CIDE

NÚMERO 91

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HOW DEAD IS THE AUGMENTED SOLOW MODEL?

Introduction

Perhaps the central empirical question in the recent literature on economic growth is whether some form of the neo-classical growth model is sufficient to explain observed behavior. Cross-sectional "conditional convergence" regressions answering this question in the affirmative have recently been effectively criticized. However, Mankiw Romer and Weil (1992 hereafter M-R-W) take a different approach by arguing that the Solow growth model, augmented to include human capital, is largely consistent with available cross-country data on per-capita income levels. They describe their results as follows, "given the inevitable imperfections in this sort of cross-country data, we consider the fit of this simple model to be remarkable. It appears that the augmented Solow model provides an almost complete explanation of why some countries are rich and other countries are poor".

Below I show that their statement is false. The results they rely on are from regressions using data that are not drawn from a common distribution. Two of the three samples used by M-R-W are inappropriately pooled, and the third (the OECD) does not support their claims. When observations that cannot be combined in a single regression are treated separately, the claimed empirical support for the augmented Solow model vanishes.

Given the problems associated with using cross-sectional regressions to test theories about growth, I go on to develop a simple, dynamic, empirical test of the neo-classical growth model. My test makes use of the fact that, in the steady state, all economies should grow at the same per-capita rate (the exogenous rate of technological progress). Therefore, we should observe absolute convergence in per-capita growth rates over time. In both a 125 country - 25 year sample and a 59 country - 35 year sample, I show that per-capita growth rates are actually diverging over time. Only the 24 OECD countries show any evidence of growth rate convergence. I also show that a small group of presumably homogeneous countries, namely 20 Latin American nations, show increasing growth rate divergence.

Since the strongest existing support for the augmented Solow model, M-R-W's work on cross-country wealth differences, is ephemeral, and since post-war growth rates around the world are diverging, I conclude that the augmented Solow model is stone dead.[1]

In what follows below, Section I discusses the development of empirical evidence attempting to discriminate between neo-classical and endogenous growth models. Section II replicates M-R-W's work on cross-country income differences and then shows that their data reject the pooling of countries used in the regressions they employ to explain steady state income differences across countries. I then show that disaggregated regressions no longer support either version of the Solow model. Section III develops a simple dynamic test of the augmented Solow model based on growth rate convergence and reports findings of significant divergence. Section IV discusses whether or not club convergence is a pervasive empirical regularity in the post war data, and section V is a brief conclusion.

Empirical testing of the Neo-Classical growth model

The bulk of empirical testing of the neo-classical growth model is unfortunately concentrated on a single concept: convergence. The fact that per-capita income differences across countries persist over time was taken by many to be a prima facia falsification of the neo-classical model and provided an impetus for work on endogenous growth. We know now that the Solow model can escape this attack by assuming that countries may differ according to their steady state income levels as well as their initial conditions. The Solow model thus predicts conditional

[1] By this I mean that the neo-classical growth model cannot be used as the basis for empirical models fit to post-war data. Solow's (1956) model is a remarkable contribution to economics and continues to be extremely useful as a way of thinking about growth.

convergence.

Conditional convergence means only that entities further away from their own steady state will grow more rapidly than entities closer to their own steadystate, irrespective of which has the poorer initial conditions.[2] A host of cross-sectional regressions where a long-term average growth rate is regressed on initial conditions and a variety of demographic, economic and political variables have been presented as confirming the hypothesis of conditional convergence.[3]

The cross-sectional regression approach to testing for conditional convergence has been strongly criticized by Friedman (1992) for being driven by measurement error in initial income, by Quah (1996) as possibly being a artifact of the unit root process found in per-capita incomes, and by Bernard and Durlaff (1996) for failing to discriminate between the Solow model and other plausible alternative hypotheses.[4]

M-R-W have largely avoided these criticisms by stressing a different test of the neo-classical growth model. While they do present cross-sectional conditional convergence regressions, their main contribution is to present and test the implications of the Solow model for cross-country differences in per-capita wealth. Thus they avoid the statistical problems inherent in interpreting the coefficient on initial conditions in a cross-sectional average growth regression.

However, there is another potentially serious problem for large sample cross-country regressions that M-R-W do not avoid, namely that data from countries as disparate as Sri Lanka, Costa Rica, and the U.S. may not belong in the same regression equation. Grier and Tullock (1989), using a cross-section / time-series experimental

[2] Conditional convergence also means that entities with the same steady states should show unconditional convergence. The OECD, the U.S. states, and Japanese prefectures are frequently offered as examples of this phenomenon.

[3] Barro and Sala-I-Martin (1992, for example) are the best known exponents of this approach. See Fuente (1997) for a recent review of the empirical convergence literature.

[4] Time series tests (e.g. Bernard and Durlauf (1995) and Quah (1996a, 1996b)) generally find little evidence in favor of convergence, though Evans and Karras (1996) is an exception.

design with multiple observations per country show that the OECD countries cannot be pooled with the rest of the world and that the non-OECD countries cannot be pooled together in a single regression. In what follows below, I show that M-R-W's two large samples cannot be pooled together, and that estimating their models on their component sub-samples does not yield much support for either the standard or augmented Solow model.

