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MARKET FORECLOSURE AND STRATEGIC ASPECTS OF VERTICAL AGREEMENTS

ABSTRACT

This paper reviews the arguments about market foreclosure -as an incentive for vertical agreements between upstream and downstream firms- and its effects on welfare. We consider that downstream firms compete both in prices and quantities in the final good market and upstream firms compete in quantities in the intermediate good market. In this context we show that a vertical agreement must not contemplate market foreclosure, but upstream firm continues buying or selling input in intermediate market. To buy or to sell depends on the competition in final market and on the magnitude of mark-ups in both markets. Regarding antitrust policy, we show that even vertical agreements aimed at increasing input price faced by other firms may be positive from the welfare viewpoint.

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MARKET FORECLOSURE AND STRATEGIC ASPECTS OF VERTICAL AGREEMENTS

I. INTRODUCTION

In a study prepared for the debate on the Economic Competition Bill in Mexico, Castañeda et al (1992) argue that one-sided or collusive vertical practices can have both pro-competitive and anti-competitive effects. For example, a vertical agreement (VA) between a manufacturer and a retailer is pro-competitive if it reduces the inefficiency of input markets when these are imperfect. But it can also be anti-competitive if it reduces buyers' access to a supplier and/or limits suppliers' access to a buyer. In this case, the input market looses one supplier and/or buyer, which could increase the input prices and trigger an increase in the price of the final good. This phenomenon is called *market foreclosure.* Therefore the new antitrust law deals with the possibility that the VAs might be used strategically as a monopolistic practice and have, therefore, anti-competitive effects.

The theory of industrial organization has made progress towards formalizing the effects of market foreclosure on the competitive structure of both downstream and upstream industries and on welfare (see Tirole, 1988). Specifically, in an industry in which there are both vertically integrated and vertically unintegrated firms, Salinger (1988) studies the effect of an increase in the number of integrated firms on prices. However, he treats integration and foreclosure as exogenous. Gaudet and Long (1993) review Salinger's model considering vertical integration (VI) incentives and characterize trade among integrated and unintegrated firms. These authors show that it may be optimal for integrated firms to purchase inputs from other manufacturers as a strategy to increase the cost of rival retailers. However, they do not analyze welfare effects and solely study the case in which firms compete "à la Cournot" in the final market.

With the exception of Gaudet and Long (1993), most authors do not give a complete explanation of why an integrated firm does not trade with unintegrated firms. Our objective is to reconsider the arguments regarding market foreclosure, as an incentive to VA, and its welfare effects. In order to fulfill this objective, we build on Gaudet and Long's model of trading among integrated and unintegrated firms with a two-part tariff contracts instead of VI. This change allows pricing the intermediate good above or below the unit production cost[1]. We also consider intermediate market inefficiency, specifically with input market price greater than marginal cost,[2] as an incentive to VA.

We then consider the conditions under which a manufacturer of an intermediate good (the "upstream firm") offers a two-part tariff contract to a retailer (the "downstream firm"), as a strategy to avoid the intermediate market inefficiency and to increase the retailer's profits. The manufacturer is able to obtain a share of the increase in the retailer's profits through the franchise fee. Acceptenance of this contract is known as VA.

The optimality of offering this contract depends on the nature of the final market interaction and on the number of manufacturers and retailers. When retailers compete "à la Cournot" we argue that there are always incentives to achieve VAs, even in the absence of double marginalization.[3] There are three reasons behind this result. First, it is optimal for the manufacturer to price the contract below the marginal cost[4]. In Cournot oligopoly, each downstream firm would like to be a Stackelberg leader but the implicit assumption in the model is that it cannot commit to such a level of output. The two-part tariff provides the necessary commitment by creating a low marginal cost and thereby making it rational for the downstream firm to produce more than the Cournot output. Second, as a strategy to increase the rival retailers' cost, the manufacturer "purchases" inputs from other

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[1] See Caillaud and Rey (1994) for a survey on delegation contracts.

[2] That is, an upstream firm sets its own mark-up without taking into account the externalities of its decision on profits of downstream firm who also sets its own mark-up unilaterally. This phenomenon is known as *double marginalization*. See Tirole (1988, ch. 4).

