SEARCH MODEL: DEBIT CARD OR CASH

TESINA

QUE PARA OBTENER EL TÍTULO DE

LICENCIADO EN ECONOMÍA

PRESENTA

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TO MY PARENTS
That always gave me love and provided me with the necessary tools to be a person of integrity.
Thank you.

TO MY BROTHER
That taught me to see life in a different way.

TO MY FRIENDS.
That helped me enjoying gracious moments even more.
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Abstract

This model addresses a strategic complementarity problem with a search-theoretic model to analyze two payment systems: cash and debit cards. The trade-off between cash and debit cards is incorporated such that the sellers incur in a cost of having a point of sale terminal and buyers receive an interest rate for holding a debit card. From four pure-strategy equilibria, two efficient equilibria where trade exist are found: where every agent has a debit card and a terminal, and where cash is used and nobody holds a terminal. This model’s result shows that there cannot be a growth of debit cards without a growth in the number of terminals. This is what we observe in the debit card and terminal growth in Mexico. Also, we find that as the cost of debit card holding decreases a debit card equilibrium is more feasible: as the cost of the terminal decreases debit cards provide enough liquidity to the economy. Finally, conditions over the discount factor showed that agents need to be more patient to achieve the cash equilibrium and need to be more impatient in the card equilibrium.

Key words: search model, payment systems, cash, debit card.
# Contents

1 Introduction ................................................. 2  

2 Motivation ................................................. 3  

3 Literature .................................................. 5  

4 The Economy ............................................... 6  

5 Bellman’s equations  
   5.1 Seller with no terminal .................................... 7  
   5.2 Seller with terminal ....................................... 8  
   5.3 Buyer with debit card .................................... 8  
   5.4 Buyer with cash ......................................... 9  

6 Equilibria .................................................. 9  
   6.1 Cash equilibrium .......................................... 11  
   6.2 Card equilibrium ......................................... 14  
   6.3 Interest rate equilibrium ................................. 16  
   6.4 Inefficient equilibrium ................................. 19  

7 Conditions on the discount factor ......................... 20  

8 Conclusion .................................................. 22  

9 Appendix ................................................... 23  

10 References ................................................. 27
1 Introduction

In the real world, many interactions have friction problems behind. In the labor market the matching is not instantaneous; different locations lead to mismatch, asymmetry in the information leads to unemployment or wage inequality. The housing market has a similar problem where buyers and sellers have a search problem. The fed funds market, being an over-the-counter (OTC) market, decentralized trading lead to inefficiencies. Macroeconomic models solve Jevons’s (1875) absence of coincidence of needs or non-storable goods with money to achieve better allocations through a monetary equilibrium. This model address a strategic complementary problem with search theory to analyze two different payment systems: cash and debit cards. In this thesis we address the following question: what are the fundamentals where debit cards are accepted when there is a more liquid asset like cash?

The payment cards were invented in 1950. They were incredibly successful after a while but, at first, there was a problem. Buyers value a card payment only if there are sufficient sellers that have a terminal. And it is only profitable to sellers having a terminal if there are enough buyers that use a card payment. The model presented gives the necessary conditions to solve this strategic complementary problem between buyers and sellers, and their decision to have a credit card and a terminal. This model shows that there are four pure-strategy possible equilibria. One where everybody holds a debit card and a terminal (card equilibrium); another where no one holds a debit card or a terminal (cash equilibrium); another with sellers that have terminal and buyers with no debit cards (inefficient equilibrium); and finally an equilibrium where there is no trade because buyers have debit cards but sellers do not have terminals (interest rate equilibrium).

Section 2 motivates the theoretical model. The model presented describes the interaction of debit cards and terminals, but it also can explain other payment problems as it will be shown in 2. In section 3 the adopted framework of Kiyotaki and Wright (1993) is explained. Section 4 contextualize the model and the environment is described. In section 5 the Bellman’s equations, or value functions, of the four types of agents are described: seller with no
terminal, seller with terminal; buyer with debit card and buyer with cash. In section 6 the four possible equilibria in pure strategies and the necessary conditions for each equilibrium are described. Finally, section 8 concludes.

2 Motivation

This model finds two plausible equilibria where efficient trade exists; the card equilibrium and the cash equilibrium. These two pure-strategy equilibria state that every seller has a terminal and every buyer a debit card, or that no one has a terminal nor a debit card. This result tells us that the growth rate of terminals and debit cards should mimic each other. When there is an increase in the terminals or debit card usage, it should follow an increase in its counterpart. The same should follow with a decrease on either of the market sides. Figure 1 shows the growth of the terminals and debit cards in Mexico; where we observe that the growth rate of terminals and debit cards mimic each other.

The motivation derived above exemplifies the problem between the usage of debit cards and terminals. But lately, other payment methods have fostered and solved the same problem. The 8 of November of 2016, the Prime Minister of India, Narendra Modi, announced that high currency notes —500 rupees (7.3 dollars) and 1,000 rupees (14.7 dollars)— were abolished and had no longer legal value. This forceful move’s purpose is to fight counterfeit money. As a result, other types of payment methods have risen. Nowadays India is the world’s fastest-growing “proximity mobile payment” market. Taxi drivers prefer electronic payment because it solves the indivisible money problem, where the user of the taxi may not have change to complete the payment.

This mobile payment problem was less costly than the debit card terminal usage problem; terminals have a cost, but who do not have a smartphone? Moreover, there is no charge for making the “proximity mobile payment”, there are even banks that give an interest rate for the money in the bank account, like Paytm Payments Bank. In India, the problem was partially

---

1Proximity mobile payment refers to a payment made with two cellphones
Figure 1: We observe that when one side of the market moves, the other side mimics the movement. Data retrieved from http://www.banxico.org.mx/SieInternet/
solved with a legal disposition where high-value notes stopped having value. In China, no law was needed to solve the problem. In China, with Alipay and WeChat Pay the joke is that beggars rather accept a mobile donation than cash.

