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Esta tesina está dedicada a mi padre, Javier López Chang, cuyo recuerdo guardo con cariño

Abstract

The present work aims to study the marketing and competition functions of an inter-temporal pricing policy on a durable product with unknown quality. Pricing policies consist of price discounts and referral payments. The product in question must be introduced by a company to a group of consumers connected through a social network. At the same time, the company faces the threat of entry of a substitute product whose quality is known by the group of prospects. Social connections allow communication via Word of Mouth between early and late consumers, so that the company can introduce incentives through its pricing policy to ensure that information is diffused optimally. In the meantime, prices and referrals send signals to entrants about the level of competition in the market. For this reason, companies of this kind of products must design their pricing policies to find the balance between being known and being competitive.

In this dissertation I propose a normative framework to design this kind of policies and assess their possible impact on the benefit of consumers and companies. To achieve this objective, I present a model of duopoly competition in social networks between two products of different quality. To simplify the analysis and impose realistic cognitive burdens over agents, mean field approximations of the proposed model are made.

In first place, this study provides guidelines for the design of these policies that take into account the level of information on product quality and the distribution of degrees (number of connections that a person has in the social network) that characterizes the social network. Secondly, a calculation is also made of the demands expected by the incumbent company and the entrant company that also depend on the degree of symmetry of information that these companies share with each other. Thirdly, the impact on the diffusion of the incumbent product is characterized according to the price of competition, whether a discount or a referral policy is implemented, the degree distribution and the degree of a specific customer.

Keywords:
Marketing in Social Networks, Competition, Word of Mouth, Mean field approach

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Chapter 1

Introduction

Companies selling durable goods of uncertain quality have a hard time. To start with, consumers are often reluctant to adopt unfamiliar products, and durability of such products can only exacerbate such reluctance. Indeed, since the purchase of durable goods commits customers for long periods of time, people may think twice before making a purchase under quality uncertainty. Secondly, consumption takes place in a highly competitive environment with both present and future substitutes. Particularly worrying are the threats of entry that big companies selling substitute products of known quality pose to these companies. Since these large companies do not necessarily offer better products, this issue is also of social interest: Companies that are not well known are potential suppliers of high quality products and therefore competition from well-known brands can reduce innovation in the marketplace. Examples of such scenarios exist in industries ranging from sales and manufacturing of clothing, footwear, furniture, etc.

This scenario provides the set-up for these companies to think of pricing strategies that have both marketing and competitive functions. On the one hand, an aggressive marketing strategy must be employed, in which incumbents must make their quality known through discounts or rewards for early use. On the other hand, they should be concerned about the strategic effects their pricing policies have on rival companies of known quality, for example, to avoid wars of attrition or to prevent these competitors from entering.

The logic behind the use of such take-up pricing lies in the phenomenon of "word-of-mouth" communication. This phenomenon is responsible for the viral marketing that companies have adopted in light of the importance of social networks in modern economies. Through this effect, early consumers become advertising agents who spread the information on the social network, increasing the demand for the product in the future. The social network on which the market interacts determines which people adopt the product in the early stages of the marketing campaign. In particular, the number of connections (friends or colleagues) that people have is a crucial determinant. Isolated people have fewer expectations of receiving information from their neighbors, while those who are more connected are more prone to receive information about previous experiences with the product in question. This creates a tension between the efficiency of the information diffusion process and the effectiveness of the incentives for early adoption. As a potential customer's number of friends increases, he or she acquires greater value as an advertising agent, but also has more incentives to wait and free-ride on her neighbors' experimentation.

In most cases it is not possible to know the specific structure of a social network, and rather certain beliefs about grade distribution are taken into account. In this thesis, I consider two types of policies that a company can implement to overcome these informational constraints: (1) Price discounts for early adoption and (2) Referral payments that reward an early customer for each neighbor who adopts the product in future stages. For people with a large number of friends, a price discount may not offset the benefits of waiting for a neighbor's opinion and making an informed decision. A referral payment, however, will be more attractive to them, given the greater number of friends who can refer them in the future. Thus, these policies turn out to be screening mechanisms in which people with few friends are more attracted to discounts, while people with more friends are encouraged by referral payments.In practice, companies using platform models, such as Uber, Didi and Rappi, have opted for a strategy that combines both discounts and referral payments.

Meanwhile, a well-known firm competing with a substitute product in the future will have reason to announce its price and quality to the market. By doing so, it may push consumers to postpone their purchase under the promise of better options in the future. Because of this, incumbent companies will have to increase price discounts and/or referral payments to meet the incentives offered by the new entrant. At the same time, the entrant must be careful in its pricing policy for two reasons. The first is simply the limitations he has in lowering his prices, as this may reduce profits. The second is that when the company is of mediocre quality, it runs the risk that an effective marketing campaign by the incumbents may reduce its market share, even if the competitor's prices are high. For example, effective rewards allow consumers to know the true quality of the incumbency, thus increasing demand for the incumbency's product through word of mouth.

In these markets, information asymmetries and network structure seem crucial: Agents must consider the trade-off between a risky but quick purchase and a safe but late one, while also considering that diffusion properties can affect payments. The characteristics of this type of markets give rise to interesting questions. What of these strategies (price discounts and referral payments) are most effective in inducing early adoption of a product of unknown quality? Can these strategies be equally effective in alleviating future wars of attrition with products of known quality? What is the impact of quality information on the consumption levels of the different products at different times? What is the impact of the network structure on the effectiveness of these strategies? Can certain policies prevent any of these firms to enter the marketplace? What should a policy maker expect in terms of welfare?

The above are normative questions that matter for both competition and marketing policies. To assess these questions, I will analyze the above scenario in a model of product adoption in which the decisions of network players, the pricing policies of companies and the referral payments of the incumbent are endogenized. The model presented in this thesis is a modification of the one presented by Leduc, Jackson, and Johari (2017), where I introduce future opportunity costs to include sequential competition. In order to evaluate the effects that information and timing have on these policies, I am explicit about the dependence on consumer impatience and product quality.

1.1 Literature review

Leduc et al's model responds to the interest in knowing the marketing strategies in social networks, social learning and pricing policies in scenarios of imperfect information.

The literature on Marketing in social networks usually focuses on the problem of a monopolist who wants to increase the demand for his product by exploiting the network externalities of his market. A seminal article in this area is that of Katz and Shapiro (1985), which evaluates price competition scenarios and where competing goods have network effects. Campbell (2013) evaluates how network externalities impact on the elasticity of demand for a product, although the focus here is on the resilience of the network to strategic attacks, in close analogy to problems such as the stability of the World Wide Web. The strategies of economic agents who possess perfect information about the web are studied in articles, such as those of Bloch and Quérou (2013) and Chen, Zenou, and Zhou (2018), which analyze competition scenarios and product complementarity, and whose results depend explicitly on measures of consumer centrality on the network. In articles such as those of Fainmesser and Galeotti (2016) and Shin (2017), by contrast, network information is imperfect and the monopolist's strategies result in mechanisms for revealing the network's structure.

The tension between delays costs and information gathering is evaluated in Social Learning papers. For example, Gul and Lundholm (1995) contrasts the processes called information cascades with the observation of grouping in collective decisions of agents who must choose not only if to execute a certain action but when and taking into account the decisions of others. Rogers (2005) is more specific and is close to our approach by evaluating the rational balances between making informed decisions and delay costs when there are informational externalities.

Also related to our analysis, there exists literature that focus on information gathering and pricing policies. Papers like those from Bar-Isaac, Caruana, and Cuñat (2010); Courty and Hao (2000); Dana (2001); Nocke, Peitz, and Rosar (2011); Lewis and Sappington (1994) study screening mechanisms as a way of discriminating among users with partial knowledge of their demand. In general, demands uncertain in the sense that quality can take high or low values.

Finally, the methods used both in this study and in Leduc et al. (2017) exploit the bounded rationality of the agents interacting in large social networks. Such methods refer to the Mean-Field approach, which is an idea taken from statistical physics, where agents form beliefs only in macroscopic parameters such as the average decisions of network agents. This gives tractability to the model at the same time that it imposes realistic restrictions in the beliefs of the agents. This method is explored in works such as those of Jackson and Yariv (2007) and Galeotti, Goyal, Jackson, Vega-Redondo, and Yariv (2010), as well as widely discussed in Leduc (2014).

1.2 Structure of the thesis

The thesis is organized as follows.

The second chapter describes the model. I start defining the actors, timing and the desired equilibria one should expect from the game proposed. I then, continue to describe the sub-game that describes the behavior of consumers. I continue by defining demands and payoffs of firms from the game. Finally, I define the corresponding welfare formulae.