[3] Our result is different from Gaudet and Long's result in the sense that theirs depends on the number of manufacturers and retailers.

[4] The same result, in a firm's internal organization context imposing a priori market foreclosure, is shown by Vickers (1985).

manufacturers. The increase in the profits of the contracted retailer compensates both, pricing the contract below the unit production cost of the input and purchasing in the market above such a cost. Third, there are gains due to the elimination of double marginalization.

When retailers compete "à la Bertrand", the optimal contract sets a price for the input above its unit cost, that is, it is suitable to maintain double marginalization.[5] If the manufacturer, in order to avoid double marginalization inefficiency, sets a price in the contract equal to the unit cost, it causes the retailer to reduce its price too much while other retailers also reduce theirs (strategic complements). This negative effect can compensate the gains due to the greater efficiency derived from the elimination of double marginalization. The suitability of setting a positive mark-up in the intermediate market may cause the VA to become unfeasible, since the optimal contract price would be greater than the market price.

Following with Bertrand competition, if the contract price does not exceed the market price, firms achieve a VA. However, unlike quantity competition, the decision of selling or purchasing input in the market depends on the number of manufacturers and retailers. In other words, it depends on the magnitude of oligopolistic rents (or mark-ups) in the intermediate and final markets. When the rents in the input market are greater than those in the product market (when there are fewer manufacturers than retailers and the products are near perfect substitutes) the rent extraction from other retailers through the strategy of purchasing the input of other manufacturers does not compensate the rent losses due to giving up selling to other retailers. On the contrary, when the oligopolic rents are greater in the product market, it is optimal for the manufacturer to purchase the input in the market.

Regarding policy, when the number of manufacturers is smaller than the

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[5] This result, without considering incentives of market foreclosure, is due to Bonano and Vickers (1988) and Lin (1988).

number of retailers, we argue that VAs are positive for welfare, whether the final market competition be in prices or quantities. In other words, the VAs are positive when the intermediate market is more inefficient than the final market. In this case, the welfare gains from an increase in the number of VAs, due to the reduction in double marginalization inefficiency, offsets the welfare losses due to the reduction in the production and number of independent retailers. When retailers compete in prices the above condition for increasing welfare is the same that for achieving a VA. Then, if the competition in the final market is Bertrand, a VA always is positive from a welfare viewpoint.

The paper is organized as follows: We begin by laying out the basic model in Section II. That Section also studies the incentives for VA when the competition in the final market is in quantities. In section III, we review the results when competition in the final market is in prices. Section IV concludes.

II. The Model

Consider a situation where there are $M \geq 2$ upstream firms (or manufacturers) who produce an intermediate homogeneous good at a constant unit cost, c, and $N \geq 2$ downstream firms (or retailers) who need only one unit of input (the intermediate good) to produce one unit of the final homogeneous good. The retailers are price-taker in the input market, where they pay a unit price, h, and compete "à la Cournot" in the final good market. In section 3 we analyze the Bertrand competition case. The manufacturers compete "a la Cournot" in the intermediate good market. Under this industrial structure the manufacturers have incentives to negotiate vertical agreements (VAs), in order to avoid double marginalization inefficiency[6].

Assume that each manufacturer offers one, and only one, retailer a

[6] It is irrelevant to contemplate the case in which the retailers offer the contract since they are price-takers in the inputs market. See Flath (1989, 1991).

two-part tariff contract (w,F) where w is the unit price of the intermediate good and F is the franchise fee[7]. The contract does not specify a priori market foreclosure, that is, the manufacturer and the retailer are able to continue purchasing and selling the input in the market. The contract must satisfy $w \leq h$, because offering a contract would not be an equilibrium strategy for the manufacturer. To see this, suppose that $w>h$. The retailer would accept to pay an input price greater than the market price if the manufacturer covers the difference. It is not an equilibrium strategy for the manufacturer to offer this contract because the retailer has incentives to purchase the input in the market. The retailer accepts the contract if he gets greater profits than purchasing the input in the market. In this case, we will say that there is a VA.

