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NúMERO 19
Juan Rosellón
RULES OF ORIGIN
AND WELFARE ANALYSIS
Introduction

There is an internal conflict in a country that is preparing for the negotiation of a free trade agreement. This conflict takes place between the domestic producer of the input to a certain final product and the "third-country" producer of that final product over the definition of the rule of origin of the final product. Intuitively, the producer of the final product will in general prefer less restrictive rules of origin than the input producers in order to have access to a large variety of inputs. The producer of the input will want more stringent rules so that the demand for his product increases.

Above this conflict, the government which is negotiating the FTA must decide which rules of origin might maximize the welfare of its country subject to the restriction of disagreements of interests in the productive chain. This, in an effort to reconcile private and public interests.

In this context, we develop in this paper a model where the government is seeking to choose an optimal rule of origin which maximizes the labor income (which measures social welfare) and solves the conflict in the productive chain by:

a) assuring adequate profits for producers of inputs and final products;

b) inducing technological improvement in the domestic input sector by transferring technology to this sector from the final product firm;

c) guaranteeing an increase in the demand for the domestic input over time.

While choosing such a rule, the government must consider that its decision will be taken once and "written in stone" from then on. That is, the selection of the rule of origin will be a one-shot, time-invariant decision.

In our model the government will try to take advantage of the new or advanced techniques of the established third-country private capital. By the example this foreign firm sets, the diffusion of technological advance will be promoted in the economy, generating external economies and enhancing the demand for domestic inputs.

This paper consists of four sections. In the first, the basic elements of the model are presented. In the second section, the method of solution is studied; in the following section, examples that help to illustrate and characterize the solution are showed. Finally, in the last section of this paper, some concluding remarks are made.

The numerical calculations in section 3 show that, in general, there is not a rule that reconciles the diverse interests for the whole economy. The optimal rule may vary from industry to industry, depending on the specifics of each one of these. In each particular industry, what is good for the domestic producers of intermediates may not always be so good for social welfare and vice versa. Even if a high rule

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appears to be better for the domestic input producers, a lower rule may be the one which maximizes the wage income of workers and reconciles the diverse interests in the productive chain. An opposite situation may also occur. It all depends on the specific market and technological restrictions faced in each industry.

1. The model

A Japanese firm based in Mexico sells its final product $Y$ in the U.S. under perfect competition. $Y$ is produced with two non-substitutable kind of inputs: labor $l_1$ and capital $z$ which can be purchased in Mexico ($z_D$) or in Japan ($z_I$).\footnote{That is, we are assuming $z = z_D + z_I$, which implies that the domestic and imported inputs are perfect substitutes. This is a similar assumption to the one of Grossman (1981). However, unlike this author, we also assume that $z$ and $l_1$ are non-substitutable.} The firm is a monopsonist in the Mexican market for $z_D$, but is a price taker in the markets of the Japanese input $z_I$ and labor $l_1$. It uses technology $F(z,l_1) = \min(z,l_1)$.

In Mexico, $z_D$ is produced by a large number of domestic firms which cannot influence its price $p_D$. A representative firm $i$ is a price taker in the labor market and produces $z_D^i$ with labor $l_1^i$ and technology $a$ using the production function $G(a,l_1^i) = a l_1^i \beta$, where $\beta \in [0,1]$.

We assume that $l_1$ and $l_2$ are two different types of labor and, as in Brito and Intriligator (1991), that one type of labor cannot be trained to become the other. The supply functions of labor are given by $w_i = l_i$, where $w_i$ is the price of labor $l_i$ and $i \in \{1,2\}$.

From the first order conditions of the input producers, we can obtain the demand for labor in their sector:

$$l_2 = \sum_{i=1}^{2} l_2^i = n \left( \frac{w_2}{p_D \alpha \beta} \right)^{\frac{1}{\phi - 1}}$$

Clearing the market for $l_2$ and using $G$, we obtain the supply function of the input sector:

$$p_D = \frac{z_D^{\frac{\rho + \beta}{\phi \beta}}}{n^{\frac{1}{\phi - \beta}} \alpha \beta}$$

(1)

Now observe that the profit function of the Japanese firm can be written as:

$$\Pi = p_C l_1 - w_1 l_1 - \left( p_D (z_D) + \frac{p_w}{\alpha} \left( 1 + \frac{1}{\alpha} \right)^{-1} l_1 \right)$$

(2)

\[ \text{where } p_C \text{ is the price of the final product, } w_1 \text{ is the price of labor, and } \alpha \text{ is the technological coefficient.} \]
where:

\[ p_c = \text{international price of the final product produced by the Japanese firm}; \]
\[ p_w = \text{international price of the Japanese input}; \]
\[ \alpha = \frac{z_p}{z_r} \]

Substituting (1) into (2), using \( z_D = \left(1 + \frac{1}{\alpha}\right)^{-1} l_i \), and from the first order conditions of the Japanese firm, we obtain:

\[ w_i = p_c - \frac{2l_1^{\beta-\gamma}h}{\beta^2 n^{2 \xi-\phi \beta} a^\phi} \left(1 + \frac{1}{\alpha}\right)^{-\gamma} - p_w (1 + \alpha)^{-1} \]

Clearing the market for labor \( l_i \), we obtain:

\[ l_1 - p_c + \frac{2l_1^{\beta-\gamma}h}{\beta^2 n^{2 \xi-\phi \beta} a^\phi} \left(1 + \frac{1}{\alpha}\right)^{-\gamma} + p_w (1 + \alpha)^{-1} = 0 \]

which is an implicit function of \( a \) and \( \alpha \). We denote this function by \( I_1(a, \alpha) \).

Therefore the set of equilibrium quantities and prices for given values of \( a \) and \( \alpha \) is:

\[ z_D^* = \left(1 + \frac{1}{\alpha}\right)^{-1} l_1(a, \alpha); \]
\[ l_1^* = I_1(a, \alpha); \]
\[ l_2^* = n^{(\beta-\gamma)h} \left(\frac{l_1(a, \alpha)}{a}\right)^{\gamma} \left(1 + \frac{1}{\alpha}\right)^{-\gamma}; \]
\[ p_D^* = n^{(\beta-\gamma)h} \frac{l_1(a, \alpha)^{\gamma-\phi} h}{a^{\phi} \beta} \left(1 + \frac{1}{\alpha}\right)^{\gamma-\phi}. \]

\(^2\) In the calculation of these general equilibrium quantities, we are implicitly applying Walras Law by making equal to one the price of the good consumed by workers and clearing the market for that good.
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\[ w_1^* = p_c - 2n^{\beta_{1-\beta} \alpha} \left( \frac{l_1(a, \alpha)^{1-\beta} \beta}{a^\gamma \beta^\gamma} \right)^{1-\beta} - p_w (1 + \alpha)^{-\gamma}; \]

\[ w_2^* = n^{\beta_{1-\beta} \alpha} \left( \frac{l_1(a, \alpha)^{1-\beta} \beta}{a^\gamma \beta^\gamma} \right)^{1-\beta} \left( 1 + \frac{1}{\alpha} \right)^{-\gamma}. \]

We have a special interest in expressing the equilibrium expressions for the quantity and price variables in the economy as functions of technology \( a \) and the ratio \( \alpha \). The reason for this is that we will substitute these equilibrium expressions in a control model (that will be developed in the next few pages) which will have \( \alpha \) as the control variable and \( a \) as the state variable. \( \alpha \) and \( a \) will be the "engines" of the economy but, as we will study, the problem will not be in a strict sense one of optimal control since \( \alpha \) will not depend on time. In the solution of such problem, \( a \) will turn out to depend on \( \alpha \).