The third chapter characterizes equilibria. Analytical solutions are difficult to obtain and the equilibria proposed need further refinements to break multiplicity. Thus, it is my work in this chapter to present a qualitative rather than a quantitative analysis on the solutions of the model. In specific, I show how consumers behave according to pricing policies and how these policies represent revelation mechanisms that screen the grade of a consumer. Comparative statics on the equilibria are shown for simple mechanisms which ilustrate the relation between information diffusion and competition parameters.

The fourth and final chapter presents the conclusions.

Chapter 2

The model

The model is a bayesian game in which exist two type of actors: consumers and firms. The game is a modification of the one proposed in Leduc et al. (2017).

Two substitute goods, labeled $i \in \{a, b\}$, can be acquired in either two periods, labeled $t \in \{0, 1\}$. Good a can be acquired in either period, but consumers are not aware of their quality, denoted by θ_a. Good b can be acquired only in the second period, but instead consumers are aware of their quality, denoted by θ_b.

Firms

There are two firms competing for customers, which we will call incumbent and entrant. The incumbent firm produces good a and the entrant firm produces good b. Alternatively, we will refer to a and b as the incumbent and the entrant good respectively.

The quality of the incumbent good can have two values H (high) or L (low). Such quality is determined by an event of nature, the result of which is only revealed to the incumbent company. Hence, there is asymmetry of information between this company and the rest of the actors.

Quality of the incumbent good, on the other hand, can have only the value B (mediocre). There is symmetrical and perfect information about the quality of the entrant good among all players.

The incumbent can set a price for early adoption of its product, P_0, and a price for late adoption, P_1. At the same time, it is able to commit to pay referral fees, η. These referral payments reward early consumers for each neighbor that adopts the product late. Moreover, the entrant, which is only competing for the late market, can only commit to set a price P_b for its product in this market. While the price fixed by the entrant only has competitive functions, the pricing scheme of the incumbent is more complex and has both competitive and marketing functions.

Consumers

Consumers are represented by a finite group of agents connected through a given network. The degree, the number of social connections that a given agent has, represents the number of communication channels a consumer has. Through these channels, information on past experiences of a product can flow without any friction from a person to her neighbors. However, the degree structure of the network is unknown to the agents, as they only know their own degree, d, but not others'. Agents nevertheless have access to a publicly known degree distribution, $f(d)$, a measure that dictates the probability that a person in the network has degree d. Thus, agents have uncertain information about the degree of their neighbors they can communicate with.

Beliefs on the quality of the goods are hold identically among agents. In specific, agents are certain that $\theta_b = B \in (L, H)$, (i.e. $\mathbb{P}(\theta_b = B) = 1$) but are uncertain that $\theta_a = H$, having instead the prior beliefs $\mathbb{P}(\theta_a = H) = p \in (0, 1)$ (and thus $\mathbb{P}(\theta_a = L) = 1 - p$). Information on quality is updated after one's use and can be shared to one's neighbors.

Consumers are forward-looking and are equally impatient, with a discount factor $\delta \in [0, 1]$.

Consumers' actions are to adopt a at $t = 0$, to adopt a or b at $t = 1$, or not to adopt neither a nor b at any t. It is assumed that goods are durable and thus agents cannot retrieve from adopting a at $t = 0$, which at the same time implies that agents can choose to adopt either a or b only if they have not adopted in the previous period. Denoting by $X_{i,1}$ the number of i's neighbors who adopt a at $t = 1$, payoffs from these actions are shown in Table 2.1.

Timing

Timing is a follows:

Table 2.1: Payoffs for different actions as a function of θ_a

	$\theta_a = H$	$\theta_a = H$
Not adopt at any t	0	0
Adopt a at $t = 0$	$H - P_0 + \eta X_{i,1}$	$L - P_0 + \eta X_{i,1}$
Adopt a at $t = 1$	$\delta H - P_1$	$\delta L - P_1$
Adopt b at $t = 1$	$\delta B - P_b$	$\delta B - P_b$

Own elaboration

1. Nature choose the quality of a: $\theta_a = \{H, L\}$. This information is revealed only to the incumbent firm. The entrant firm and consumers form a belief $p = \mathbb{P}(\theta_a = H)$ instead.
2. The incumbent announces a dynamic pricing policy $\{P_0, P_1, \eta\}$. Meanwhile, the entrant announces its price at t=1, P_b.
3. Consumers observe $\{P_0, P_1, \eta\}$ and P_b, and update their beliefs to $\mathbb{P}(\theta_a = H | P_0, P_1, \eta, P_b)$.
4. Consumption takes place:
   - At t=0: Some consumers adopt a and communicate θ_a to their neighbours.
   - At t=1: Consumers who didn't adopt at t=0 decide whether to buy a or b

Desired equilibria

Although we have not yet specified the payouts for all players in this game, we can begin to glimpse the kind of equilibrium that should be expected from rational players. These equilibria should consist of the following:

1. A set $\{P_0, P_1, \eta\}_{\theta_a}$, which prescribes the pricing policy chosen by the incumbent firm for each realization of θ_a .
2. A price P_b that is chosen by the entrant firm given the prior beliefs p that $\theta_a = H$.
3. A set of actions for each agent in the network, which consists of an early action (at $t = 0$) and a late action (at $t = 1$).
4. A consistent set of beliefs that $\theta_a = H$, for each agent in the network.

In the rest of this chapter it is my goal to define properly this equilibrium.

2.1 The mean field sub-game of adoption of substitute goods

In the final stage, consumers must decide on their adoption by taking the price and referral payments as established. Such decisions are described through a game of adopting substitute goods with exogenous pricing

First, let us denote by $\bar{A} := pH + (1-p)L$ the expected quality of the incumbent good under the prior beliefs. Policies announced by firms effectively send a signal of the quality of the incumbent good, $\sigma(\theta_a = \bar{A})$, to all agents equally. By adopting the incumbent good at $t = 0$, (an action denoted by a) an agent learn its quality and *teaches* this value to her neighbors before $t = 1$. On the other hand, if she decides not to adopt at $t = 0$ (denoted by $not\ a$), she can *learn* this value from neighbors who have adopted a at $t = 0$. If at least one neighbor adopted in $t = 0$, she can receive signals $\sigma(\theta_a = L)$ or $\sigma(\theta_a = H)$. In case not, she maintains its prior beliefs (effectively receiving the same signal $\sigma(\theta_a = \bar{A})$). In this case, she finally decides at $t = 1$ whether to adopt from the incumbent (a), from the entrant (b) or neither one ($none$)). This process resembles a percolation process where information of a group of agents who adopt a at $t = 0$ filters or diffuses to this group's neighborhood. Diffusion properties of this game then leads to local externalities of adoption from the early stage to the late one. This process is commonly referred as *Word-Of-Mouth* communication.

To capture the incentives of adoption that these externalities give to agents, we need to make some informational assumptions.

Assumption 1 (Forcing the incentives of adoption):

$\theta_a > 0$

$pH + (1-p)L < B$ ***Assumption 2 (Mediocre entrant):*** *The good b has a quality $B \in (L, H)$.*

Assumptions 1 and 2 capture the situation where, in the absence of competition, it is in an agent's interest to adopt a at $t = 0$. On the other hand, the fact that the expected quality of the good a does not reaches the quality of the good b ensures that an uninformed agent at $t = 1$ will prefer b over a. At the same time, b is a mediocre good, in the sense that its quality lies between

L and H. Altogether, these assumptions imply that in the absence of information and prices, agents are not interested in adopting a instead of b. However, when information of a's quality is available, an agent will be interested in doing the opposite, i.e. adopting a instead of b. Hence, an agent who has not adopted a in $t = 1$ will adopt only when she is informed that the quality of a is high ($\theta_a = H$) , and will adopt b otherwise.

Best response at t=1

Optimal response of an agent who has not adopted a at $t = 0$ must arise from comparing the benefits of purchasing a ($U_a^1(\theta_a) := \delta\theta_a - P_1$) versus purchasing b ($U_b^1 := \delta B - P_b$). Clearly, an agent will strictly prefer to adopt a whenever $U_a^1(\theta_a) > U_b^1$, strictly prefer to adopt b whenever $U_a^1(\theta_a) < U_b^1$, and mix otherwise. While $\{P_{1,}P_b, B\}$ is public information, θ_a is not, and thus the consumption trade-offs will depend on what information each agent has, prior to taking her $t = 1$ decision, which itself depends on whether a neighbor has adopted at $t = 0$. The following observation formalizes these decisions.