Assume that there are r VAs (that is, r couples of firms have achieved a VA), n independent retailers and m independent manufacturers (that is, firms that except through the market do not have any vertical relationship). We will group them, respectively, in the sets I, J and K. Then $M=m+r$ and $N=n+r$. We will save indexes i, j and k for variables related to firms in sets I, J and K, respectively.

We consider a three stage game. In the first stage the manufacturers decide to offer a VA. In the second stage, manufacturers decide their production to sell in the market and the production to sell to the retailer with whom they have a VA. In the third stage the retailers compete in the final good market.

2.1 Final Market

The final good (inverse) demand is given by:

[7] A manufacturer may have incentives to offer a contract to more than one retailer and increase the price of the intermediate good in order to reduce competition in the final market. This is obviously a horizontal integration strategy and it is negative for final consumers. Therefore, we assume that the law forbids agreements with more than one retailer.

$$p(X) = 1 - X \tag{1}$$

where $p(X)$ is the product price and X is the quantity. Facing this demand, the retailer $i \in I$ chooses x_i to maximize its profits:

$$\Pi D_i = (p(X) - w_i)x_i - F_i \tag{2}$$

The FOC is given by:

$$x_i = 1 - X - w_i \tag{3}$$

The retailer $j \in J$ chooses x_j to maximize:

$$\Pi D_j = (p(X) - h)x_j \tag{4}$$

From FOCs and given that all the independent retailers face the same cost, the total output produced by them, $X_J = \sum_{j \in J} x_j = nx_j$, must satisfy:

$$X_J = n(1 - X - h) \tag{5}$$

2.2 Input Market

The demand of the input market is given by the production decision of independent retailers, X_J. This demand is satisfied by independent manufacturers whose total production is $Z = \sum_{k \in K} z_k$, and by manufactures in VAs whose total production is $\Theta = \sum_{i \in I} \theta_i$. So,

$$X_J = Z + \Theta \tag{6}$$

Then from (5) and (6) we obtain the (inverse) input demand:

$$h = 1 - (1+1/n)Z - (1+1/n)\Theta - X_I \tag{7}$$

where $X_I = \sum_{i \in I} x_i$ is the total output produced by retailers into I.

The independent manufacturer $k \in K$ faces demand (7) and chooses z_k to maximize:

$$\Pi U_k = (h - c)z_k \tag{8}$$

From the FOCs of the m independent producers and given that $Z=mz_k$ we get:

$$h - c = (1+1/n)Z/m \tag{9}$$

The manufacturer $i \in I$ faces the market demand given by (7) and the demand from retailer $i \in I$ obtained from (3) and (6):

$$w_i = 1 - Z - \Theta - X_{I-i} - 2x_i \qquad i \in I \tag{10}$$

where X_{I-i} is the total production of retailers into I except the i-th. Facing demands (7) and (10) the manufacturer $i \in I$ chooses θ_i to sell in market and x_i to sell to its contracted retailer to maximize:

$$\Pi_i = (w_i - c)x_i + (h - c)\theta_i + F_i \tag{11}$$

given the retailer participation condition:

$$\Pi D_i = (p(X)-w_i)x_i - F_i \geq \Pi D_j \tag{12}$$

where ΠD_j, given by (4), is the retailer opportunity cost[8]. This cost is a constant for the manufacturer, so it solves:

$$\text{Max}_{x_i, \theta_i} \; (w_i - c)x_i + (h - c)\theta_i + (p(X) - w_i)x_i, \tag{13}$$

that is, it maximizes the relation value. The FOCs are given by:

[8] If there are n independent retailers, then when one of the retailers accepts a VA proposed by a manufacturer, it forgoes the equilibrium profit of an independent retailer with n independent retailers

$$w_i - c = \theta_i \tag{14.1}$$
$$h - c = (1+1/n)\theta_i + x_i \tag{14.2}$$

Given that manufacturing costs are equal, then $Z=mz_k$, $X_I=rx_i$ y $\Theta=r\theta_i$. From these equalities, (7), (9), (10) and (14), we obtain the following equation system:

$$\begin{aligned}
(1+1/r)X_I + (1+1/r)\Theta + Z &= 1-c \\
(1+1/r)X_I + (1+1/r)(1+1/n)\Theta + (1+1/n)Z &= 1-c \\
X_I + (1+1/n)\Theta + (1+1/n)(1+1/m)Z &= 1-c
\end{aligned}$$