Let \( \alpha \) now be a variable called the "rule of origin". We will be interested in how movements of \( \alpha \) may affect the equilibrium prices and quantities and also the technology. At this moment, we would like to know how the demand for the domestic input behaves when the rule of origin becomes more restrictive. The following proposition implies that an increase in \( \alpha \) affects the demand for the domestic input positively if the elasticity of the technology \( a \) with respect to the rule of origin is large enough.

**Proposition:** If the elasticity of the domestic sector technology \( a \) with respect to the rule of origin is greater than one, then the elasticity of the equilibrium labor quantity \( l_1^* \) of the Japanese sector is positive.

**Proof:** From the equilibrium expression for \( l_1^* \), we can implicitly differentiate with respect to \( \alpha \) and get

\[
\frac{\partial l_1^*}{\partial \alpha} = 2 \left( 1 + \frac{1}{\alpha} \right)^{-\gamma} \beta l_1^{\alpha-\beta} \left( \frac{2}{\beta} a^\gamma \right) \left( a^{-1} \frac{\partial a}{\partial \alpha} - (1 + \frac{1}{\alpha})^{-1} \alpha^{-2} \right) + p_w (1 + \alpha)^{-\gamma} \beta^\gamma n^{\beta_{1-\beta} \alpha} \left( \frac{2}{\beta} \right) l_1^{\alpha-\beta} \beta^\gamma + 2(1 + \frac{1}{\alpha})^{-\gamma} \alpha^{-2} \beta \left( \frac{2}{\beta} \right) l_1^{\alpha-\beta} \beta^\gamma \]

Then if

\[ \beta > \frac{2}{\alpha} \]

\[ Musa (1984) \text{ finds in his model that the equilibrium amount of domestic input increases with an increase in content protection if the elasticity for the final product is less than a critical value. We, in a previous paper, found the effects that influence the way in which the rule of origin alters the equilibrium amount of domestic input.} \]
\[
\frac{\partial a}{\partial a} \alpha \geq (1 + \alpha)^{-1}
\]

we will have

\[
\frac{\partial l^*}{\partial a} > 0.
\]

Since we are only interested in non-negative values of the rule of origin \(\alpha\), then the proposition follows. \textit{Q.E.D.}

From this proposition it follows immediately that

\[
\frac{\partial z^*}{\partial a} = (1 + 1/\alpha)^{-1} \frac{\partial l_1}{\partial a} + l_1(\alpha)(1 + 1/\alpha)^{-2} \alpha^2 \geq 0
\]

if \(\frac{\partial a}{\partial a} \alpha \geq 1\).

Now assume time \(t\) is continuous and \(t \in [0, \infty)\). Suppose there is a Mexican policy maker interested in choosing an \(\alpha\) which maximizes

\[
\int_0^\infty e^{-\delta t} \left[ w_1^* l_1^* + w_2^* l_2^* \right] dt
\]

subject to

\[
\bar{a}(t) = e^{-\delta t} \left[ a(t) l_2^* \right]^{\delta}
\]

\[
\Pi^* = R_k l_1^* - w_1^* l_1^* - \left( p_2^* + \frac{p_k}{\alpha} \right)(1 + 1/\alpha)^{-1} l_1^* \geq K
\]

\[
a(0) = a_0 (a_0 > 0)
\]

\[
\alpha \geq 0
\]

where \(p, \xi \in (0,1)\) are discount factors, \(\delta \in (0,1)\) and \(K\) is the non-negative amount of profits that the Japanese firm would obtain, at any time \(t\), without complying with the rule of origin \(\alpha\) and without receiving tariff preferences on its exports.\(^4\)

\(^4\) \(K\) will depend on the technology used in the input sector and on the ratio \(\alpha_0\) of domestic to imported inputs used by the Japanese firm in the equilibrium without government intervention. \(K\) will also depend, as we studied in chapter 2, on the tariffs that the firm should pay for exporting to the U.S.
In other words, the policy maker is trying to find a value of the rule of origin which maximizes labor income over time. This rule of origin must maintain a level of profits for the Japanese firm which will make this firm comply with such a rule. Also, the rule of origin chosen has to insure a certain technological progress in the Mexican input sector. This technological progress is achieved through the transfer of technology from the final-product sector to the input sector through a learning-by-doing process. The evolution of technology has a very particular form which depends on the production function of the domestic input sector as in Rivero-Batiz and Romer (1991). This means that, through the process which produces the domestic input, technological progress is also "produced" in a smaller proportion ($\delta \in (0,1)$). This is the basic idea at the heart of the typical learning-by-doing argument (see, for example, Mookherjee and Ray, 1989).

In making his choice of $\alpha$, the policy maker must take into account that he is making a "one-shot" decision; he will not be able to make later adjustments to the optimal rule chosen. This situation is similar to the one in Brito and Oakland (1977), where the policy maker is trying to find the optimal quantity of public goods when the expenditure on these goods is financed by an optimal income tax. As we will see later, the mathematical method to solve system (3) when $\alpha$ is time invariant is provided by Kokotovic and Heller (1967).

Using the equilibrium expressions for prices and quantities, we can rewrite (3) as:

$$\max_{\alpha} \int_0^{\infty} e^{-\rho t} \left[ P_0 \frac{\partial (a,\alpha)}{\partial a} + \frac{\partial (a,\alpha)}{\partial a} \right] dt$$

subject to

$$\dot{\alpha}(t) = e^{-\rho t} \left[ \frac{n^{\beta-1} (a,\alpha) (1 + 1/\alpha)^{-1}}{n^{\gamma} \alpha^{\gamma}} \right]$$

$$\Pi^*(\alpha, t) = \frac{\partial (a,\alpha)}{\partial a} \left( \frac{2}{\beta^2} - \frac{1}{\beta} \right) \geq K$$

(4)

$$a(0) = a_0 (a_0 > 0)$$

$$\alpha \geq 0$$

Observe that what the policy maker is doing in (4) is picking, at each time $t$, one of an infinite number of general equilibria (one for each $\alpha$) expressed by the equilibrium quantities and prices $z_0^*, l_1^*, l_2^*, \rho_0, w_1^*$ and $w_2^*$. That is, the policy maker chooses a continuous path of general equilibria which maximizes the labor income in the econ-
omony, assures a certain technological progress in the domestic sector and maintains a level of profits for the Japanese firm greater than \( K \) over time.\(^5\) Other examples where the optimizer takes the maximizing behaviour of the other agents in the economy as a constraint in the selection of an optimal policy are given in the optimal income-tax papers by Mirlees (1971) and Brito and Oakland (1977).

Our model can also be understood as a situation in which the Japanese firm is a Stackelberg leader that establishes, at each time \( t \), the amount of domestic input to be produced by the Mexican followers. The model captures the behavior of this dynamic monopsony over a continuous time framework, where the quantity of the input produced increases with time.

2. Solution

The solution for \( \alpha \) may be found by solving the differential equation for \( a \) in (4) (using the initial condition \( a(0) = a_0 \ (a_0 > 0) \)), substituting the optimal value \( a^*(\alpha,t) \) in the objective function and the remaining constraint \( \Pi^* \geq K \) in (4), and solving the new optimization problem for \( \alpha \), as in Kokotovic and Heller (1967).

In order to find \( a^*(\alpha,t) \), note that we can rewrite

\[
a^*(t) = e^{-y} \left[ n^{B-1} l_1 (a,\alpha) (1 + 1/\alpha)^{-1} \right]^B
\]

as

\[
-e^{-y} n^{B-1} (1 + 1/\alpha)^{-4} \ dx + l_1 (a,\alpha)^{-2} \ da = 0
\]

Integrating both sides of equation (5), we get

\[
\frac{-e^{-y}}{\xi} n^{B-1} (1 + 1/\alpha)^{-3} + \int l_1 (a^*,\alpha)^{-2} \ da^* = c
\]

Then \( a^*(\alpha,t) \), with the initial condition \( a(0) = a_0 \), must satisfy (6).