Replication and specification testing of M-R-W

M-R-W begin with regressions using the Solow model to explain cross-country differences in per-capita wealth. They use a sample of 98 non-oil producing countries, a 75 country subset of that sample which excludes countries with small populations or low quality data, and a final subset of the 22 largest OECD countries. They regress the Log of GDP per worker in 1985 on a constant, the Log of Investment as a fraction of GDP, and the Log of the sum of the depreciation rate, the growth of the labor force and the exogenous rate of technological progress. The Solow model predicts that the coefficient on investment should be positive, the coefficient on population negative, and that they should be about 1/2 and -1/2 respectively. This last prediction is equivalent to a capital share of 1/3 in a Cobb-Douglass production function.

M-R-W report that "the coefficients on saving and population have the predicted signs and, for two of the three samples, are highly significant". They also argue that the high adjusted R^2's of their regressions support the Solow model. The only problem they find for the Solow model is that the implied capital share is much higher that 1/3; in the two larger samples the implicit capital share is 0.6.

However, the one sample that does not have significant coefficients or a good fit is the one reasonably homogenous sample M-R-W use, namely the 22 largest OECD countries. A simple F-test of the hypothesis that the OECD countries, the 53 "intermediate" but non OECD countries, and the 23 small size or poor data countries can all be pooled into a single sample produces a statistic of 10.81 which allows rejection at the 0.01 level. This result invalidates any inferences drawn from the 98 country sample. Further, an F-test of the hypothesis that the OECD and non-OECD intermediate countries can be pooled into a single sample produces a statistic of 3.57 which rejects the null at the 0.05 level. Valid inferences cannot be drawn from either M-R-W's 98 country or 75 country samples.

Table 1 presents disaggregated M-R-W regressions on the 23 small size or poor data countries, the 53 intermediate but non-OECD countries and the 22 largest OECD countries (which is the third sample used by M-R-W). Here there is no supportive evidence for the Solow model. Investment is positive and significant in two of the three regressions, but population growth is insignificant in all three specifications. Further, the adjusted R^2's for the three regressions are .29, .35 and .01 respectively.[5] There is no good news for the Solow model in these regressions.

M-R-W go on to add a measure of human capital to the Solow model, calling the new version the augmented Solow model. They then repeat their analysis of cross-country income differences using the augmented model on the same three samples. They find that investment and population growth are still properly signed and significant in the 2 larger samples while the schooling variable is positive and significant in all three samples. Further the implicit physical capital share is now much closer to the desired 1/3 level and the implicit human capital share is also close to the desired 1/3 level in the two larger samples. M-R-W also point to the fact that the adjusted R^2's for the 2 large sample models are .78 and .79 as strong support for their augmented Solow model.

Again though, the two largest samples cannot legitimately be pooled

[5] I use M-R-W's data from the appendix of their paper. The only discrepancies between their results and my replications are in the third or fourth digit of coefficients. I exactly replicate their adjusted R^2's and their reported s.e.e.'s.

together. When I repeat the F-tests described above using the augmented regression model, putting all 98 countries in a single sample is rejected at the 0.01 level with an F-statistic of 6.06. Similarly, putting the 75 non-small, non-poor data countries into a single sample is rejected at the 0.05 level with an F-statistic of 3.21. Table 2 presents disaggregated tests of the augmented Solow model using separate regressions for the 23 small or poor data countries, the 53 intermediate but non-OECD countries, and the 22 largest OECD countries.

The disagregated results do not support the augmented Solow model. In the 23 small or poor data country sample, the variables are correctly signed but only investment is significant at the 0.05 level. As shown in column 1 of Table 2, the adjusted R^2 is .36. I also use nonlinear least squares to obtain unique estimates of the coefficients of physical capital and human capital in the underlying production function. The estimated physical capital coefficient is .27 and the estimated humancapital coefficient is .12. The null hypothesis that both equal 1/3 is rejected at the 0.01 level.

In the 53 intermediate, but non-OECD, country sample (column 2 of Table 2), the investment and schooling coefficients are properly signed and significant at the 0.01 level, but the population growth coefficient is completely insignificant. The adjusted R^2 is .65. When I use nonlinear least squares to estimate the production function coefficients, I obtain coefficients of .25 for physical capital and .30 for human capital. The null hypothesis that both equal 1/3 can be rejected at the 0.01 level.

As M-R-W's own results show, the augmented Solow model does not explain the 22 largest OECD countries income differences well (replicated in column 3 of Table 2). The adjusted R^2 is low, and only schooling is properly signed and significant at the 0.05 level. Ironically though this sample produces the only case where the hypothesis that the production function coefficients for human and physical capital both equal 1/3 cannot be rejected.

Accounting for the fact that heterogeneous countries cannot legitimately be combined into a single regression vitiates the support M-R-W claim for the augmented Solow models. Population growth is never a significant regressor, production function coefficients are incorrectly sized in two of three cases, and there is a large amount of variation in incomes left unexplained by the model. Since M-R-W are estimating coefficients in an aggregate national production function, my non-pooling results directly reject the textbook neo-classical growth model's assumption of a common technology across countries.