Equilibrium is characterized by this system. Its solution is given by (outputs are proportional to 1-c):

$$x_i = \frac{1}{1+r} \qquad Z = \frac{mn}{(n+1)(m+r+1)} \qquad \theta_i = -\frac{Z}{1+r} \tag{15}$$

From (15) and (14) it is easy to check that $h>c>w_i$. Then the retailer $i\in I$ does not have incentives to purchase the input in the market. We can enunciate the next result:

Proposition 1: *If retailers compete "à la Cournot", an optimal contract specifies an input price lower than the production cost ($w_i\leq c$). Under this contract, the manufacturer "purchases" the input in the intermediate market $\theta_i\leq 0$).*

The intuition behind the result is that the contract acts as a commitment device. In Cournot oligopoly, each retailer would like to be a Stackelberg leader but the implicit assumption in the model is that it cannot commit to such a level of output. The two part tariff provides the necessary commitment by creating a low marginal cost and thereby making it rational for the retailer to sell more than the Cournot output.

Alternatively, the optimality of pricing in the contract below the unit production cost can be explained in terms of the slopes of the reaction function. A reduction in the contract price induces an increase in retailer production. Because the slopes of the reaction function are negative (strategic substitutes) the other retailers reduce their production, so the retailer under a contract increases its market share and profits and this profit increment covers the cost of setting the contract price below production cost.[9]

Regarding market foreclosure, proposition 1 indicates that the optimal relation of firms in a VA with independent firms is to "purchase" the input. This result may be understood as a strategy to increase the rivals' cost (other retailers) and indicates that the increase in profits induced by this strategy offsets the cost of purchasing the input at a greater price than the internal production cost.

With vertical integration (which implies that w=c) instead of a two-part tariff contract, Gaudet and Long (1993) also show the optimality of purchasing the input in the case of a double duopoly bilateral (N=M=2). In the general case, however, these authors obtain that the sign of θ_1 is ambiguous and is positive if the number of independent firms is small with respect to the number of integrated firms. In this case, when there are relatively few independent rivals, taking away their profits by increasing their costs does not compensate paying a higher price for the input than its production cost. In our case, however, the possibility of fixing a contract price below its production cost, raises the positive effect on profits of firms in a VA induced by the strategy of increasing rivals' cost. Then, this positive effect always offsets the cost of purchasing the input at a higher price than its production cost.

2.3 Vertical Agreements

Now, we analyze whether it is optimal to offer a contract such as the one defined in proposition 1. The condition needed to achieve an agreement is

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[9] This is a classical result in literature on pre-commitment effects which assumes that delegation contracts, if any, are publicly observed. See Caillaud and Rey (1994).

that joint profits exceed the sum of profits of one independent manufacturer and one independent retailer. We must consider that a new VA reduces the number of independent firms. Let $\Pi^{i}(N,M,r)$ be the profits of firms in a VA and $\Pi^{ni}(N,M,r) = \Pi U_k(N,M,r) + \Pi D_j(N,M,r)$ be the profits of independent firms. If there are r VAs, there will be r+1 VAs if:

$$\Pi^{i}(N,M,r+1) \geq \Pi^{ni}(N,M,r) \tag{16}$$

is satisfied where $r=0,1,\ldots,\mathrm{Min}\{N,M\}-1$. Reviewing this inequality we obtain the following result:

Proposition 2: *For any number of vertical agreements achieved, if the retailers compete "à la Cournot", a manufacturer always has incentives to offer a vertical agreement. If there are no costs related to signing a vertical agreement, the intermediate market disappears and the Cournot equilibrium is achieved with Min(N,M) firm.*

Proof: See Appendix.