Observation 1 (best response at t=1): *Suppose an agent i has not adopted at $t = 0$. Denote by $X_{i,0}$ the number of i's neighbors who have adopted a at $t = 0$. Then:*

(i) If $P_1 - P_b > \delta(H - B)$: Adopts b in any case.

(ii) If $\delta(H - B) > P_1 - P_b > \delta(\bar{A} - B)$: Adopts a if $X_{i,0} > 0$ and $\theta_a = H$. Otherwise she adopts b.

(iii) If $\delta(\bar{A} - B) > P_1 - P_b > \delta(L - B)$: Adopts b if $X_{i,0} > 0$ and $\theta_a = L$. Otherwise she adopts a.

(iv) If $\delta(L - B) > P_1 - P_b$: Adopts a in any case.

Thus, observation 1 shows that whether the difference in prices in the second period is above or below the difference in (discounted) qualities, agents will respond differently to their neighbors' prior decisions.

After having determined the best response of the agents in $t = 1$, the decision in $t = 0$ is simplified. In $t = 0$, one must compare the benefits of acquiring (a) versus the benefits of deferring the purchase to $t = 1$ ($not\ a$) and obtain any of the payments described before. Table

2.2 summarizes the payments for actions taken at $t = 0$ for each of the cases (i-iv) of observation 1.

Table 2.2: Payoffs for actions at $t = 0$ as a function of θ_a

(i) $P_1 - P_b > \delta(H - B)$

	$\theta_a = H$	$\theta_a = L$
Buy a	$H - P_0 + \eta X_{i,1}$	$L - P_0 + \eta X_{i,1}$
Wait and $X_{i,0} > 0$	$\delta B - P_b$	$\delta B - P_b$
Wait and $X_{i,0} = 0$	$\delta B - P_b$	$\delta B - P_b$

(ii) $\delta(H - B) > P_1 - P_b > \delta(\bar{A} - B)$

	$\theta_a = H$	$\theta_a = L$
Buy a	$H - P_0 + \eta X_{i,1}$	$L - P_0 + \eta X_{i,1}$
Wait and $X_{i,0} > 0$	$\delta H - P_1$	$\delta B - P_b$
Wait and $X_{i,0} = 0$	$\delta B - P_b$	$\delta B - P_b$

(iii) $\delta(\bar{A} - B) > P_1 - P_b > \delta(L - B)$

	$\theta_a = H$	$\theta_a = L$
Buy a	$H - P_0 + \eta X_{i,1}$	$L - P_0 + \eta X_{i,1}$
Wait and $X_{i,0} > 0$	$\delta H - P_1$	$\delta B - P_b$
Wait and $X_{i,0} = 0$	$\delta H - P_1$	$\delta L - P_1$

(iv) $\delta(L - B) > P_1 - P_b$

	$\theta_a = H$	$\theta_a = L$
Buy a	$H - P_0 + \eta X_{i,1}$	$L - P_0 + \eta X_{i,1}$
Wait and $X_{i,0} > 0$	$\delta H - P_1$	$\delta L - P_1$
Wait and $X_{i,0} = 0$	$\delta H - P_1$	$\delta L - P_1$

Own elaboration

The game described is a network game (Galeotti et al. (2010)). In this game an agent must respond optimally to the actions of her neighbors, which are uncertain ex-ante. This implies that she must form beliefs about the actions of each of her neighbors, who at the same time must form beliefs about the actions of her. This requirement puts a cognitive burden on the agents, as the problem of computing these responses becomes intractable as a function of the network.

An alternative to solve this problem is to consider a large population of agents with bounded rationality. Such an assumption transforms this game into its mean field approach, a considerably less complex problem. A large number of small agents are considered in this approach. In addition, agents simply form beliefs about the average behavior of their neighbors. This idea comes from statistical physics, where the interaction dynamics of a large population of particles can be described by macroscopic parameters.

The following assumptions capture how an agent reasons about the network and the actions of her neighbors.

Assumption 3 (Neighborhood degree distribution): *An agent reasons about the degree* d *of their neighbors as a random variable distributed under the edge-perspective degree distribution:*

$$\bar{f}(d) = \frac{f(d)d}{\sum_x f(x)x}$$

Assumption 4 (Beliefs of neighbors' actions): *An agent* i *forecasts that each of her neighbors have a probability* $\alpha \in [0, 1]$ *of adopting* a *at* $t = 0$ *independently among her neighbors.*

Assumption 3 is a standard in defining the distribution of neighbors and maintains consistency with degree distribution $f(d)$. Assumption 4, on the other hand, simplifies the beliefs that an agent has on the actions of their neighbors, considering only the probability that neighbors adopt in average, α, that distributes i.i.d. on the population of her neighborhood.

Best response at t=0

Taking into account assumptions 3 and 4, the best response of an agent in $t = 0$ is simplified. Again, each response will depend on the level observed in the price difference in the second period, $P_1 - P_b$. Let us examine each case (i) to (iv) of observation 1.

(i) $P_1 - P_b > \delta(H - B)$

This case implies a strict preference of b over a in the second period. Therefore, in $t = 0$, an agent must expect a payment $U(not\ a) = U_b^1 = \delta B - P_b$. On the other hand, if he decides to buy a, he must wait for a payment $U(a) = U_a^0(\bar{A}) = \bar{A} - P_0$. Note that there are no referral payments resulting from this decision, as no neighbor is expected to adopt a in $t = 1$, i.e. $X_{i,0} = 0$. The best response will depend on the sign of $\Delta U = U(a) - U(not\ a)$: he will choose a if ΔU is positive, $(not\ a)$ if ΔU is negative, and a stochastic mixture if $\Delta U = 0$.

(ii) $\delta(H - B) > P_1 - P_b > \delta(\bar{A} - B)$

If an agent of degree d defers the use of a at $t = 0$, then at $t = 1$ she will adopt a if at least one of her neighbors already adopted a and have communicated a high value on θ_a. Agents adopt b in any other case. Given the belief α, the first event can occur with a probability $pG(\alpha, d)$, where $G(\alpha, d)$ stands for the probability that at least one neighbor adopts at $t = 0$. Since α distributes iid across neighbors, $G(\alpha, d) = (1 - (1 - \alpha)^d)$. Thus, deferring the use of a ($not\ a$) reaps a payment $U(not\ a) = pG(\alpha, d)\{U_a^1(H) - U_b^1\} + U_b^1$. On the other hand, if she decides to adopt, she will reap $U(a) = \bar{A} - P_0 + \eta p(1 - \alpha)d$. Note that in taking this action, a referral payment is expected. The expected number of neighbors adopting a at t=1 ($X_{i,1}$) is a binomial variable determined by the probability that an agent in the neighborhood does not adopt at t=0 $(1 - \alpha)$ and the probability that a high quality is communicated p. Thus, $\mathbb{E}_d(X_{i,1}) = p(1 - \alpha)d$. Again, this agent will choose her action depending on the sign of ΔU.

(iii) $\delta(\bar{A} - B) > P_1 - P_b > \delta(L - B)$

This case is similar to (ii). In this case, an agent will always adopt a at $t = 1$, except if a low quality is communicated to her. Payments will be $U(not\ a) = (1 - p)G(\alpha, d)\{U_b^1 - U_a^1(L)\} + U_a^1(\bar{A})$ and $U(a) = \bar{A} - P_0 + \eta p(1 - \alpha)d$.

(iv) $\delta(L - B) > P_1 - P_b$

This case is the opposite to (i). In this case an agent will strictly adopt a at $t = 1$, thus, $U(not\ a) = U_a^1(\bar{A}) = \delta\bar{A} - P_1$. In case she decides to adopt at $t = 0$, she will reap a payment $U(a) = \bar{A} - P_0$

Note that in all cases (i) to (iv), payments of an agent do not depend neither on the exact structure of her neighborhood nor on complex beliefs of the actions on neighbors. Instead, actions simply depend on each agent's degree and the expected fraction of neighbors adopting at $t = 0$, α. As α is identical among agents, they can only be distinguished by their degree, d, meaning that other agents having the same degree should be seen as identical. Consequently, agents with same degree will respond symmetrically. At $t = 0$, each agent having degree d , will strictly prefer to adopt when $U(a) > U(not\ a)$, strictly prefer to defer when $U(a) < U(not\ a)$

and willing to mix when $U(a) = U(not\ a)$. The concept of *mean field strategy,* which accounts for this behavior, is defined as follows.

