000. *Disequilibrium, Growth and Labor Market Dynamics: Macroperspectives*. Springer-Verlag, Heidelberg.
- Chiarella, C.P., Flaschel, G., 1996. Real and Monetary cycles in models of Keynes-Wicksell type. *Journal of Economic Behavior & Organization* 30, 327-351.
- Daveri, F., Tabellini, G., 2000. Unemployment, Growth and Taxation in Industrial Countries. *Economic Policy* 30, 49-104.
- Desai, M., Henry, B., Mosley, A., Pemberton, M., 2003. *On an Inconsistency in the Goodwin Model of the Growth Cycle*. London School of Economics.
- Galor, O., Weil, D.N., 2000. Population, technology, and growth: From the Malthusian regime to the demographic transition and beyond. *American Economic Review* 90, 806-828.

- Goodwin, R.M., 1967. A Growth Cycle, in: Feinstein, C.H. (Ed), *Socialism, Capitalism and Economic Growth*, Cambridge University Press, pp.54-58.
- Grossman, G. M., Helpman E., 1991. *Innovation and Growth in the Global Economy*. Cambridge MA, MIT Press.
- Ito, T., 1978. A note on Disequilibrium Growth Theory. *Economics Letters* 1, 45-49.
- Ito, T., 1980. Disequilibrium Growth Theory. *Journal of Economic Theory* 23, 380-409.
- Jones, I.C., 2001. Was a Industrial Revolution Inevitable? *Economic Growth over the Very Long Run*. *Advances in Macroeconomics* 1, n<sup>o</sup> 2, art. 1.
- Keynes, J.M., 1936. *The General Theory of Employment, Interest, and Money*. Macmillan, Londres.
- Kremer, M., 1993. Population Growth and Technological Change: One Million B.C. to 1990. *Quarterly Journal of Economics* 108, 681-716.
- Laxton, D., Meredith, G., Rose, D. 1995. Asymmetric effects of economic activity on Inflation: Evidence and policy implications. *IMF Staff Papers*, 42, 2.
- Layard, R., Nickell, S., 1985. The Causes of British Unemployment. *National Institute Economic Review* 111, 62-85.
- Layard, R., Nickell, S., 1986. Unemployment in Britain. *Economica* 53 (Supplement), S121-S169.
- Li, T.Y., Yorke, J.A., 1975. Period Three Implies Chaos. *American Mathematical Monthly* 82, 271-271.
- Lucas, R.E. Jr., 1988. On the Mechanics of Development Planning. *Journal of Monetary Economics* 22, 3-42.
- Lipsey, R.G., 1960. The Relation between Unemployment and the Rate of Change of Money Wage Rates in the United Kingdom, 1861-1957: A Further Analysis. *Economica*

27, 1-31.

Macfadyen, A., 1963. *Animal Ecology*. London: Pitman.

Mankiw, N.G., Romer, D., (Eds), 1991. *New Keynesian Economics*, Vol. I, II. MIT Press, Cambridge.

May, R.M., 1975. Biological Populations Obeying Difference Equations: Stable Points, Stable Cycles, and Chaos. *Journal of Theoretical Biology* 51, 511-524.

May, R.M., Oster, G.F., 1976. Bifurcations and dynamic complexity in simple ecological models. *American Naturalist* 110, 573-599.

Moran, P.A.P., 1950. Some Remarks on Animal Population Dynamics. *Biometrics* 6, 250-258.

Pissarides, C. A., 1990. *Equilibrium unemployment theory*. Basil Blackwell, Oxford.

Pohjola, M.J., 1981. Stable and Chaotic Growth: the Dynamics of a Discrete Version of Goodwin's Growth Cycle Model. *Zeitschrift für Nationalökonomie* 41, 27-38.

Ricker, W.E., 1954. Stock and Recruitment. *Journal Fisheries Research Board of Canada* 11, 559-623.

Romer, P., 1990. Endogenous Technological Change. *Journal of Political Economy* 98, part II, S71-S102.

Rose, H., 1967. On the non-linear theory of the employment cycle. *Review of Economic Studies* 34, 153-173.

