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Chocolate Money

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Abstract

Why did people exert considerable eort on gold production when chocolate is cheaper and it can serve as money as well? Production costs determine a commodity's use as money, as a consumption good, or as both. Three results emerge: rst, the production must be costly enough for a commodity to become money. Second, the relationships between costs and both, monetary supply and monetary trade, are non-monotonic. Third, production costs determine the transaction value of a commodity when it is used as money only. This result supports a conjecture of Jevons that a good must be valued to function as money.

Keywords: production costs, transaction value

Resumen

¿Por qué las personas ejercieron un esfuerzo considerable en la producción de oro cuando el chocolate es más barato y también puede servir como dinero? Los costos de producción determinan el uso de un *commodity* como dinero, como bien de consumo o como ambos. Surgen tres resultados: primero, la producción debe ser lo suficientemente costosa para que un bien se convierta en dinero. En segundo lugar, que las relaciones entre los costos, la oferta monetaria y el comercio monetario, no sean monotónicas. Tercero, los costos de producción determinan el valor de transacción de un producto cuando se usa sólo como dinero. Este resultado respalda la conjetura de Jevons de que un bien debe ser valorado para funcionar como dinero.

Palabras claves: costos de producción, valor de transacción

Chocolate Money

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Abstract

Why did people exert considerable effort on gold production when chocolate is cheaper and it can serve as money as well? Production costs determine a commodity's use as money, as a consumption good, or as both. Three results emerge: first, the production must be costly enough for a commodity to become money. Second, the relationships between costs and both, monetary supply and monetary trade, are non-monotonic. Third, production costs determine the transaction value of a commodity when it is used as money only. This result supports a conjecture of Jevons that a good must be valued to function as money.

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1 Introduction

1.1 Contribution

In a world that is bound to use a commodity as money, why choose gold over chocolate? Gold is hard to mine, melt, and mint. Chocolate, in contrast, is cheaper to produce, and both commodities are, to various degrees, portable, recognizable, divisible, durable, and homogeneous. Hence, both can serve as money, and nothing disqualifies chocolate to serve as a medium of exchange. There are two possible motivations to choose gold over chocolate: first, the extra effort is justified by differences in physical qualities alone. For example, gold is more durable than chocolate. Second, the production costs themselves are crucial for a commodity to become money. The latter motivation would be noteworthy because we currently use paper money which is almost costless to the government. Therefore, it appears feasible to use a cheap-to-produce commodity as money. But nobody enjoys paper money directly while almost everybody can appreciate gold and chocolate for their intrinsic consumption possibilities. The central question becomes whether production costs determine a commodity's use as money, as a consumption good, or as both. In other words, this paper contributes to the question why gold became the gold standard, and chocolate did not.

There are three results; first, the production must be costly enough so that a commodity can become money. Simply put, a commodity does not inspire an exchange if it is cheap to acquire outside a trading opportunity.[1] Second, the relationships between costs and monetary variables are non-monotonic: money supply and monetary trade initially increase in cost. The higher replacement efforts imply a higher transaction value which entices more trade. Eventually,

[1]I conjecture an economy with fiat money works not because paper money is cheap to produce for the government but because it is intrinsically useless for an individual.

money supply and monetary trade taper down as costs increase because the production efforts must be matched by the gains from trade. Third, production or replacement costs determine the transaction value of a commodity when it is used as money only. Wallace (1998) conjectured that the determinacy issue in fiat money economies does not extend to commodity money economies. Zhou (2003) employs a counterexample where the transaction value of a dividend-paying asset is derived from future gains from trade. Multiple price equilibria can emerge similar to a version of the same economy with fiat money. Here, the production costs become a disciplining device for the transaction value and it solves the determinacy issue.

A monetary model with indivisible goods and storage constraints based on that of Kiyotaki and Wright (1989, KW from hereon) is augmented with a production and consumption decision for a homogeneous commodity that can also serve as money. The rationale of the results is as follows: a commodity will not be money if the fundamental consumption value exceeds the expected gains from holding onto the commodity and using it in a trade later. I refer to the latter as the transaction value. The commodity is produced if the costs are smaller than the consumption value. With moderate costs agents use a fraction of their production as money. The necessary increase in the transaction value derives from the fact that it is pinned by the cost to obtain the commodity. Increasing the cost further can crowd out consumption altogether even when consumption value is below the production costs. Agents produce the commodity for monetary purposes, even when the cost to produce exceeds the consumption value. As costs increase further, the money supply drops, as agents now match trading gains and production pains. All production ceases once the cost exceed the transaction value.

To answer the original question of why gold became synonymous with money, and chocolate did not: chocolate was indeed used as a medium of exchange and a store of value by the Aztecs in Pre-Columbian Mexico (compare Kurtz (1974); Crown and Hurst (2009)). A laborer would obtain the catch of a fisherman in exchange for dried cacao beans. The fisherman could then buy new fishing rods or save up to purchase a boat. But the beans were also consumed in the form of an indulgent chocolate drink.[2] The model here predicts such an outcome because chocolate production is relatively cheap: the laborer would simply harvest cacao pods from the widespread cacao trees, and she dries or toasts the pods to produce cacao beans.[3] Gold, on the other hand, is costly to come by: it became money thousands of years ago, and remained synonymous with money ever since. Nonetheless, chocolate is the commodity of choice for the following exposition.

