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ASSET PRICING UNDER HABIT FORMATION AND DISASTER RISK

TESINA

QUE PARA OBTENER EL GRADO DE

MAESTRO EN ECONOMÍA

PRESENTA

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Para mis papás y para Lin.

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Abstract

I modify the Campbell and Cochrane (1999) by adding disaster risk from Barro (2006). Through a numerical solution I find that adding disaster risk improves the (Campbell and Cochrane, 1999) equity premium estimation with a more reasonable level of surplus consumption. The drawback is an overstated risk-free rate.

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Chapter 1

Introduction

Macroeconomic models have difficulties explaining the high equity premia of the last century, a problem known as "the equity premium puzzle". Mehra and Prescott (1985) found that a frictionless standard model generates an equity premium of .35%, inconsistent with the 6% observed in the data. A vast amount of literature in the next 30 years has been devoted to solving it, proposing modifications to neoclassical general equilibrium models. Among the possible explanations, two have been particularly helpful: risk of disaster and habit formation.

This work proposes to combine the framework of the Campbell and Cochrane (1999) habit formation model with the disaster risk from Barro (2006). The central goal is to improve the results from Campbell and Cochrane (1999): (i) to improve the equity premium estimation, and (ii) to obtain a higher surplus consumption value. The main result is that the estimation of the equity premium improves while producing a higher and more reasonable surplus consumption level. The drawback is an overstated risk-free interest rate. This work also accounts for features such as the high volatility of the stock market and a low risk-free rate.

Since the 90's habit formation has had a major role in several papers in the area of economic finance. Habit formation shows the intuitive psychologic fact that people get used to stimuli. If you eat a lot one week, for instance, the next week you might need more food to be satiated.The habit produces a complementarity between present and future consumption, making agents more sensitive to consumption changes. This form of modeling habit disconnects the risk aversion coefficient from the intertemporal substitution coefficient, generating a smoother consumption and therefore a higher rate of return.

Disaster risk models can provide a plausible and straightforward explanation of the equity premium puzzle. The main mechanism is that risk-averse agents demand a greater return to offset the potential losses they would incur in the event of a disaster.

The economy modeled in this work is based on the Campbell and Cochrane (1999) model, whose main contribution to the habit literature is the incorporation of a slow-moving external habit in a power utility function that has a non-linear dynamic with respect to past consumption. This non-linearity keeps consumption above the habit avoiding a negative marginal utility and produce a constant risk-free rate. They also use two simplifications: the equity is modeled as a claim to the consumption process and they use an endowment economy.[1]

Campbell and Cochrane (1999) state that adding bad states of the economy to their model could improve their results, particularly the low surplus consumption value in the steady state. Motivated by this argument, I introduce to their model the possibility of a disaster as a stochastic shock to the process of the endowment, which I modeled with a Normal distribution with negative mean μ_d. With this process, I calculate the intertemporal marginal rate of substitution, which allows me to derive a recursive equation with the price/dividend function. I prove that this equation generates a contraction under the parameters used. Finally, I compute the solution numerically because of the complexity of the Euler equation. With this function, I can simulate the model and

[1] Campbell and Cochrane (1999) argue that any standard concave technology with easy opportunities for intertemporal transformation will not change much the asset pricing implications of an endowment economy

obtain returns, volatilities of returns and other moments.

My model preserves the main conclusions of Campbell and Cochrane (1999). It fits the equity premium and a low risk-free rate and it generates high stock volatility despite a smooth and undpredictable dividend stream. The addition of disasters improves the accuracy of the excess return of stock and volatility estimation. The original model implies that on average habits are 5 percent lower than consumption, and my model produces a value of around 10 percent. However the introduction of disasters prevent me from obtaining a constant risk-free rate, so calibrate the model to replicate the behavior of the risk-free rate obtained by Wachter (2005). She set the risk-free rate to match the upward-sloping yield curve for nominal Treasury bonds. This produce an average risk-free rate of around 3%, 2% higher than in the data. These results imply that the framework of Campbell and Cochrane (1999) can handle severe states of the economy with plausible results.

The remaining of the work is organized as follows. Section 2 relates the existing literature to this work.Section 3 presents the model setup and the equations to be solved numerically. Section 4 shows the disaster data used for the disaster distribution and section 5 the parameters of the model. In section 6 I present the numerical results. In section 7 I discuss the conclusions.

Chapter 2

Literature Review

The best known Asset Pricing Puzzle is identified by Mehra and Prescott (1985), they find that in standard neoclassical models, for example, Lucas Jr (1978) or Hansen and Singleton (1982), the risk aversion coefficient required to explain the returns of stocks in the last century is totally incompatible with the literature. Mankiw et al. (1985) and Campbell and Shiller (1988) find that stock volatility is very high if dividends are discounted at a constant rate. Some authors such as Weil (1989) argue that the aversion coefficient is much higher than previously thought, but this results in a very high-interest rate, because agents want to borrow to smooth consumption, this creates the Risk-free rate puzzle.

Mehra et al. (2007) present a comprehensive review of the equity premium puzzle Literature. On the one hand, he analyses the risk and preference based explanations, here we can find a review of the two explanations used in this paper: habit formation and disaster states of the economy. He also discuss uninsurable income risk of Constantinides and Duffie (1996) and behavioral explanations like Barberis et al. (2001). On the other hand, he analyses explanations that are not based on risk, like borrowing constraints (Constantinidies et al. (1998)) and liquidity premium (Bansal and Coleman (1996)). He concludes that no single explanation has fully resolved the puzzle, but considerable progress has been made.

Rietz (1988) is the first to propose the idea of disasters (like wars or financial crises) as a solution to the equity premium puzzle, but it was discredited in the following years because the parameters of the disasters used in the calibration seemed implausible. Barro (2006) undertakes a deeper analysis of the international disaster probabilities, which support a level of risk sufficient to change the economic cycle. However, the development of the model uses an endowment economy, leaving aside feedback of some macroeconomic variables. This study together with the recent crises of 2008 and 2011 have intensified interest in economic models whose main explanatory channel is disaster risk. I use the disaster calibration of Barro (2006) in this work.

Further contributions are from Bloom (2009), who study the effect of the uncertainty of shocks on total factor productivity and Gourio (2012) which solves a model of economic cycles introducing time variable risk and prolonged disasters, allowing a reasonable connection between macroeconomic variables (including capital and labor) and prices.

Constantinides (1990) is one of the first authors to try to explain the equity premium by relaxing the temporal separability of the Neumann-Morgenstern preferences, using habit to allow complementarity between consumption. By increasing the present consumption, the marginal utility of future consumption is increased. Intuitively the more you buy today, the more you want to buy tomorrow. Abel (1990) incorporates into the habit the concept of "catching up with the Joneses," where the utility depends on past aggregated consumption, so it is known as external habit formation. Subsequently Campbell and Cochrane (1999) makes an effort to model the low and constant interest rate along with high and volatile risk asset return rates, using a convenient sensitivity function used to define the habit. I extend this model with the possibility of disasters.