Samuelson, P.A., Solow R.M., 1960. Analytical Aspects of Anti-Inflation Policy. *The American Economic Review* 50, 177-194.

Schumpeter, J.A., 1934. *The Theory of Economic Development*. Cambridge MA, Harvard University Press.

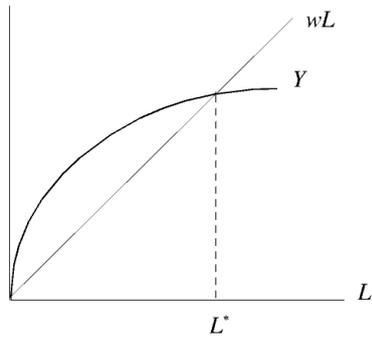


Figure 1: Production and cost functions

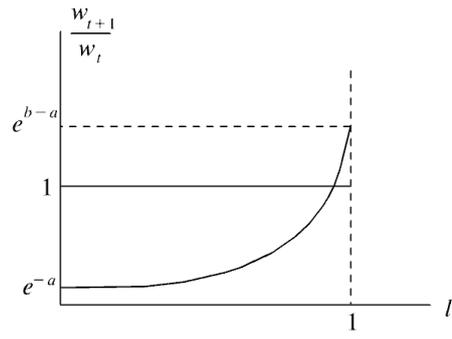


Figure 2: Wage dynamics and rate of employment

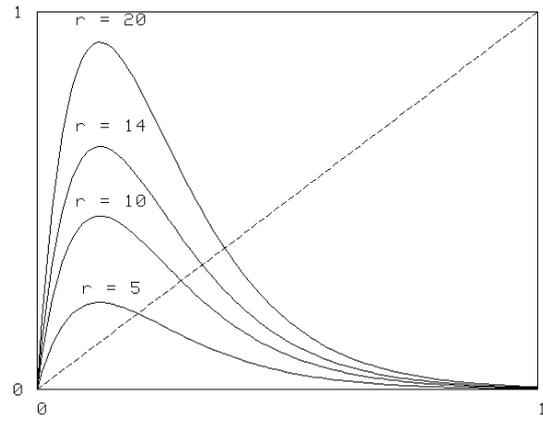


Figure 3:  $f$  for different values of  $r$  and  $s = 9$

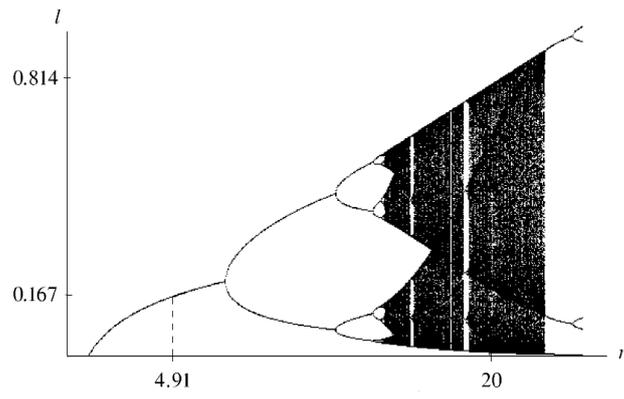


Figure 4: Bifurcation diagram  $s = 9$

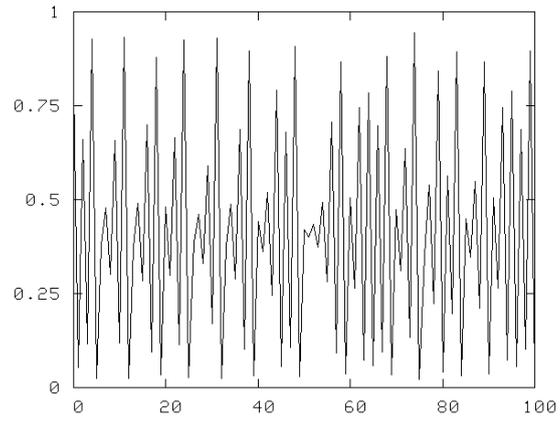


Figure 5: Chaotic time series of  $l$

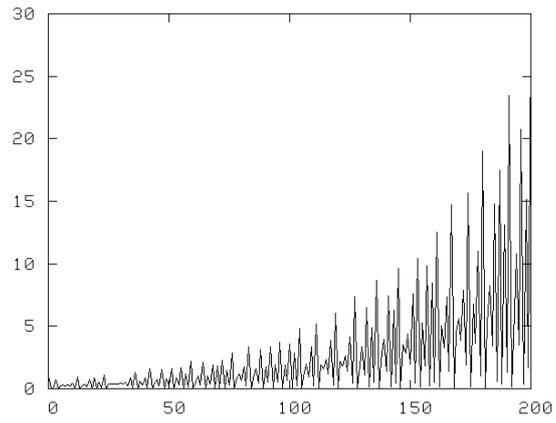


Figure 6: Chaotic time serie of  $y$