1.2 Literature

Sargent and Wallace (1983) use an overlapping generation model where money is essential in the sense of Hahn (1989). Employing a commodity as a medium of exchange forgoes consumption value. KW started a distinct branch of the literature on money using search theory.[4] A monetary equilibrium always exists when (holding) costs are sufficiently low. But production in KW is costless and direct consumption not a viable option: agents who accept a commodity as money have no direct use for it.

[2]In fact, the term "chocolate" and it's variations in different languages stems from the classical Nahuatl word "xocolatl". It is also noteworthy that chocolate even had a ceremonial function which elevates the utility derived from it even more. Compare Weatherford (1998).

[3]Cacao trees grow between the circles of latitude that are 20 degrees north and south of the equatorial plane. This area enveloped the Aztec capital of Tenochtitlan. There is some evidence that chocolate was used as money outside this area. E.g., archaeologists found traces of chocolate in urns found in New Mexico about 2.000 km north of the upper limit where it can be grown.

[4] A recent review of related work can be found in Lagos et al. (2017).

The setup used here is similar to Burdett et al. (2001, BTW from hereon). Prisoners use cigarettes as commodity money in a P.O.W. environment described in Radford (1945). Cigarettes are delivered by the Red Cross. Everybody loves to smoke but can decide not to consume a cigarette and use it as a medium of exchange later. The dichotomy of provision and consumption is also internalized here, but the production decision yields additional insights in the form of equilibria not discussed by BTW. In particular, the commodity must be costly enough to qualify as money.

This paper addresses the endogenous supply of money, and in particular, the private and decentralized production and provision of costly commodity money. Araujo and Camargo (2008) address the dynamic inconsistency problem associated with the issuance of cost-less but valued fiat money by a single, self-interested agent. With her choice quasi private due to the nature of decentralized exchange, and in the absence of commitment, she has an incentive to over-issue. Berentsen (2006) finds that public record keeping of an issuer's trading history can make the private provision of fiat currency feasible but sub-optimal. Here, production costs discipline the quantity of money effectively in a feasible sense, at least for some parameter spaces. The storage constraint in KW implies a social optimum of money-holdings at $M = 1/2$. This number maximizes the number of meetings between agents that hold zero and one unit of currency. Deviatov and Wallace (2001) allows for households to hold one or two units so that individual money withdrawal and injection can be beneficial. This relaxation is not required here as the money supply is endogenously chosen against production costs.

Li (1995a) addresses social optimality when the search intensity in a decentralized economy is a choice, and proposes taxing money balances to recover search externalities. There are several reasons why production here is at a sub-optimal level. First, the issuer cannot recover all welfare gains the commodity will facilitate in future exchanges, and hence, production rents spill into the economy. Second, the ex-ante production generates sunk-cost considerations. Compare Julien et al. (2008) for an economy with fiat money and on-the-spot production of the good of interest where sunk-cost considerations play no role. Dutu and Julien (2008) consider ex-ante production of the special good, which generates sunk-cost losses. Finally, there is a congestion issue in the Kiyotaki-Wright style economy where agents can only hold one unit of the commodity.

What makes good money is at the center of a long historical debate. Jevons (1875) lists good properties of commodity money, and among them are intrinsic value, portability, indestructibility, homogeneity, divisibility, stability of value, and cognizability. The paper emphasizes intrinsic value through an opportunity cost channel. In particular, the replacement cost drives a wedge between the valuation of having chocolate and being without. This difference in valuation must be matched when the commodity is eaten. Cuadras-Morató (1997) demonstrates that even perishable goods, such as ice cream, can serve as money. Cuadras-Morató (1994) and Li (1995b) show that goods with heterogeneous quality can also emerge as money. Dubey et al. (2003) acknowledge that gold pays real dividends (like an asset) while tobacco perishes upon consumption. Gold is inefficient because of a mismatch of savings and appreciation motives when a rental market is missing which usually is the case when gold is used as a store of value.

2 A model of production, consumption, and exchange

2.1 Environment

The following model leans on the specification in BTW. I focus on steady state symmetric equilibria, and therefore drop indices of time and individuality. Time is continuous and continuous forever. There exists a unit mass of agents that discounts time at rate r. All goods are indivisible. While there are many different types of goods they can be divided into two groups.

The goods of one group are heterogeneous and nonstorable. I will refer to them as special goods, as this category captures specialization in production and variety of tastes. Specialization implies that only $0 < x \leq 1$ of the population derives $u_s > 0$ of utility from consuming whatever a particular agent can produce. The remaining agents obtain nothing. Agents do not like their own production which precludes the production of special goods for self-sustenance. The (conditional) probability that two agents enjoy each other's production, given that one agent enjoys the production of the other, is $0 \leq y \leq 1$.[5] The production of one unit of the special goods is on the spot[6] and costs $c_s > 0$.