The calibration of Barro (2006) is criticized by Julliard and Ghosh (2012), they state that accumulating the total disaster as one observation overstates the effect of a disaster.

They find that given a better calibration (where the total disaster is not accumulated in a single point in time) of disaster data, the risk aversion factor necessary to explain some empirical observations remains too high. Therefore as explained in the calibration part, I also use a calibration where the disasters are not accumulated in single observation

Finally, Wachter (2005) improves the numerical method used in the model solution by using a finer grid. This grid with the Campbell and Cochrane (1999) parametrization leads to a poor estimation of the equity moments. I use this finer grid in this work, and I obtain better estimations. The next section presents the habit model with the the addition of the disaster risk.

Chapter 3

Model

3.1 Preferences and Habits

The model is based on the habit model by Campbell and Cochrane (1999). The main objective is to calculate equity returns by solving the price-dividend ratio of this economy. There is a continuum of representative agents; they have preferences related to the habit level X_t given by past consumption of the form:

$$\sum_{t=0}^{\infty} \delta^t \frac{(C_t - X_t)^{1-\gamma} - 1}{1 - \gamma} \tag{3.1}$$

where δ is the subjective discount factor, C_t the consumption and γ is the risk aversion coefficient. A traditional linear (on past consumption) specification of the habit implies an unstable risk-free rate. Therefore Campbell and Cochrane (1999) introduce a non-linear dynamic of the habit that allows a constant risk-free rate. This is achieved by assuming a dynamic over the surplus consumption, defined by:

$$S_t \equiv \frac{C_t^a - X_t}{C_t^a} \tag{3.2}$$

Here C_t^a is the average consumption of all individuals in the economy. S_t indicates the percentage that habit is below consumption, a S_t near zero means a unpleasant consumption state. This implies an external habit specification, where the habit of every agent depends on the aggregate consumption. I assume that the aggregate log-surplus consumption $\log(S_t) = s_t$ (I use lowercase letters to indicate logs) follows an AR(1) process:

$$s_{t+1}^a = (1-\phi)\bar{s} + \phi s_t^a + \lambda(s_t^a)(c_{t+1}^a - c_t^a - g) \tag{3.3}$$

Where g is the mean consumption growth, $\bar{s}$ the aggregate log-surplus consumption in steady state and $\lambda(s_t)$ is the sensibility function (specified below) that determines how the change in consumption affects the habit. Besides the constant risk-free rate, this AR(1) specification also prevent the surplus from falling below 0, a fact that would result in a negative marginal utility. In Appendix A I show that near the steady state and using a log-linear approximation, this specification is approximately the standard habit equation:

$$x_{t+1} \approx [h + \frac{g}{(1-\phi)}] + (1-\phi)\sum_{j=0}^{\infty} \phi^j c_{t-j}$$

Where $h = ln(1-\bar{S})$ is the steady state of $x - c$ and the habit depends linearly on past consumption at a decreasing rate. $\lambda(s_t)$ is defined to accomplish two properties: habit predetermined at steady state $s_t = \bar{s}$ and habit predetermined near the steady state[1]. These restrictions lead them to the following sensibility function defined by parts:

$$\lambda(s_t) = \begin{cases} \frac{1}{\bar{s}}\sqrt{1-2(s_t-\bar{s})} - 1 \; if \; s_t < s_{max} \\ 0 \; if \; s_t > s_{max} \end{cases} \tag{3.4}$$

$s_{max} = \bar{s} + 1/2(s_t - \bar{S^2})$ is the value where $\lambda(s_t)$ is equal to zero and $\bar{s}$ is the logarithm of the steady-state surplus consumption defined later.

[1]See Campbell and Cochrane (1999) section Choosing the sensitivity function.

3.2 Disasters

Campbell and Cochrane (1999) show that a linear technology in this model generates the same asset-pricing results as an endowment economy. They argue that any standard concave technology will not change much of the asset pricing implications in an endowment economy.[2] Following this reasoning, I use an endowment economy in this exercise.

The endowment process in Campbell and Cochrane (1999) is assumed to follow the standard log-normal process with usual shocks v_{t+1} . To this process, I add the possibility of a disaster. Like Gourio (2012) and for simplicity, I represent a disaster with a normal distribution ψ_t with a large negative mean μ. The occurrence of a disaster is given by a Bernoulli distribution η_t with probability p.

$$
\begin{aligned}
\log(Y_{t+1}) - \log(Y_t) = g + v_{t+1} + \eta_{t+1}\psi_{t+1} \\
v_{t+1} \sim \mathcal{N}(0, \sigma_c^2) \\
\psi_{t+1} \sim \mathcal{N}(\mu, \sigma_d^2) \\
\eta_{t+1} \sim \mathcal{B}(p)
\end{aligned}
\tag{3.5}
$$

For example if η_t is one, and the ψ_t distribution takes a value of $-.20$ the endowment process would fall 20% independently of the usual v_{t+1} shock.

3.3 Maximization problem

In this economy, agents can invest in risk-free bonds b_t or on stocks e_t with return R_t modeled as a claim to the endowment process as this is the definition of the "wealth

[2]See preferences and technology section Campbell and Cochrane (1999)

portfolio" in finance theory. So the problem of the agent is:

$$
\begin{aligned}
& max_{c_t,b_t,e_t} \sum_{t=0}^{\infty} \delta^t \frac{(C_t - X_t)^{1-\gamma} - 1}{1-\gamma} \\
& s.t.\ C_t = Y_t + b_t R^f_{t+1} + e_t R_{t+1} - e_{t+1} - b_{t+1}
\end{aligned}
\tag{3.6}
$$

Where e_t is the quantity of equity and b_t the quantity of bonds.Given the power utility used, the marginal utility is:

$$
U_c(C_t, X_t) = (C_t - X_t)^{-\gamma} = S_t^{-\gamma} C_t^{-\gamma} \tag{3.7}
$$

Therefore the intertemporal rate of substitution or as known in the finance literature, the stochastic discount factor is:

$$
M_{t+1} = \delta \frac{U_c(C_{t+1}, X_{t+1})}{U_c(C_t, X_t)} = \delta \left(\frac{S_{t+1}}{S_t} \frac{C_{t+1}}{C_t} \right)^{-\gamma} \tag{3.8}
$$

Solving the lagrangian associated to problem (6), I obtain classic asset pricing equations:

$$
\mathbb{E}_t(M_{t+1} R^f_t) = 1 \tag{3.9}
$$

$$
\mathbb{E}_t(M_{t+1} R_t) = 1 \tag{3.10}
$$

3.4 Equilibrium

In equilibrium markets clear, so $Y_t = C_t$ and individuals choose the same level of consumption $C_t = C^a_t$ and $S_t = S^a_t$. Substituting in M_{t+1} the ratio $\frac{C_{t+1}}{C_t}$ with the exponential of process (3.5):

$$
M_{t+1} = \delta e^{(-\gamma g - \gamma(s_{t+1} - s_t + v_{t+1} + \eta_{t+1}\psi_{t+1}))}
$$