Definition 1 (Mean Field Strategy): *A mean field strategy (MFS) is a function* $\mu : \mathbb{Z}_+ \to [0, 1]$ *which specifies the probability of adoption of agents depending on its degree* d*.*

With this definition, the best responses can be easily defined. For each agent of degree d, if $\Delta U > 0$ then $\mu(d) = 1$, if $\Delta U < 0$, $\mu(d) = 1$, and $\mu(d)$ can be any $q \in [0, 1]$ if $\Delta U = 0$.

To define a mean field equilibrium, let us start saying that it must be composed by a MFS and a belief α, the probability that an agent adopts a at $t = 0$. In order to this duple to be a plausible equilibrium we must require two conditions. The first must be optimality: that a MFS is a best response of the action of an agent d having beliefs α. The second one must be consistency: that beliefs α actually arise from adding up the strategies of agents in a given neighborhood, according to the edge-perspective distribution.

Definition 2 (Mean Field Equilibrium): *A MFS* μ^* *together with* α^* *is a mean field equilibrium (MFE) if*

(i) For each d*:* $$\mu^*(d|\alpha) = \begin{cases} 1 & if\ \Delta U > 0 \\ 0 & if\ \Delta U < 0 \cdot \\ some\ q \in [0, 1] & if\ \Delta U = 0 \end{cases}$$

(ii) $\alpha^* = \sum_d \bar{f}(d)\mu^*(d|\alpha^*)$

The next theorem shows the existence of a MFE.

Theorem 1 (Existence and Uniqueness): *There exists a MFE to the dynamic adoption game described. Moreover, if* μ^* *and* μ' *are MFE then* $\alpha(\mu^*) = \alpha(\mu')$*.*

The proof of theorem 1 is beyond the scope of this thesis. However, the interested reader is referred to consult the proof carried out in Jackson and Yariv (2007).

Demands

The MFEs give us the fraction of neighbors that a randomly selected individual expects to adopt the incumbent property in the first period. They also provide us with a measure of the actions taken by an agent depending on his degree, the policies of the companies, and the other exogenous parameters such as the edge-perspective distribution $\bar{f}$ and the expected qualities of the companies $\{\theta_a, B\}$. To simplify the exposition, let us call $S := \{P_0, P_1, \eta, P_b\}$ to he set of policies announced by the companies, and C to the set that includes the rest of the primitive characteristics of the model. The MFEs for the game will then have an explicit dependency on the previous parameters: $\alpha^* = \alpha^*(S, C)$ and $\mu^*(d) = \mu^*(d|S, C)$. With this information from the decisions of d-agents, the associated demands of each good in each period, as well as the number of referral payments made by the incumbent, can also be described as a function of S and C. Indeed, these demands and payments will result simply from adding the actions of all agents under the network measure $f(d)$. Unlike $\bar{f}$, f aggregates the actions not in a random neighborhood of the network, but in the entire network, giving us an adequate average from the perspective of the entire market.

Normalize the mass of agents to 1 ($\sum_d f(d) = 1$). The following observation defines these demands in a formal way.

Observation 2 (Demands and Number of referral payments): *Denote by* $\alpha^* = \alpha^*(S, C)$ *and* $\mu^*(d) = \mu^*(d|S, C)$.

(a) Denote by β *the demand for good* a *in* $t = 0$*,* γ *the demand for* a *at* $t = 1$*, and* ϕ *the number of referral payments paid by the incumbent. For each type of the incumbent's quality:*

(i) If $P_1 - P_b > \delta(H - B)$ *and* $P_0 - P_b > \bar{A} - \delta B$

	$\theta_a = H$	$\theta_a = L$
β	*0*	*0*
γ	*0*	*0*
ϕ	*0*	*0*

(ii) $P_1 - P_b > \delta(H - B)$ *and* $P_0 - P_b < \bar{A} - \delta B$

	$\theta_a = H$	$\theta_a = L$
β	*1*	*1*
γ	*0*	*0*
ϕ	*0*	*0*

(iii) $\delta(H - B) > P_1 - P_b > \delta(L - B)$

	$\theta_a = H$	$\theta_a = L$
β	$\sum_d f(d)\mu^*(d)$	$\sum_d f(d)\mu^*(d)$
γ	$\sum_d f(d)(1-\mu^*(d))(1-(1-\alpha(\mu^*))^d)$	0
ϕ	$(1-\alpha(\mu^*))\sum_d f(d)\mu^*(d)d$	0

(iv) $\delta(L - B) > P_1 - P_b$

	$\theta_a = H$	$\theta_a = L$
β	1	1
γ	0	0
ϕ	0	0

(b) Denote by ψ *the demand for good* b *in* $t = 0$*. For each type of the incumbent's quality,* $\theta_a \in \{H, L\}$*:*

$$\psi(\theta_a) = 1 - \beta(\theta_a) - \gamma(\theta_a)$$

■

$\beta(\mu^*)$ is the mass of agents adopting a at $t = 0$. Note that the difference between $\alpha(\mu^*)$ and $\beta(\mu^*)$ comes from the distribution on the degree, $f(d)$ which by construction first-stochastically dominates $\bar{f}(d)$. In this sense, agents are myopic as they only use their priors to interact with their neighbors. An incumbent firm who internalizes a MFE μ^* will then aggregate the agent's reaction in equilibrium and weight with the distribution of the degree rather that on the edge-perspective distribution.

$\gamma(\mu^*)$ is the mass of agents who adopt a at $t = 1$. The event that a d-agent will adopt $t = 0$ with a probability equal the probability that at least one neighbor adopts at $t = 0$. Weighting with $f(d)$, the aggregate quantity is as stated.

$\phi(\mu^*)$ is the expected mass of neighbors of early adopters who are not early adopters as well. results from multiplying the expected number of neighbors of early adopters $\sum_d f(d)\mu^*(d)d$ by the probability that these neighbors have not adopted early as well $(1 - \alpha(\mu^*))$.

$\psi(\mu^*)$ is the mass of agents that acquiring b at $t = 1$. By construction, all agents who did not adopt a at either period adopt b at $t = 1$, which explains the expression on observation 2.

2.2 Firm competition

There is problem of signaling that affects agents' beliefs. From the point of view of consumers, the incumbent firm has two types $\theta_a \in \{H, L\}$. However, only pooling equilibria are relevant. Indeed, if we let the incumbent to choose freely its marketing strategy $\{P_0, P_1, \eta\}_{\theta_a}$, an agent could infer the quality of a. However, if agents can update their priors before taking an action, the equilibrium properties of the mean field game of subsection 2.1 will no longer hold[1]. This problem can ben solved by observing that the low quality type entrant will have no incentives in taking a different action from the high quality type as this will signal agents of its quality, and then everybody will wait to buy b at $t = 1$. Hence, the proper equilibrium considered is a pooling strategy where the high quality type maximizes its profits and the low quality type

[1]Recall that the dynamic network game studied in chapter 2 assumes that prior beliefs are hold.

mimics. Later in the text I will state formally this result.

Firms compete simultaneously taken into account the demands and the cost of referrals, described in Observation 2.

Assuming zero costs of production, Incumbent's profit is defined as follows:

$$\Pi_a(P_0, P_1, \eta, \theta_a) = \beta_{\theta_a}(\mu^*)P_0 + \gamma_{\theta_a}(\mu^*)P_1 - \phi_{\theta_a}(\mu^*)\eta$$

Similarly, the entrant firm will compute the expected fraction of adopters of b, $\psi(\mu^*)$. Again, assuming no costs of production, the entrant's profit is:

$$\Pi_b(P_b) = \mathbb{E}_{\theta_a}\psi(\mu^*)P_b = \mathbb{E}_{\theta_a}[1 - \beta_{\theta_a}(\mu^*) - \gamma_{\theta_a}(\mu^*)]P_b$$

Note that since the entrant does not know the quality of the incumbent with certainty, her expected profits must take into account her prior beliefs about θ_a. In specific, the expected demand of b is $\mathbb{E}_{\theta_a}\psi(\mu^*) = q(\psi_H(\mu^*)) + (1-q)(\psi_L(\mu^*))$, where q stands for the entrant's prior beliefs that the incumbent product has a high quality.

The proper equilibrium is a Pooling Perfect Bayesian Equilibrium (PPBE), defined as follows.