The goods of the other group are homogeneous and enjoyed by everyone. I will call this good chocolate. Chocolate is storable, and, as mentioned above, indivisible. Any agent can hold at most one unit, and the fraction of agents that hold chocolate is G, the social state. A unit of chocolate rots according to a Poisson process with a parameter $\delta > 0$. There are two ways to obtain chocolate as follows: home production and trade. Once a unit of chocolate is obtained, the agent has to decide whether to consume it ($\theta = 1$) or to hold onto it to trade it against a special good later ($\theta = 0$). She can also play a randomization strategy where an agent sometimes eats it and sometimes holds onto it ($\theta \in (0, 1)$). Eating a unit of chocolate yields $u_c > 0$ of utility, which is the fundamental consumption value of chocolate. An agent without chocolate

[5] We observe a double coincidence of needs in every match when $y = 1$, and $y < 1$ motivates the use of money as an equilibrium solution, or money becomes essential. KW lives on $x = \frac{1}{3}$ and $y = 0$.

[6] One could think of this good as a service. Julien et al. (2008) show that a monetary equilibrium with fiat money always exists while introducing an ex-ante production decision introduces sunk cost considerations as shown by Dutu and Julien (2008). The specification chosen here simply avoids sunk cost considerations for the special good.

chooses intensity $\sigma \geq 0$, which defines another Poisson process, resulting in the production of one unit of chocolate. This production decision creates cost $c(\sigma) = \gamma\sigma$, and there is an upper bound for the production $\sigma \leq \overline{\sigma}$. Agents only produce if it improves their welfare in a strict sense.[7]

All trades occur between matches of two agents. All agents meet bilaterally and randomly according to yet another Poisson process with parameter $\bar{\alpha}$. A meeting has the following two steps: first, the two agents bargain under full information. Then, they produce, exchange, consume special goods and part ways after exchanging goods (or not). No records are kept in private or public. Agents exchange goods only when both parties are strictly better off. $c_s < u_c < u_s$ ensures gains from trade and takes decisions regarding the exchange out of the equilibrium definition. In particular, it is always rational to trade chocolate for a special good because $u_c < u_s$. Further, an agent will always accept chocolate for his production as $u_c > c_s$. Agents can not consume chocolate during a meeting. Therefore, two agents with chocolate cannot trade in a single coincidence of needs event, as the producer could not be compensated.[8] For illustrative purposes, I set $y = 0$ and denote $\alpha = \bar{\alpha}x\,(1 - y)$. That is, there is no barter involving only special goods, and α becomes the rate for single-coincidence meetings.

2.2 Valuations and law of motion

There are only two types of agents. One type has one unit of chocolate and her valuation is denoted V_1. The other has nothing, and her valuation is V_0. The wedge between the two valuations is denoted by $\Delta = V_1 - V_0$. This can be interpreted as the transaction value of chocolate.

[7]This restriction avoids a multiplicity of solutions when cost and benefit are equal.

[8]Allowing agent in possession of chocolate to consume one unit of chocolate during a meeting would circumvent the congestion issue of a Kiyotaki-Wright economy. But it would also require some modification with respect of the consumption decision when an agent holds a unit of chocolate. Overall, this would not contribute to the exposition of the original problem so I decided against this modification.

An agent without chocolate values her existence in a steady state with the following Bellman equation

$$\begin{aligned} rV_0 = &\alpha G \left[\theta u_c + (1-\theta)\Delta - c_s\right] \\ &+ \sigma \left[\theta u_c + (1-\theta)\Delta - \gamma\right] \end{aligned} \tag{1}$$

The first line displays utility drawn from a single coincidence opportunity, in which the agent produces a special good and obtains chocolate in exchange. The second line is the result of the pain and gain from home production.

Living with one unit of chocolate in a steady state yields the following payoff

$$\begin{aligned} rV_1 = &\alpha (1-G) \left[u_s - \Delta\right] \\ &- \delta\Delta \end{aligned} \tag{2}$$

The first line mirrors potential trading activity where the chocolate owner obtains a special good, consumes it and separates from her unit of chocolate. The second line reflects the loss of her chocolate because it went bad.

Given θ and σ, the equalized flows between obtaining chocolate and separating from it present the following steady state condition

$$\theta = \frac{\sigma - \delta\frac{G}{1-G}}{\sigma + \alpha G} \tag{3}$$

2.3 Consumption and production

An agent essentially solves two problems. The decision to consume or provide is determined by the following condition

$$\begin{aligned}
\Delta - u_c < 0 \quad &\Rightarrow \quad \theta = 1 \\
\Delta - u_c = 0 \quad &\Rightarrow \quad \theta \in (0,1) \\
\Delta - u_c > 0 \quad &\Rightarrow \quad \theta = 0
\end{aligned} \tag{4}$$

Non-consumption implies the provision of chocolate in a later exchange; hence, I refer to this condition as the provision incentive condition. The consumption decision categorizes an equilibrium as either non-monetary ($\theta^* = 1$), monetized ($\theta^* = 0$), or dual-role ($\theta^* \in (0,1)$).