3 Literature

As an introduction to the literature section, some history about monetary search models. First generation search models were mostly developed in Kiyotaki and Wright (1989, 1993). In these models, as in this thesis, money is indivisible and agents can only hold one unit of consumption goods or money. These models describe a decentralized environment where fiat money is accepted to alleviate the double coincidence of wants problem. In all these models it is assumed imperfect record keeping of trading histories, so credit is not feasible. Because of this double coincidence of wants problem and the imperfect record keeping, first generation search models create a monetary equilibrium were money achieve better allocations.

Kiyotaki and Wright (1993) motivate the use of money in an economy where agents have two decisions, to incur in a production cost now and produce a real commodity in exchange for an asset they deem useless; and to be willing to exchange the asset for utility. The latter decision takes relevance when the asset gives you a real flow, like an interest rate. The model presented in section 4 adds two decisions more. The decision to have a terminal and the decision to have a debit card. Kiyotaki and Wright (1989) use a search model to determine which commodities would become media of exchange. The paper finds that the commodities that will be used as media of exchange will be those with lest holding cost. We observe something similar in the conclusions of the present work. There will be a usage of debit cards only when the cost of the terminals are low enough, giving the debit card the necessary liquidity to become a medium of exchange.

Since the search approach is very tractable it has a lot of uses. Craig and Rocheteau (2008) use a search model to evaluate the welfare cost of inflation. Lagos and Rocheteau (2007) use
a search model where real assets compete with fiat money; answering the questions of how can fiat money be valued when real assets can be used as means of payment. Something very similar is answered in Burdett et al. (2001). Aiyagari and Wallace (1992) show how fiat money can be used in an economy where there are other assets that give a rate of return; getting to the result that it always exist steady state where everyone accepts the least costly-to-store asset. In particular, they show how if one of those assets is money, then there will always exist a monetary equilibrium. This model shows what the conditions for cash and debit cards are to exist as a medium of exchange; where a low terminal cost enables the debit card equilibrium, making the holding cost of debit cards lower. In plain words, we show that there are robust equilibria where cash and debit cards exist. The coexistence of assets has been also studied from other perspectives. Telyukova and Wright (2008) study the credit card debt puzzle; where there is a high-interest rate on credit cards and low rates on bank accounts, but there is no payment of the debt. They show that that puzzle exists because agents know they might need the liquid asset later on.

4 The Economy

As said above, this model combines elements from standard macroeconomics and search theory. In this model time is discrete and continues forever. There is a continuum of infinitely lived agents that discount time at rate $\beta$. Agents are either buyers or sellers of one many non-storable, non-divisible goods. So, there is a probability $\alpha$ that an agent likes your product or that you like theirs.

Agents can have 1 unit of money in cash or in a debit card. Those agents with an asset are the buyers and have a mass or size of $M$, and can use the asset to get a utility $u$. Those with no money are producers and can sell their consumption good. Those agents that do not have money have a mass of $(1 - M)$ and can produce a real good with cost $c$. Buyers and producers make two decisions each. Buyers choose whether to exchange their asset for a
utility \( u \); via the decision variable \( \tau_1 \). Second, they decide whether to have a debit card that gives a constant flow of \( i \) but to be able to buy only from producers with a terminal, or to be a buyer with cash that gives no interest rate, and buy from all sellers; via the decision variable \( \sigma \).

Sellers first decide if they produce a real good with a cost \( c \) in exchange of a fiduciary asset; via de decision variable \( \tau_0 \). Second, producers decide whether to incur in a cost \( CT \) to have a terminal so they can accept debit cards and cash, or to be a seller that only accepts cash with no cost involved; via the decision variable \( \theta \). Also, if the sellers only accept cash they avoid paying taxes (\( \tau \)) with an exogenous probability \( \mu \) that is the government monitoring.

5 Bellman’s equations

Let \( V_j \) denote the value function of an agent in state \( j \), where \( j = S, T, C, D \). Where \( S \) indicates a seller with no terminal, \( T \) indicates a seller with terminal; \( C \) indicates a buyer with cash, and \( D \) indicates a buyer with a debit card. The value functions satisfy the following Bellman’s equations, that describe how exchange is given and how agents make decisions.

5.1 Seller with no terminal

Sellers with no terminal can only sell to a subset of agents that fulfill five characteristics. The portion of agents that with probability \( \alpha \) like the product, to the portion of agents that have money \( M \), to the portion of agents that do not have debit card \((1 - \theta)\) —they cannot sell to agents with debit card because they do not have terminal—, to the portion of buyers that will exchange the asset for the consumption good \( \tau_1 \), and finally the seller himself must be willing to produce in exchange of the asset \( \tau_0 \). If the seller with no terminal finds an agent with those characteristics, then he will produce with a cost \( c \), and since that transaction was made in cash, there is an exogenous probability that allows the agent to evade taxes \((1 - \mu) \tau\). Finally, the agent that produced and exchanged will go to the next period with money discounted.
by beta, either as a buyer with debit card or only cash; what gives the agent more utility
\( \beta \text{Max} \{V_C, V_D\} \). The last part of the equation shows the probability of not finding an agent with
the necessary conditions to make the exchange, and in that case, he will go to the next
period as a seller with no terminal or with terminal \( \beta \text{Max} \{V_S, V_T\} \).

\[
V_S = \alpha M (1 - \theta) \tau_0 \tau_1 [-c + (1 - \mu) \tau + \beta \text{Max} \{V_C, V_D\}] + (1 - \alpha M (1 - \theta) \tau_0 \tau_1) [\beta \text{Max} \{V_S, V_T\}]
\]

\[\text{5.2 Seller with terminal}\]