Substituting $\frac{S_{t+1}}{S_t}$ with the exponential of process (3.3) gives:

$$M_{t+1} = \delta e^{(-\gamma g - \gamma[(\phi-1)(s_t-\bar{s})+(1+\lambda(s_t)(v_{t+1}+\eta_{t+1}\psi_{t+1}))]} \quad (3.11)$$

Using the relation $R_t^f = \frac{1}{\mathbb{E}_t(M_{t+1})}$ the interest risk free rate can be computed directly [3]:

$$\begin{aligned} r(s_t) &= -\log(M_{t+1}) = -\log[\delta e^{-\gamma g - \gamma(1-\phi)(\bar{s}-s_t)} \\ &((1-p)e^{\frac{\gamma^2\sigma_v^2}{2}(1+\lambda(s_t))^2} + pe^{-\gamma(1+\lambda(s_t))(\mu)+\frac{\gamma^2(\sigma_v^2+\sigma_\psi^2)}{2}(1+\lambda(s_t))^2})] \end{aligned} \quad (3.12)$$

Campbell and Cochrane (1999) obtain a simpler equation for the risk free rate, this allows them to obtain a constant risk-free rate by setting the steady state in terms of the parameters $\bar{s} = \sigma_c\sqrt{\frac{\gamma}{1-\phi}}$ and $b = 0$:

$$r_t^f = -ln(\delta) + \gamma g - \left(\frac{\gamma}{\bar{S}}\right)^2 \frac{\sigma^2}{2}$$

This equation provides the intuition behind the explanation of the equity premium with habit formation; the risk-free-rate is not so sensitive to changes in consumption growth g (γ=2). This allows having a low risk-free rate not very sensitive to g with a high Sharpe ratio. [4] In Mehra and Prescott (1985) model the risk aversion coefficient needed to rationalize the equity premium is around 41 resulting in a contrafactual sensitive response of the risk-free rate to consumption growth changes g.

A possible generalization is to let the risk-free rate be a linear function of s_t:

$$r_t^f = r_0^f - B(s_t - \bar{s})$$

This modification implies a change in $\bar{s} = \sigma_c\sqrt{\frac{\gamma}{1-\phi-\frac{B}{\gamma}}}$ with B a parameter. Wachter

[3] I use the fact that $\mathbb{E}[e^x] = e^{\mu+\frac{1}{2}\sigma^2}$ if $x \sim \mathcal{N}(\mu, \sigma^2)$ and that η_{t+1} is independent of ψ_{t+1} and v_{t+1}

[4] see Campbell and Cochrane (1999) Model intuition section

(2006) calibrate the risk-free rate with $B \neq 0$ to be a linear function of the surplus consumption to obtain a upward-sloping yield of nominal bonds. She finds in the data that $B > 0$ implying that the intertemporal smoothing effect dominates the precautionary savings effect. In my case, it is not possible to achieve constant behavior of the risk-free rate. As explained in the calibration section, I set $\bar{s}$ to replicate Wachter (2006) risk-free rate behavior.

To solve the return of the equity, I derive the price-dividend ratio, that depends on the only state variable of the economy(s_t). Given the relation $R_{t+1} = \frac{P_{t+1}+D_{t+1}}{P_t}$ equation (3.10) can be written:

$$\mathbb{E}_t\left[M_{t+1}\frac{P_{t+1}+D_{t+1}}{P_t}\right] = 1 \tag{3.13}$$

Recalling that in this economy $D_t = C_t$ this can be rewritten as:

$$\frac{P_t}{C_t}(s_t) = \mathbb{E}_t[M_{t+1}\frac{C_{t+1}}{C_t}[1+\frac{P_{t+1}}{C_{t+1}}(s_{t+1})]] \tag{3.14}$$

Plugging in process (5) and (3) like in the risk-free rate yields:

$$\begin{aligned}\frac{P_t}{C_t}(s_t) &= \delta e^{(1-\gamma)g-\gamma(1-\phi)(\bar{s}-s_t)} \\ &\cdot \mathbb{E}_t\Bigg[e^{(1-\gamma)(v+\eta\psi)-\gamma\lambda(s_t)(v+\eta\psi)} \\ &\left(\frac{P_{t+1}}{C_{t+1}}((1-\phi)\bar{s}+\psi s_t+\lambda(s_t)(v+\eta\psi))+1\right)\Bigg]\end{aligned} \tag{3.15}$$

Given the independence of η, v and ψ I compute the expectation over the bernoulli distribution η and obtain:

$$\begin{aligned}
\frac{P_t}{C_t}(s_t) = {} & \delta e^{(1-\gamma)g-\gamma(1-\phi)(\bar{s}-s_t)} \\
& \left[p \int_{-\infty}^{\infty} e^{(1-\gamma)(v+\psi)-\gamma\lambda(s_t)(v+\psi)} p(v+\psi) \right. \\
& \left(\frac{P_{t+1}}{C_{t+1}}((1-\phi)\bar{s}+\phi s_t+\lambda(s_t)(v+\psi))+1 \right) d(v+\psi) \\
& + \ (1-p) \int_{-\infty}^{\infty} e^{(1-\gamma)v-\gamma\lambda(s_t)v} p(v) \\
& \left. \left(\frac{P_{t+1}}{C_{t+1}}((1-\phi)\bar{s}+\phi s_t+\lambda(s_t)v)+1 \right) dv \right]
\end{aligned} \tag{3.16}$$

Where $d(v+\psi)$ is a normal distribution with mean μ and variance $\sigma_c^2 + \sigma_d^2$ This price-dividend as a function of s_t does not have an obvious analytical solution, so I use a numerical approximation using a fixed point iteration method to solve it. Once solved, I can calculate in the return in the simulations with:

$$R_{t+1} = \frac{P_{t+1}/C_{t+1}+1}{P_t/C_t} \frac{C_{t+1}}{C_t} \tag{3.17}$$

Chen et al. (2008) proved the existence and uniqueness of the price-dividend function in the model of Campbell and Cochrane (1999). I prove the existence with the addition of disasters in Appendix B. Next I analyze the disaster data used to estimate the parameters of the disaster distribution ψ_t.

Chapter 4

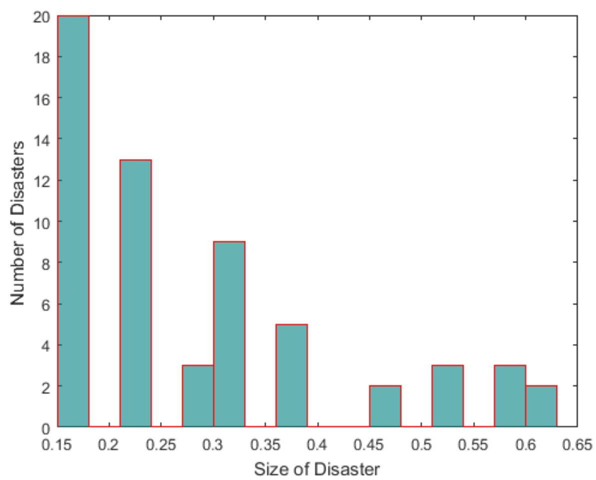
Disaster data

The disasters are any unusual event that causes a fall in consumption, like wars, financial crises or natural disasters. Disasters considered are twentieth-century events that caused a per capita GDP fall by more than 15 percent. I use the database used by Barro (2006) and provided by Angus (2003), which gives us observations of these phenomena for OECD countries, Latin America and Asia.