The lack of coefficient stability across countries illustrates that a major problem with the neo-classical model is the rarely discussed assumption that complex, idiosyncratic national economics can be aggregated up into a simple national production function that is invariant across countries. In the Solow model, the only difference between the U.S. economy and the Guatemalan economy is the amount of capital per worker, and not the form of the production function. The way that labor and capital in the US combine to produce computer chips, automobiles and litigation services is exactly the same way labor and capital combine in Guatemala to produce coffee and bananas. A model with such heroic homogeneity assumptions is unlikely to be able to explain much about observed behavior.[6]

A dynamic test of neo-classical growth models

In the augmented Solow model, whatever steady state an economy has, steady state per-capita income grows at the exogenous and constant across countries rate of labor-augmented technological progress. Thus over time, per-capita growth rates should converge to the rate of technological progress as countries approach their steady states. I examine this implication of the augmented Solow model by

[6] Andes, Domenech and Molinas (1996) study an OECD panel dataset and conclude that macroeconomic factors are equally important and temporally more stable that the augmented Solow model's regressors.

calculating the standard deviation of per capita growth rates over time for several groups of countries and then testing the hypothesis that the dispersion of growth rates decreases over time.

This test has several nice properties. It is not tied to a specific variant of the neo-classical growth model, as long as a model predicts steady state growth at the level of technological progress, cross country growth rates will converge. The test is not weakened by problems of poorly measured explanatory variables such as the stock of human capital, nor by potential problems with endogenous regressors. Finally, the test is extremely simple. The only maintained assumption is that over the last 25 or 35 years, countries have gotten closer to their steady state levels of income per effective worker.

Since the steady state growth rate is shared by all countries, regardless of their possibly unique steady state level of income per effective worker, the proper sample size here is the entire world. However, since equal growth rates occur only in the steady state, a long time period is required to fairly test the hypothesis. As is well known, there is a frustrating trade off between the number of countries considered and the length of the time series of data available.

I deal with this by presenting results for 125 countries from the Summers-Heston data over the period 1961 - 1985 as well as results for 59 countries from the same source over the 1951 - 1989 period. I also report results for the 24 OECD countries and 20 Latin American countries from 1951 - 1990.

Figure 1 shows the evolution of the standard deviation of per-capita GDP growth across 125 countries from 1961 - 1985. The relationship between the dispersion of growth rates and time is unambiguously positive. A regression of the dispersion on a linear trend produces a coefficient of 0.067 with a robust t-statistic of 3.32 as shown in equation A1 of Table 3. Figure 2 depicts the same relationship for the 59 countries having complete data from 1951 through 1989. The relationship is still positive, but weaker than in the larger country - shorter time span A regression of the dispersion of growth rates on a linear trend in this sample produces a coefficient of 0.038 with a robust t-statistic of 2.19 as shown in equation B1 of Table 3. A second degree polynomial in time fits the data significantly better as shown in equation B2 of Table 3 and Figure 3. There is thus some evidence for these 59 countries that the rate of increase in growth dispersion is itself increasing.

In the two large country samples examined above, there is no evidence that growth rates are converging to the common exogenous rate of technological progress as predicted by any version of the Solow model. However, the OECD countries do exhibit growth rate convergence. Figure 3 shows the evolution of growth rate dispersion from 1951 - 1990 in the OECD. Equation C1 of Table 3 confirms that growth rates have significantly converged over time in the OECD. However, Equation C2 of Table 3 and Figure 5 show that since 1980, OECD growth rates have begun to diverge.

Do the data show Club Convergence?

It is possible to interpret the above results as favorable to the club convergence hypothesis (see Galor 1996 for an overview). Here an appropriately chosen set of countries exhibit convergence given some similar characteristics such as initial conditions. However, it tempting to define club convergence tautologically; if a sub-group of countries converge, then they were in a convergence club. In particular, there is a degree of circular reasoning required to take the OECD as an example of club convergence.[7]

From the perspective of 1951, there are few similarities between the countries in the OECD. There are huge differences in culture, institutions,

[7] This circularity extends to the theory of club convergence as well. For example, Galor (1996) lists similarity of initial income levels, human capital levels and income distributions as three factors that may create a convergence club. We are thus dangerously close to defining a convergence club as a group of countries that have already converged!

languages, religions and tremendous physical distances between the OECD countries. They are a most unlikely ex-ante convergence club, yet they have converged.

Now consider the countries of Latin America. Equations D1 and D2 in Table 3 along with figures 5 and 6 show these countries's growth rates are significantly diverging over the sample period. Yet, from an ex-ante perspective, these Latin American countries were far more likely to have exhibited club convergence than the OECD. The countries largely share a common language, culture, religion and colonial heritage and are closer to the US than any OECD country save Canada.[8]
I consider both the convergence of the OECD and the non-convergence of Latin America as evidence against the empirical relevance of club convergence theories, though a comprehensive examination of the theory remains to be conducted.

Discussion

In this paper, I argue that there is no general empirical support for the neo-classical growth model. In particular, I show that Mankiw, Romer and Weil's (1992) influential evidence does not survive simple tests for sample pooling and that, except for the OECD countries, world wide per-capita income growth rates are significantly diverging over time.