$\Delta - u_c$ simplifies due to proportionality so that

$$\begin{aligned}
\Delta - u_c \propto \alpha \left(u_s - G\left(u_s - c_s\right) - u_c\right) \\
- \left(r + \delta + \sigma\right) u_c + \sigma\gamma
\end{aligned} \tag{5}$$

Note that the net gain from trade, $\Delta - u_c$, is positively correlated to the cost, γ. This reflects the sunk cost associated with the production; agents with chocolate can eat it but then must face the same effort to get back into the ownership state.[9]

The optimal production level by an agent without chocolate is given by

$$\sigma^* = \arg \max_{0 \leq \sigma \leq \overline{\sigma}} \left\{\sigma \left(\theta u_c + (1-\theta)\,\Delta - \gamma\right)\right\} \tag{6}$$

The production decision (6) can be simplified into two cases conditional on

[9] It is also noteworthy that the money supply in a Kiyotaki-Wright type model is not neutral. This is reflected in (5). The social state G affects the decision to consume (positively) or provide (negatively). The mechanism is straightforward: the more chocolate is in circulation the less likely an agent will meet someone who can accept it as payment. Hence, money in circulation congests trading opportunities by design. While this is a consequence of modeling choices, in particular the restriction that agents can only hold one unit of chocolate, it also reflects an economic observation: wealthy agents produce less in bilateral meetings. Compare Molico (2006) or, more recently, Rocheteau et al. (2015) for models where a non-degenerate distribution of money holdings influences bilateral bargaining situations in such a way.

the consumption decision. First, if $\theta = 0$ no agent eats chocolate and $\Delta > u_c$, and we can observe both, corner and interior solutions for σ^*. Second, when $\theta > 0$ agents consume chocolate and $\Delta \leq u_c$ must hold, which simplifies (6) with two possible corner solutions. If $\gamma < u_c$, an agent produces at her limit so that $\sigma^* = \bar{\sigma}$. If costs weakly exceed the benefit of chocolate, $\gamma \geq u_c$, we observe $\sigma^* = 0$. Similar to the provision decision, the production choice categorizes an equilibrium as either at capacity ($\sigma^* = \overline{\sigma}$), partially producing ($0 < \sigma^* < \overline{\sigma}$), or non-producing ($\sigma^* = 0$). The classes of possible equilibria below are derived from the categorizations by the consumption and production decisions.

2.4 Decentralized equilibria

First, a definition for equilibria in this environment:

Definition 1. A steady-state symmetric equilibrium is defined by the list $\{V_0, V_1, \theta, \sigma, G\}$ satisfying (1), (2), (3), (4), and (6).

In practice, there are three unknowns, namely, (σ, θ, G). The equilibria are found by dividing the parameter space. First, if $\gamma < u_c$, agents always produce the maximum, and the parameter conditions can be pinned down with a conjecture on θ. On the other hand, if $\gamma \geq u_c$, no agent will eat chocolate in equilibrium so that $\theta = 0$, and we can conjecture interior and corner solutions for the production intensity σ, which pins down the remaining parameter conditions.

This leads to the following proposition where $\Phi = r + \alpha + \delta + \overline{\sigma}$, $\Psi = \left(\delta u_s + \overline{\sigma} c_s\right) / \left(\delta + \overline{\sigma}\right)$, $\gamma_1 = \max\left\{0, \min\left\{\left(\Phi u_c - \alpha u_s\right) / \overline{\sigma}, u_c\right\}\right\}$, $\gamma_2 = \max\left\{0, \min\left\{\left(\Phi u_c - \Psi \alpha\right) / \overline{\sigma}, u_c\right\}\right\}$, $\gamma_3 = \max\left\{\alpha \Psi / \left(r + \alpha + \delta\right), u_c\right\}$, and $\gamma_4 = \max\left\{\alpha u_s / \left(r + \alpha + \delta\right), u_c\right\}$.

Proposition 1. *For the environment described in definition 1 there exists 6 classes of equilibria. If $\gamma < u_c$ all agents produce at their limit, $\sigma^* = \overline{\sigma}$. Then,*

$0 \leq \gamma \leq \gamma_1$ *creates an **au**tark equilibrium where $\theta^* = 1$.*

$\gamma_1 < \gamma < \gamma_2$ *produces a **d**ual-**r**ole equilibrium where $\theta^* \in (0,1)$, and*

$\gamma_2 \leq \gamma < u_c$ *yields a **c**onsumption-**b**acked monetized equilibrium where $\theta^* = 0$.*

If, on the other hand, $\gamma \geq u_c$ no agent consumes, $\theta = 0$, and

$u_c \leq \gamma \leq \gamma_3$ *creates a **c**apacity-**c**onstrained monetized equilibrium where $\sigma^* = \overline{\sigma}$.*

$\gamma_3 < \gamma < \gamma_4$ *produces a **c**ost-**m**atching monetized equilibrium where $\sigma^* \in (0, \overline{\sigma})$, and*

$\gamma_4 \leq \gamma$ *yields a **n**on-**p**roducing **e**quilibrium where $\sigma^* = 0$.*

Proposition 1 orders equilibria along the cost dimension. If costs are low enough ($0 \leq \gamma \leq \gamma_1$), agents do not trade with each other but produce only for their own direct consumption. They are essentially autarkic. Increasing the cost ($\gamma_1 < \gamma < \gamma_2$) causes the commodity to serve dual roles in this economy. The monetary use can crowd out consumption altogether when cost increase further, even when the fundamental consumption value of the commodity covers the cost ($\gamma_2 \leq \gamma < u_c$). Production for monetary purposes is fully backed by the consumption value. Production at the capacity limit is possible even when the costs exceed u_c ($u_c \leq \gamma \leq \gamma_3$) but it is throttled beyond the upper threshold when costs start to match monetary use. Finally, production ceases when costs exceed γ_4.