Definition 4 (Pooling Perfect Bayesian Equilibrium (PPPE)): *The set of policies* $\{P_0, P_1, \eta, P_b\}$, *beliefs* $p = \mathbb{P}(\theta_a = H|\{P_0, P_1, \eta, P_b)$ *and a MFE* $\mu^* = \mu^*(P_0, P_1, \eta, P_b)$ *constitute a PPBE if:*

(1) The incumbent best responds to P_b*:*

$$\begin{aligned} BR_a(P_b) &= \underset{P_0,P_1,\eta}{argmax}\Pi_a(P_0, P_1, \eta, P_b) \\ &= \underset{P_0,P_1,\eta}{argmax}\{\beta_H(\mu^*)P_0 + \gamma_H(\mu^*)P_1 - \phi_H(\mu^*)\eta\} \end{aligned}$$

(2) The entrant best responds to (P_0, P_1, η)*:*

$$\begin{aligned} BR_b(P_0, P_1, \eta) &= \underset{P_b}{argmax} \Pi_b(P_0, P_1, \eta, P_b) \\ &= \underset{P_b}{argmax} \mathbb{E}_{\theta_a}[\psi_{\theta_a}(\mu^*) P_b] \end{aligned}$$

(3) $(P_0, P_1, \eta, P_b) = BR_a(P_b) \cap BR_b(P_0, P_1, \eta)$

2.3 Welfare

In a normative framework, it is desirable to have formulae for the consumer, industry and society's surpluses. To do this, we take account of the mean-field demands and costs. Note, however, that the definition of welfare depends on the actor and on the moment in time when this estimate is made. Thus, for example, an ex-ante social regulator (who faces quality uncertainty) will estimate a different social benefit than an ex-post one(when quality information is perfectly known). In order to make an analysis of market efficiencies that takes into account information asymmetries, we define surpluses according to the type of quality of the incumbent. An ex-ante assessment will then simply add the measures for quality according to the prior beliefs. The following observation formalizes this definition.

Observation 3 (Welfare formulae): *For each* $\theta_a \in \{H, L\}$,

(i) Consumer surplus is $CS\theta_a) := U_b + \beta_{\theta_a}\{U_a^0(\theta_a) - U_b^1\} + \gamma_{\theta_a}\{U_a^1(\theta_a) - U_b^1\} + \phi_{\theta_a}\eta$

(ii) Producer surplus is $PS(\theta_a) := P_b + \beta_{\theta_a}\{P_0 - P_b\} + \gamma_{\theta_a}\{P_1 - P_b\} - \phi_{\theta_a}\eta$

(i) Social welfare is $W(\theta_a) := CS(\theta_a) + PS(\theta_a) = \delta B + \beta_{\theta_a}\{\theta_a - \delta B\} + \gamma_{\theta_a}\{\delta\theta_a - \delta B\}$

Chapter 3

Characterization of equilibria

The model proposed is complex enough to permit the analytical derivation of the equilibria. Instead, I present a qualitative analysis of the properties of the "partial" equilibrium (the consumption game) to examine the dynamics of the expected demands faced by firms. This analysis shows us the performance of price policies as incentive schemes for the diffusion of a product and how competition characteristics affect this process.

3.1 Qualitative analysis

In the dynamic game of consumption, agents best respond to policies announced by the firms, according to primitive characteristics such as the edge-perspective density function of the network $\bar{f}(d)$, the grade of impatience δ, and the perceived quality level of the entrant product θ_b. For clarity of exposition, denote by S and C the sets of firm policies and primitive characteristics, respectively. Under the mean-field approach, these best responses are characterized by a MFE $\{\mu^*(x), \alpha^*\}$ that satisfies both optimality, $\mu^*(x) \in BR(\alpha^*|S, C)$, and consistency, $\alpha^* = \sum_x \bar{f}(x)\mu^*(x)$.

Double threshold characterization

When an agent is confronted with the decision to buy a or wait, the number of social connections he maintains will represent an important factor. The rationale behind this is that, given that there is uncertainty about the quality of the incumbent good, the benefit of waiting comes from the possibility that a neighbor adopts first and provides him with information about this product. The probability that this event happens is greater when the individual in question has more friends. Therefore, those individuals with fewer friends will have more incentives to adopt early. On the other hand, when reference policies are in place, the decision to advance the purchase also brings benefits that depend on the number of friends to whom the product is recommended and who adopt the product in the future. Here we will have the opposite effect, and it will be the people with more friends the ones who have more incentives to adopt early. Intuitively, the combination of prices and referral payments offered by companies will have a marketing effect on products, where individuals with enough few and enough many friends will adopt early. Alternatively, those who have an intermediate number of friends will choose to wait until time clears the uncertainty about the quality of a, and then decide whether to buy a or b. The following definition defines formally such a strategy.

Definition 3 (Double threshold strategy): *A MFS is a double-threshold strategy (DTS) if there exists a lower threshold d_L and an upper threshold $d_U \in \mathbb{N} \cup \{\infty\}$ such that:*

$$\mu(d) = \begin{cases} 1 & if\ d < d_L \\ some\ p \in [0,1] & if\ d = d_L \\ 0 & if\ d \in (d_L, d_U) \\ some\ q \in [0,1] & if\ d = d_U \\ 1 & if\ d > d_U \end{cases}$$

In particular, if a DTS is such that $d_U = \infty$ we will say that it is a lower threshold strategy (LTS), while if it is such that$d_L = 0$ we will say that it is an upper threshold strategy (UTS).

The following theorem shows that in equilibrium, one should expect consumers to behave according to a DTS.

Theorem 2 (Double threshold equilibrium): *Suppose $\delta H - P_1 \leq \delta B - P_b$. A MFE is a DTS. In this case, any MFE can be characterized by uniques d_L and d_U.*

Proof. Let us consider a MFE, $\{\mu^*(x), \alpha^*\}$. To achieve optimality, $\mu^*(x)$ must satisfy:

$$\mu(x) = \begin{cases} 1 & if\ \Delta U_0 > 0 \\ any\ q \in [0,1] & if\ \Delta U_0 = 0 \\ 0 & if\ \Delta U_0 < 0 \end{cases}$$

Which d-agents adopt, depends on the sign of $\Delta U_0 = \Delta U_0(d)$. Therefore, the inverse image of $I := (-\infty, 0)$ under ΔU_0 gives us the set of degrees of those agents that strictly prefer not to adopt. Consequently, the grade of those agents that do adopt will be strictly out of this set. Let us examine ΔU_0 for each case (i-iv) of the best response in $t = 1$.

(i) $P_1 - P_b > \delta(H - B)$

In this case, all agents, regardless of their grade, will choose b should they postpone their purchase. Then, if $U_a^0(\bar{A}) > U_b^1$, all agents will choose a in $t = 0$. On the other hand, if $U_a^0(\bar{A}) < U_b^1$, all agents will choose $not\ a$ in $t = 0$. In the first case we choose $d_L = d_U = 1$; in the second, $d_L = 1$ and $d_U = \infty$.

(ii)$\delta(H - B) > P_1 - P_b > \delta(\bar{A} - B)$

In this case, $U_a^1(H) > U_b^1 > U_a^1(\bar{A})$

$$\begin{aligned} \Delta U_0 &= U_a^0(\bar{A}) + \eta p(1-\alpha)d \\ &\quad - p(1-(1-\alpha)^d)\{U_a^1(H) - U_b^1\} - U_b^1 \end{aligned}$$

Note that ΔU_0 is a convex function in $d \in [1, \infty)$ (as its second partial derivative from d is positive throughout this interval). Thus, it is also quasiconvex in $d \in [1, \infty)$. Hence, the inverse image of $I := (-\infty, 0)$ is a convex set. Several cases are followed:

(a) $\Delta U_0^{-1}(I) = \emptyset$

Then, for any d, no agent strictly prefers $not\ a$ over a in $t = 0$. Therefore, we can set $d_L = 1$ and $d_U = \infty$.

(b) $\Delta U_0^{-1}(I) = [1, y)$

Thus, for $d < y$, agents strictly prefers $not\ a$ over a in $t = 0$. Therefore, we can set $d_L = 1$

and $d_U > y$.

(c) $\Delta U_0^{-1}(I) = (x, y)$

Similarly, for any $d \in (x, y)$, agents strictly prefers $not\ a$ over a in $t = 0$. Therefore, we can set $d_L < x$ and $d_U > y$.

(iii) If $\delta(\bar{A} - B) > P_1 - P_b > \delta(L - B)$

$$\begin{aligned}\Delta U_0 &= U_a^0(\bar{A}) + \eta p(1-\alpha)d \\ &\quad -(1-p)(1-(1-\alpha)^d)\{U_b^1 - U_a^1(L)\} - U_a^1(\bar{A})\end{aligned}$$

This case is analogous to (ii), and thresholds are guaranteed by similar arguments.