My results also illustrate the proposition that large groupings of disparate countries cannot meaningfully be placed in a single regression equation. This finding casts considerable doubt on the statistical validity of the findings contained in scores of papers using cross-sectional growth regressions.

Finally I argue that the club convergence hypothesis is not obviously supported by the data. The OECD countries converge, but were not very likely to

[8] Brazil's Portugese background and Guyana's French and British background are the exceptions with regard to language and colonial heritage. Dropping these countries from the analysis does not affect the finding of significant growth rate divergence.

have done so from an ex-ante perspective. The Latin American Countries diverge, but seem far more homogeneous a group than the OECD.

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Table 1: Disaggregate Tests of the Solow Model using M-R-W's Dataset

Variable	Eq.1 23 Small or Poor Data Counties	Eq.2 53 Intermediate but non-OECD Countries	Eq.3 22 Largest OECD Countries
Constant	8.916 (2.71)	10.749 (4.13)	8.021 (3.18)
Ln(I/GNP)	0.579 (3.28)	1.083 (5.35)	0.500 (1.15)
Ln(n+g+δ)	0.279 (0.22)	0.286 (0.28)	-0.742 (0.87)
R2	0.290	0.349	0.01
s.e.e.	0.436	0.627	0.377

Numbers in parenthesis are t-statistics. Dependent variable is Ln of GDP per working age person in 1985. Data are from M-R-W (1992).

Table 2: Disaggregate Tests of the Augmented Solow Model using M-R-W's Dataset

Variable	Eq.1 23 Small or Poor Data Counties	Eq.2 53 Intermediate but non-OECD Countries	Eq.3 22 Largest OECD Countries
Constant	7.841 (2.48)	12.068 (6.29)	8.637 (3.90)
Ln(I/GNP)	0.453 (2.51)	0.561 (3.34)	0.276 (0.71)
Ln(n+g+δ)	-0.345 (0.28)	0.357 (0.48)	-1.076 (1.42)
Ln(School)	0.201 (1.83)	0.672 (6.64)	0.767 (2.62)
R2	0.365	0.650	0.244
s.e.e.	0.412	0.460	0.330

Numbers in parenthesis are t-statistics. Dependent variable is Ln of GDP per working age person in 1985. Data are from M-R-W (1992).

Table 3. The Dispersion of Cross-Country Growth Rates Over Time

A. 125 Countries, 1961 - 1985 (N=25)

Std. Deviation of Growth Rates = 5.588 + 0.067*Time R^2= .239
(18)   (3.32)

B. 59 Countries, 1951 - 1989 (N=39)

Std. Deviation of Growth Rates = 4.150 + 0.038*Time R^2= .114
(10)   (2.18)

Std. Deviation of Growth Rates = 4.905 - 0.067*Time + 0.003*Time2 R^2= .176
(10)   (1.34)   (1.83)

C. 24 OECD Countries, 1951 - 1990 (N=40)

Std. Deviation of Growth Rates = 4.161 - 0.060*Time R^2= .448
(11)   (3.77)

Std. Deviation of Growth Rates = 4.987 - 0.178*Time + 0.003*Time2 R^2= .558
(11)   (3.67)   (2.55)

D. 20 Latin-American Countries, 1951 - 1990 (N=40)

Std. Deviation of Growth Rates = 3.478 + 0.058*Time R^2= .178
(6.74)   (2.59)

Std. Deviation of Growth Rates = 4.793 - 0.091*Time + 0.004*Time2 R^2= .253
(6.91)   (1.04)   (1.83)

Numbers in parentheses are HAC T-statistics. Dependent variable is the cross-country standard deviation of the growth rate of per-capita GDP each year. Data are from Heston and Summers.