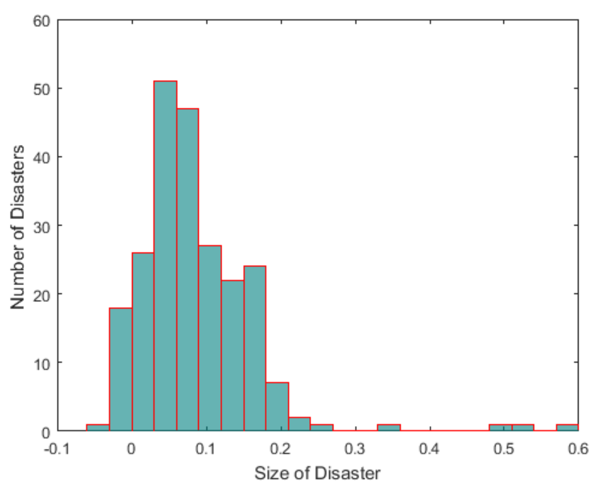
Barro (2006) finds 60 events in a sample of 35 countries over 100 years, resulting in a 1.7 % percent annual disaster probability. Accumulating the total fall over the duration of the disaster in one observation for every disaster, he finds that the average fall is 29 %. Julliard and Ghosh (2012) criticize this procedure because it overestimates the effect of disasters, an adverse risk agent fears more a large contraction of consumption at a single point in time than the same contraction distributed throughout the duration of the disaster. For example the fall of Second World War in Germany (1944-1446) could be 1 observation with a fall of 64% or 3 observations with a fall of around 21% For this reason, I consider two ways of calibrating disasters: accumulating (as Barro) the disaster in one observation, and taking each year of the disaster as an observation. Tables I and II in Appendix C summarize the main statistics. It can be noticed that most contractions are related to events such as the first and second world war or the

Great Depression, events that occurred over several years, with the average duration of events considered around four years. This generates that the distributions of the two approaches mentioned above are considerably different, these are illustrated in Figure 1.

Figure 4.1: Disaster Distribution



Panel A: Histogram using accumulated disasters fall real % GDP



Panel B: Histogram using non-accumulated disasters fall real % GDP

Chapter 5

Calibration

5.1 Choice of parameters

For the calibration of the model, I use the parameters of Campbell and Cochrane (1999) except the free parameter δ. All parameters other than γ are taken directly from the data. They consider postwar data (1947-1995) for consumption of non-durable goods and services. The consumption growth parameters g and σ_c, are taken form the first two moments of the consumption series. The parameter ϕ is defined to match the serial correlation of the log price-dividend ratios. The curvature coefficient of the utility function γ is calibrated to match the Sharpe ratio in the data. To facilitate the risk-free calibration and given than the results are not very sensitive to changes in γ[1] I use the value from Campbell and Cochrane (1999) $\gamma = 2$. The δ parameter in Campbell and Cochrane (1999) is set to match the risk-free fixed rate to .94 % (3-Month Treasury Bill rate in the period studied). In my case, I use δ to match this value in the Steady State. These parameters are presented in Table 5.1.

In the case of disasters, I assume like Gourio (2012) that disasters have a normal distribution with mean μ and variance σ_d. I use three calibrations for this parameters; I first use the moments obtained by Barro (2006), then the distributed disasters of the data

[1]See Choosing parameters section Campbell and Cochrane (1995)

Table 5.1: Parameters

Parameter	Variable	Value
Consumption Growth	g	1.8 %
Consumption standard deviation	σ_c	1.5 %
log-risk free rate	r	.94 %
s persistent coefficient	ϕ	.87
Uitility curvature	γ	2

section and finally I also calculate the moments using only disaster data of developed economies. Table 5.2 summarizes these moments:

Table 5.2: Disaster Parameters

Parameter	Variable	Value
Disasters with Barro calibration:		
Mean	μ	-29%
Disaster standard deviation:	σ_d	13%
Probability:	p	1.8%
Disasters with distributed calibration:		
Mean	μ	-8%
Disaster standard deviation:	σ_d	8%
Probability:	p	6.5%
Disasters of developed countries calibration:		
Mean	μ	-24%
Disaster standard deviation:	σ_d	18%
Probability:	p	1.5%

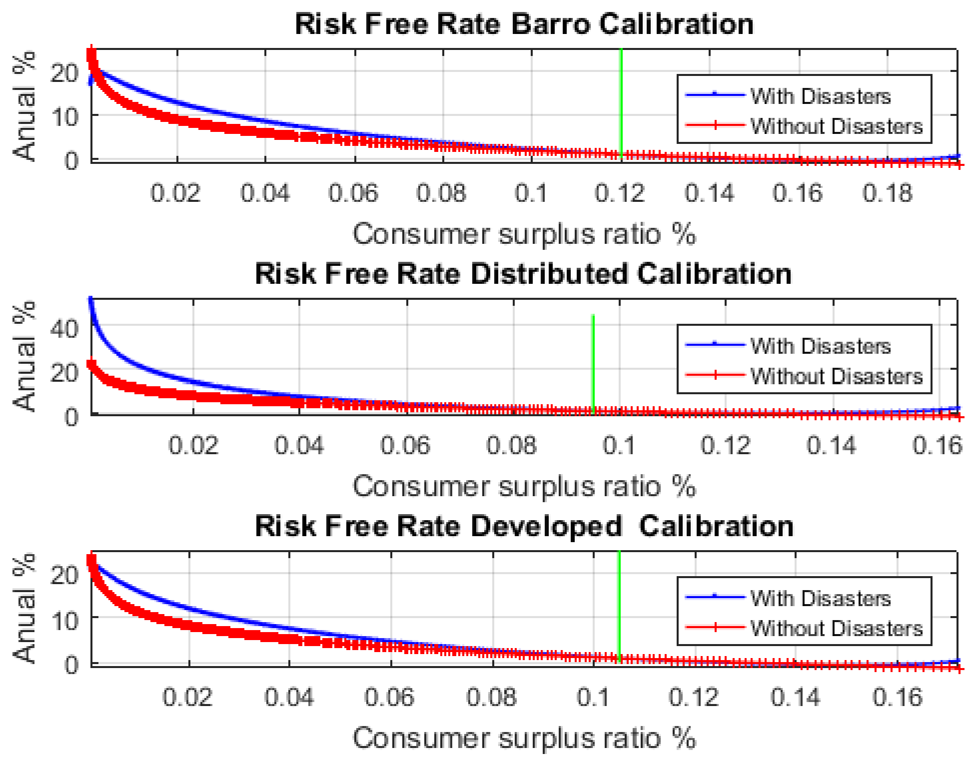
Aditionallty I compute a multi mass distribution of two points with the disasters of the sample, one representing small disasters with $\mu = -22\%$ and $p' = .77$ and the other, big disasters with $\mu = -51\%$ and $1 - p' = .23$, where p' is the probability of a small disaster given that there is a disaster. As mentioned in the model section, I calibrate numerically the value of $\bar{s}$ to replicate the risk-free rate obtained by Wachter (2006). She obtains the risk-free rate without disasters by setting $B = .011$ in the formula for $\bar{s}$. This adjustment produce a value of $\bar{s}$ for each parametrization of disaster; I presented it in Table 5.3. The adjustment is illustrated in figure 2. The vertical lines indicate the steady state of the economy. The disaster models present a good fit of the risk-free rate in the