(iv) $\delta(L - B) > P_1 - P_b$

In this case $U_a^1(L) > U_b^1$, so whatever the priors, agents who postpone their purchase will opt for a in $t = 1$. Therefore in $t = 0$, if $U_a^0(\bar{A}) > U_a^1(\bar{A})$, all agents will choose a regardless of their grade, and choose $not\ a$ otherwise. Similarly to case (i), we choose $d_L = d_U = 1$ in the first case, and $d_L = 1$ and $d_U = \infty$ in the second.

■

Thus, Theorem 2 shows that each MFE is defined by two thresholds, the lower threshold d^D and the upper threshold d^U. Hence, we can classify the behavior of the consumers according to their degree: The low degree consumers ($d < d^D$) and the high degree ones ($d > d^U$) adopt in $t = 0$, while the medium degree agents ($d^U > d > d^D$) postpone (and adopt a or b in $t = 1$).

It is worth noting that this classification is not trivial only in the case where the price differential $P_1 - P_b$ is between $\delta(H - B)$ and $\delta(L - B)$. Outside this range, referral payments have no effect on agents' behaviour, and prices act symmetrically on all agents regardless of their grade. For this reason, in the rest of the analysis we restrict ourselves precisely to this case.

The next proposition shows that a MFE can be classified as either an LTS or a DTS depending on the value of η. The intuition behind this result is that referral payments provide incentives for high-grade individuals to adopt. In the absence of these incentives, it will be the lower-graders who take up, as they are the least likely to get information in the future.

Proposition 1 (Relation between referrals and thresholds): *Let $\bar{d}$ be the maximal degree such that $f(d) > 0$ iff $0 < d \leq \bar{d}$. Let us suppose that $\delta(H - B) > P_1 - P_b > \delta(L - B)$. Then,*

(a) If $\delta(H - B) > P_1 - P_b > \delta(\bar{A} - B)$, there exist positive $\eta_1 = \alpha(1-\alpha)^{\bar{d}-2}\{U_a^1(H) - U_b^1\}$ and $\eta_2 = U_a^1(H) - U_b^1$ such that:

$\eta < \eta_1 \implies d_U > \bar{d}$;

$\eta > \eta_2 \implies d_L = 0$;

$\eta_1 < \eta < \eta_2 \implies d_L \geq 1 \; and \; d_U \leq \bar{d}$. *(b) If $\delta(\bar{A} - B) > P_1 - P_b > \delta(\bar{L} - B)$, then $\eta_1 = \frac{1-p}{p}\alpha(1-\alpha)^{\bar{d}-2}\{U_b^1 - U_a^1(L)\}$ and $\eta_2 = U_b^1 - U_a^1(L)$.*

Proof. Supppose $\delta(H - B) > P_1 - P_b > \delta(\bar{A} - B)$. We have $\Delta U(\alpha, d) = U_a^0(\bar{A}) + \eta p(1 - \alpha)d - pG(\alpha, d)\{U_a^1(H) - U_b^1\} - U_b^1$, where $G(\alpha, d) = 1 - (1 - \alpha)^d$.

(1) Consider $\Delta U(\alpha, \bar{d}) - \Delta U(\alpha, \bar{d}-1) = \eta p(1-\alpha) - \alpha p(1-\alpha)^{\bar{d}-1}\{U_a^1(H) - U_b^1\}$. We have for each $\alpha \in (0, 1)$: if $\eta < \eta_1 := \alpha(1-\alpha)^{\bar{d}-2}\{U_a^1(H) - U_b^1\}$ we get $\Delta U(\alpha, \bar{d}) < \Delta U(\alpha, \bar{d}-1)$. By convexity of $\Delta U(\alpha, d)$ in d, it follows by induction that $\Delta U(\alpha, d) < \Delta U(\alpha, d + 1)$, for all $1 \leq d < \bar{d}$. Hence, if $\eta < \eta_1$ any MFS is an LTS.

(2) Consider $\Delta U(\alpha, 2) - \Delta U(\alpha, 1) = (1 - \alpha)[\eta p - \alpha p\{U_a^1(H) - U_b^1\}]$. We have for each $\alpha \in (0, 1)$: if $\eta > \eta_2 := U_a^1(H) - U_b^1$ we get $\Delta U(\alpha, 1) < \Delta U(\alpha, 2)$. Similarly, by convexity, it follows by induction that $\Delta U(\alpha, d) > \Delta U(\alpha, d - 1)$, for each $1 \leq d < \bar{d}$. Thus, if $\eta > \eta_2$ any MFS is a UTS.

(3) From (1) and (2) it follows that if $\eta_1 < \eta < \eta_2$, any MFS is a non-trivial DTS, in which $d_L \geq 1 \; and \; d_U \leq \bar{d}$.

On the other hand, if $\delta(\bar{A} - B) > P_1 - P_b > \delta(L - B)$ we can reason similarly to the previous case. Here, $\Delta U(\alpha, d) = U_a^0(\bar{A}) + \eta p(1-\alpha)d - (1-p)G(\alpha, d)\{U_b^1 - U_a^1(L)\} - U_a^1(\bar{A})$, and $\eta_1 = \frac{1-p}{p}\alpha(1 - \alpha)^{\bar{d}-2}\{U_b^1 - U_a^1(L)\}$ and $\eta_2 = U_b^1 - U_a^1(L)$.

■

Let us look carefully at proposition 1. First, it shows us that there are two "thresholds" in the referral payment: a low one and a high one. If the rewards are below the low threshold, there is not enough incentive for high-grade agents to adopt the good a early. As we increase this payment, above this threshold, the pricing scheme will induce early use in both isolated and highly connected agents. However, if the referral payments exceed the high threshold, there will be so many high-grade agents adopting the product that the low-grade agents will increase their chances of learning about the incumbent's quality through the former, thus choosing to postpone its purchase to the next period.

This capacity to regulate the level of late adopters has marketing implications for both companies. We can understand the referral payment as a mechanism that can segment the market in three different ways. When the referral payment is small the early market is composed of socially isolated agents and the remaining market is divided between the two companies in the future. When the referral payment is high, this market is divided in the opposite way, with lone consumers being divided by the companies in the future. Finally, when the referral payment is moderate the future market is composed of 'medium' grade players.

With proposition 1, two mechanisms are natural to analyze.

Screening mechanisms

The first is a policy where referral payments are not feasible. This policy captures pure price competition, where the only mechanism that influences agents' take-up is given by the price difference (between the prices of a for different periods, and between the early price of a and the price of b). Here the market segmentation leads to the most connected agents postponing their purchase (and deciding between a or b). The following corollary describes this mechanism in a formal way.

Corollary 1 (Price discount screening): *Suppose* $\delta(H-B) > P_1 - P_b > \delta(L-B)$. *Under a price discount policy* $(P_0,\ P_1,\ 0)$*, a MFE is characterized by a single threshold* d_L *(*$d_U > \bar{d}$*). Thus, such a policy constitutes a screening mechanism under which lower-degree agents adopt early and higher-degree agents free-ride on the information gathered by the former.*

Proof. Suppose that the incumbent chooses a pricing policy without referral payments, i.e. $(P_0,\ P_1,\ 0)$. We got $\eta = 0 < min\{\alpha(1-\alpha)^{\bar{d}-2}\{U_a^1(H) - U_b^1\}, \frac{1-p}{p}\alpha(1-\alpha)^{\bar{d}-2}\{U_b^1 - U_a^1(L)\}\}$. By proposition 1, any corresponding MFE is an LTS. Therefore, such pricing policy constitutes a revelation mechanism in which agents of grade $d < d_L$ adopt a in $t = 0$, and otherwise wait.

■

The second mechanism results from adding a sufficiently high reference payment to these price policies. As we saw earlier, this mechanism reverses market segmentation. This mechanism is described in the following corollary.

Corollary 2 (Referral screening): *Suppose $\delta(H - B) > P_1 - P_b > \delta(L - B)$. For any pricing P_0, P_1 there exists a referral $\eta' < \infty$ such that under any policy $(P_0,\ P_1,\ \eta)$ with $\eta > \eta'$ any MFE can be characterized by a single upper-threshold d_U ($d_L = 1$). Thus, such a policy constitutes a screening mechanism under which higher-degree agents adopt early and lower-degree agents free-ride on the information gathered by the former.*

Proof. Suppose $\delta(H - B) > P_1 - P_b > \delta(L - B)$. Choose $\eta > max\{U_a^1(H) - U_b^1, U_b^1 - U_a^1(L)\}$. Then, by proposition 1 we get that any MFE is a UTS. Therefore, such pricing policy constitutes a revelation mechanism in which agents of grade $d > d_L$ adopt a in $t = 0$, and otherwise wait.