Fig. 1: The Dispersion of Growth Rates in a Sample of 125 Countries, 1961 - 85

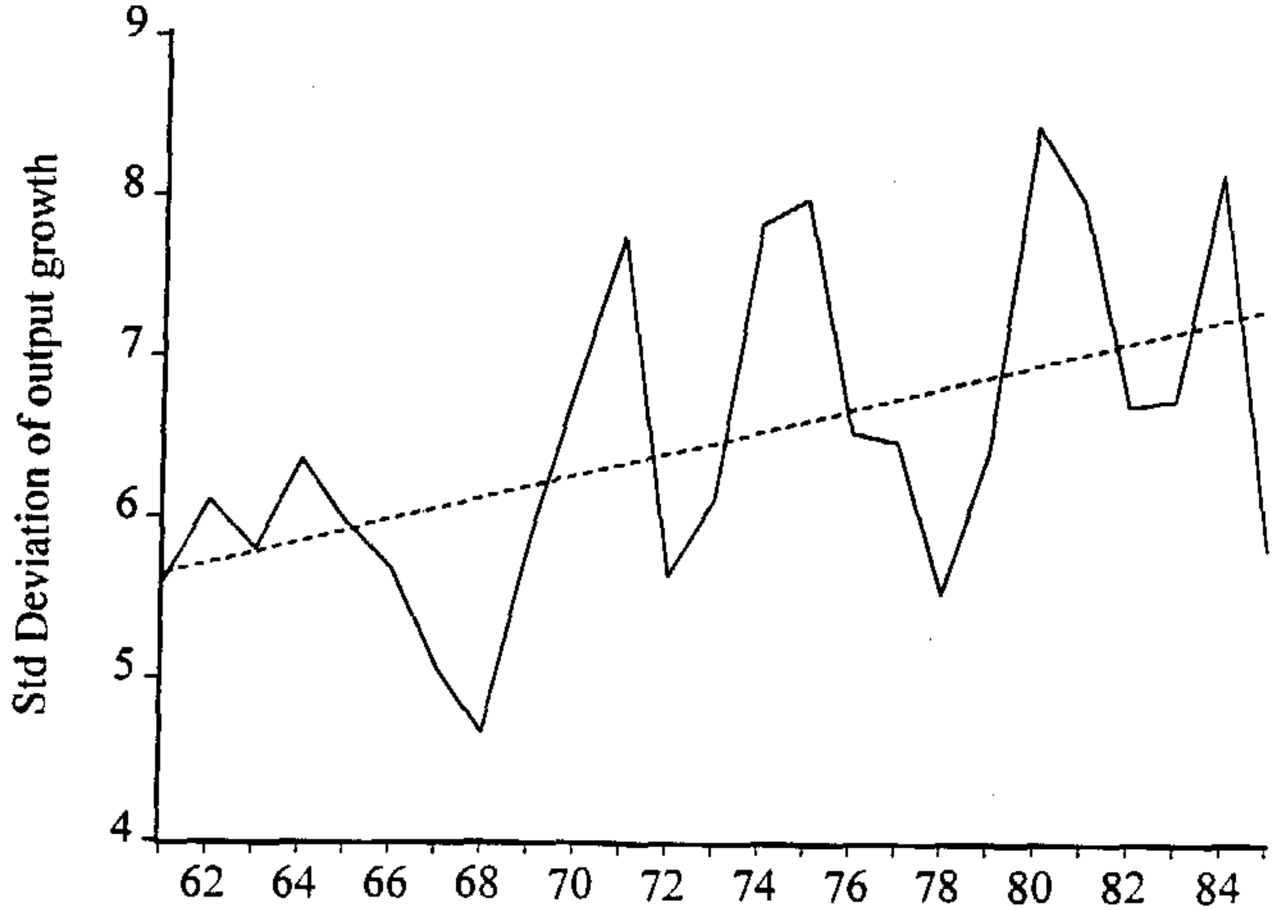

Fig. 2. The Dispersion of Growth Rates in a Sample of 59 Countries, 1951 - 89

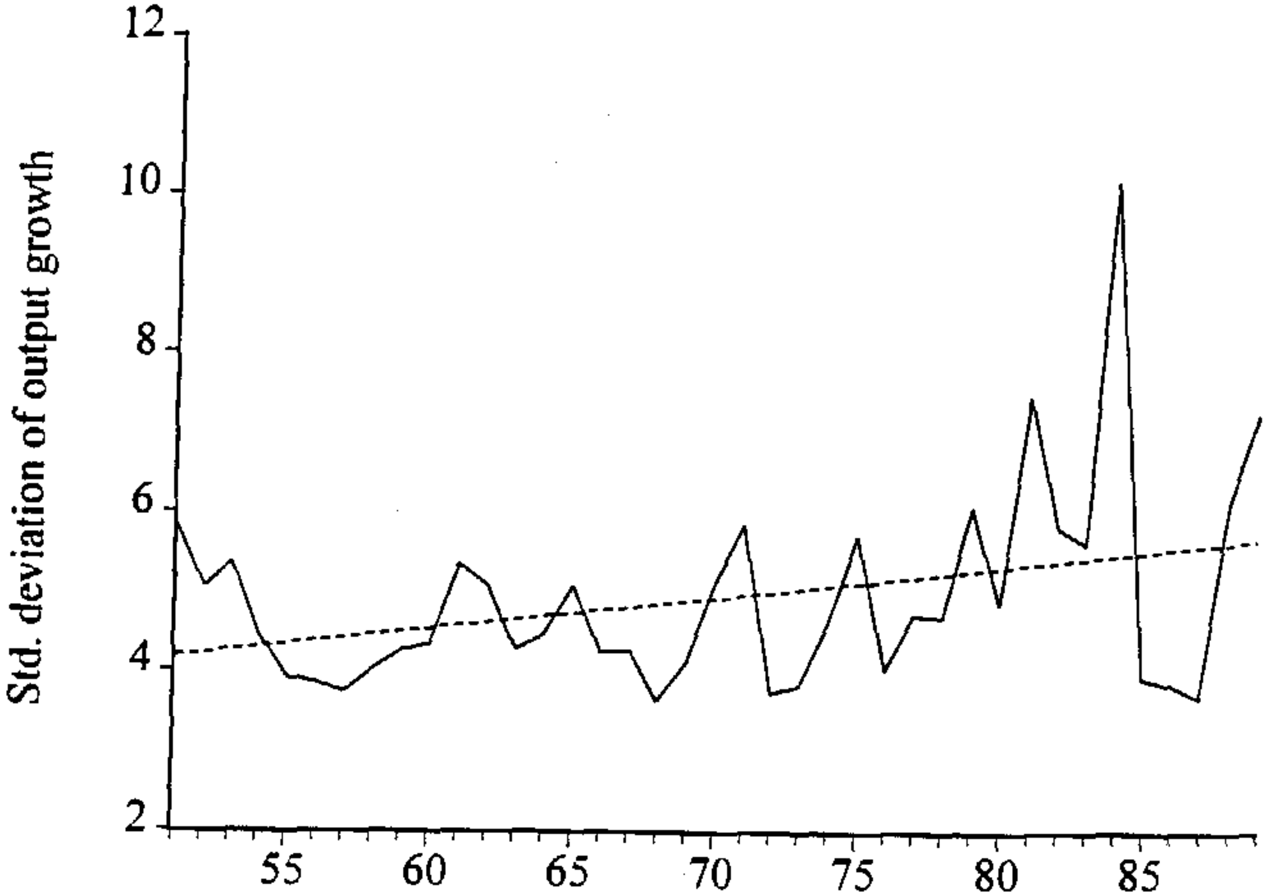