Substituting \( a^*(\alpha,t) \) into the objective function and constraint \( \Pi^* \geq K \) of (4), we obtain the optimization problem

\(^5\) Or, in other words, he picks one of an infinite number of vector functions. The domain of each of these functions is the set of points in time \( t \) (where \( t \) is in the interval \( [0,\infty) \)) and the countdomain is a vector space in \( R^{6 \times} \). Each element in this vector space is a general equilibrium of the economy.
\[
\text{Max}_{a} \int_{0}^{\infty} \left\{ p_{t} l_{1}(a, \alpha) - \frac{l_{1}(a, \alpha)^{2h}(1 + 1/\alpha)^{3h}}{n^{2+3h} a^{3h}} \left( \frac{2}{\beta^{2}} - 1 \right) - p_{w} l_{1}(a, \alpha)(1 + \alpha)^{-1} \right\} dt \tag{7}
\]

subject to

\[
\Pi^*(\alpha, t) = \frac{l_{1}(a, \alpha)^{2h}(1 + 1/\alpha)^{3h}}{n^{2+3h} a^{3h}} \left( \frac{2}{\beta^{2}} - 1 \right) \geq K, \quad \forall t \in [0, \infty); \quad \alpha \geq 0
\]

Observe that, after integrating with respect to \(t\), the objective function will depend only on \(\alpha\). Let us analyze the constraint \(\Pi^* \geq K\).

\(\Pi^*\) is a function in the \(\alpha, t\) plane. The plane \(K\) "slices" \(\Pi^*\), and we will only be interested in the restrictions on \(\alpha\) implied by the portions of \(\Pi^*\) which are greater than or equal to \(K\) (see figure 1).

![Figure 1](image-url)
In fact, since the objective function in (7) depends on $\alpha$ (and not on time $t$), we will only be interested in the one variable function implied by $\Pi^*$ which is given by:

$$\alpha \rightarrow \min_i \frac{t_1 (a^*, \alpha)^{2p} (1 + 1/\alpha)^{2q}}{n^{2n-3k} a^{2k}} \left(\frac{2}{\beta^2} - \frac{1}{\beta}\right).$$

(8)

The restrictions on $\alpha$, implied by (8) $\geq K$, may be convex or non-convex (see figure 2).

Then we finally end up with the one variable optimization problem:

$$\max_a \int_0^\infty e^{-\alpha t} \left[ p_c \pi \left( a^*, \alpha \right) - \frac{t_1 (a^*, \alpha)^{2p} (1 + 1/\alpha)^{2q}}{n^{2n-3k} a^{2k}} \left(\frac{2}{\beta^2} - 1\right) - p_w \pi \left( a^*, \alpha \right)(1 + \alpha)^{-1} \right] dt$$

subject to

$$\min_i \frac{t_1 (a^*, \alpha)^{2p} (1 + 1/\alpha)^{2q}}{n^{2n-3k} a^{2k}} \left(\frac{2}{\beta^2} - \frac{1}{\beta}\right) \geq K$$

(9)

$$\alpha \geq 0$$

The Kuhn-Tucker (necessary) conditions are:

6 (8) considers the minimum amount of profits that the Japanese firm may obtain. In case $\Pi^*(\alpha, t)$ is decreasing in $t$, the minimum of $\Pi^*$ with respect to $t$ will occur at $\lim_{t \to \infty} \Pi^*(\alpha, t)$.
\[
\frac{\partial \Lambda}{\partial \alpha} = \gamma \left( \frac{\partial V}{\partial a^*} \frac{\partial a^*}{\partial \alpha} + \frac{\partial V}{\partial \alpha} \right) + \lambda \frac{\partial \text{Const}}{\partial \alpha} \leq 0, \alpha \geq 0, \frac{\partial \Lambda}{\partial \lambda} = 0; \\
\frac{\partial \Lambda}{\partial \lambda} \geq 0, \lambda \geq 0, \frac{\partial \Lambda}{\partial \lambda} \lambda = 0
\]

(10)

where \( \Lambda \) is the lagrangian, \( \lambda \) the lagrange multiplier and

\[
V = e^{-\gamma t} \left[ p_c l_1 (a^*, \alpha) - \frac{l_1 (a^*, \omega)^{2h} (1 + 1/\alpha)^{2h}}{n^{20-20h} a^{1/\beta h}} \left( \frac{2}{\beta^2} - 1 \right) - p_w l_1 (a^*, \alpha)(1 + \alpha)^{-1} \right]
\]

\[
\text{Const} = \min \left\{ \frac{l_1 (a^*, \alpha)^{2h} (1 + 1/\alpha)^{2h}}{n^{20-20h} a^{1/\beta h}} \left( \frac{2}{\beta^2} - 1 \right) - K; \right\}
\]

\[
\frac{\partial V}{\partial a^*} = e^{-\gamma t} \left[ \frac{\partial l_1}{\partial a^*} (p_c - p_w(1 + \alpha)^{-1}) \right] - \\
- e^{-\gamma t} \left[ \frac{(1 + 1/\alpha)^{-1} (2 - 1)}{n^{20-20h} a^{1/\beta h}} \left( \frac{2}{\beta^2} - 1 \right) \right] \\
\frac{\partial V}{\partial \alpha} = e^{-\gamma t} \left[ \frac{\partial l_1}{\partial \alpha} - \frac{(2 - 1)}{n^{20-20h}} \left( 1 + 1/\alpha \right)^{-1} \right] \left( 2/\beta \right) l_1^{2h-1} \left( \frac{\partial l_1}{\partial a^*} \frac{\partial a^*}{\partial \alpha} + \frac{\partial l_1}{\partial \alpha} \right) \\
+ l_1^{2h} \left( 1 + 1/\alpha \right)^{-1} \left( -2/\beta \right) a^{1/2h-1} \frac{\partial a^*}{\partial \alpha} + a^{2h} \left( -2/\beta \right) \left( 1 + 1/\alpha \right)^{-1} \left( -\alpha^{-2} \right) \right] \\
- p_w \left[ l_1 (1 + \alpha)^{-1} + (1 + \alpha)^{-1} \frac{\partial l_1}{\partial \alpha} \right];
\]

\[
\frac{\partial a^*}{\partial \alpha} = t_1 \left[ -\delta n^{2(1-1)h} e^{-\delta t} \alpha^{-1} a^{-2} (1 + 1/\alpha)^{2h-1} - \left( \frac{\partial}{\partial \alpha} \int \frac{\partial \delta}{\partial \alpha} a^* + \frac{\partial l_1}{\partial \alpha} \right) \right]
\]

\[
\frac{\partial \text{Const}}{\partial \alpha} = \min \left\{ \frac{(2 - 1)}{n^{20-20h}} \left( 1 + 1/\alpha \right)^{-1} \right\} \left( 2/\beta \right) l_1^{2h-1} \left( \frac{\partial l_1}{\partial a^*} \frac{\partial a^*}{\partial \alpha} + \frac{\partial l_1}{\partial \alpha} \right); 
\]
Let us analyze the implications of conditions (10) on the extreme value $\alpha^*$ of $\alpha$. If $\alpha^*=0$, then $\frac{\partial \Lambda}{\partial \lambda} \geq 0$ implies $-K \geq 0$, which is a contradiction. Then we must have $\alpha^* > 0$.

$\alpha^* > 0$ implies $\frac{\partial \Lambda}{\partial \alpha} \geq 0$, which is an equation in $\alpha$ and $\lambda$. $\lambda$ may be greater than or equal to zero. If $\lambda=0$, then from $\frac{\partial \Lambda}{\partial \alpha} \geq 0$ we obtain a value $\hat{\alpha}^*$ of $\alpha^*$. If $\hat{\alpha}^*$ makes sense with $\frac{\partial \Lambda}{\partial \lambda} \geq 0$, then $\hat{\alpha}^*$ is the solution. The case $\lambda = 0$ corresponds to the case when the constraint in (9) is not binding (see figure 3).  