■

Comparative Statics

In principle, we are interested in finding out how demands and referral payments change as a function of both pricing policies and model primitives. The first exercise results in a sort of calculation of the own and cross elasticities of the incumbent and the incoming good. This calculation would give us an insight into what we would expect from competitive behavior among firms. The second exercise gives us the statics that occur in equilibrium when we vary exogenous characteristics of the model which can change the analysis depending on the type of market we are studying. In this analysis, features such as the degree distribution of the consumer network, impatience, and the ex-ante perceived quality levels of the substitute goods come into play. Unfortunately, this exercise is quite complicated, although important, and is therefore beyond the scope of this dissertation.

Instead of exploring demands and referral payments, I will do comparative statics on the probability of a neighbor adopting early α^* that results in equilibrium. We can interpret this probability as the level of information penetration that policies effectively have in the population (Leduc et al. (2017)). As an alternative, α^* can also be interpreted as the demand for the incumbent good as expected by consumers. In addition, through α^*, demands and reward costs can be calculated, as expressed in observation 2. This exercise, then, is a first step to a more complete analysis of the problem.

Let us start by exploring o first-order stochastic shifht of $\bar{f}(d)$ on α^*.

Proposition 2 (Comparative statics on $\bar{f}(d)$***):*** *Let* $f(d)$ *and* $f'(d)$ *be two degree distribution such that* $f(d) \neq f'(d)$ *and such that* $\bar{f}'(d)$ *first order stochastically dominates* $\bar{f}(d)$*. Let* $\mu^*(d)$ *and* $\mu'(d)$ *two MFE relative to* $f(d)$ *and* $f'(d)$ *respectively. Then:*

1. *If MFE is an LTS then* $\alpha(\mu') \leq \alpha(\mu*)$*, and*

2. *If MFE is a UTS then* $\alpha(\mu') \geq \alpha(\mu*)$*.*

Proof. Let us examine the cases and proceed by contradiction.

(1)Suppose $\alpha(\mu') > \alpha(\mu*)$. Recall that optimality of $\mu^*(\alpha^*)$ implies $\mu^*(\alpha^*) \geq \mu'(\alpha')$. Then:

$$\sum_d \bar{f}(d)\mu^*(\alpha^*) \geq \sum_d \bar{f}'(d)\mu'(\alpha') = \alpha(\mu')$$

On the other hand, suppose that μ^* must be an LTS. Then $\mu^*(d)$ is a decreasing function on d. Considering that $\bar{f}'$ first-order stochastically dominates $\bar{f}$, we have:

$$\alpha(\mu^*) = \sum_d \bar{f}(d)\mu^*(\alpha^*) \geq \sum_d \bar{f}'(d)\mu^*(\alpha^*).$$

Thus, $\alpha(\mu^*) \geq \alpha(\mu')$, which is a contradiction.

Therefore, $\eta < \eta_U$ implies $\alpha(\mu') \leq \alpha(\mu^*)$.

(2) This case is analogous to (1). Suppose $\alpha(\mu') < \alpha(\mu*)$. It follows that $\mu^*(\alpha^*) \leq \mu'(\alpha')$, which implies:

$$\sum_d \bar{f}(d)\mu^*(\alpha^*) \leq \sum_d \bar{f}'(d)\mu'(\alpha') = \alpha(\mu').$$

Similarly, if $\mu^*(d)$ is a UTS, we have an incresing function of d^*. Then, taking into account the first order stochastical dominance of $\bar{f}$ over f:

$$\alpha(\mu^*) = \sum_d \bar{f}(d)\mu^*(\alpha^*) \leq \sum_d \bar{f}'(d)\mu^*(\alpha^*)$$

Thus, a contradiction follows: $\alpha(\mu^*) \leq \alpha(\mu')$, so we proof (b).

■

Proposition 2 says that depending on the structure of consumer interaction a policy of discounts or referrals can have opposite effects. If we have a policy of discounts on prices (low referral rewards), it will be the lesser degree agents who will adopt the product early. Hence, a population that has a larger mass of high-grade consumers will cause fewer agents to adopt in equilibrium. In this population this pricing policy will have less information penetration. On the other hand, if the pricing policy contains sufficiently high referral payments, those who adopt will be the best connected agents. Consequently, a population with more of these individuals will have higher information penetration.

The following proposition shows that when we have discount or large referral policies, α^* is related to $\bar{f}$ in a simple way.

Proposition 3: *Suppose (α^*, μ^*) is MFE. Restrict to the case where this MFE is either an LTS or a UTS. Denote by d^* the corresponding threshold.*

(a) If (α^*, μ^*) is an LTS:

$$\alpha^* = \bar{F}(d^*) \tag{3.1}$$

(b) If (α^*, μ^*) *is a UTS:*

$$\alpha^* = 1 - \bar{F}(d^*) \tag{3.2}$$

Proof. In a MFE, we have:

$$\alpha^* = \sum_d \bar{f}(d)\mu^*(d)$$

Suppose this MFE is an LTS. Then,

$$\begin{aligned}
\alpha^* &= \sum_d \bar{f}(d) \begin{cases} 1 & if\ d < d^* \\ 0 & if\ d > d^* \end{cases} \\
\alpha^* &= \sum_d^{d^*} \bar{f}(d) \\
\alpha^* &= \bar{F}(d^*)
\end{aligned}$$

This proves (a).

Now, suppose this MFE is a UTS. Then,

$$\begin{aligned}
\alpha^* &= \sum_d \bar{f}(d) \begin{cases} 0 & if\ d < d^* \\ 1 & if\ d > d^* \end{cases} \\
\alpha^* &= 1 - \sum_d^{d*} \bar{f}(d) \\
\alpha^* &= 1 - \bar{F}(d^*)
\end{aligned}$$

This proves (b).

■

Proposition 3 may be useful in making comparative statics on exogenous parameters. To see this more clearly, consider the threshold d^* in the case of an LTS. As we have seen, there are two cases, which depend on the price difference between the incumbent and the entrant goods in $t = 1$.

(i) $\delta(H - B) > P_1 - P_b > \delta(\bar{A} - B)$

$$p(1 - (1 - \alpha^*)^d) = \frac{U_a^0(\bar{A}) - U_b^1}{U_a^1(H) - U_b^1}$$

$$d^* = ln\left(1 - \frac{1}{p}\left(\frac{U_a^0(\bar{A}) - U_b^1}{U_a^1(H) - U_b^1}\right)\right)\frac{1}{ln(1 - \alpha^*)} \tag{3.3}$$

(ii) $\delta(\bar{A} - B) > P_1 - P_b > \delta(L - B)$

$$d^* = ln\left(1 - \frac{1}{(1 - p)}\left(\frac{U_a^0(\bar{A}) - U_a^1(\bar{A})}{U_b^1 - U_a^1(L)}\right)\right)\frac{1}{ln(1 - \alpha^*)} \tag{3.4}$$

The equations 3.3 and 3.4 give us an expression of d as a function of the exogenous parameters along with α^*, which is also a function of the exogenous parameters. The chain rule applied to these exogenous parameters, along with equation 3.1, enables us to make comparative statics in the exogeneous variables as well as in the pricing policies. The following proposition work under the case of interest and for all $\bar{f}(d)$.

Proposition 4: Let α^* conform a MFE for a given $\bar{f}(d)$. Denote by x any variable other than α in the function $d^* = d^*(\alpha, x, ...)$.

(a) If α^* conform an LTS:

$$\frac{\partial \alpha^*}{\partial x} = \left(\frac{\bar{f}(d^*)}{1 - \bar{f}(d^*)\frac{\partial d*}{\partial \alpha^*}}\right)\frac{\partial d^*}{\partial x} \tag{3.5}$$

(b) If α^* conform a UTS:

$$\frac{\partial \alpha^*}{\partial x} = -\left(\frac{\bar{f}(d^*)}{1 + \bar{f}(d^*)\frac{\partial d*}{\partial \alpha^*}}\right)\frac{\partial d^*}{\partial x} \tag{3.6}$$

Proof.- Let α^* conform a LTS-MFE. Then, $\mu^*(d) = \begin{cases} 1 & if\ d < d^* \\ 0 & if\ d > d^* \end{cases}$, and by proposition 3, $\alpha^* = \bar{F}(d^*)$. Applying the chain rule:

$$\frac{\partial \alpha^*}{\partial x} = \bar{f}(d^*)\left(\frac{\partial d*}{\partial \alpha^*}\frac{\partial \alpha^*}{\partial x} + \frac{\partial d*}{\partial x}\right) \tag{3.7}$$

Rearranging terms, we obtain equation 3.5, which proves (a).