This result indicates that firms not only have incentives to eliminate double marginalization but also have incentives to price below marginal cost in contracts. This last fact explains that, contrary to Gaudet and Long (1993), achieving a VA does not depend on the number of firms. In fact, even though there is no double marginalization (that happens when the input market is competitive, that is, if $m \to \infty$), firms achieve VA since this is the only way they can sell the input below marginal cost and recover, through the franchise, the increase in the retailer's profits.

It is not difficult to find real-world examples of this. Some industrial corporations, with a credit institution among their firms, customarily give preferential credits to other firms within the corporation so that the latter can be more competitive in their industrial sector. In international trade, some industrial corporations have gone into the United States market purchasing retail chains and selling below production cost, a strategy which has provoked dumping lawsuits.

2.4 Welfare

In order to obtain some suggestions for antitrust policy, we review the effects of VAs on welfare which is defined by:

$$\Omega = X^2/2 + r\Pi_i + (N-r)\Pi D_j + (M-r)\Pi U_k, \tag{17}$$

that is, by the consumer surplus plus industry profits. An increase in the number of VAs is positive, from a welfare viewpoint, if

$$\Omega(N,M,r+1) \geq \Omega(N,M,r) \tag{18}$$

is satisfied, for $r=0,1,\ldots,\mathrm{Min}\{N,M\}-1$. We review this inequality in the next lemma:

Lemma 1: *Inequality (18) is true iff $(r+1)(M-N-1) < (M-r)(N-r)$ for $r=0,1,2,\ldots,\min\{M,N\}-1$.*

Proof: See Appendix.

We can derive some conclusions from this lemma. Note that when M<N, that is, when the number of retailers is greater than the number of manufacturers, an increase in the number of VAs has a positive effect on welfare. However, when M>N+1, welfare can be reduced. Although a VA has a positive effect, since the retailer increases its production and therefore the consumer price is reduced, it also has a negative effect since the input market price increases and the number and production of independent retailers decreases, thus increasing the final consumer price. This last effect can be dominant.

III. Price Competition in the Final Market.

In this section we examine the above results when retailers compete in prices. As is usually done in the literature, to prevent the model from collapsing into perfect competition, we assume that the final goods are differentiated. The input's still homogeneous and the manufacturers compete "a la Cournot". The i-th retailer faces the demand:

$$x_i = 1 - p_i + \alpha\bar{p}_{-i} \tag{19}$$

where $\bar{p}_{-i}$ is the average of the prices of all the retailers except for the i-th and α is a parameter that indicates the grade of differentiation.[10] As α grows the products are more similar.

Since the method for solving the model is the same as the one developed above we relegate it to Appendix B and focus on the results. The characteristics of the vertical contract are given in the following result:

Proposition 3: *If the retailers compete "à la Bertrand", an optimal contract fixes a price greater than input unit cost, that is, the manufacturer does not eliminate double marginalization. When the number of manufacturers is less than the number of retailers, the manufacturer "sells" the input in the market. If final products are gross substitutes and the number of manufacturers is not less than the number of retailers, the manufacturer "purchases" the input in the market.*

Proof: See Appendix B.

In agreement with this result, the manufacturer fixes a positive mark-up in the contract which can be explained by the nature of market interactions of price competition. If the manufacturer, with the objective of eliminating double marginalization inefficiency, fixes $w=c$, it induces the retailer to

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[10]This demand system comes from the utility function:

$$U = \sum_{i=1}^{N} x_i\left(a - \frac{f}{2}x_i - \frac{d}{2}\sum_{j=1}^{N} x_j\right) + m$$

where $a = \frac{1}{1-\alpha}$, $f = \frac{N-1}{N-1+\alpha}$, $d = \frac{\alpha a}{N-1+\alpha}$. This function is concave if $f\sum_i h_i^2 + d(\sum_i h_i)^2 \geq 0 \ \forall h_1, h_2, \ldots h_N$, that occurs when $0 \leq \alpha < 1$.

reduce its price too much. Since the slopes of the reaction function are positive (strategic complements) the other retailers also reduce their prices. So, the VA profits are reduced and this negative effect on profits can compensate the gains due to the greater efficiency from the elimination of double marginalization.[11]

Regarding market foreclosure, proposition 3 indicates that when there are more retailers than manufacturers, a manufacturer in a VA "sells" the input in the market. That is, the manufacturer does not use the strategy of increasing rivals' cost by purchasing the input in the market. The reason is simple. The greater the number of retailers, the greater the input market demand and the smaller the oligopolic rents in the final market. It is not profitable for the manufacturer to give this demand up by not selling in the input market.