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**Figure 3**

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7 The solution pictures and discussions from now on are done assuming the first order conditions (10) are necessary and sufficient or, in other words, assuming the objective function in (9) is concave in $\alpha$. 

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If \( \lambda > 0 \), then \( \frac{\partial \lambda}{\partial \alpha} = 0 \), which gives us a value \( \hat{\alpha}^* \) of \( \alpha^* \). If \( \hat{\alpha}^* \) is consistent with \( \frac{\partial \lambda}{\partial \alpha} = 0 \), then \( \hat{\alpha}^* \) is the solution, and this case corresponds to the situation when the constraint in (9) is binding (figure 4).

Assume \( \alpha^* \) is the optimal solution to (9). Then since \( \frac{\partial \alpha^*}{\partial t} \geq 0 \) and

\[
\frac{\partial l^*}{\partial \alpha^*} = \frac{2(1 + 1/\alpha^*)^{-2} \beta^{\alpha^*+1} \beta^{1-1/\alpha^*}}{2(1 + 1/\alpha^*)^{-1} \beta^{1-1/\alpha^*}} \geq 0
\]

the demand by the Japanese firm for the domestic input \( z_D \) will increase with time. This process will imply the dynamization of the domestic sector, and the evolution of the state variable will express the technological progress in the production of \( z_D \). This technological progress is achieved through the transfer of technology from the Japanese to the domestic sector through a "learning-by-doing" process.

As we next illustrate with two examples, the technological change induced by \( \alpha^* \) will also have an influence on the distribution of labor income through time. Although in a very different context, this analysis addresses a similar issue as that in Brito and Intrilligator (1991). However, unlike that study, our model does not suggest the elimination of any firm or group or workers.
3. Examples

Example 1: \( \ell \) fixed. Decreasing returns to scale

Assume we have a perfectly inelastic supply function of labor in the final product sector such that \( \ell_f(\alpha, \alpha) = \ell_f \) and let \( \beta \in (0, 1) \). Then for given \( \alpha \) and \( \alpha \), the general equilibrium set of quantities and prices is given by:

\[ \ell^*_1 = \frac{\ell_f}{1 + \alpha} \]

\[ \ell^*_2 = \frac{\beta}{\alpha} \left( \frac{1}{1 + \alpha} \right)^{\frac{1}{\beta}} \]

\[ P^*_d = \frac{\beta - \beta}{\alpha} \left( \frac{1}{1 + \alpha} \right)^{\frac{1}{\beta}} \]

\[ w^*_1 = P_C - \frac{n}{\beta} \left( \frac{1}{1 + \alpha} \right)^{\frac{1}{\beta}} - P_w (1 + \alpha)^{-1} \]

\[ w^*_2 = \frac{\beta}{\alpha} \left( \frac{1}{1 + \alpha} \right)^{\frac{1}{\beta}} \]

Therefore, optimization problem (4) becomes:

\[
\max_{\alpha} \int_0^\infty e^{-\alpha t} \left\{ P_C \ell_1 - \frac{\beta}{\alpha} \left( \frac{1}{1 + \alpha} \right)^{\frac{1}{\beta}} - P_w (1 + \alpha)^{-1} \right\} dt
\]

subject to

\[ a(t) = e^{\alpha t} \left[ n^{-1} \ell_1 (1 + 1/\alpha)^{-1} \right] \]

\[ \Pi^*(\alpha, t) = \frac{\beta}{\alpha} \left( \frac{1}{1 + \alpha} \right)^{\frac{1}{\beta}} \left( \frac{2}{\beta^2} - \frac{1}{\beta} \right) \geq K, \quad \forall t \in [0, \infty) \quad (4') \]
\[ a(0) = a_g(a_0 > 0); \quad \alpha \geq 0 \]

Solving the differential equation for \( a \), with initial condition \( a(0) = a_0 \ (a_0 > 0) \), we have

\[ a^*(\alpha, t) = \frac{n^{(1-1-\beta)} \bar{I}_1 (1 + 1/\alpha)^{-\delta} (1 - e^{-\delta}) \zeta^{-1} + a_0}. \]

Substituting \( a^*(\alpha, t) \) into (4'), we obtain the optimization problem:

\[
\text{Max}_{\alpha} \int_0^\infty e^{-\alpha t} \left[ p_C \bar{I}_1 - \frac{\bar{I}_1^{1-b} (1 + 1/\alpha)^{-b}}{n^{1-b} \Phi_b} \left( \frac{2}{\beta^2} - 1 \right) n^{(\beta-1)} \bar{I}_1^{1-b} (1 + 1/\alpha)^{-b} (1 - e^{-\delta}) \xi^{-1} + a_0 \right] \, dt
\]

subject to

\[
\Pi^*(\alpha, t) = \frac{\bar{I}_1^{1-b} (1 + 1/\alpha)^{-b}}{n^{1-b} \Phi_b} \left( \frac{2}{\beta^2} - 1 \right) n^{(\beta-1)} \bar{I}_1^{1-b} (1 + 1/\alpha)^{-b} (1 - e^{-\delta}) \xi^{-1} + a_0 \geq K,
\]

\[ \forall t \in [0, \infty); \quad \alpha \geq 0 \]

Now observe that, for any \( \alpha \), the minimum with respect to \( t \) of \( \Pi^*(\alpha, t) \) occurs at

\[
\lim_{t \to \infty} \Pi^*(\alpha, t) = \frac{\bar{I}_1^{1-b} (1 + 1/\alpha)^{-b}}{n^{1-b} \Phi_b} \left( \frac{2}{\beta^2} - 1 \right) n^{(\beta-1)} \bar{I}_1^{1-b} (1 + 1/\alpha)^{-b} (1 - e^{-\delta}) \xi^{-1} + a_0 \rightarrow \infty
\]

Then we finally have the one variable optimization problem:

\[
\text{Max}_{\alpha} \int_0^\infty e^{-\alpha t} \left[ p_C \bar{I}_1 - \frac{\bar{I}_1^{1-b} (1 + 1/\alpha)^{-b}}{n^{1-b} \Phi_b} \left( \frac{2}{\beta^2} - 1 \right) n^{(\beta-1)} \bar{I}_1^{1-b} (1 + 1/\alpha)^{-b} (1 - e^{-\delta}) \xi^{-1} + a_0 \right] \, dt
\]

subject to

\[
\Pi^*(\alpha, t) = \frac{\bar{I}_1^{1-b} (1 + 1/\alpha)^{-b}}{n^{1-b} \Phi_b} \left( \frac{2}{\beta^2} - 1 \right) n^{(\beta-1)} \bar{I}_1^{1-b} (1 + 1/\alpha)^{-b} (1 - e^{-\delta}) \xi^{-1} + a_0 \geq K,
\]

\[ \forall t \in [0, \infty); \quad \alpha \geq 0 \]
\[
\Pi^*(\alpha, \infty) = \frac{\ell_1^{2/3}(1 + 1/\alpha)^{-2/3}}{n^{-1/3} \alpha_0^{2/3}} \frac{2}{\beta^2 - 1} \left( n^{4/3 - 1} \ell_1^{2/3} (1 + 1/\alpha)^{-2/3} \alpha_0^{2/3} \right) \geq K \quad (9')
\]
\[
\alpha \geq 0
\]

The constraint for \(\Pi^*\) in (9') will imply the feasible set \(\{\alpha/\alpha > \tilde{\alpha}\}\) where \(\tilde{\alpha}\) satisfies:

\[
\tilde{\alpha} \left[ \frac{\ell_1}{n^{1-\delta}} \left( \frac{2}{\beta^2 - 1} \right)^{\delta} \right] - \frac{\ell_1}{n^{1-\delta}} \tilde{\alpha} (1 + \tilde{\alpha})^{1-\delta} - \alpha_0 = 0 \quad (12)
\]

(see figure 5). \(^8\)

Set \(\{\alpha/\alpha > \tilde{\alpha}^*\}\) which defines the feasible rules of origin for example 1.