The case of a seller with terminal is very similar to the one just mentioned. There are two
things that change. First, the seller can produce for the agents with cash, \((1 - \theta)\); and to
agents with credit card \((\theta)\). Second, there is a cost of having the terminal \(CT\) that the seller
pays no matter what.

\[
V_T = \alpha M (1 - \theta) \tau_0 \tau_1 [-c + (1 - \mu) \tau + \beta \text{Max} \{V_C, V_D\}] + \alpha M (\theta) \tau_0 \tau_1 [-c + \beta \text{Max} \{V_C, V_D\}] + (1 - \alpha M \tau_0 \tau_1) [\beta \text{Max} \{V_S, V_T\}] - CT
\]

\[\text{5.3 Buyer with debit card}\]

The Bellman’s equations for the buyer with debit card has the same logic. The agent needs
to find his counterpart with certain characteristics: agent with no money \((1 - M)\) and seller
with terminal \((\sigma)\). If that is the case, then he receives a utility \(u\) and goes to the next period
without money whether as a seller with or without terminal \(\text{Max} \{V_S, V_T\}\). Lastly, no matter
what, the agent receives an interest rate \((i)\).
\[ V_D = \alpha(1-M)(\sigma)\tau_1\tau_0[u+\beta Max\{V_S, V_T\}] + \\
(1-\alpha(1-M)(\sigma)\tau_1\tau_0)[\beta Max\{V_C, V_D\}] + i \quad (3) \]

5.4 Buyer with cash

The value function of a buyer with cash is very similar to the one above. Two things change. First, the buyer with cash can only exchange with agents that have a terminal \((\sigma)\), and with sellers with no terminal \((1-\sigma)\). Second, there is no interest rate \((i)\).

\[ V_C = \alpha(1-M)(1-\sigma)\tau_1\tau_0[u+\beta Max\{V_S, V_T\}] + \\
\alpha(1-M)(\sigma)\tau_1\tau_0[u+\beta Max\{V_S, V_T\}] + (1-\alpha(1-M)\tau_1\tau_0)[\beta Max\{V_C, V_D\}] \quad (4) \]

6 Equilibria

This section analyses the possible equilibria in pure strategies. To start with, in this model there are heterogeneous agents and commodities; and in equilibrium a specific medium of exchange is chosen to maximize the valuation of the agents. A steady-state equilibrium in this model will constitute a list \(\{V_S, V_T, V_D, V_C, \tau_0, \tau_1, \sigma, \theta\}\) and the maximization of both the buyers and sellers \(\{Max\{V_C, V_D\}, Max\{V_S, V_T\}\}\) given the exogenous parameters \(\{\alpha, M, u, \beta, i, \tau, c\}\) satisfying (1), (2), (3) and (4). In particular, this model use pure-strategy equilibria, so they can be interpreted as “long run” solutions where agents adapt over time and decant for debit cards or cash.

Technically, there are \(2^4\) possible strategy equilibria. But taking order into consideration and denying repetition, table 1 shows the four possible equilibria. The equilibria that are marked with an \(X\) are the ones where there is an efficient trade. These equilibria are where
there are no terminals and no debit cards ($\sigma = \theta = 0$) and where everybody has a terminal and credit cards ($\sigma = \theta = 1$). In the case where there are terminals but no debit cards ($\sigma = 1, \theta = 0$) there is no form of exchange, but agents that decide to hold a debit card receive an interest $i$. The last case where everybody has debit cards and no one holds terminals ($\sigma = 0, \theta = 1$) there are transactions, but in an inefficient way because there is a cost of having terminals, $CT$, and no one holds a debit card that could give the interest rate $i$.

The fact that there are four possible equilibria in pure strategies allow us to test one by one and show what are the necessary conditions to attain that equilibrium. In general, the conditions necessary to generate trade are:

\[
\frac{1}{r}(\hat{V}_i - \hat{V}_j) > c \Rightarrow \tau_0 = 1
\]

\[
\frac{1}{r}(\hat{V}_i - \hat{V}_j) < u \Rightarrow \tau_1 = 1
\]

Where $i = D, C$ and $j = T, S$. The first condition state that there will always be production by the sellers, or $\tau_0=1$. The second condition state that there will always be an exchange of the asset for the utility given by the consumable good, or $\tau_1 = 1$. In other words, the discounted difference between being rich, $i = D, C$, and being poor, $j = T, S$, or the real value of the asset, needs to be larger than the cost of the consumable good, and needs to be smaller than the utility given by consumable good. The following definitions are necessary and will be used for each equilibrium $\hat{V}_i \equiv \frac{r}{1+r}V_i$, $\hat{V}_j \equiv \frac{r}{1+r}V_j$, $\beta = \frac{1}{1+r}$, $Max\{V_S, V_T\} \equiv V_0$, $Max\{V_C, V_D\} \equiv V_1$, $\Delta_D \equiv \hat{V}_D - \hat{V}_T$ and $\Delta_C \equiv \hat{V}_C - \hat{V}_S$. Like in debit card net asset value ($\Delta_D$) and cash net asset value ($\Delta_C$). So, the economic problems agents face is a maximization of $V_j$ and $V_i$ given the exogenous parameters $\{\alpha, M, u, \beta, i, \tau, c\}$ choosing $\{\tau_0, \tau_1, \sigma, \theta\}$. Given

<table>
<thead>
<tr>
<th>$\sigma$</th>
<th>$\theta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>$X$</td>
</tr>
<tr>
<td>1</td>
<td>$X$</td>
</tr>
</tbody>
</table>

Table 1: Pure Strategies possible equilibria.
this economic problem, the parameter conditions can be pinned down with a conjecture on \( \sigma \) and \( \theta \). With that conjecture agents are able to decide between having a debit card or cash, and from having or not a terminal. Finally, forcing \( \frac{1}{r} \Delta_i < u \) and \( \frac{1}{r} \Delta_i > c \) we are able to find conditions on the discount rate \( \beta \).