On the other hand, the fact that the optimal contract fixes a price greater than the marginal cost, is able to cause that $w_i>h$ and the contract is not feasible. This occurs when there are relatively too many manufacturers (as an extreme case, when the input market is competitive then $h=c<w_i$) and in this case, offering a contract is not an equilibrium strategy for the manufacturer because the retailer has incentives to purchase the input in the market.

When m is such that $h>w_i$ (that is, when the contract is feasible), it is difficult to derive general propositions regarding the optimality of the VA and its effects on welfare, since the solution of the model depends on parameters α, M, N and r. For this reason, in order to obtain some conclusions, we have made some numerical exercises giving values to N and M. The results are given in table 1, from which, in general terms, we conclude that when $w_i<h$ and retailers compete in prices, there will always be VAs. This is explained by the reduction in double marginalization.

[11] This is also a classical result in the literature on pre-commitment effects.

Finally, we review the effects of the VAS on welfare. From numerical exercises we find that when retailers compete in prices, and a VA is achieved, this is socially desirable. A condition to achieve a VA is $h>w_i$, so a VA reduces the retailer's cost which in turn induces a reduction in the final product prices for all the retailers. As a consequence, welfare increases.

Table 1: Vertical agreements in equilibrium

N\M	2	3	4	5	6
2	r=2	r=2	α>0.89, r=0	α>0.81, r=0	α>0.75, r=0
3	r=2	r=3	α>0.97, r=1	α>0.89, r=1	α>0.93, r=0 α>0.83, r=1
4	r=2	r=3	r=4	α>0.97, r=2	α>0.98, r=1 α>0.90, r=2
5	r=2	r=3	r=4	r=5	α>0.97, r=3
6	r=2	r=3	r=4	r=5	r=6

The r values indicate the number of VAs.
For α values greater than indicated,
VAs are not achieved since $w_i>h$.

IV. Conclusions

In this work we reconsidered the arguments regarding foreclosure and according to our results, at least under the industrial structure assumed by other similar models, there are no reasons for firms in a VA to deny participation in the intermediate market. In fact, trade among firms in a VA with independent firms is an incentive to achieve it, since it can be used strategically goals to gain greater profits.

Regarding antitrust policy, we show that even monopolization attempts can be positive from a welfare viewpoint, since a manufacturer, who desires a greater market share sets prices in a contract below market price. The VAs can be negative for welfare when there are few retailers with respect to the number of manufacturers and competition in the final market is in quantities.

When retailers compete in prices, a VA is always positive.

This work is related with Hart and Tirole (1990) and Ordover, Saloner and Salop (1990) who focus on foreclosure rather than on efficiency, with a model in which double marginalization does not arise as a motive for VI[12]. However, Carlton[13] argues that if the relevance of their results for policy making is to be considered, double marginalization must be taken into account because two-part tariff may not be in use and the price may exceed marginal cost. Furthermore, as we have pointed out above, it is not necessarily suitable that the price in a two-part tariff contract be equal to unit cost, as Hart and Tirole assume. Furthermore, these authors assume implicitly that the trade of integrated firms with unintegrated firms is reduced to selling the input, that is, they do not review the suitability of purchasing input from other manufacturers.

This paper is also related with Comanor and Frech (1985, 1987), Mathewson and Winter (1987) and Schwartz (1987) who analyze the effects of exclusively dealing on welfare. These authors do not consider the strategic interactions in the final market, since they assume that retailers sell in independent markets. Under these conditions, in agreement with our model, the VAs trigger, necessarily, market foreclosure.

Our conclusions recover some arguments, frequently associated with the University of Chicago (See Tirole, 1988, pp 170, Mathewson and Winter, 1987) that hold that the observed vertical controls have as a unique goal to increase the efficiency of the vertical relations -not monopolization. In this work we establish the conditions that support these arguments allowing final market competition.