**Figure 5**

---

\(^8\) Note that we are implicitly using:

\[
\Pi^*(\infty, \alpha) = \frac{\ell_1^{2/3}(2 \beta^2 - 1)}{n^{-1/3} \alpha_0^{2/3}} \left( n^{4/3 - 1} \ell_1^{2/3} (1 + 1/\alpha)^{-2/3} \alpha_0^{2/3} \right) \geq K \quad (11)
\]

which is derived from the assumption that, for any \(t\), the profits \(\Pi^*(\alpha, t)\) are greater than \(K\) (where \(\alpha_0\) is the ratio of Mexican to Japanese inputs that the Japanese firm would voluntarily choose without complying with the rule of origin and without receiving any tariff preference). This assumption is valid since the profits achieved by participating in the FTA must be greater than the profits obtained without such participation, when the non-restrictive ratio \(\alpha_0\) is used.
Before going into the solution discussion, it is pertinent to note that the form of the technological progress \( \alpha(t) \) in (4') implies that the demand for the domestic input, the labor income in the Mexican sector, the level of technology in the domestic sector and the minimum profits of the Japanese firm all increase as the rule of origin \( \alpha \) is larger. Therefore we may be led to think that the optimal rule should be as large as possible. However, we must also consider the effects of an increase of \( \alpha \) on the salaries in the Japanese sector and on the price of the domestic input. As can be shown, the change in \( w_i^* \) and \( p_D^* \) as \( \alpha \) increases is not always positive. For example if \( \beta = 1 \) and \( \delta = 1/2 \), then \( \frac{\partial p_D^*}{\partial \alpha} \) may be negative.  

The Kuhn-Tucker (necessary) conditions of problem (9') are given by (10), where now:

\[
V = e^{-\rho t} \left[ p_C T_1 - \frac{T_i}{n^{2+o_{1/2}}} \left( \frac{2}{\beta - 1} \right) \left( n^{(\beta-1) - (1 + \alpha)} (1 - e^{-\delta}) x_{\alpha}^{-1} + a_0 \right)^{1/\beta} \right]
\]

\[
\text{Const} = \Pi^*(\alpha, \dot{\alpha}) - K_i
\]

\[
\frac{\partial V}{\partial \alpha} = -e^{-\rho t} \frac{T_i}{n^{2+o_{1/2}}} \left( \frac{2}{\beta - 1} \right) \left( n^{(\beta-1) - (1 + \alpha)} (1 - e^{-\delta}) x_{\alpha}^{-1} + a_0 \right)^{1/\beta - 1}
\]

\[
\frac{\partial V}{\partial \alpha} = -e^{-\rho t} \frac{T_i}{n^{2+o_{1/2}}} \left( \frac{2}{\beta - 1} \right) \left( n^{(\beta-1) - (1 + \alpha)} (1 - e^{-\delta}) x_{\alpha}^{-1} + a_0 \right)^{1/\beta - 1}
\]

\[
10 \quad \frac{\partial p_D^*}{\partial \alpha} = \frac{1}{\beta n^{2+o_{1/2}}} \left( \frac{2}{\beta - 1} \right) \left( n^{(\beta-1) - (1 + \alpha)} (1 - e^{-\delta}) x_{\alpha}^{-1} + a_0 \right)^{1/\beta - 1} \times
\]

\[
\times \left[ \left( \frac{2 - \beta}{2} - \delta \right) (1 + \alpha)^{2 - \beta} - \left( \alpha^{-2} + \frac{2 - \beta}{2} (1 + \alpha)^{2 - \beta} \right) \left( \alpha^{-2} \right) \right]
\]

\[
9 \quad \text{That is} \quad \left. \frac{\partial \Pi^*(\alpha, \dot{\alpha})}{\partial \alpha} \right|_{\alpha^*} \quad \left. \frac{\partial w_i^*}{\partial \alpha} \right|_{\alpha^*} \quad \left. \frac{\partial T_i}{\partial \alpha} \right|_{\alpha^*} \quad \left. \frac{\partial \rho}{\partial \alpha} \right|_{\alpha^*} \quad \left. \frac{\partial \Pi^*(\alpha, \dot{\alpha})}{\partial \alpha} \right|_{\alpha^*} \quad \text{are positive. This can be easily shown.}
\]
The solution discussion goes as follows. When the constraint is not binding,
we will get the solution \( \hat{\alpha}^* \) from \( \frac{\partial \Lambda}{\partial \alpha} = 0 \) (see figure 3a). When the constraint is
binding, we will get the solution \( \bar{\alpha}^* \) from (12) (see figure 4a).

Now assume \( \alpha^* \) is the optimal solution to (9'). Then \( z^*_2 \big|_{\alpha^*} \) will remain constant
over time, but, since technology increases with time \( \left( \frac{\partial \alpha^*}{\partial t} \Big|_{\alpha^*} > 0 \right) \), the domestic pro-
ducers of inputs will be able to produce \( z^*_2 \big|_{\alpha^*} \) more efficiently, causing the input price
\( p^*_D \big|_{\alpha^*} \) and the domestic-sector labor income \( w^*_2 \big|_{\alpha^*} \) to decrease over time (i.e.
\( \frac{\partial p^*_D}{\partial t} \bigg|_{\alpha^*} < 0 \) and \( \frac{\partial w^*_2}{\partial t} \bigg|_{\alpha^*} < 0 \)). On the other hand, the labor income in the Japanese
sector increases as time passes \( \left( \frac{\partial w^*_1}{\partial t} \bigg|_{\alpha^*} > 0 \right) \), more than compensating for the drop
in the cost of input \( z \). Thus, the profits of the Japanese firm decrease over time.

On the other hand, observe the role of \( K \) in (9'). From figure 5, we can see that
as \( K \) increases, the feasible set becomes smaller. In other words as the profits that
the Japanese firm can obtain by not complying with the rule of origin are larger,
there is less room for the policy maker to choose an adequate rule. This due to the
fact that, in the case of our specific example, the Japanese profits diminish with
time. Then, as \( K \) becomes larger, it becomes more and more difficult to find an \( \alpha \)
that can maintain these profits larger than \( K \) over time.
Comparative Statics.

In order to analyze some other economic implications of example 1, we proceed to change some parameter values slightly. Table 1 shows the equilibrium values of the endogenous variables of interest when parameter values are changed. This (cross section) analysis is done for a particular point in time since, with the exception of $\alpha$, all the endogenous variables in the model depend on time.