### 6.1 Cash equilibrium

This equilibrium is the one where no buyer holds a debit card and no seller holds a terminal and the one that is known in literature. In other words, \( \theta = \sigma = 0 \).

**Buyer**

The value functions of the buyer with debit card and with cash replacing \( \sigma = 0 \) are:

\[
V_D = [\beta V_1] + i
\]

\[
V_C = \alpha (1 - M) \tau_1 \tau_0 [u + \beta V_0] + + (1 - \alpha (1 - M) \tau_1 \tau_0) [\beta V_1]
\]

A buyer with debit card in a world with agents with no terminals cannot at all. So if the agent do not change to a buyer with cash he will never be able to get the utility for his asset; he only receives the interest rate \( i \). It is straightforward that \( V_C > V_D \). The condition that suffices is:

\[
\alpha (1 - M) \tau_1 \tau_0 [u + \beta (V_0 - V_1)] > i
\]

With the condition \( V_C > V_D \) it can be shown that the value function of a seller goes to:

\[
\hat{V}_S = \frac{r}{r + \alpha M \tau_1 \tau_0} \left\{ \alpha M \tau_1 \tau_0 \left[ -c + (1 - \mu) \tau + \frac{1}{r} V_C \right] \right\}
\]

In the appendix, the whole derivation is given.
Seller

The value functions of the seller with and without a terminal replacing $\theta = 0$ are:

$$V_T = \alpha M \tau_0 \tau_1 [-c + (1 - \mu) \tau + \beta V_1] + (1 - \alpha M \tau_0 \tau_1) [\beta V_0] - CT$$

$$V_S = \alpha M \tau_0 \tau_1 [-c + (1 - \mu) \tau + \beta V_1] + (1 - \alpha M \tau_0 \tau_1) [\beta V_0]$$

The two value functions are practically the same, but when the agent holds a terminal there is a cost $CT > 0$ that makes $V_S > V_T$. It can be shown that the value function of a buyer without a terminal goes to:

$$\hat{V}_C = \frac{r}{r + \alpha (1 - M) \tau_1 \tau_0} \left\{ \alpha (1 - M) \tau_1 \tau_0 \left[ u + \frac{1}{r} (\hat{V}_S) \right] \right\}$$

In the appendix, the whole derivation is given.

Best responses

Until now, we know that agents do choose cash and being a seller without a terminal. But we do not know what are the conditions on the primitive parameters that make the sellers produce with a cost $c$, and the buyers give their asset for a utility $u$. To define the conditions of this equilibrium we need to impose $\tau_1 = \tau_0 = 1$ and see what are the conditions needed for $\frac{1}{r} (\Delta C) > c$ and $\frac{1}{r} (\Delta D) < u$. With the two conditions found above, $V_C > V_D$ and $V_S > V_T$, we obtain a two by two system of linear equations.

$$\hat{V}_C = \frac{r}{r + \alpha (1 - M) \tau_1 \tau_0} \left\{ \alpha (1 - M) \tau_1 \tau_0 \left[ u + \frac{1}{r} (\hat{V}_S) \right] \right\}$$

$$\hat{V}_S = \frac{r}{r + \alpha M} \left\{ \alpha M \left[ -c + (1 - \mu) \tau + \frac{1}{r} (\hat{V}_C) \right] \right\}$$

Solving the two by two system and using the necessary conditions to create exchange we
obtain for $\tau_0 = 1$:

$$r < \frac{\alpha [(u - c)(1 - M) - M(1 - \mu)\tau]}{c}$$

If this equation is fulfilled, then the seller always produce. But since the producer becomes a buyer in the next period, and the tax evasion $(1 - \mu)\tau$ only benefits the seller, then if there is a bigger tax evasion it is harder to fulfill this equation. We also can observe that if the linear utility $u$ or $\alpha$ (linear cost $c$) go up (goes down) then it is easier to fulfill this condition. Lastly, if the impatience goes up ($r$ goes down) then it is easier to fulfill this equation. Because agent that produce is becoming a seller. So you want to produce to become that seller.

The conditions on the primitives for $\tau_1 = 1$ are:

$$r > \frac{\alpha M[-(u - c) - (1 - \mu)\tau]}{u}$$

If this equation is fulfilled, then the agent always buys. But since the buyer becomes a producer in the next period, and the tax evasion $(1 - \mu)\tau$ only benefits the seller, then if there is a bigger tax evasion, it is easier to fulfill this equation. We also can observe that if the linear utility $u$ or $\alpha$ (linear cost $c$) go up (goes down) then it is easier to fulfill this condition. Lastly, if the impatience goes up ($r$ goes down) then it is harder to fulfill this equation. Because the agent that exchanges his asset is becoming a producer with no money, and the agent has to wait more to have utility again. With these conditions, now we know the conditions on $r$ to achieve a card equilibrium:

$$\frac{\alpha M[-(u - c) - (1 - \mu)\tau]}{u} < \mu^{\text{cash}} < \frac{\alpha [(u - c)(1 - M) - M(1 - \mu)\tau]}{c}$$
6.2 Card equilibrium

This equilibrium is the one where every buyer holds a debit card and every seller holds a terminal. So, we are in the case where $\theta = \sigma = 1$.