Proposition 1 supports the first result, namely that a commodity must be costly enough to serve as money. In particular, below γ_1 the commodity will not circulate. The proposition also extends the results of KW, where holding costs determine whether and which type(s) of good is used as money. An agent in KW first obtains a commodity to use it in an exchange later for what she really wants, but never to consume it right away. In contrast, agents here consume the commodity directly when the costs are sufficiently low, as seen in the class of autark and dual-role equilibria. The equilibria here for $\gamma < u_c$ correspond to proposition 1 in BTW where production is exogenous. Proposition 1 extends their results with equilibria where $u_c < \gamma$.

All equilibria are not only Nash but also unique. Multiplicity of equilibria often occurs in monetary models; fiat money is accepted by an agent because she believes it will provide utility to her when it is accepted by the next agent through an exchange for something else. Without this individual creed, fiat money is not valued. There are always at least two equilibria, a monetized one and a non-monetary one. Wallace (1998) speculates that this is a feature found only in fiat money economies. Zhou (2003) shows that multiple equilibria can continue to exist if the dividend paid by an object used as money is not too large. Here, agents accept chocolate because the reward for the consumption of the general good exceeds the production cost of the special good, $c_s < u_c$. The flooring ensures uniqueness for the existence of monetized equilibria. But the uniqueness in the third result is stricter in the sense that it pins down the transaction value uniquely. This is addressed below.

Figure (1) summarizes where these equilibria live in the parameter space. For illustrative purposes, I describe the thresholds γ_1 and γ_4. Rearranging the

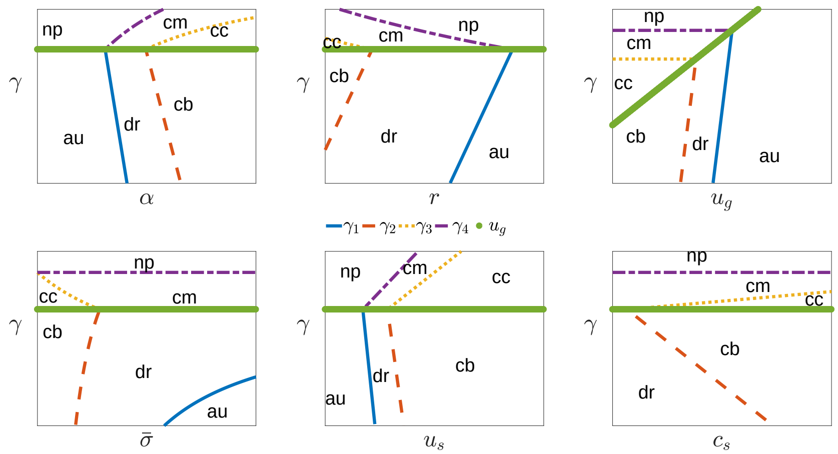

Figure 1: Equilibria regions in parameter space. The regions are marked by a two-letter abbreviation referring to the classifications in proposition 1: **au** are autark equilibria, **dr** are dual-role equilibria, **cb** are consumption-backed monetized equilibria, **cc** are capacity-constrained monetized equilibria, **cm** are cost-matching monetized equilibria, and **np** are non-producing equilibria.

(partial) condition for autarky yields

$$(r+\delta)\,u_c + (u_c-\gamma)\,\overline{\sigma} > \alpha\,(u_s-u_c)$$

so that, conditional on the fundamental value exceeding the cost, agents produce and fully consume when, first, agents are sufficiently impatient. Second, chocolate rots easily. Third, the net gains of self-sustenance, $u-\gamma$, are sufficiently high. Fourth, the maximal production is high enough. Fifth, the probability of a single coincidence is relatively low. In addition, sixth, the net gains from a trade are relatively low. This is largely in line with the literature (compare BTW). The condition for γ_2 is similar, but u_s is exchanged for the weighted average Ψ.

Rearranging the condition for the lower bound of the non-producing equilibria yields

$$\alpha\left(u_s-\gamma\right)\leq\gamma\left(r+\delta\right)$$

In other words, production ceases when, first, the chance of trade is small enough. Second, the net gains from trade must be small. Third, the cost must be high. Fourth, agents need to be sufficiently impatient, and, fifth, chocolate must spoil at a high rate. The condition for γ_3 is similar, but u_s is again exchanged for the weighted average Ψ.

3 Non - monotonicities

3.1 Money supply

Production is constant for $\gamma\leq\gamma_3$ because production is incentivized by either the fundamental value of consuming chocolate ($0\leq\gamma\leq\gamma_2$) or by the liquidity it provides ($\gamma_1\leq\gamma\leq\gamma_3$). An exchange is still ensured by the fundamental consumption value beyond this point because $c_s<u_c$. However, agents produce chocolate because it facilitates an exchange of specialized goods. Hence, harvesting would cease without trade. The production decision can then be summarized as follows

$$\sigma^*=\begin{cases}\overline{\sigma} & 0\leq\gamma\leq\gamma_3\\ \delta\frac{\alpha u_s-\gamma(r+\alpha+\delta)}{\gamma(r+\alpha+\delta)-\alpha c_s} & \gamma_3<\gamma<\gamma_4\\ 0 & \gamma_4\leq\gamma\end{cases}$$

The top left panel of figure (2) summarizes this form.