**Table 1.**

Comparative Statics of Example 1

| Benchmark: $\beta = .5, \delta = .1, K = 5 \times 10^5, \xi = .02, p = .02$ |
|---------------------------------------------|---------|----------------|-----------|--------------|---------------|-------------|
| $\alpha^*$                                | $\alpha^*$ | $z^*_N(10^8)$ | $z_N(10^8)$ | $p^*_M(10^8)$ | $w^*_M(10^8)$ |
| base                                      | 4.0973   | .0419         | 8.0382    | 4.9674      | 1.3489        | 4.9674      |
| $\beta = .49$                             | 3.1597   | .0415         | 7.5960    | 5.0008      | 1.8397        | 5.0008      |
| $\beta = .51$                             | 5.3186   | .0422         | 8.4174    | 4.8450      | 9.7242        | 4.8450      |
| $\delta = .09$                            | 2.8500   | .0386         | 7.4026    | 4.8206      | 1.4646        | 4.8206      |
| $\delta = .11$                            | 6.6290   | .0455         | 8.6892    | 5.0611      | 1.2201        | 5.0611      |
| $K = 4(10^5)$                            | 3.0901   | .0416         | 7.5551    | 4.4430      | 1.1481        | 4.4430      |
| $K = 6(10^5)$                            | 5.4756   | .0421         | 8.4557    | 5.4415      | 1.5387        | 5.4415      |
| $\xi = .019$                             | 5.7092   | .0400         | 8.5095    | 5.4984      | 1.5612        | 5.4984      |
| $\xi = .021$                             | 3.1909   | .0437         | 7.6139    | 4.5099      | 1.1738        | 4.5099      |
| $p = .019$                               | 4.0973   | .0419         | 8.0382    | 4.9674      | 1.3489        | 4.9674      |
| $p = .021$                               | 4.0973   | .0419         | 8.0382    | 4.9674      | 1.3489        | 4.9674      |

The results of table 1 are reached after assuming $p_C = 18,000, p_o = 50, \bar{L}_t = 10,000$, $n = 20, t = 1$, and $a_0 = .01$. When choosing these numbers, we are thinking, for example, of a Japanese car firm based in Mexico which faces a fixed supply of labor and consumes batteries from several low-technology domestic producers.

We also suppose that $K = 5 \times 10^5$. Under the assumed parameter values $\Pi^*(\alpha, \xi)$ is greater than $10^6$ so that (11) (in footnote 8) is satisfied. The constraint $\Pi^*(\alpha, \xi) - K$ and the implied feasible set are as in figure 5. $\hat{\alpha}$ is calculated from (12) and, in the benchmark case, is equal to 4.0973. The shape of the objective function is as in figure 6.
For the particular parameter values chosen, the constraint $\Pi'(\alpha, \infty) - K \geq 0$ will be binding (as in figure 4a), and therefore the solution $\alpha^*$ will be given by the value $\bar{\alpha}$ computed from (12).

The results of table 1 show how a larger labor share in the domestic sector is associated with a larger value rule of origin $\alpha$. Also, slight increases in $\delta$ are associated with relatively large increases in the rule of origin and increases (at time $t = 1$) in the state of technology, the demand for the domestic input, the equilibrium amount of labor and wages in the domestic sector, and the productivity of the domestic sector (expressed by a decrease in the equilibrium level of $p_d^*$). These results suggest the importance of the domestic sector of capacity for “absorbing” technology through learning-by-doing. The existence of a sufficiently qualified domestic labor force could help to achieve this goal. Therefore, a government policy of broad support to education is suggested.

Table 1 also shows how a higher convergence of the technological improvement (expressed by an increase in $\xi$) results in a lower value of the rule of origin $\alpha$, but a higher convergence of the wage income (expressed by an increase in $\rho$) does not affect the equilibrium level of this rule. Another interesting result is that the rule of origin becomes larger as $K$ increases. This confirms that the constraint $\Pi'(\alpha, \infty) - K \geq 0$ is binding.

Example 1 illustrates how a policy maker behaves when seeking an adequate rule of origin that: a) resolves the conflicts in the productive chain; b) achieves transfer of technology from the Japanese to the Mexican sector; c) maximizes wage
income. The policy maker must take into account these diverse interests and find the best rule which reconciles them.

In general, what is good for the domestic producers of intermediates may not always be so good as far as social welfare is concerned. In this sense, even if intuitively a high rule would be better for the domestic input producers, a lower rule may be the one which maximizes the wage income of workers and reconciles the diverse interests in the economy.

However, it is possible that the optimal rule takes place at a bigger value than the rule which would maximize the total wage income in the economy. This is illustrated in figure 6 and is the situation in example 1. On the other hand, figure 7 depicts the shape of the function of technological evolution for the base value of \( \alpha^* \) in example 1, for the particular parameter values chosen.

![Graph](image)

**Figure 7**

**Example 2: \( l_t \) Variable. Constant Returns to Scale**

Assume now that the supply function in the labor market is given by \( w_t = l_t \), and let \( \beta = 1 \). Then for given \( a \) and \( \alpha \), the clearing amount of labor used in the final product sector will be

\[
l_t(a, \alpha) = \frac{R - P_w (1 + \alpha)^{-1}}{1 + \frac{\alpha}{2} (1 + 1/\alpha)^{-2}}
\]
Then the set of equilibrium quantities and prices for given values of \( a \) and \( \alpha \) is

\[
\begin{align*}
\bar{z}_D^* &= \frac{P_C - P_w (1 + \alpha)^{-1}}{(1 + 1/\alpha) + \frac{2}{a} (1 + 1/\alpha)^{-1}}, \\
I_i^* &= (1 + 1/\alpha) \bar{z}_D^*, \\
I_2^* &= \frac{1}{a} \bar{z}_D^*, \\
P_D^* &= \frac{1}{a^2} \bar{z}_D^*, \\
w_i^* &= P_C - \frac{2}{a} (1 + 1/\alpha)^{-1} \bar{z}_D^* - P_w (1 + \alpha)^{-1}, \\
w_2^* &= \frac{1}{a^2} \bar{z}_D^*.
\end{align*}
\]

Therefore, optimization problem (4) becomes:\(^{11}\)

\[
\text{Max}_{\bar{a}} \int_0^\infty e^{-\bar{a} t} \left\{ \frac{[P_C - P_w (1 + \alpha)^{-1}]^2}{1 + \frac{2}{a^2} (1 + 1/\alpha)^{-1}} \left[ 1 - \frac{1}{a^2 (1 + 1/\alpha)^2 + 2} \right] \right\} \, dt
\]

subject to

\[
\bar{a}(t) = e^{-\bar{a} t} \left[ \frac{P_C - P_w (1 + \alpha)^{-1}}{(1 + 1/\alpha) + \frac{2}{a^2} (1 + 1/\alpha)^{-1}} \right]^\delta
\]

\[
\Pi^*(\bar{a}, \delta) = \frac{1}{a^2} \frac{[P_C - P_w (1 + \alpha)^{-1}]^2}{\left[ (1 + 1/\alpha) + \frac{2}{a^2} (1 + 1/\alpha)^{-1} \right]^2} \geq K, \quad \forall t \in [0, \infty) \quad (4^{11})
\]

\(^{11}\) Note that \( \Pi^*(\bar{a}, \delta) \) is equal in this example to \( P_D^* \bar{z}_D^* \) and that \( \bar{a}(t) = e^{-\bar{a} t} (\bar{z}_D^*)^\delta \). Note also that we (of course) have \( P_D^* \bar{z}_D^* = w_2^* I_2^* \), which implies that the profits in the domestic sector are zero due to the constant returns to scale technology.
Let us now solve the differential equation for $a$. Note that from (4''), we can rewrite the expression for $a(t)$ as

$$-e^{-\delta t}(1 + 1/\alpha)^{-\delta} \, dt + \int R(a, \alpha)^{-\delta} \, da = 0 \tag{5'}$$

Integrating both sides of (5'), we get

$$e^{-\delta t} \xi^{-1}(1 + 1/\alpha)^{-\delta} + \int l^* (a, \alpha)^{-\delta} \, da = c.$$

Using the initial condition $a(0) = a_0$, we obtain the value of $c$ and the solution $a^*(\alpha, t)$ of (5').