Fig. 3: The Dispersion - Time Relation is Significantly Non Linear in the 59 Country Sample, 1951 - 89

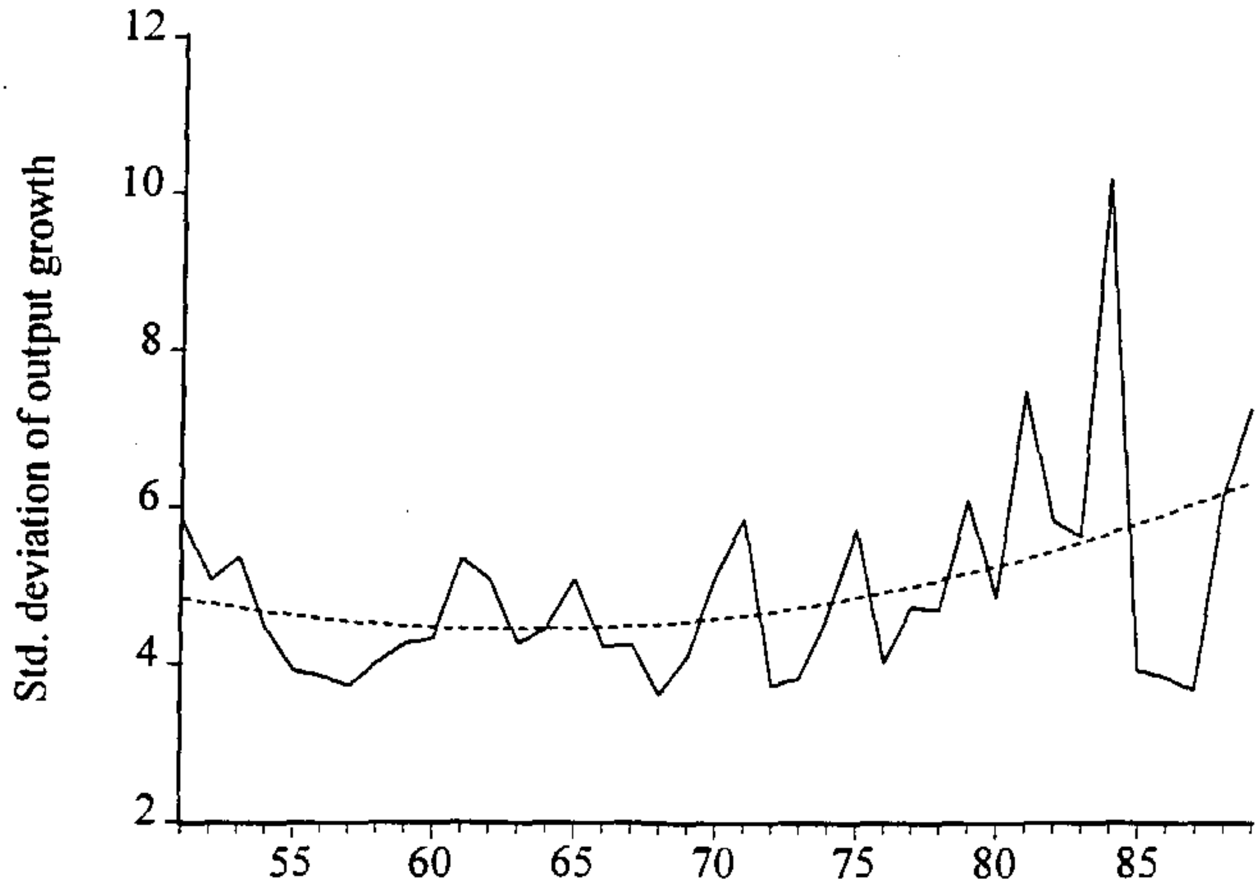

Fig. 4: The Dispersion of Growth Rates in a Sample of 24 OECD Countries, 1951 - 90