On the other hand, agents are autarkic and consume all chocolate until the cost increases beyond γ_1. Increasing the cost further increases the wedge be-

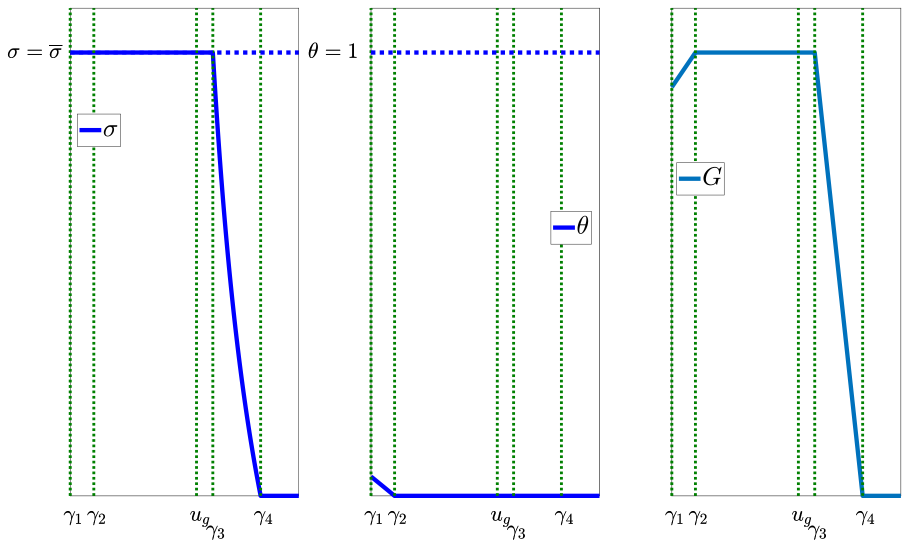

Figure 2: Policies and the social state.

tween having chocolate and going without it. Agents hold onto the commodity, first partially ($\gamma_1 < \gamma < \gamma_2$) and later fully ($\gamma_2 \leq \gamma$). Hence, the consumption decision is

$$\theta^* = \begin{cases} 1 & 0 \leq \gamma \leq \gamma_1 \\ \widehat{\theta} & \gamma_1 < \gamma < \gamma_2 \\ 0 & \gamma_2 \leq \gamma \end{cases}$$

where

$$\widehat{\theta} = \frac{(u_s - c_s)\left(\alpha\left(\delta u_s + \overline{\sigma} c_s\right) + (\delta + \overline{\sigma})\left(\gamma\overline{\sigma} - u_c\Phi\right)\right)}{\left(\alpha c_s + \gamma\overline{\sigma} - u_c\Phi\right)\left(\sigma\left(\gamma - c_s\right) + u_s\left(\alpha + \overline{\sigma}\right) - u_c\Phi\right)}.$$

The center panel of figure (2) captures this pattern.

This yields the mapping for the social state, G, capturing the holding of the commodity to be used as a medium of exchange, i.e., the money supply.

$$G^* = \begin{cases} 0 & 0 \leq \gamma \leq \gamma_1 \\ \frac{\alpha u_s + \gamma \overline{\sigma} - u_c \Phi}{\alpha(u_s - c_s)} & \gamma_1 < \gamma < \gamma_2 \\ \frac{\overline{\sigma}}{\overline{\sigma} + \delta} & \gamma_2 \leq \gamma \leq \gamma_3 \\ \frac{\alpha u_s - \gamma(\alpha + \delta + r)}{\alpha(u_s - c_s)} & \gamma_3 < \gamma < \gamma_4 \\ 0 & \gamma_4 \leq \gamma \end{cases}$$

which in turn yields the following corollary:

Corollary 1. *The social state is non-monotonic in cost. In particular,*

$$sign\left(\frac{\partial G^*}{\partial \gamma}\right) = \begin{cases} 0 & 0 \leq \gamma \leq \gamma_1 \\ + & \gamma_1 < \gamma < \gamma_2 \\ 0 & \gamma_2 \leq \gamma \leq \gamma_3 \\ - & \gamma_3 < \gamma < \gamma_4 \\ 0 & \gamma_4 \leq \gamma \end{cases}$$

In other words, the cost to produce will first ensure that monetary use crowds out consumption as provision incentives increase. While the transaction value can ensure production even when the cost exceeds the fundamental value of consumption production decreases and ceases eventually. The right panel of figure (2) captures this pattern.

3.2 Trades

The quantity of trades in this economy is given by

$$T = \alpha G (1 - G)$$

In other words, it is the mass of agents with chocolate, G, meeting agents without chocolate, 1-G, at the frequency of a single coincidence of needs, α. Hence, for $0 \leq G < 1/2$, T is strictly monotonically increasing in G, while it is strictly decreasing for $\frac{1}{2} < G \leq 1$. Obviously, $\sup_{\gamma \geq 0} \{G^* (\gamma)\} = \overline{\sigma} / (\overline{\sigma} + \delta)$, $\underset{\gamma \downarrow \gamma_1}{G^* (\gamma)} \leq \underset{\gamma \uparrow \gamma_2}{G^* (\gamma)}$ and $\underset{\gamma \downarrow \gamma_3}{G^* (\gamma)} \geq \underset{\gamma \uparrow \gamma_4}{G^* (\gamma)}$, and note that $\overline{\sigma} / (\overline{\sigma} + \delta) \leq 1/2 \Leftrightarrow \overline{\sigma} \leq \delta$. Then, we can state the following corollary.