Substituting $a^*(\alpha, t)$ into (4''), we get the optimization problem:

$$\text{Max}_{\alpha} \int_{0}^{\infty} e^{-\mu t} \left\{ \frac{[R_0 - P_0 (1 + \alpha)^{-1}]^2}{1 + \frac{2}{a^*(\alpha, t)^2} (1 + 1/\alpha)^{-2}} \right\} dt$$

subject to

$$\Pi^*(\alpha, t) = \frac{[R_0 - P_0 (1 + \alpha)^{-1}]^2}{a^*(\alpha, t)^2 (1 + 1/\alpha)^2 + 2} \geq K, \; \forall \alpha \in [0, \infty)$$

$$\alpha \geq 0$$

Now observe that, as in example 1, the minimum of $\Pi^*(\alpha, t)$ with respect to $t$ occurs at

$$\lim_{t \to \infty} \Pi^*(\alpha, t) = \frac{[R_0 - P_0 (1 + \alpha)^{-1}]^2}{a^*(\alpha, \infty)^2 (1 + 1/\alpha)^2 + 2}$$

Then we end up with the one variable optimization problem:
\[
\text{Max } \alpha \int_0^{\infty} e^{-\alpha t} \left\{ \frac{[p_c - p_w(1 + \alpha)^{-1}]^2}{1 + \frac{2}{a^*(\alpha, t)^2} (1 + 1/\alpha)^2} \left[ 1 - \frac{1}{a^*(\alpha, t)^2 (1 + 1/\alpha)^2 + 2} \right] \right\} \, dt
\]

subject to

\[
\Pi^*(\alpha, \infty) = \frac{[p_c - p_w(1 + \alpha)^{-1}]^2}{a^*(\alpha, \infty)^2 (1 + 1/\alpha)^2 + 2} \geq K \tag{9''}
\]

\[\alpha \geq 0\]

As before the constraint for \(\Pi^*\) in (9'') will define the set of feasible rules of origin (see figure 8).\(^{12}\)

\[\text{Set of feasible rules of origin in example 2, given by ".........."}\]

Figure 8

The Kuhn-Tucker (necessary) conditions are given by (10) where now:

\[\Pi^*(\tilde{\alpha}, \infty) > K \tag{11'}\]

\(^{12}\) Note that we are implicitly using that there is at least one \(\tilde{\alpha}\) such that

As in example 1, this is derived from the assumption \(\Pi^*(\alpha_0, \tilde{t}) > K\) for any \(\tilde{t}\) (where \(\alpha_0\) is the Mexican to Japanese input ratio voluntarily chosen by the Japanese firm).
\[ \nu = e^{-\alpha} \left\{ \frac{[p_c - p_w(1 + \alpha)^{-1}]^2}{1 + \frac{2}{a^*} (1 + 1/\alpha)^{-2}} \left[ 1 - \frac{1}{a^*(\alpha, \beta)^2 (1 + \beta)^2 + 2} \right] \right\}; \]

\[ \text{Const} = \Pi^*(\alpha, \infty) - K; \]

\[ \frac{\partial \nu}{\partial a^*} = e^{-\alpha} \left\{ \frac{[p_c - p_w(1 + \alpha)^{-1}]^2}{1 + \frac{2}{a^*} (1 + 1/\alpha)^{-2}} \left[ (a^2(1 + 1/\alpha)^2 + 2)^{-2} 2a^* (1 + 1/\alpha)^2 \right. \right. \]

\[ + \left. \left( 1 - \frac{1}{a^2 (1 + 1/\alpha)^2 + 2} \right) \left[ 1 + \frac{2}{a^*} (1 + 1/\alpha)^{-2} \right]^{-1} \frac{4(1 + 1/\alpha)^{-2}}{a^3} \right\}; \]

\[ \frac{\partial \nu}{\partial \alpha} = e^{-\alpha} \left\{ \frac{[p_c - p_w(1 + \alpha)^{-1}]^2}{1 + \frac{2}{a^*} (1 + 1/\alpha)^{-2}} \left[ (a^2(1 + 1/\alpha)^2 + 2)^{-2} 2a^* (1 + 1/\alpha)(-\alpha^{-2}) \right. \right. \]

\[ - \left( 1 - \frac{1}{a^2 (1 + 1/\alpha)^2 + 2} \right) \left[ 1 + \frac{2}{a^*} (1 + 1/\alpha)^{-2} \right]^{-1} \frac{4(1 + 1/\alpha)^{-2} \alpha^{-2}}{a^2} \]

\[ - 2(p_c - p_w(1 + \alpha)^{-1})^{-1} p_w(1 + \alpha)^{-2} \right\}; \]

\[ \frac{\partial a^*}{\partial \alpha} = -\delta l_j (a^*, \alpha)^5 e^{-\psi \xi^{-1}} \alpha^{-2} (1 + 1/\alpha)^{\xi^{-1}} \]

\[ - \left( \frac{d}{d\alpha} \int l_j (a^*, \alpha)^5 \, da^* \right) l_j (a^*, \alpha)^5 + \frac{dC}{d\alpha} l_j (a^*, \alpha)^5; \]

\[ \frac{\partial \text{Const}}{\partial \alpha} = \min_i \frac{2l_i^*}{a^2 (1 + 1/\alpha)^2} \left[ \left( \frac{\partial l_i^*}{\partial a^*} \frac{\partial a^*}{\partial \alpha} + \frac{\partial l_i^*}{\partial \alpha} \right) - l_i^* \left( \frac{\partial a^*}{\partial \alpha} \right) a^{-1} - (1 + 1/\alpha)^{-2} \right]; \]

\[ \frac{\partial l_i^*}{\partial a^*} = (p_c - p_w(1 + \alpha)^{-1})(1 + \frac{2}{a^2} (1 + 1/\alpha)^{-2})^{-2} \frac{4(1 + 1/\alpha)^{-2}}{a^3}; \]
The characterization of the solution corresponding to these conditions is as on section 2 (see figures 3 and 4).

Now assume \( \alpha^* \) is the optimal solution to (91). Then the demand by the Japanese firm for the domestic input \( z^*_{D} \) will increase through time \( \frac{\partial z^*_{D}}{\partial t} \geq 0 \), implying a learning-by-doing transfer of technology from the Japanese to the Mexican sector and resulting in a technological progress expressed by \( \dot{\alpha}(t) \). This technological progress is such that it implies an overtime decrease of the price \( \frac{\partial p^*_{D}}{\partial t} < 0 \).

The decrease of the price of the domestic input is caused by the every time increasing efficiency of the domestic producers. However, the technological progress over time will be such that it will also imply a decrease of the domestic sector's revenue

\[
\left. \frac{\partial p^*_{D}}{\partial t} \right|_{\alpha^*} \leq 0
\]

Consequently, the sector's labor income will decline

\[
\left. \frac{\partial \omega^*_{2}}{\partial t} \right|_{\alpha^*} \leq 0
\]

keeping the sector's profits equal to zero. This is due to the constant returns to scale technology in the domestic sector. Also, as in example 1, the labor income in the Japanese sector becomes larger as time passes \( \left. \frac{\partial w^*_{1}}{\partial t} \right|_{\alpha^*} > 0 \) and the
profits \( \Pi'(\alpha, t) \) of this sector will diminish over time since they behave identically to the revenue \( p^* \) of the domestic sector.

Also, note how all the behavior of the economy across time is mainly induced by the behaviour of \( z^*_D \), while in example 1 movement is only induced by the progress of technology. On the other hand, \( K \) will again play an important role in the definition of the choice set of the policy maker. The bigger the \( K \), the less room for choice.

**Comparative Statics**

In table 2 we present the results of an analysis for example 2 analogous to the one showed in table 1. The results of table 2 are reached after assuming \( \beta = 1, \delta = 1, n = 20, t = 1, \) and \( a_0 = 1 \).