Seller

The value functions of the seller with and without a terminal replacing $\theta = 1$ take the following form:

$$V_S = \beta \text{Max} \{V_S, V_T\}$$

$$V_T = \alpha M \tau_0 \tau_1 [-c + \beta V_1] + (1 - \alpha M \tau_0 \tau_1) [\beta \text{Max} \{V_S, V_T\}] - CT$$

A seller without terminal in a world with buyers with debit card cannot exchange at all. So, if the agent does not change to a seller with terminal he will never be able to sell his production. The seller with a terminal can clearly sell his production but needs to pay the cost for the terminal.

It is straightforward that $V_T > V_S$. The condition that suffices is:

$$\alpha M \tau_0 \tau_1 [-c + \beta (V_1 - V_0)] > CT$$

With the condition $V_T > V_S$ it can be shown that the value function of a seller with terminal goes to:

$$\dot{V}_T = \alpha M(\theta) \tau_0 \tau_1 \left[-c + \frac{1}{\frac{1}{D}}\right] - CT$$

In the appendix, the whole derivation is given.
Buyer

The value functions of the buyer with cash and with debit card replacing \( \sigma = 1 \) are:

\[
V_C = \alpha (1 - M) \tau_1 \tau_0 [u + \beta V_0] + (1 - \alpha (1 - M) \tau_1 \tau_0) [\beta V_1]
\]

\[
V_D = \alpha (1 - M) \tau_1 \tau_0 [u + \beta V_0] + (1 - \alpha (1 - M) \tau_0) [\beta V_1] + i
\]

The two value functions are practically the same, but when the agent holds a debit card there is an interest rate \( i > 0 \) that makes \( V_D > V_C \). It can be shown that the value function of a buyer with debit card goes to:

\[
\hat{V}_D = \alpha (1 - M)(\sigma) \tau_1 \tau_0 \left[u - \frac{1}{r} \Delta_D\right] + i
\]

In the appendix, the whole derivation is given.

Best responses

Until now, we know that agents do choose debit cards and terminals. But we do not know what are the conditions on the primitive parameters that make the sellers produce with a cost \( c \), and the buyers give their asset for a utility \( u \). To force we need to impose \( \tau_1 = \tau_0 = 1 \) and to see what are the conditions needed for \( \frac{1}{r} \Delta_D > c \) and \( \frac{1}{r} \Delta_D < u \).

Replacing \( \hat{V}_D \) and \( \hat{V}_T \) in \( \frac{1}{r}(\hat{V}_D - \hat{V}_T) > c \) and solving for \( \frac{1}{r} \Delta_D \) we obtain:

\[
\frac{1}{r} \Delta_D = \frac{\alpha (1 - M) u + \alpha M c + i + CT}{r + \alpha (1 - M) + \alpha M}
\]

The necessary conditions to create exchange we need to impose \( \tau_0 = 1 \), so:

\[
\frac{u (1 - M) \alpha + \alpha M c + CT + i}{r + \alpha} > c
\]
\[
\frac{i + CT + \alpha (u - c)(1 - M)}{c} > r
\]

From the last equation, we observe that a higher cost makes it harder to fulfill the condition, and a higher \( u \) makes it easier. Something that someone would expect from the model is that if \( \alpha \) increase, then the conditions fulfill easier. We observe that from the last equation.

The conditions on the primitives for \( \tau_1 = 1 \) to force exchange are:

\[
\frac{\alpha Mc + i + CT}{r + \alpha M} < u
\]

\[
\frac{i + CT - \alpha (u - c)M}{u} < r
\]

From the last equation, we can see that if the cost of producing goes up it is harder to fulfill this condition. Again, if meeting probability \( \alpha \) goes up, either in the buyer or the seller's case, it is easier to fulfill the conditions. Something that is worth noticing is that if the interest rate or the terminal cost goes up, then it is harder for the buyer to trade the asset. This makes sense because if the flow of the asset that you have increments, then you will not trade it away that easy. Also, the fact that an increment in the terminal cost makes it harder to fulfill the condition makes sense. If for the seller is more expensive to get a terminal cost, then there will be less terminals, so for the buyer, it will be harder to trade their asset. With these conditions, now we know the conditions on the discount rate to achieve a card equilibrium:

\[
\frac{i + CT - \alpha (u - c)M}{u} < r_{\text{card}} < \frac{i + CT + \alpha (u - c)(1 - M)}{c}
\]

### 6.3 Interest rate equilibrium

This equilibrium is the one where the buyers hold only cash and the sellers hold terminal. In other words, \( \sigma = 1, \theta = 0 \).
Buyer

The value functions of the buyers with debit card and with cash replacing $\sigma = 1$ are:

\[
V_D = \alpha (1 - M) \tau_1 \tau_0 [u + \beta V_0] + (1 - \alpha (1 - M) \tau_1 \tau_0) [\beta V_1] + i
\]

\[
V_C = \alpha (1 - M) \tau_1 \tau_0 [u + \beta V_0] + (1 - \alpha (1 - M) \tau_1 \tau_0) [\beta V_1]
\]

The decision of the buyer whether to have a debit card or hold cash depends on the assumption made on the terminals; and since in this equilibrium it is supposed that every seller hold a terminals, the buyers decide to hold a credit card because it gives an extra $i$. It is straightforward that $V_D > V_C$. The condition that suffices is $i > 0$.

With the condition $V_D > V_C$ it can be shown that the value function of the seller goes to:

\[
\hat{V}_S = \frac{r}{r + \alpha M \tau_1 \tau_0} \left\{ \alpha M \tau_1 \tau_0 \left[ -c + (1 - \mu) \tau + \frac{1}{r} \hat{V}_D \right] \right\}
\]

In the appendix, the whole derivation is given.