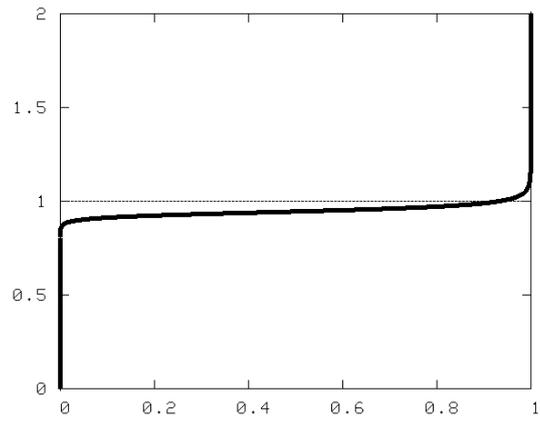


Figure 7: Phillips Curve (9)

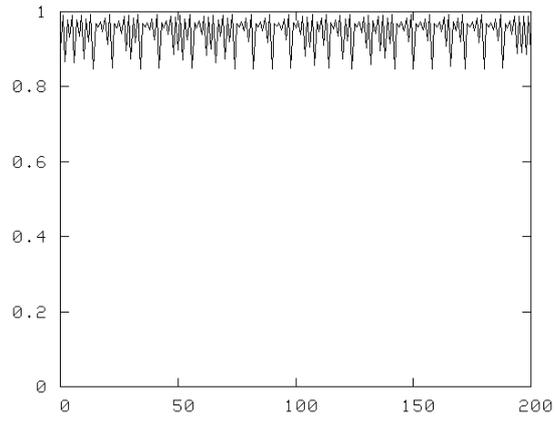


Figure 8: Intertemporal evolution of  $l$  with Phillips Curve (9)

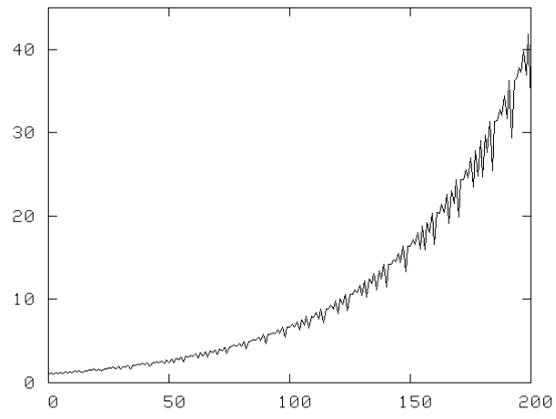


Figure 9: Intertemporal evolution of  $y$  with Phillips Curve (9)

Dynamics Behavior	Values of $r$
<i>Stable equilibrium point</i>	$1 < r < e^2$
<i>Stable cycle of period 2</i>	$e^2 < r < 12.5039$
<i>Stable cycle of period 4</i>	$12.5039 < r < 14.2392$
<i>Stable cycle of period 8</i>	$14.2392 < r < 14.6582$
<i>Stable cycles of period 16, 32, 64,...</i>	$14.6582 < r < 14.7611$
<i>Chaos</i>	$r > 14.7611$

Table 1. Dynamics behaviour of system (7)

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