To prove (b), use $\alpha^* = 1 - \bar{F}(d^*)$. Applying the chain rule:

$$\frac{\partial \alpha^*}{\partial P_b} = -\bar{f}(d^*)\left(\frac{\partial d*}{\partial \alpha^*}\frac{\partial \alpha^*}{\partial x} + \frac{\partial d*}{\partial x}\right) \tag{3.8}$$

Rearranging terms, we get the result.

■

Proposition 5. *Let α^* conform a MFE under a price discount policy.*

If $\delta(H - B) > P_1 - P_b > \delta(\bar{A} - B)$ then $\frac{\partial \alpha^*}{\partial P_b} > 0$

If $\delta(\bar{A} - B) > P_1 - P_b > \delta(L - B)$ then $\frac{\partial \alpha^*}{\partial P_b} < 0$ *Proof.* To prove (a), compute the partial derivatives of $\frac{\partial d*}{\partial \alpha^*}$ and $\frac{\partial d*}{\partial P_b *}$ using equation 3.3. This leads to:

$$\frac{\partial d^*}{\partial \alpha^*} = ln\left(1 - \frac{1}{p}\left(\frac{U_a^0(\bar{A}) - U_b^1}{U_a^1(H) - U_b^1}\right)\right)\frac{1}{(1-\alpha^*)ln^2(1-\alpha^*)} \tag{3.9}$$

and

$$\frac{\partial d^*}{\partial P_b} = \frac{1}{pln(1-\alpha^*)}\left(\frac{1}{1 - \frac{1}{p}\left(\frac{U_a^0(\bar{A}) - U_b^1}{U_a^1(H) - U_b^1}\right)}\right)\left(\frac{U_a^0(\bar{A}) - U_a^1(H)}{\left(U_a^1(H) - U_b^1\right)^2}\right) \tag{3.10}$$

Observe that under any case, 3.9 has a negative sign, as the argument in the first logarithm is below one. On the other hand, noting that $ln(1-\alpha^*) < 0$, we see that equation 3.10 is positive, as $U_a^0(\bar{A}) < U_a^1(H)$. Taking x as P_b and substituting eqs. 3.9 and 3.10 into 3.5, we see that the sign of this equation depends only on the sign of equation 3.10, which proves the point.

To prove (b), proceed in a similar manner. We have that:

$$\frac{\partial d^*}{\partial \alpha^*} = ln\left(1 - \frac{1}{(1-p)}\left(\frac{U_a^0(\bar{A}) - U_a^1(\bar{A})}{U_b^1 - U_a^1(L)}\right)\right)\frac{1}{(1-\alpha^*)ln^2(1-\alpha^*)} \tag{3.11}$$

$$\frac{\partial d^*}{\partial P_b} = \frac{1}{(1-p)ln(1-\alpha^*)}\left(\frac{1}{1 - \frac{1}{(1-p)}\left(\frac{U_a^0(\bar{A}) - U_a^1(\bar{A})}{U_b^1 - U_a^1(L)}\right)}\right)\left(\frac{U_a^1(\bar{A}) - U_a^0(\bar{A})}{\left(U_b^1 - U_a^1(L)\right)^2}\right) \tag{3.12}$$

Again, we have that 3.11 is negative. Likewise, 3.12 is also negative, since $U_a^1(\bar{A}) > U_a^0(\bar{A})$. Therefore, taking x as P_b and substituting in 3.5, the result follows.

■

Proposition 5 provides insight into the response of consumers to an increase in P_b when companies compete only on price. We observe that when $\delta(H - B) > P_1 - P_b > \delta(\bar{A} - B)$, the effect on α^* is positive. The intuition of this result is that when the price difference is in this range, the level of information penetration benefits the incumbent company, as agents are hesitant to purchase the good a due to lack of information. In this situation, an increase in P_b gives greater incentives to adopt the incumbent good early. On the other hand, when $\delta(\bar{A} - B) > P_1 - P_b > \delta(L - B)$, the effect on α^* is negative. Here, agents are willing to buy unless they learn that it is of inferior quality, in which case they will prefer to adopt b. Then, an increase in P_b has no other purpose than to signal that a is of poor quality. Therefore, consumers reduce their early adoption.

Chapter 4

Conclusions

Companies that produce durable goods of uncertain quality have two major concerns. On the one hand they have to publicize their product and often the best way to do this is to have their product tested and let it " speak" for itself, through word of mouth communication from early consumers to future prospects. But, at the same time, how profitable can the entry be when familiar firms announce their entry into this same market? Here a marketing strategy seems to go in the opposite direction to a strategy that addresses these entrants. In this dissertation I have investigated the competitive and marketing functions that a dynamic pricing policy may have on the objectives of this type of companies. To this end, I have proposed a normative framework that makes use of a two-period substitute product adoption model, which is an extension of the work of Leduc et al. (2017), where future competition is included.

To illustrate the role of Word of Mouth communication, consumers of this model exhibit network externalities that are informational in nature. We see that companies of this type of product can benefit from these externalities by using an incentive scheme that maximizes the use of the "best connected" agents depending on the social network where the market is located. When there is imperfect information about the network structure of the market, a company can implement mechanisms that optimize the process of diffusion of its product by using a pricing policy that includes discounts and referral payments. The rationale behind this strategy is that by encouraging the early use of the best positioned consumers, they can efficiently communicate quality information to their neighbors at the lowest possible cost.

At the same time, this information diffusion strategy must be designed so that a company that enters in the second period with a product of known quality does not end up reducing the profits of the incumbent company to zero. Fortunately, the incumbent can also design this strategy so that the entrant company simply decides not to compete in the future.

The equilibria of the proposed dynamic game yield several results that provide us with information on the competitive dynamics that these companies will develop. It also gives us an insight into the shortcomings of implementing a policy of discounts and referrals in this context. The results are as follows:

1. The design of an optimal policy of discounts and referrals has to take into account the quality and price of future options. In particular, the difference between the future price and the price of the competition is a determinant of the type of competition that will take place. Namely, the position of this price difference in relation to the perceived difference in quality of the respective products determines whether or not a referral policy affects the take-up, and even whether entry by one of the competitors is feasible.

2. Consumer behaviour can be classified as a two-threshold strategy. An agent with a grade lower than d^D (the low threshold) will adopt the product early. An agent with a higher grade than d^U (the high threshold) will also adopt the product at this early stage. Such agents constitute the number of marketing agents of the incumbent company. This market needs to be stimulated and represents a cost for the incumbent.

3. On the other hand, those agents with a grade between d^D and d^U will wait for the next period, where they will opt for a or b. This market is willing to pay and therefore represents profits for both companies.

4. The above thresholds contain information on the type of competition between the companies. In particular, they are sensitive to prices, referral payments and the price difference

(referred to in point 1) relative to the difference in perceived qualities.

5. This characterization helps to design two screening mechanisms that have opposite effects on the population. The first is a discount policy (where there are no referrals) and has the effect of encouraging the most isolated individuals in the network. The other is a policy of "high" referrals that encourages the most connected individuals.

6. The future market is divided between incumbent and entrant companies indirectly through the demands expected by the firms. I emphasize here that the demand used by the entrant company for its profit maximization problem would have to be one that depends directly on the incumbent's quality that this company perceives. This demand is sensitive to this quality information, and can lead to a market share of 0 to 100 percent. In other words, a policy of discounts and referrals can have opposite effects on competition, from preventing the entry of future competitors to discouraging the entry of the incumbent counterparts

7. The welfare resulting from these policies can be assessed from a regulator's point of view. It is argued that the objectives of a utilitarian regulator will depend on its position in time and on the level of information it has about the quality of products. Therefore, depending on this, two regulators may set competition rules that can range from preventing the entry by competitors harmful to future welfare to lowering quality levels in the market.

8. How effective a pricing policy is in terms of information can be measured by α^*. Furthermore, the sensitivity of α^* to network degree distribution and competitor prices is demonstrated. This illustrates the marketing performance of a pricing and referral policy depending on the communication structure of consumers and the level of competition in the market.

The model also has implicitly an algorithm for the calculation of the pooling equilibria defined in Chapter 2, which are intended to yield the pricing policies that result from simultaneous competition between firms. A future stage of this work should involve numerical simulations to obtain the payments and the equilibrium policies. This exercise would allow to know in detail the impact that the perceived qualities and the network structure have on the relevant welfare measures (as defined also in chapter 2). In this dissertation, however, I merely showed the effects that prices have on demands, as a first step in developing this analysis.

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