Table 5.3: Surplus consumption and discount rate

Disasters with Barro calibration:		
Subjective discount factor	δ	.9
Consumption surplus ratio in ss	$\bar{S}$	.12
Consumption surplus limit in ss	S_{max}	.2
Disasters with distributed calibration:		
Subjective discount factor	δ	.87
Consumption surplus ratio in ss	$\bar{S}$	.1
Consumption surplus limit in ss	S_{max}	.16
Disasters of developed countries calibration:		
Subjective discount factor	δ	.9
Consumption surplus ratio in ss	$\bar{S}$	.105
Consumption surplus limit in ss	S_{max}	.17
Disasters of multi mass distribution:		
Subjective discount factor	δ	.87
Consumption surplus ratio in ss	$\bar{S}$	.11
Consumption surplus limit in ss	S_{max}	.18

neighborhood of the ss, but overstate its value near 0 and S_{max}.

Figure 5.1: Risk-Free Rate Adjustment



5.2 Numeric procedure

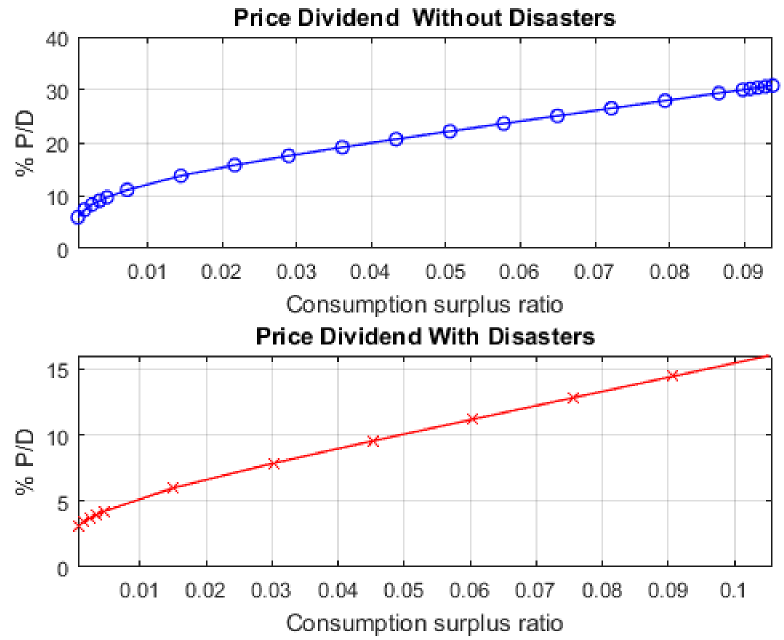
Like Campbell and Cochrane (1999) I solve the price-dividend ratio using a fixed point iteration method, I use a Gauss-Hermite quadrature to estimate the integrals of the Euler equation with ten nodes. I use a grid of 22 points, the first part is defined between 0 and S_{max} using 13 equidistant points, and the other 9 points are used to adjust the non-linear behavior near S_{max} and 0. With points .0005, .0015, .0025, .0035 and .0045 and 4 points at intervals of .01 from S_{max}. This choice for the grid is based on the accuracy analysis from Wachter (2005), where it is proven that adding points to the grid near 0 and S_{max} improves the accuracy of the price-dividend ratio estimation.

Chapter 6

Results

Figure 6.1 illustrates the comparison between the price-dividend ratio of the original model and the one with disasters calibrated by Barro (2006). In a recession, as a result of the low consumption relative to the habit, the marginal utility is high. These agents push prices of equity down, resulting in a low price dividend-ratio. As the surplus consumption ratio improves, the prices and the price-dividend ratio rise. The dividend ratio with disasters has the same behavior, but at a lower level by cause of the higher risk of the equity.

Figure 6.1: Price-Dividend Function



In the next figures, I present the conditional moments of the returns as a function of the consumer surplus ratio. Figure 6.2 displays how the expected return $\mathbb{E}_t(r_{t+1})$of the equity rises exponentially as the surplus consumption approaches 0. The Barro calibration produces the higher expected return, and accordingly to Julliard and Ghosh (2012) the distributed calibration lowers this return. This fact is due to the argument that investors fear more one big drop in consumption in one year, than the same drop over several years.

Figure 6.2: Expected Returns

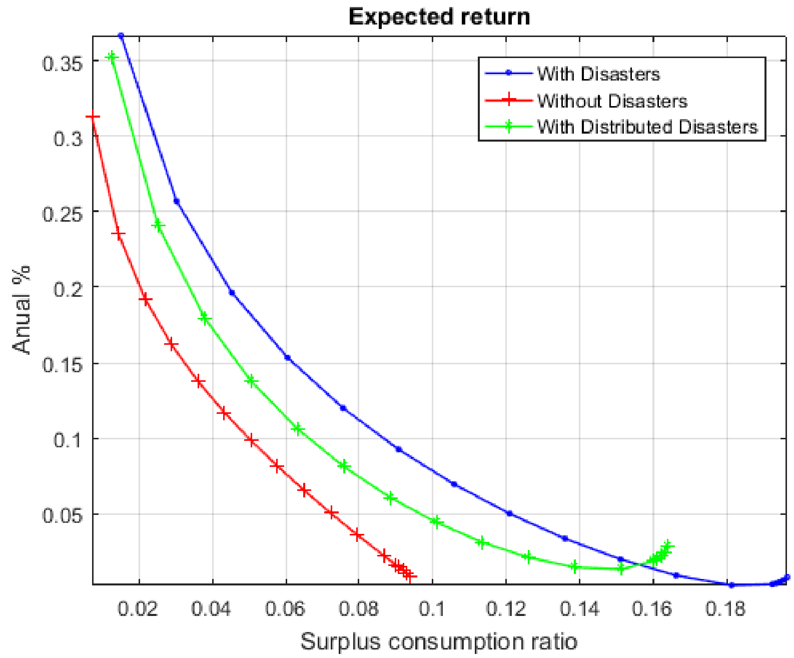


Figure 6.3 displays the conditional volatility $\sigma_t(r_{t+1})$ of the returns. The volatility increases near 0 for both models as a result of the shape of function $\lambda(s_t)$, it increases when the surplus consumption moves towards 0. The sensibility functions of the two models are represented in figure 6.4. Although the volatility of the disaster is implied in the volatility of the endowment equation 3.5, the non-disaster model has a bigger $\lambda(s_t)$ and this effect dominates, therefore the volatility is lower for the disaster model near 0.