Seller

The value functions of the seller with and without a terminal are:

\[
V_T = \alpha M \tau_0 \tau_1 [-c + (1 - \mu) \tau + \beta V_1] + (1 - \alpha M \tau_0 \tau_1) [\beta V_0] - CT
\]

\[
V_S = \alpha M \tau_0 \tau_1 [-c + (1 - \mu) \tau + \beta V_1] + (1 - \alpha M \tau_0 \tau_1) [\beta V_0]
\]

The same logic applies here. The decision of the seller to have a terminal or not depends on the supposition made on the debit cars; and since in this equilibrium it is supposed they the agents hold only cash, the sellers decide not to hold terminals because of the cost of terminals $CT$. So, $V_S > V_T$ and the condition that suffices is $CT > 0$. 
With the condition $V_S > V_T$ it can be shown that the value function of a buyer goes to:

$$\hat{V}_D = \left( \frac{r}{r + \alpha (1 - M) \tau_1 \tau_0} \right) \left\{ \alpha (1 - M) \tau_1 \tau_0 \left[ u + \frac{1}{r} \hat{V}_S \right] + i \right\}$$

In the appendix, the whole derivation is given.

**Best responses**

In this economy where $V_S > V_T$ and $V_D > V_C$ there are no transactions. Buyers hold debit cards, but sellers do not hold terminals. Sellers would like to sell their production, so $\tau_0 = 1$. Buyers given that cannot buy anything prefer to hold to their asset and receive $i$, so $\tau_1 = 0$. Then in this equilibrium we need to force: $\frac{1}{r}(\hat{V}_D - \hat{V}_S) > c$ and $\frac{1}{r}(\hat{V}_D - \hat{V}_S) > u$. Again we have a two by two system of linear equations.

$$\hat{V}_D = i$$

$$\hat{V}_S = 0$$

So the conditions necessary for this equilibrium to hold the asset are:

$$\frac{i}{r} > c$$

$$\frac{i}{r} > u$$

$$\frac{i}{u} > \mu_{interest}$$

Given that $u > c$ the only condition that operates is the first one. The interest rate has to be so high that, even discounted by $r$, it has to be bigger than the utility given by the
consumable good. From the latest equation, we can observe that if the linear utility goes up, then it is harder to fulfill this condition; because the agent rather exchanges the asset for the consumable good.

6.4 Inefficient equilibrium

In this last equilibrium it is supposed that buyers have debit cards but sellers have no terminals. In other words, $\sigma = 0, \theta = 1$.

**Buyer**

The value functions of the buyers with debit card and with cash replacing $\sigma = 0$ are:

$$V_D = i$$

$$V_C = \alpha (1 - M) (\tau_1 \tau_0 [u + \beta V_0] + + (1 - \alpha (1 - M) \tau_1 \tau_0) [\beta V_1]$$

In this equilibrium, it is supposed that no seller hold a terminal, so the buyers decide to hold cash, clearly.

**Seller**

The value functions of the seller without a terminal and with a terminal are:

$$V_T = \alpha M(\theta) \tau_0 \tau_1 [-c + \beta V_1] + (1 - \alpha M \tau_0 \tau_1) [\beta V_0] - CT$$

$$V_S = 0$$

The same logic applies here. The decision of the seller to have a terminal or not depends on the supposition made on the debit cars; and since in this equilibrium it is supposed that the
agents hold credit cards, the sellers decide to hold terminals. So, $V_S > V_T$ and the condition that suffices is $CT > 0$.

This equilibrium is clearly an inefficient one. Buyers decide to hold only cash, buy sellers hold terminals. So buyers do not receive the interest rate $i$ but sellers have the terminal cost, $CT$.

**Best responses**

In this economy both the seller and the produce want to exchange, so $\tau_0 = 1$ and $\tau_1 = 1$.

$$\frac{CT + \alpha (u - c) (1 - M)}{c} > \frac{CT - \alpha (u - c) M}{u}$$

These best response conditions are very similar to those found in the card and cash equilibria. They are the same as the card best responses, but with no $i$. So, for the producer it is harder to fulfill the condition, but for the buyers it is easier to give away their assets since there is no interest rate that the asset gives.

**7 Conditions on the discount factor**

The conditions to generate trade show us how patient (or impatient) the agent has to be to achieve that equilibrium. Remembering that $\beta = \frac{1}{1+r}$, the bigger the $r$ the more impatient the agent is. The brackets of the conditions on the discount factor for each equilibrium are:

**Cash**

$$\frac{\alpha M [- (u - c) - (1 - \mu) \tau]}{u} < \mu^{\text{cash}} < \frac{\alpha [(u - c) (1 - M) - M (1 - \mu) \tau]}{c}$$

**Card**

$$\frac{i + CT - \alpha (u - c) M}{u} < \mu^{\text{card}} < \frac{i + CT + \alpha (u - c) (1 - M)}{c}$$

19
Inefficient

\[ \frac{CT - \alpha (u - c) M}{u} < r_{\text{inefficient}} \leq \frac{CT + \alpha (u - c)(1 - M)}{c} \]

Interest rate

\[ r_{\text{interest}} < \frac{i}{u} \]

In this model it is assumed an interest rate \( i > 0 \), so the condition \( \frac{1}{u}(\Delta_i) < u \). takes relevance, giving a lower bound as well as the normal upper bound found in the literature. It is straightforward that \( r_{\text{cash}} < r_{\text{inefficient}} < r_{\text{card}} \). So, agents are more impatient in the card equilibrium and are less impatient in the cash one. The fact that agents need to be more patient in the cash equilibrium comes from the fact that the evadable taxes are collected by the seller. And given the definition of \( \Delta_D = \hat{V}_D - \hat{V}_T \), the seller valuation comes in the next period. So, if agents want to evade taxes they have to wait until the next period, hence be patient. That is why if the amount of evadable taxes \( (1 - \mu) \tau \) goes up, then the agent has to be more patient to get to the next period and to that equilibrium.