Corollary 2. *The number of trades is constant for* $0 < \gamma \leq \gamma_1$*,* $\gamma_2 \leq \gamma \leq \gamma_3$*, and* $\gamma_4 \leq \gamma$*. If* $\overline{\sigma} \leq \delta$*, then*

$$sign\left(\frac{\partial T}{\partial \gamma}\right) = \begin{cases} + & \gamma_1 < \gamma < \gamma_2 \\ - & \gamma_3 \leq \gamma \leq \gamma_4 \end{cases}$$

while $\overline{\sigma} > \delta$ *implies the possible existence of two maxima at* γ_a *and* γ_b*. In particular, if* $\gamma_1 < \gamma_a = (2u_c\Phi - \alpha (c_s + u_s)) / 2\overline{\sigma} < \gamma_2$*, we observe a maximum at* γ_a *so that*

$$sign\left(\frac{\partial T}{\partial \gamma}\right) = \begin{cases} + & \gamma_1 < \gamma < \gamma_a \\ - & \gamma_a \leq \gamma < \gamma_2 \end{cases}$$

and if $\gamma_3 < \gamma_b = \alpha (c_s + u_s) / (2 (\alpha + \delta + r)) < \gamma_4$*, we find a maximum at* γ_b *and*

$$sign\left(\frac{\partial T}{\partial \gamma}\right) = \begin{cases} + & \gamma_3 < \gamma < \gamma_b \\ - & \gamma_b \leq \gamma \leq \gamma_4 \end{cases}$$

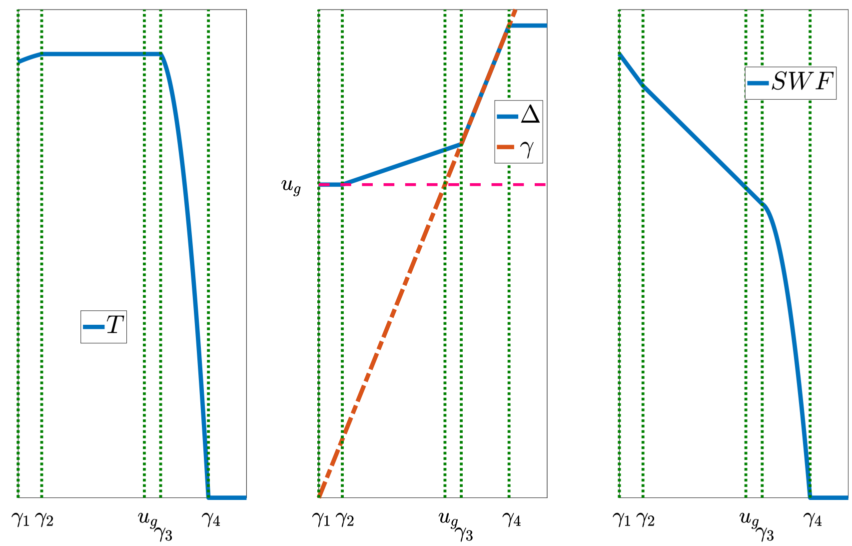

Figure 3: Trades, valuation difference and social welfare.

The top left panel of figure (3) captures the case when $\overline{\sigma} \leq \delta$.

The corollaries 1 and 2 provide the second result. The following is a note on the exact shape of the non-monotonicity and the external validity of corollary (2): the number of trades increases when the cost parameter γ moves past γ_1, and decreases as it approaches γ_4. This is a fairly general result; agents consume less when cost increases in a dual-role economy and produce less when the commodity is used solely as money. The double hump shape reflects the congestion issues innate to the Kiyotaki-Wright framework when too many agents hold cash. Hence, it is an artifact of the modeling assumption that agents can only hold a limited amount of chocolate.

3.3 Welfare

Corollary 3. *The transaction value is*

$$\Delta = \begin{cases} \frac{\overline{\sigma}(\gamma - u_c) + \alpha u_s}{\alpha + \delta + r} & 0 \leq \gamma \leq \gamma_1 \\ u_c & \gamma_1 < \gamma < \gamma_2 \\ \frac{\gamma\overline{\sigma}(\delta + \overline{\sigma}) + \alpha(\delta u_s + \overline{\sigma} c_s)}{(\delta + \overline{\sigma})\Phi} & \gamma_2 \leq \gamma \leq \gamma_3 \\ \gamma & \gamma_3 \leq \gamma \leq \gamma_4 \\ \frac{\alpha u_s}{\alpha + \delta + r} & \gamma_4 \leq \gamma \end{cases}$$

The transaction value is determined by the production costs when chocolate is not used as a medium of exchange. The off-equilibrium valuation is below the consumption value, or $\Delta < u_c$ when $0 \leq \gamma \leq \gamma_1$. As the cost drive up the transaction value it levels with the consumption value, $\Delta = u_c$ when $\gamma_1 < \gamma < \gamma_2$. Costs determine indirectly through a definition of θ that keeps those two value identical. For the region $\gamma_2 < \gamma < \gamma_3$ the transaction value can exceed both, the consumption value as well as the transaction value. But this is the result of the production constraint. Otherwise, production would equalize one or the other. Once production is unconstrained it matches with the cost directly, so that $\Delta = \gamma$ for $\gamma_3 \leq \gamma \leq \gamma_4$. This is the third result.