Figure 6.3: Conditional Volatility

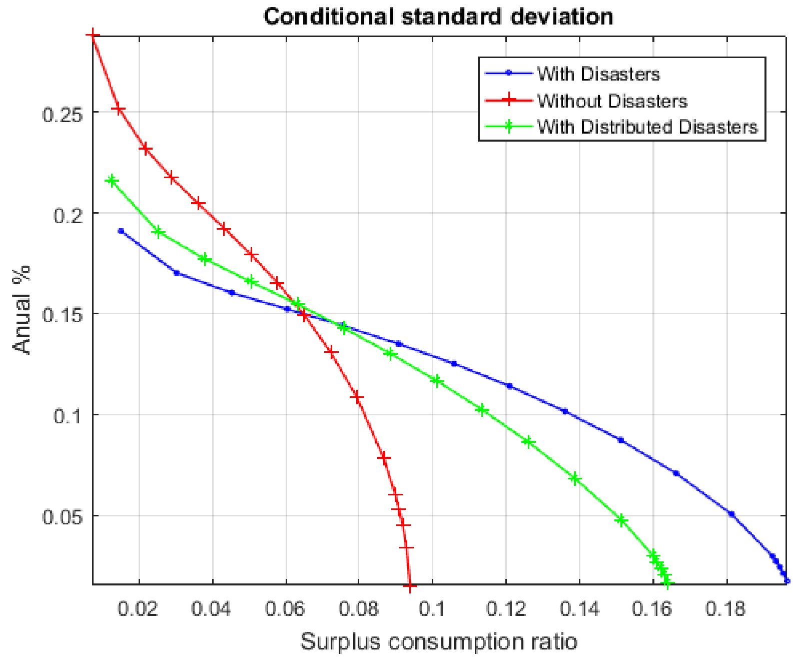
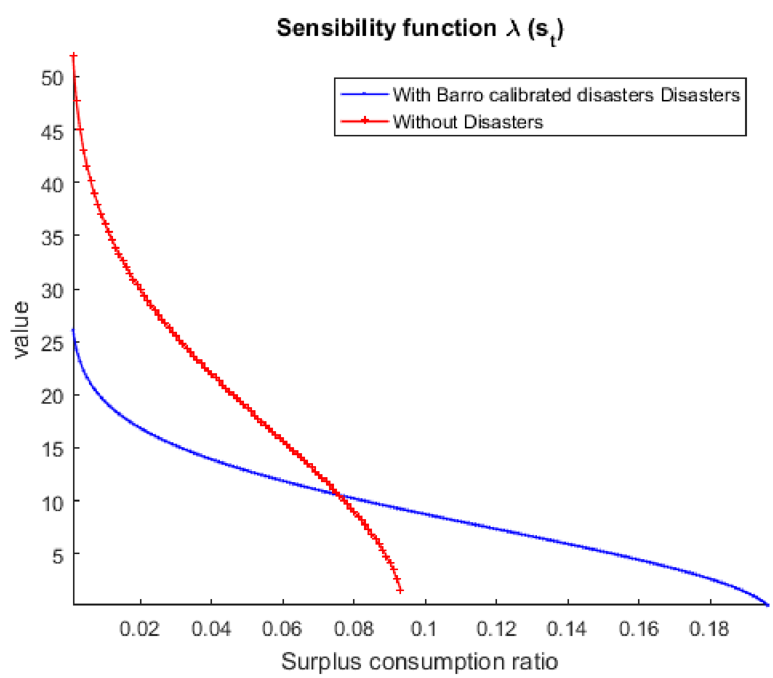


Figure 6.4: Sensitivity function $\lambda(s_t)$



The conditional Sharpe ratio $\mathbb{E}_t(R_{t+1})/\sigma_t(R_{t+1})$ is described in figure 6.5, the large expected return near the lowest surplus consumption increases the Sharpe ratio (the return increases faster than the volatility) in all the models. The Barro calibration presents a bigger Sharpe ratio than the calibration without disasters over all the grid.

Figure 6.5: Sharpe ratio

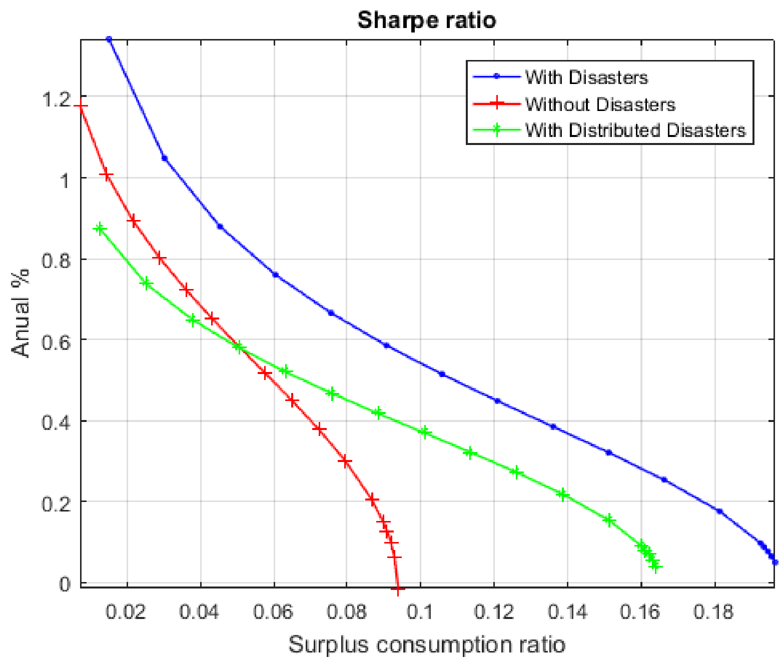


Table 6.1 shows the moments obtained when 120000 months of data are simulated, where $E(r^m - r^f)$ represents the equity premium. The first column is parametrized like Campbell and Cochrane (1999), as noted by Wachter (2005), with the finer grid the equity premium and the volatility is considerably smaller than the data values. The second column incorporates disasters with the parameters from Barro; the third one uses the distributed distribution of disasters derived in the data section.Column four reports the moments when we only consider developed countries; this calibration provides very similar results with a lower risk-free rate. The sixth column has the results with the two point distribution, this distribution implies less volatility due to

the less volatile distribution of the disasters and a bigger equity premium.

Table 6.1: Results of the Simulations

Simulation result of my model with different calibrations						
Moment	Campbell	Barro	Distributed	Developed	Multi mass	Data
$E(r^m - r^f)$ %	4.8	5.99	6.73	6.02	6.47	6.69
$\sigma(r^m - r^f)$%	11.1	12.88	13.97	13.6	12.6	15.7
Sharpe ratio	0.44	0.46	.48	.43	.51	0.43
Skewness	0.11	-0.55	-.16	-.44	-0.42	-0.53
Kurtosis	3.41	4.09	3.4	4.07	3.9	3.35
$E(r^f)$%	.94	3.39	4.03	2.9	3	.94
$\bar{S}$%	5.7	12	9.5	10.5	11	-

The equity premium with the Barro calibration is larger than the simulation without disasters, one of the objectives of this work. It is important to note that the $\bar{S}$ is considerably bigger that the one obtained by Barro (2006), corroborating the intuition that a boost mechanism to the equity premium (like disasters) could increase the surplus consumption value. The skewness of the returns is closer to the data as returns turn negative with disasters. The modified models also have better volatilities, with the distributed calibration volatility closer to the data. The expected return is higher as a result of the higher risk-free rates near 0 surplus consumption.