In the card equilibrium, agents must be less patient because the seller cannot evade taxes in the next period. In the card equilibrium the interest rate \( i \) is given that first period to the buyer, and if the agent is impatient then the cost of the terminal \( CT \) will be lower relatively. That is why if the interest rate \( i \) or the terminal cost \( CT \) goes up then you need to be less patient to get to this equilibrium.

One interesting thing to see would be to have the evadable taxes in the buyer side instead of the seller. This would be interesting because in that way the conditions for trade in the cash equilibrium would be:

\[ \frac{\alpha M [-(u - c) + (1 - \mu) \tau]}{u} < r_{\text{cash}} < \frac{\alpha [(u - c)(1 - M) + M(1 - \mu) \tau]}{c} \]

In that case, the government monitoring \((\mu)\) would play an interesting role over the evadable taxes. If the government monitoring is deficient, then it would follow that \( (1 - \mu) \tau > \)
$i + CT$, because agents would be able to evade a lot of taxes. In that case, only the patient agents would achieve the card equilibrium. On the other hand, if the government monitoring is good enough, then it would follow that $(1 - \mu) \tau < i + CT$. In that case, the impatient agents would get to the card equilibrium.

8 Conclusion

This model shows that there are four possible pure strategy equilibria. Each one of them with best responses that define them. In the interest rate equilibrium, we found that the interest rate weighted by the utility has to be bigger than the discount factor $r$. In the cash equilibrium, we saw that the tax evasion had a different effect on buyers and sellers, helping only the sellers to fulfill their conditions. The inefficient equilibrium is very similar to the financial one but gives no interest rate $i$ to the buyers. In this equilibrium and in the card equilibrium we observed that a lower cost of the terminal helps buyers to fulfill their conditions, making trade more feasible. This is something that mirrors something happening in India. India has very little bank branches in rural areas, yet this is where half of 1.32 billion people live. Having access to the financial system is close to impossible, but with mobile wallets that have a very low terminal cost, India now is the fastest mobile payment market.

This model has important extensions ahead. For example, Lee (2013) analyses the impact on welfare when a different price is charged for a commodity paid with cash or debit card. In the U.K retailers differentiate prices given the payment method, charging more for payments in card. In the U.S retailers cannot differentiate. Their results suggest that a uniform pricing increase the consumption dispersion between the poor and rich. This is partially explained in this model; debit card payment is more expensive because there is a cost of the terminal and you cannot evade taxes. As a retailer, you will increase the price of paying with cash if you can only charge one price for both payment methods. To analyze the welfare impact in our model, we would have to move our first generation model to a second generation model;
where the indivisibility restriction in goods and money is loosen and prices are determined endogenously, maintaining the decentralized trading assumption. With this loosen restriction, we could see how the constrained and unconstrained price method affects welfare in our model. Another plausible extension would be making a distribution of agents that have access to ATM’s more easily than others achieving mixed strategies.

This model finds two efficient equilibria where trade exist: where every agent has a debit card and a terminal and where cash is used and nobody holds a terminal. This result shows that there cannot be a growth of debit cards without a growth in the number of terminals. This is what we observe in the debit card and terminal growth in Mexico. Finally, conditions over the discount factor showed that agents need to be more patient to achieve the cash equilibrium because tax evasion comes discounted periods after. On the other hand, agents need to be less patient in the card equilibrium because the positive interest rate is given that same period and the negative terminal cost comes discounted periods after. In few words, this model showed the primitive conditions where debit cards provide enough liquidity to the economy, finding that the terminal cost needs to be low enough to achieve this equilibrium.

9 Appendix

With $\max \{ \hat{V}_S, \hat{V}_T \} \equiv \hat{V}_0$ and $\max \{ \hat{V}_C, \hat{V}_D \} \equiv \hat{V}_1$

No terminal

$$V_S = \alpha M (1 - \theta) \tau_0 \tau_1 \left[ -c + (1 - \mu) \tau + \beta V_1 \right] + (1 - \alpha M (1 - \theta) \tau_0 \tau_1) [\beta V_0]$$

Knowing $V_S > V_T$ and with $\hat{V}_S = \frac{r}{1 + r} V_S$.

$$\hat{V}_S = \alpha M (1 - \theta) \tau_0 \tau_1 \left[ -c + (1 - \mu) \tau + \beta (\hat{V}_1 - V_S) \right]$$
\[
\hat{V}_S = \alpha M (1 - \theta) \tau_0 \tau_1 \left[-c + (1 - \mu) \tau + \frac{1}{r} (\hat{V}_1 - \hat{V}_S)\right]
\]

\[
\hat{V}_S \left(\frac{r + \alpha M (1 - \theta) \tau_1 \tau_0}{r}\right) = \alpha M (1 - \theta) \tau_1 \tau_0 \left[-c + (1 - \mu) \tau + \frac{1}{r} \hat{V}_1\right]
\]

\[
\hat{V}_S = \frac{r}{r + \alpha M (1 - \theta) \tau_1 \tau_0} \left\{ \alpha M (1 - \theta) \tau_1 \tau_0 \left[-c + (1 - \mu) \tau + \frac{1}{r} \hat{V}_1\right]\right\}
\]

**Terminal**

\[
V_T = \alpha M (1 - \theta) \tau_0 \tau_1 \left[-c + (1 - \mu) \tau + \beta V_1\right] + \\
\alpha M (\theta) \tau_0 \tau_1 \left[-c + \beta V_1\right] + (1 - \alpha M \tau_0 \tau_1) [\beta V_0] - CT
\]