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[12] For that purpose, Hart and Tirole assume the possibility of two-part tariff in the input's market and Ordover, Saloner and Salop assume input price competition and one homogeneous input.

[13] See Hart and Tirole (1990, pp. 279).

APPENDIX A

Proof Proposition 2. From (12) and (11) we obtain

$$\Pi_i = x_i^2 + 2x_i\theta_i + (1+1/n)\theta_i^2$$

and from (14):

$$\Pi_i(n,m,r) = \frac{n(r+1)^2 + (r+m+1)^2}{(n+1)(r+1)^2(r+m+1)^2} \qquad \text{A1}$$

From (10), (9), (14) and $mz_k = Z$ we obtain:

$$\Pi U_k = (1+1/n)Z^2/m^2 = \frac{n}{(n+1)(m+r+1)^2}$$

From (5) and (4), y using $nx_j = X_J$, we obtain:

$$\Pi D_j = X_J^2/n^2 = \frac{m^2}{(1+r)^2(n+1)^2(n+r+1)^2}$$

The unintegrated profits are:

$$\Pi_{ni}(n,m,r) = (1+1/n)Z^2/m^2 + X_J^2/n^2 = \frac{m^2 + n(r+1)^2(n+1)}{(n+1)^2(r+1)^2(m+r+1)^2} \qquad \text{A2}$$

The firms integrate if the next expression is positive:

$$\Pi_i(N,M,r+1) - \Pi_{ni}(N,M,r)$$

for r=0,1,2,..,Min{N,M}-1. Using A1 y A2, this expression is proportional to:

$$\frac{(m-1)(m+2r+3)(r+1)}{(r+2)^2(m+r+1)^2} + \frac{n(n(r+1)^2 + r^2 - 2)}{(r+2)^2(r+1)} + \frac{2mn}{(m+r+1)^2}$$

It is straightforward that this expression is positive $\forall$ m,n,r.■

Proof lemma 1. In this case we compute:

$$\Delta\Omega = \Omega(N,M,r+1)-\Omega(N,M,r)$$

Developing we obtain that $\Delta\Omega$ is proportional to:

$$\frac{(M-r)^2}{(r+1)^2(N-r+1)^2} - \frac{(M-r-1)^2}{(r+2)^2(N-r)^2} + \frac{2(M-r)}{(r+1)(N-r+1)} - \frac{2(M-r-1)}{(r+2)(N-r)}$$

It is easy to see that this expression is positive if $(M-r)(N-r)>(M-N-1)(r+1)$.■

APPENDIX B:

Price Competition in the final market. Each retailer faces the demand

$$x_i = 1 - p_i + \alpha\bar{p}_{-i} \qquad \text{B1}$$

where $\bar{p}_{-i}$ is the average of all the retailers prices, except for the i-th. The inverse demand is:

$$p_i = a - bx_i - dx_{-i} \qquad \text{B2}$$

where $a=\frac{1}{1-\alpha}$; $b=\frac{(N-1)(1-\alpha)+\alpha}{(N-1+\alpha)(1-\alpha)}$; $d=\frac{\alpha}{(N-1+\alpha)(1-\alpha)}$ and x_{-i} is the production of all the retailers except for the i-th. The retailer $i\in I$ chooses the price p_i that maximizes its profits given by:

$$\Pi D_i = (p_i-w_i)x_i \qquad \text{B3}$$

The FOCs are:

$$p_i = w_i + x_i \qquad \text{B4}$$

The retailer $j\in J$ chooses the price p_j to maximize its profits:

$$\Pi D_j = (p_j - h)x_j \tag{B5}$$

The FOCs are:

$$p_j = h + x_j \tag{B6}$$

From B2, B4 and B6, the intermediate input demands become:

$$w_i = a - (b+1)x_i - d(Z + \theta + X_{I-i}) \tag{B7.1}$$
$$h = a - \xi(Z + \theta) - dX_I \tag{B7.2}$$