Chapter 7

Conclusion

The inclusion of reasonable distributions of disasters to the Campbell and Cochrane (1999) improves the equity premium estimation and the value of the consumption surplus ratio. This with the drawback of a higher risk-free rate. The volatility of returns and the third moment also improves. This work supports the robustness of the Campbell and Cochrane (1999), since even with its non-linear habit specification can handle big drops in consumption. Given that the two theories are not excluding, the framework of this work provides a more realistic environment for the explanation of the equity premium puzzle. A possible extension is to model the dividends as an independent process of consumption; this specification would need a more complex calibration since the effect of a disaster on stocks is different than on consumption.

Appendix A

Log-Linearization

s_t can be written as $s_t = \log\left(\frac{e^{c_t} - e^{x_t}}{e^{c_t}}\right)$ taking the log-linearization around $s_t = \bar{s}$ and $c_{t+1} - c_t = g$:

$$\frac{e^g(c_t - g)}{e^g - e^{\bar{x}}} - (c_t - g) - \frac{e^{\bar{x}}(x - \bar{x})}{e^g - e^{\bar{x}}}$$

$$\frac{c_t - g}{\bar{S}} - (c_t - g) + \left(1 - \frac{1}{\bar{S}}\right)(x_t - \bar{x})$$

so:

$$s_t - \bar{s} \approx \left(1 - \frac{1}{\bar{S}}\right)(x_t - c_t - \bar{x} - g)$$

for $\lambda(s_t)$:

$$\lambda(s_t)(c_{t+1} - c_t - g) \approx (\lambda(\bar{s}) + \lambda'(\bar{s})(s_t - \bar{s}))(c_{t+1} - c_t - g)$$

using $\lambda'(\bar{s}) = 0$

$$\lambda(s_t)(c_{t+1} - c_t - g) \approx \lambda(\bar{s})(c_{t+1} - c_t - g)$$

Substituting these approximation in equation :

$$\left(1 - \frac{1}{\bar{S}}\right)(x_{t+1} - c_{t+1} - h) + \bar{s} = (1 - \phi)\bar{s} +$$
$$\phi\left(1 - \frac{1}{\bar{S}}\right)(x_t - c_t - h) + \bar{s}$$
$$+ \lambda(\bar{s})(c_{t+1} - c_t - g)$$

Using $\lambda(\bar{s}) = \left(\frac{1}{\bar{S}} - 1\right)$ and reducing:

$$x_{t+1} = (1-\phi)c_t + \phi x_t + (1 - \phi h + g)$$

Iterating x_t we obtain the equation:

$$x_{t+1} \approx [h + \frac{g}{(1-\phi)}] + (1-\phi)\sum_{j=0}^{\infty} \phi^j c_{t-j}$$

Appendix B

Contraction Proof

We rewrite the price-dividend function as:

$$\begin{aligned}\frac{P_t}{C_t}(s_t) &= \delta e^{-\gamma(g+(1-\phi)(\bar{s}-s_t))} \\ \mathbb{E}_t &\left[e^{(1-\gamma)(v_{t+1}+\eta_{t+1}\psi_{t+1})-\gamma\lambda(s_t)(v_{t+1}+\eta_{t+1}\psi_{t+1})} \right. \\ &\left. \left(\frac{P_{t+1}}{C_{t+1}}(1-\phi)\bar{s} + \phi s_t + \lambda(s_t)(c_{t+1}-c_t-g) \right) \right]\end{aligned} \tag{B.1}$$

Since ψ and η are independent, the expectation over the bernoulli distribution can be computed:

$$\begin{aligned}&\frac{P_t}{C_t}(s_t) = \delta e^{-\gamma(g+(1-\phi)(\bar{s}-s_t))} \\ &\left((1-p) * \mathbb{E}_t \left[e^{(1-\gamma)(v_{t+1})-\gamma\lambda(s_t)(v_{t+1})} \left(\frac{P_{t+1}}{C_{t+1}}(1-\phi)\bar{s} + \phi s_t + \lambda(s_t)(v_{t+1}) \right) \right] \right. \\ &\left. + p * \mathbb{E}_t \left[e^{(1-\gamma)(v_{t+1}+\psi_{t+1})-\gamma\lambda(s_t)(v_{t+1}+\psi_{t+1})} \left(\frac{P_{t+1}}{C_{t+1}}^{k-1} (1-\phi)\bar{s} + \psi s_t + \lambda(s_t)(v_{t+1}+\psi_{t+1}) \right) \right] \right)\end{aligned} \tag{B.2}$$

Where $\xi_{t+1} = v_{t+1} + \psi_{t+1} \sim \mathcal{N}(\mu, \sigma_d^2 + \sigma_c^2)$

To facilitate the test we make the following variable changes: $x = s_t - \bar{s}, k_0 = \delta e^{g(1-\gamma)}, k_1 = (1-\psi), \frac{P_t}{C_t} = G$

Thus, the intergral version of the expectation is

$$k_0 e^{xk_1}\left[\frac{(1-p)}{\sqrt{2\pi}\sigma_v}\int_{-\infty}^{\infty} e^{(1-\gamma(1+\lambda(x)))v - \frac{v^2}{2\sigma_v^2}}[1+G(\psi x+\lambda(x)v)]dv \right. \\ \left. + \frac{(p)}{\sqrt{2\pi}\sigma_v}\int_{-\infty}^{\infty} e^{(1-\gamma(1+\lambda(x)))\xi - \frac{(\xi-\mu)^2}{2\sigma_\xi^2}}[1+G(\psi x+\lambda(x)\xi)]d\xi\right] \tag{B.3}$$

By completing the squares in the exponentials, for the first integral:

$$-\frac{v^2}{2\sigma_v^2} + (1-\gamma(1-+\lambda(x)))v = -\frac{1}{2\sigma_v^2}[v-\sigma_v^2(1-\gamma(1+\lambda(x))]^2 + \\ \frac{\sigma_v^2}{2}(1-\gamma(1+\lambda(x)))^2 \tag{B.4}$$

Defining $M(x) = k_0 e^{xk_1 + \frac{\sigma_v^2}{2}(1-\gamma(1+\lambda(x)))^2}$, $y = v - \sigma_v^2(1-\gamma(1+\lambda))$ and $\chi(y,x) = \lambda(x)y + \sigma_v^2\lambda(x)(1-\gamma(1+\lambda(x))) + \psi x$

We obtain:

$$M(x)\frac{1}{\sqrt{2\pi}\sigma_v}\int_{-\infty}^{\infty} e^{\frac{y^2}{2\sigma_v^2}} G(1+\chi(y,x))dy \tag{B.5}$$