The social welfare function in this economy is a weighted average of the valuation of the two different types of agents, or

$$SWF = V_0 + G\,\Delta \tag{7}$$

While the social welfare function is a cumbersome animal, we can state the following corollary regarding the first derivative.

Corollary 4. *The derivative of the social welfare is*

$$\frac{\partial SWF}{\partial \gamma} = \begin{cases} -\overline{\sigma} & 0 \leq \gamma \leq \gamma_1 \\ \frac{\overline{\sigma}}{\alpha(u_s - c_s)} \left((\alpha + 1) u_c - \alpha u_s \right) & \gamma_1 < \gamma < \gamma_2 \\ \frac{\overline{\sigma}}{(\delta + \overline{\sigma})\Phi} \left(\overline{\sigma} (1 - r) - \delta \Phi \right) & \gamma_2 \leq \gamma \leq \gamma_3 \\ \frac{(\alpha+1)\alpha u_s - ((\alpha+1)2\gamma - \alpha c_s)(\alpha + \delta + r)}{\alpha(u_s - c_s)} & \gamma_3 \leq \gamma \leq \gamma_4 \\ 0 & \gamma_4 \leq \gamma \end{cases}$$

There are three cases when social welfare can increase with increasing cost. First, welfare can increase in cost in the dual-role equilibrium when the consumption benefits are high enough ($u_c > \alpha u_s / (\alpha + 1)$). Second, welfare can also increase when the maximal production intensity is large enough ($\overline{\sigma} > \delta\Phi / (1 - r)$). Finally, welfare can also increase in costs when the cost are small enough ($2\gamma < \alpha (u_s + (\alpha + \delta + r) c_s / (\delta + r)) / (\delta + r)$).

4 Conclusion

The costs of production determine whether a commodity can be used as money, as a consumption good, or as both. The commodity can not be too cheap so that it is used as a medium of exchange. The effect of cost on money supply and trade volume is not monotonic and can generate a hump-shape. And costs determine the transaction value of a commodity when it is used as money only. Production costs become a disciplining device for the value of a commodity.

The underlying mechanics using production costs can rationalize other economic phenomena. For example, Bitcoin has recently become a popular electronic currency with a annual net rate of return of 127% between December of 2014 and August 2018. I conjecture that part of this steep price increase is due to an increased media exposure. But a part of the valuation increase can be ascribed to increased costs of producing new Bitcoins which is called mining. In particular, the bookkeeping of transfers is decentralized to ensure correctness, stability, and independence. The bookkeeping requires that many participants provide processing power resources. In order to entice participants the algorithm presents a computational puzzle which contains the bookkeeping process, and the participant who solves it first is rewarded with newly issued Bitcoins. Naturally, the more people participate the quicker the solution is found. In order to keep the growth rate of Bitcoins stable the difficulty of the puzzle is adjusted. With an increased interest in Bitcoins more people participate which requires the puzzle to become harder to solve. The model here predicts that the higher difficulty to obtain new Bitcoins directly translates into a higher transaction value.

Take as another example initial public offerings (IPOs). They entail higher costs and higher trading volume on secondary markets in the United States compared to their European counterparts.[10] This observation is in line with the non-monotone relationship of production costs and trading volume mentioned above and described below. How do IPOs map into the model presented here? The commodity production can reflect IPOs of financial assets, consumption can be considered the long-term holding of an asset, and traders hold onto newly issued assets to trade them for specific hedging needs. The aforementioned stylized fact of higher cost associated with higher trade volume can be explained if we assume that the costs to issue, γ^{IPO}, are $\gamma_1 < \gamma^{IPO} < \gamma_2$. If the cost

[10] Chen and Ritter (2000) document that IPOs in the United States entail costs of seven percent of the IPO value independent of size, and hence, a much larger fraction than their European counterparts as documented by Abrahamson et al. (2011).

would be lower, the issuer would just keep the asset and hold it forever. Hence, we would not observe any IPOs. If the cost would be above γ_2, nobody would ever hold any asset for a long period of time. If $\gamma_1 < \gamma^{IPO} < \gamma_2$ then $\partial T/\partial\gamma > 0$ which explains the stylized fact.

But we can also stay in Mexico to make some transfer: during the eighteenth century the Spanish crown reduced the cost of mercury supplied to its colonies in the new world. Mercury was an important ingredient in the mining process of silver. Dobado and Marrero (2011) documents that the production of silver went up. González and Montero (2010) use meat as a proxy for real wages and document that labour wages increased and were higher in Mexico than in all European cities in their sample which is in stark contrast to the "reversal of fortune" hypothesis of Acemoglu et al. (2002). The model here predicts an increase of production for silver and an increase in Δ in line with the observations by Dobado and Marrero (2011) and González and Montero (2010).

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