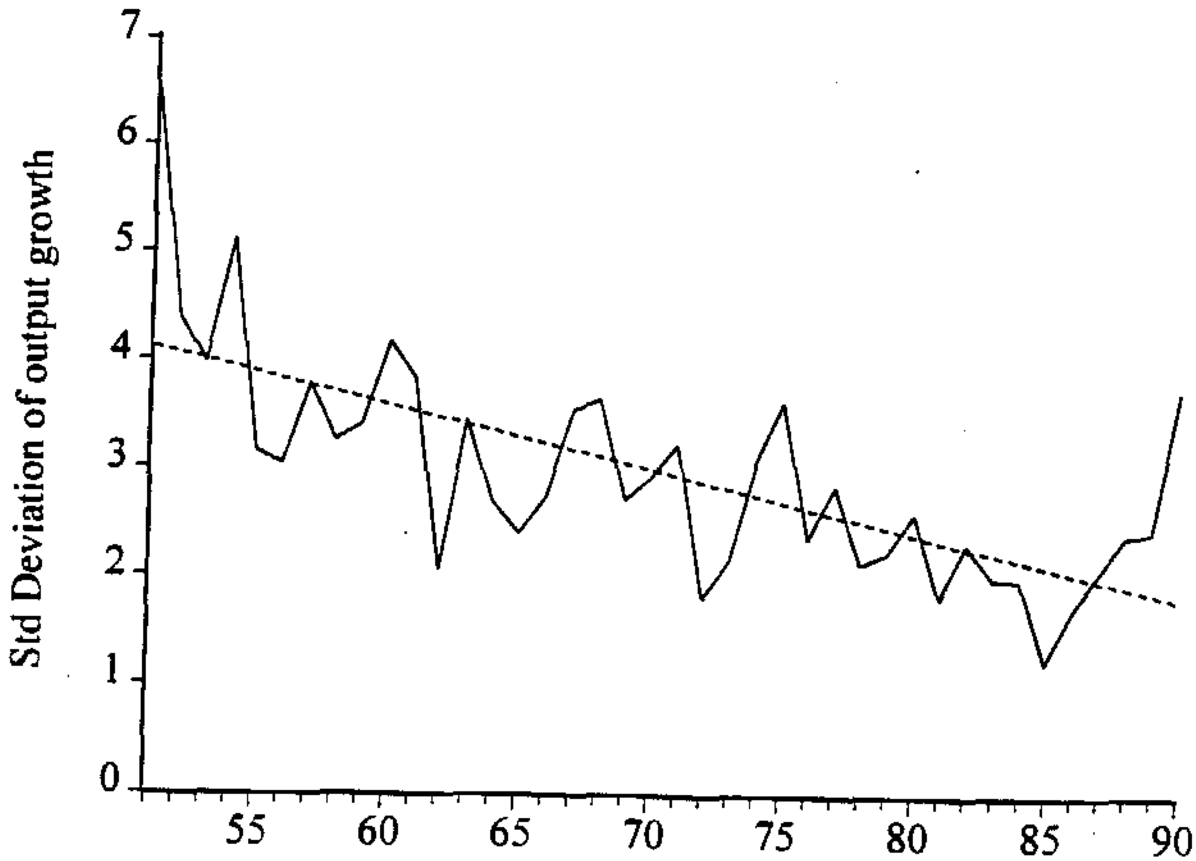

Fig. 5: The Dispersion - Time Relation is Significantly Non Linear in the 24 OECD Country Sample, 1951 - 90