Knowing \(V_T > V_S\) and with \(\hat{V}_T = \frac{r}{1 + r} V_T\)

\[
\hat{V}_T = \alpha M (1 - \theta) \tau_0 \tau_1 \left[-c + (1 - \mu) \tau + \beta (\hat{V}_1 - V_T)\right] + \\
\alpha M (\theta) \tau_0 \tau_1 \left[-c + \beta (\hat{V}_1 - V_T)\right] - CT
\]

\[
\hat{V}_T = \alpha M (1 - \theta) \tau_0 \tau_1 \left[-c + (1 - \mu) \tau + \frac{1}{r} (\hat{V}_1 - \hat{V}_T)\right] + \\
\alpha M (\theta) \tau_0 \tau_1 \left[-c + \frac{1}{r} (\hat{V}_1 - \hat{V}_T)\right] - CT
\]

With \(\Delta_F = Max \{\hat{V}_C, \hat{V}_D\} - \hat{V}_T\)

23
\[ \hat{V}_T = \alpha M(1 - \theta) \tau_0 \tau_1 \left[ -c + (1 - \mu) \tau + \frac{1}{r} \Delta F \right] + \alpha M(\theta) \tau_0 \tau_1 \left[ -c + \frac{1}{r} \Delta F \right] - CT \]

For the derivation of the card equilibrium the last equation suffices.

\[ \hat{V}_T \left( \frac{r + \alpha M \tau_0}{r} \right) = \alpha M(1 - \theta) \tau_0 \tau_1 \left[ -c + (1 - \mu) \tau + \frac{1}{r} \hat{V}_1 \right] + \alpha M(\theta) \tau_0 \tau_1 \left[ -c + \frac{1}{r} \hat{V}_1 \right] - CT \]

\[ \hat{V}_T = \left( \frac{r}{r + \alpha M \tau_0 \tau_1} \right) \left\{ \alpha M(1 - \theta) \tau_0 \tau_1 \left[ -c + (1 - \mu) \tau + \frac{1}{r} \hat{V}_1 \right] + \alpha M(\theta) \tau_0 \tau_1 \left[ -c + \frac{1}{r} \hat{V}_1 \right] - CT \right\} \]

**Debit card**

\[ V_D = \alpha (1 - M)(\sigma) \tau_1 \tau_0 [u + \beta V_0] + (1 - \alpha (1 - M)(\sigma) \tau_1 \tau_0) [\beta V_1] + i \]

Knowing \( V_D > V_C \) and with \( \hat{V}_D = \frac{r}{1+r} V_D \)

\[ \hat{V}_D = \frac{r}{1+r} V_D = \alpha (1 - M)(\sigma) \tau_1 \tau_0 [u + \beta (\hat{V}_0 - V_D)] + i \]

\[ \hat{V}_D = \alpha (1 - M)(\sigma) \tau_1 \tau_0 \left[ u + \frac{1}{r} (\hat{V}_0 - \hat{V}_D) \right] + i \]

With \( \Delta F = \hat{V}_1 - \hat{V}_T \)
\[ \hat{V}_D = \alpha (1 - M)(\sigma) \tau_1 \tau_0 \left[ u - \frac{1}{r} \Delta_F \right] + i \]

For the derivation of the card equilibrium the last equation suffices.

\[ \hat{V}_D \left( \frac{r + \alpha (1 - M)(\sigma) \tau_1 \tau_0}{r} \right) = \alpha (1 - M)(\sigma) \tau_1 \tau_0 \left[ u + \frac{1}{r} \left( \hat{V}_0 \right) \right] + i \]

\[ \hat{V}_D = \left( \frac{r}{r + \alpha (1 - M)(\sigma) \tau_1 \tau_0} \right) \left\{ \alpha (1 - M)(\sigma) \tau_1 \tau_0 \left[ u + \frac{1}{r} \left( \hat{V}_0 \right) \right] + i \right\} \]

**Cash**

\[ V_C = \alpha (1 - M)(1 - \sigma) \tau_1 \tau_0 \left[ u + \beta V_0 \right] + \alpha (1 - M)(\sigma) \tau_1 \tau_0 \left[ u + \beta V_0 \right] \]

\[ + (1 - \alpha (1 - M) \tau_1 \tau_0) [\beta V_1] \]

Knowing \( V_C > V_D \) and with \( \hat{V}_C = \frac{r}{1 + r} V_C \)

\[ \hat{V}_C = \alpha (1 - M)(1 - \sigma) \tau_1 \tau_0 \left[ u + \beta (\hat{V}_0 - V_C) \right] + \alpha (1 - M)(\sigma) \tau_1 \tau_0 \left[ u + \beta (\hat{V}_0 - V_C) \right] \]

\[ \hat{V}_C = \alpha (1 - M)(1 - \sigma) \tau_1 \tau_0 \left[ u + \frac{1}{r} (\hat{V}_0 - \hat{V}_C) \right] + \alpha (1 - M)(\sigma) \tau_1 \tau_0 \left[ u + \frac{1}{r} (\hat{V}_0 - \hat{V}_C) \right] \]

\[ \hat{V}_C \left( \frac{r + \alpha (1 - M) \tau_1 \tau_0}{r} \right) = \alpha (1 - M)(1 - \sigma) \tau_1 \tau_0 \left[ u + \frac{1}{r} \left( \hat{V}_S \right) \right] \]

\[ + \alpha (1 - M)(\sigma) \tau_1 \tau_0 \left[ u + \frac{1}{r} \left( \hat{V}_S \right) \right] \]
\[ \hat{V}_C = \frac{r}{r + \alpha(1-M) \tau_1 \tau_0} \left\{ \alpha(1-M)(1-\sigma)\tau_1 \tau_0 \left[ u + \frac{1}{r}(\hat{V}_S) \right] + \alpha(1-M)(\sigma)\tau_1 \tau_0 \left[ u + \frac{1}{r}(\hat{V}_0) \right] \right\} \]
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