where $\xi = \frac{b+1+d(n-1)}{n}$. The independent manufacturer $k \in K$ chooses z_k to maximize:

$$\Pi U_k = (h-c)z_k \tag{B8}$$

from the FOCs of the m independent manufacturers, and $Z=mz_k$, we get:

$$h-c = \xi Z/m \tag{B9}$$

The manufacturer $i \in I$ chooses x_i and θ_i to maximize:

$$\Pi_i = (w_i - c)x_i + (h-c)\theta_i + \Pi D_i \tag{B10}$$

therefore:

$$w_i - c = d\theta_i + (b-1)x_i \tag{B11}$$
$$h - c = dx_i + \xi\theta_i \tag{B12}$$

Using B8, B9 y B11, the equilibrium is characterized by:

$$\begin{aligned} 2bx_i + dX_{I-i} + 2d\theta_i + d\theta_{-i} + dZ &= a - c \\ 2dx_i + dX_{I-i} + 2\xi\theta_i + \xi\theta_{-i} + \xi Z &= a - c \\ dX_I + \xi\theta + \xi(1+1/m)Z &= a - c \end{aligned} \tag{B13}$$

As the production costs are the same, it is satisfied that $\theta_i = r\Theta$, $x_i = rX_I$. So, the system B13 becomes:

$$\frac{2b+d(r-1)}{r}X_I + d(1+\frac{1}{r})\Theta + dZ = a - c$$
$$d(1+\frac{1}{r})X_I + \xi(1+\frac{1}{r})\Theta + \xi Z = a - c \qquad \text{B13'}$$
$$dX_I + \xi\Theta + \xi(1+1/m)Z = a - c$$

Solving this system we get the equilibrium:

$$x_i = \frac{\xi - d}{\Delta} = \frac{b-d+1}{\Delta n} \qquad \text{B14}$$

$$Z = \frac{m}{\xi(m+r+1)} = \frac{mn}{(b+d(n-1)+1)(M+1)} \qquad \text{B15}$$

$$\theta_i = \frac{2\xi(b-d) - dm(\xi-d)}{\xi\Delta(M+1)} \qquad \text{B16}$$

where $\Delta = 2\xi(b-d) + d(r+1)(\xi-d)$.

Proof of Proposition 3. In order to show proposition 3 we check the signs of B11 and B16. From B16, the sign of θ_i is given by the sign of

$$2\xi(b-d) - dm(\xi-d)$$

Using B2 and B7, the sign of this expression is equal to the sign of:

$$2[2(N-1) + \alpha] + \alpha a[2(N-M) - \alpha\frac{M-r}{N-1}] \qquad \text{B17}$$

When N>M, expression B17 is positive if:

$$2(N-M) > \alpha(M-r)/(M-1) \qquad \text{B18}$$

The left hand side can be bounded below as follows:

$$2(N-M) > 2 \qquad \text{B19}$$

The right hand side can be bounded above as follows:

$$\alpha\frac{M-r}{N-1} < \frac{M-r}{N-1} \leq 1 \text{ for } r \geq 1 \qquad \text{B20}$$

From B19 and B20 it results that if $N>M$ then $\theta_i>0\ \forall\ \alpha$.
When $N\leq M$, a necessary condition for B17 to be negative is:

$$\frac{\alpha}{1-\alpha} > \frac{2[2(N-1)+\alpha]}{2(M-N)+\alpha(M-r)/(N-1)} \qquad \text{B21}$$

the left hand side can be bounded above by $2(N-1)/(M-N)$. So it is easy to see that exit α satisfies B21. In order to check the sign of w_i-c, we substitute B14 and B16 into B11 and simplifying we obtain:

$$w_i-c = \frac{d}{\xi(m+r+1)} + \frac{(\xi-d)(\xi(b-1)-d^2)}{\xi r\Delta} \qquad \text{B22}$$

Using ξ, d and b, defined in B2, the sign of the second term of B22 becomes:

$$\xi(b-1)-d^2 = \frac{\alpha^2 a[2(N-1)+\alpha-n]}{n(N-1+\alpha)^2}>0 \qquad \text{B23}$$

So, $w_i-c>0$. ■

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