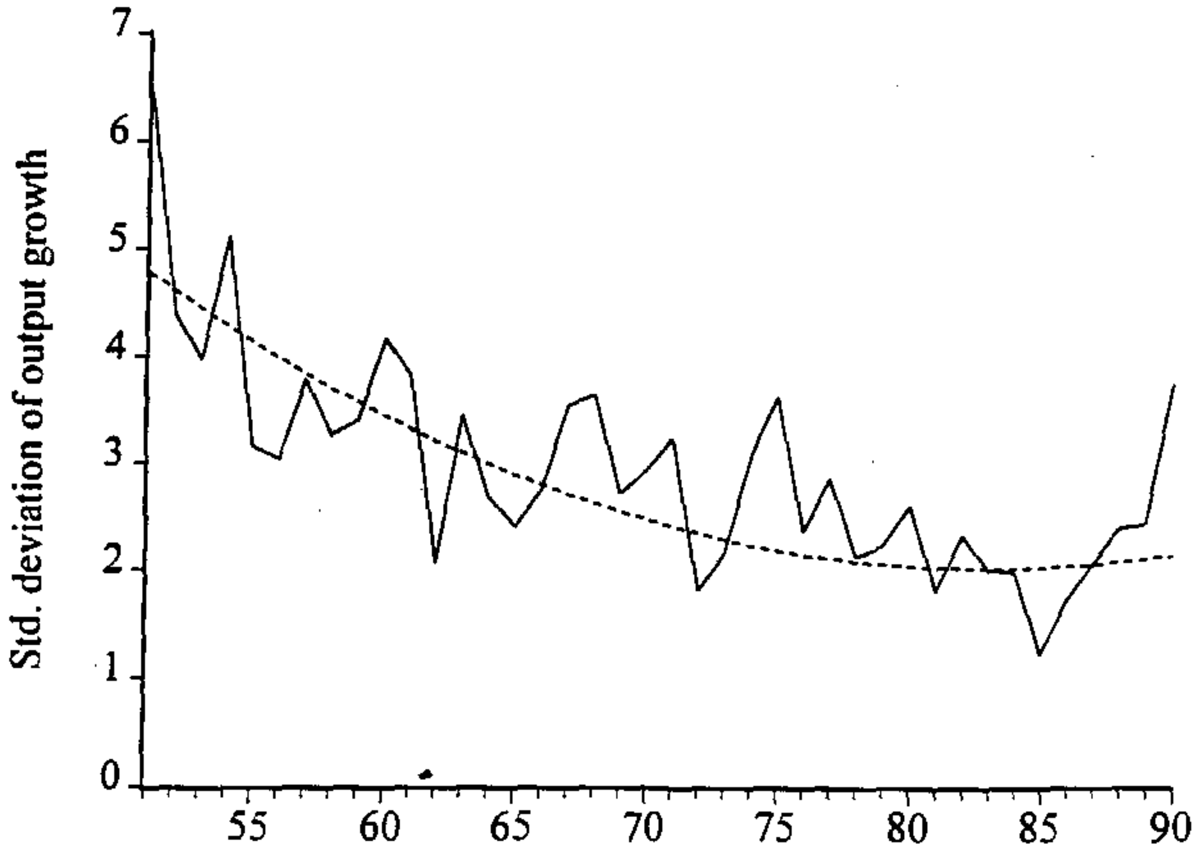

Fig. 6: The Dispersion of Growth Rates in a Sample of 20 Latin American Countries, 1951 - 90

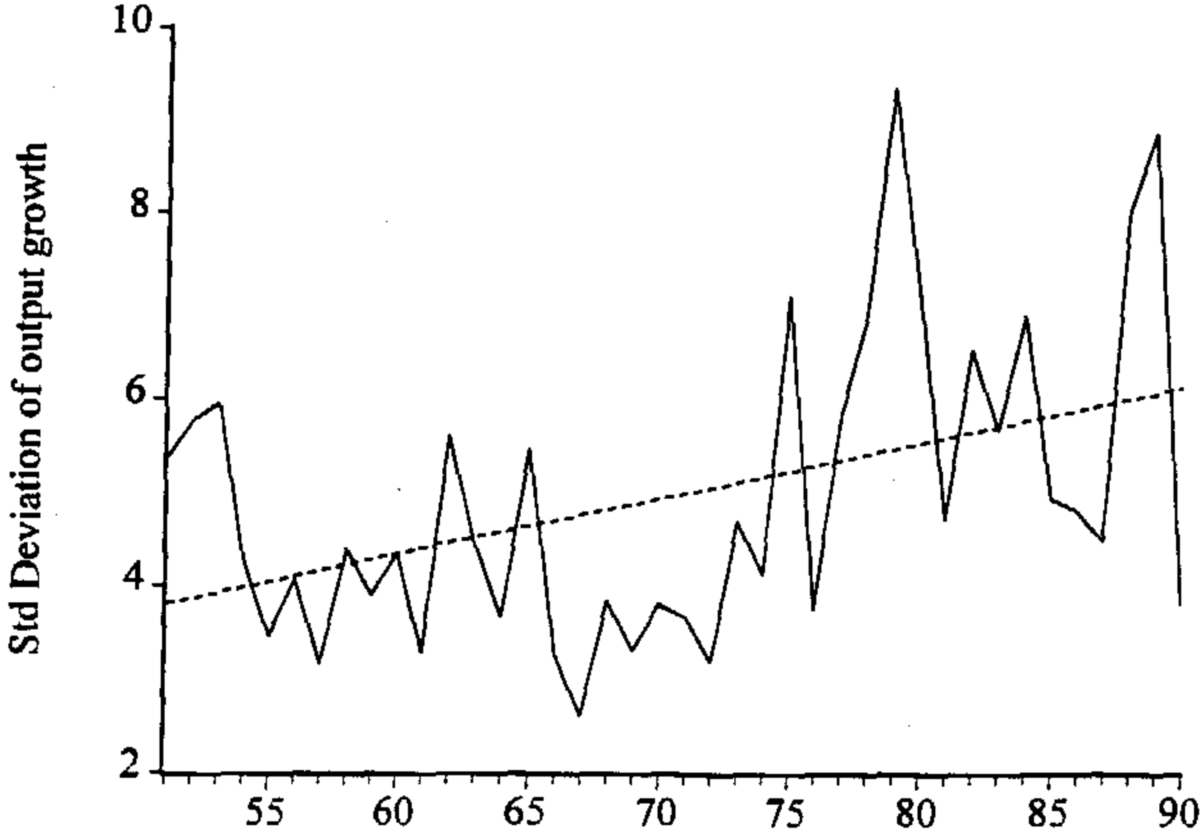

Fig. 7: The Dispersion - Time Relation is Significantly
Non Linear in the Latin American Sample, 1951 - 90