**Table 2.**

Comparative Statics of Example 2

| benchmark: \( \xi = .02, p_c = 18(10^3), p_w = 50, \rho = .02, K = 5 \times 10^5 \) |
|---|---|---|---|---|---|---|
| \( a^*(10^4) \) | \( \alpha^*(10^5) \) | \( z^*_D(10^5) \) | \( \ell^*_W(10^4) \) | \( \ell^*_Z = \ell^*_W \) | \( p^*_D = 10^{-5} \) | \( w^*_l(10^4) \) |
| base | 1.5351 | 3.1718 | 1.7988 | 1.8000 | .5671 | 1.7880 | 1.8000 |
| \( \xi = .018 \) | | | | | | |
| \( \xi = .022 \) | | | | | | |
| \( p_c = 15 (10^3) \) | 1.1537 | 2.2018 | 1.4987 | 1.5000 | .6807 | 3.0915 | 1.5000 |
| \( p_c = 20 (10^3) \) | 1.7259 | 4.3178 | 2.0988 | 2.1000 | .4861 | 1.1258 | 2.1000 |
| \( p_w = 40 \) | .9629 | 3.1694 | 1.7981 | 1.8000 | .5673 | 1.7900 | 1.8000 |
| \( p_w = 60 \) | 1.9166 | 3.1726 | 1.7991 | 1.8000 | .5671 | 1.7873 | 1.8000 |
| \( \rho = .016 \) | | | | | | |
| \( \rho = .024 \) | | | | | | |
| \( K = 4 (10^5) \) | 1.1537 | 3.1705 | 1.7984 | 1.8000 | .5672 | 1.7891 | 1.8000 |
| \( K = 6 (10^5) \) | 1.5351 | 3.1718 | 1.7988 | 1.8000 | .5671 | 1.7880 | 1.8000 |

The assumption \( \delta = 1 \) implies that the domestic sector is very successful in achieving technological progress by a learning-by-doing process. The production in the
domestic sector is such that it "produces" batteries and technology in the same proportion. As is pointed out in Rivera-Batiz and Romer (1991), this is an extreme case of technological evolution. When $\delta = 1$, the solution $a^*(\alpha, t)$ of (5') is given by

$$a^*(t) = \frac{1}{2} \left( p_c - p_w (1 + \alpha)^{-1} \right) \left( 1 + 1/\alpha \right)^{-1} (1 - e^{-\eta}) + a_0 - 2 \frac{(1 + 1/\alpha)^{-2}}{a_0}$$

$$+ (1/2) \left[ \left( p_c - p_w (1 + \alpha)^{-1} \right) \left( 1 + 1/\alpha \right)^{-1} (1 - e^{-\eta}) + a_0 - 2 \frac{(1 + 1/\alpha)^{-2}}{a_0} \right]^2 + 8 \left( 1 + 1/\alpha \right)^{-2}$$

For the assumed parameter values, the shape of the constraint $\Pi^* (\alpha, \infty)$ is as in figure 9. Then, since $K$ is by assumption equal to $5 \times 10^5$ and under the assumed parameter values $\Pi^* (\infty, \infty)$ is greater than $10^6$, (11'), in footnote 12, is satisfied. The shape of the objective function is as in figure 10.
For the particular parameter values chosen, the constraint $\Pi^*(\alpha, \kappa) - K \geq 0$ will not be binding. Therefore, the solution $\alpha^*$ will be given by the value $\hat{\alpha}^*$ computed from $\frac{\partial \Lambda}{\partial \alpha} = 0$.

Table 2 shows how a different situation from example 1 ($l_1$ variable and constant returns) implies that the value of the optimal rule can be much larger. This suggests that the rule that reconciles the diverse interests of the economy may vary from industry to industry, depending on the specifics of each industry.\(^\text{13}\)

The results in table 2 show that an increase in the international price $p_C$ is associated with an increase in the equilibrium level of the rule of origin as well as with increases (at time $t=1$) of the state of technology, the wage income in the Japanese sector, the demand for domestic inputs and the productivity in the domestic sector. However, as $p_C$ becomes larger the wage income in the domestic sector decreases.

Another result which is consistent with intuition is the one that relates an increase in the price $p_w$ of the Japanese input with a larger demand for the domestic input and a decrease of the price $p_D$ of the product of the domestic sector. On the other hand, higher convergences of the technological improvement (expressed by an increase in $\xi$) and the wage income (expressed by an increase in $\rho$) result in a lower value of the rule of origin. Finally, the fact that the rule of origin remains unaltered

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\(^{13}\) This result supports the position taken by the Mexican delegation in the actual negotiations for the North American Free Trade Agreement. During the internal negotiations, each industry was analyzed separately and diverse positions on rule of origin (one for each industry) were presented to Americans and Canadians in the table of negotiations.
as $K$ increases is in accordance with the fact that the constraint $1^* (\alpha, \infty) - K \geq 0$ is not binding for the particular parameter values chosen. On the other hand, the shape of the function of technological evolution for the base value of $\alpha^*$ in example 2 is depicted in figure 11.

![Figure 11](image)

4. Conclusions

The model of this paper aimed to analyze the problem of definition of adequate rules of origin in a country that is preparing itself for the negotiation of a free trade agreement. The government of such a country would like to take sensible rules to the negotiation table. These rules must be such that they insure domestic welfare maximization, resolve the antagonisms in the productive chain, and induce domestic technological progress.

With this purpose, we developed a dynamic model of a policy maker who is making a one-shot decision regarding rules of origin. In making such a decision, he must take into account that he seeks to maximize labor income over time in a domestic productive chain with special characteristics. In this productive chain, a "third-country" firm which exports to another country within the free trade zone may buy its inputs from domestic or foreign suppliers. The rule selected must be such that it maintains the level of profits that the third-country could obtain without participating in the agreement. Therefore, the bigger these profits are, the less room for choice of rules there is. Also, the rule chosen has to insure a certain level of technological progress in the Mexican input sector and an increase of the domestic
demand overtime. The advance in technology is achieved by the transfer of technology from the third-country firm to the domestic sector.

The solution to the maximization problem of the model provides an optimal vector function which assigns to each point in time a general equilibrium of the economy. With two examples we showed that this solution may vary according to the specific characteristics of each industry. In this sense it could happen that even if an a priori intuition suggests a high rule for the good of the economy, a lower rule could be the one which maximizes labor income and reconciles the opposing economic interests. An opposite case could also be possible.

In example 1 we supposed a perfectly inelastic supply-of-labor function in the final product sector and decreasing returns to scale in the domestic input sector. With these assumptions we showed that once the optimal rule of origin is chosen the technology will improve over time. This will let the domestic producers of inputs produce every time more efficiently, causing the input price to decrease with time. The labor income will diminish in the domestic sector, but will grow in the final product sector as time passes. The assumption of perfect inelasticity of labor in the Japanese sector implies that the demand for domestic inputs will remain constant through time. Then the dynamic behaviour of the economy in example 1 is only explained by the intertemporal progress of technology.

A comparative statics (numerical) analysis provided more information about the characteristics of a particular solution to example 1. An interesting result is that slight increases in the value of the parameter of technology convergence implies large increases in the equilibrium rule of origin and increases (at a particular point in time) in the state of technology, the demand for the domestic input, the productivity of the domestic sector, and the equilibrium amount of labor and wages in the domestic sector. These results suggest, as a policy recommendation, that the government should support the formation of a qualified domestic labor force that facilitates the process of technology absorption in the domestic sector.

In example 2, we assumed an elastic supply of labor in the Japanese product sector and constant returns to scale in the domestic sector. Given this setting we showed, assuming that the optimal rule was chosen, that the demand by the Japanese firm for the domestic input would increase through time. Therefore, in this example, the dynamics of the system are basically driven by the increase of this demand, which contributes to the transfer of technology to the domestic producers. The growing efficiency in this sector leads to a decrease in the price of the input across time. Also, as in example 1, the two labor incomes of the economy observe opposite intertemporal behaviour. Namely, labor income decreases in the domestic sector but increases in the Japanese sector.

The comparative statics analysis of example 2 showed, among other things, that an increase in the international price of the Japanese final product causes increases in the equilibrium level of the rule of origin and an increase, at a certain point in time, in the state of technology, the wage income in the Japanese sector, the demand for domestic inputs, and the productivity in the domestic sector.
References