For the second:

$$-\frac{(\xi-\mu)^2}{2\sigma_\xi^2} + (1-\gamma(1-+\lambda(x)))\xi = \\ -\frac{1}{2\sigma_\xi^2}[\xi - \sigma_\xi^2(1-\gamma(1+\lambda) + \frac{\mu}{\sigma_\xi^2})]^2 + \\ \frac{\sigma_\xi^2}{2}(1-\gamma(1+\lambda)+\frac{\mu}{\sigma_\xi^2}))^2 + \frac{\mu^2}{2\sigma_\xi^2} \tag{B.6}$$

Defining $M'(x) = k_0 e^{xk_1 + \frac{\sigma_v^2}{2}(1-\gamma(1+\lambda(x))+\frac{\mu}{\sigma_\xi^2}))^2 + \frac{\mu^2}{2\sigma_\xi^2}}$, $z = \xi - \sigma_\xi^2(1-\gamma(1+\lambda(x)) + \frac{\mu}{\sigma_\xi^2})$ and $\tau(y,x) = \lambda(x)z + \sigma_\xi^2\lambda(x)(1-\gamma(1+\lambda(x)) + \frac{\mu}{\sigma_\xi^2}) + \psi x$

We obtain:

$$M'(x)\frac{1}{\sqrt{2\pi}\sigma_\xi}\int_{-\infty}^{\infty} e^{\frac{z^2}{2\sigma_\xi^2}} G(1+\tau(z,x))dz \tag{B.7}$$

Now we define the following mapping to prove it generates a contraction over the grid.

$$G_{n+1}(x) = \begin{cases} (1-p)M(x)\frac{1}{\sqrt{2\pi}\sigma_v}\int_{-\infty}^{\infty} e^{\frac{y^2}{2\sigma_v^2}}(1+G_n(\chi(y,x)))dy \\ +pM'(x)\frac{1}{\sqrt{2\pi}\sigma_\xi}\int_{-\infty}^{\infty} e^{\frac{z^2}{2\sigma_\xi^2}}(1+G_n(\tau(z,x)))dz \; if \; \underline{x} \leq x \leq \bar{x} \\ G_{n+1}(\bar{x}) \; if \; x > \bar{x} \\ G_{n+1}(\underline{x}) \; if \; x < \underline{x} \end{cases} \tag{B.8}$$

We set $G^0 = 0$, therefore G^1 is:

$$G_1(x) = \begin{cases} (1-p)M(x) + (p)M'(x) \\ G_1(\bar{x}) \; if \; x > \bar{x} \\ G_1(\underline{x}) \; if \; x < \underline{x} \end{cases} \tag{B.9}$$

We have:$\|G_1 - G_0\|_\infty = \|G_1\|_\infty < (1-p)M^* + pM'^*$ for $\underline{x} \leq x \leq \bar{x}$

Taking the difference:

$$\begin{aligned} G_{k+1} - G_k = \\ (1-p)M(x)\frac{1}{\sqrt{2\pi}\sigma_v}\int_{-\infty}^{\infty} e^{\frac{y^2}{2\sigma_v^2}}(G_n(\chi(y,x) - G_{n-1}(\chi(y,x)))dy \\ +pM'(x)\frac{1}{\sqrt{2\pi}\sigma_\xi}\int_{-\infty}^{\infty} e^{\frac{z^2}{2\sigma_\xi^2}}(G_n(\tau(z,x)) - G_{n-1}(\tau(z,x)))dz \end{aligned} \tag{B.10}$$

By the triangle inequality:

$$
\begin{aligned}
|G_{k+1} - G_k| \leq \\
(1-p)M(x)\frac{1}{\sqrt{2\pi}\sigma_v}\int_{-\infty}^{\infty} e^{\frac{y^2}{2\sigma_v^2}}|G_n(\chi(y,x) - G_{n-1}(\chi(y,x))|dy \\
+pM'(x)\frac{1}{\sqrt{2\pi}\sigma_\xi}\int_{-\infty}^{\infty} e^{\frac{z^2}{2\sigma_\xi^2}}|G_n(\tau(z,x)) - G_{n-1}(\tau(z,x))|dz \\
\leq \|G_n - G_{n-1}\|_\infty((1-p)M^*(x) + pM'^*)
\end{aligned}
\tag{B.11}
$$

$\|G_{n+1} - G_n\|_\infty \leq \|G_n - G_{n-1}\|_\infty \leq ((1-p)M^*(x) + pM'^*)^n((1-p)M^*(x) + pM'^*)$
Taking the limit, we have that by the test M of Weistrass, G converges and the sufficient condition is $((1-p)M^*(x) + pM'^*) < 1$. This proof shows that the necessary condition fo the existence of the price-dividend ratio is $((1-p)M^*(x) + pM'^*) < 1$,where M and M' are functions of the parameters and the consumption surplus. The parameters used in our standard calibration with disasters satisfy this condition over the grid with values of $((1-p)M^*(x) + pM'^*)$ between .94 and .95.

Appendix C

Disaster Data

Table C.1: Disasters Developed Countries

Tabla I			
Country	Years	GDP % Real Fall	Total GDP % Average Real Fall
Australia	1928-1931	20	7
Austria	1913-1919	35	6.
	1944-1945	58	58
	1929-1933	23	6
Belgium	1916-1918	30	16
	1939-1943	24	7
Canada	1929-1933	33	10
Denmark	1914-1918	16	4
	1939-1941	24	13
Finland	1913-1918	35	7
France	1916-1918	31	17
	1929-1932	16	6
	1939-1944	49	13
Germany	1914-1919	29	5
	1928-1932	18	5
	1944-1946	64	38
Greece	1939-1945	64	15
Italy	1940-1945	45	11
Japan	1943-1945	52	28
Netherlands	1913-1918	17	4
	1929-1934	16	3
	1939-1945	52	11
New Zealand	1929-1932	18	6
Norway	1940-1944	20	4
Portugal	1934-1936	15	8
Spain	1935-1938	31	11
Sweden	1913-1918	18	1
United States	1929-1933	31	9

Table C.2: Disasters Latin America and Asia

Tabla II

Country	Years	GDP % Real Total Fall	GDP % Average Real Fall
Argentina	1912-1917	29	6
	1929-1932	19	7
	1979-1985	17	3
	1998-2002	21	6
Chile	1912-1915	16	7
	1917-1919	23	8
	1929-1932	33	18
	1971-1975	24	6
	1983-1985	18	10
Indonesia	1941-1949	36	5
	1929-1932	15	8
Malaysia	1929-1932	17	6
	1942-1947	36	9
Mexico	1926-1932	31	6
Peru	1929-1932	29	9
	1941-1943	18	2
	1981-1983	17	9
	1987-1992	30	7
Philippines	1940-1946	60	13
	1982-1985	18	7
Sri Lanka	1929-1932	15	5
	1943-1946	21	6
South Korea	1938-1945	59	8
Taiwan	1942-1945	51	17
Uruguay	1912-1915	30	11
	1917-1919	36	14
	1981-1984	17	7
Venezuela	1913-1916	17	6
	1929-1932	24	8
	1939-1942	22	8
	1977-1985	24	3

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