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NÚMERO 98

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**TEMPORARY STABILIZATION:
A STOCHASTIC ANALYSIS**

Abstract

Although much knowledge has been gained about temporary inflation stabilization from the experience of South America and the Middle East in the 1970s and 1980s, much remains to be learned. The Mexican episode of 1989-1994 brings the issue to our attention once again. The purpose of this paper is to extend Calvo's (1986) work on temporary stabilization into a richer stochastic environment. After all, what produces expected temporariness is uncertainty. It is assumed that, inside an adjustable band, the expected rate of devaluation is driven by a mixed diffusion-jump process. The paper provides an explanation of the response of consumption to stabilization plans under political uncertainty, as occurred in Mexico during 1989-1994. An important aspect of this paper is that Calvo's (1986) deterministic result on booms in consumption under temporary stabilization, in a cash-in-advance economy, is locally preserved despite stochastic considerations. However, in a Sidrauski-type stochastic model, we show that Pareto-Edgeworth independence does not lead to exchange-rate policy neutrality, contrary to Calvo's deterministic conclusion. We also assess welfare consequences, carry out simulation experiments, and examine the effects of stochastic stabilization horizons.

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Introduction

Even though very much knowledge has been gained on temporary stabilization from Argentina, Brazil, Chile, Uruguay, and Israel in the 1970s and 1980s¹, much still remains to be learned. The Mexican experience of 1989-1994 (up until the December 1994 financial debacle) brings the issue to our attention once again. Let us highlight some of the key features of the Mexican economy between 1989 and 1994. During this period Mexico had an intense trade opening combined with a high degree of capital mobility, while the government built up a large debt relying on a privatization process and a stabilization program. By the beginning of 1989, the government predetermined the rate of devaluation as a nominal anchor to stabilize inflation. From 1990 to 1992 there were several slight modifications to the original exchange-rate policy, including a modest increase in the rate of devaluation and the adoption of an adjustable band that was gradually widened keeping constant the lower limit. During 1993-1994, a climate of political uncertainty (risk) developed as a result of the following events: the Zapatista guerrilla uprising in Chiapas, the assassination of a presidential candidate, a weak "Pacto", a slowdown in exports, recommendations from specialists to devalue in order to ease an overvalued currency, an increase in the interest of US government bonds, and approaching elections. As a consequence, the intended exchange-rate policy to stabilize inflation was not expected to continue. The public was unsure about the government's commitment to defend the exchange-rate policy, and future devaluations, including devaluations, were expected. The anticipated temporariness of the stabilization plan resulted in an increase in imports that worsened considerably the existing current account deficit. In fact, in 1994, Mexico ran its worst trade deficit of the century. By the beginning of 1995, Mexico had abandoned the adjustable band system, and shifted to a managed floating exchange-rate regime².

Calvo's (1986) influential paper on temporary stabilization has been extended in various deterministic directions. For instance, Matsuyama (1991), and De Gregorio, Guidotti and Végh (1994) have included durable goods, Calvo and Végh (1993), and Reinhart and Végh (1993a, b) have considered imperfect credibility, Rebelo and Végh (1995) have proposed a unifying approach to assess real effects,

¹ For surveys on Argentina, Chile and Uruguay we direct the reader to the references contained in Obstfeld (1985). For studies on Israel, see Bufman and Leiderman (1995), and on Brazil Edwards (1994).

² For additional studies on Mexico, see Sachs, Tornell and Velasco (1996), Gil-Díaz and Carstens (1996), Calvo and Mendoza (1996a, b), Edwards (1996), Solís (1996), Feltestein and Shah (1995), Dornbusch, Goldfajn and Valdéz (1995), Dornbusch and Werner (1994), and Aspe (1993).

Roldos (1995) has analyzed supply-side effects of disinflation programs, and, more recently, Uribe (1997) has studied the initial real effects of credible plans. While this large theoretical and empirical literature has provided considerable insights into temporary stabilization, there has not been attempt to include stochastic considerations in the dynamics of the expected rate of devaluation. The purpose of this paper is to extend Calvo's work into a richer stochastic environment. After all, what produces expected temporariness is uncertainty. We think that a stochastic formulation is the appropriate one to study temporary stabilization plans. In our approach, the random variations in the expected rate of devaluation are modelled through a mixed diffusion-jump process; the Brownian component governs the devaluation rate, and the Poisson component drives possible devaluations. There is an increasing amount of literature in international economics using expected utility maximization with a diffusion-jump process driving the dynamics of the exchange rate. Examples are Svensson (1992), and Penati and Pennacchi (1989).

Our stochastic extension is, by itself, capable of dealing with a range of interesting issues. However, we restrict ourselves to study inflation stabilization plans in which, initially, not only the expected rate of devaluation is set at a low level, as in Calvo (1986), but also the inverse of volatility, the probability of devaluation, and the expected size of a devaluation are small. Subsequently, all of them are higher. We identify significant qualitative and quantitative deviations from Calvo's deterministic results on the effects of temporary stabilization. Our model is capable of reproducing stylized facts via simulation experiments as well as supplying an explanation of the response of consumption to stabilization plans for the Mexican economy during 1989-1994. Finally, our analysis also considers the impact of temporary stabilization on economic welfare, and examines the effects of stochastic stabilization horizons on consumption.

The structure of this paper is as follows: In the next section, we work out Calvo's Ramsey-type one-good, cash-in-advance, model with a logarithmic utility function under the additional assumption that, within an adjustable band, the expected rate of devaluation is driven by a mixed diffusion-jump process. We show that if the agent's time-preference rate equals the world's interest rate, there is no steady-state consumption behavior. Surprisingly, Calvo's deterministic result on the response of consumption to temporary stabilization is locally maintained in our stochastic analysis. We also examine the impact of temporary stabilization on welfare, carry out simulation experiments, and examine the effects of stochastic stabilization horizons. In section 3, we study the corresponding stochastic extension to Calvo's Sidrauski-type, one-good model. We show that Pareto-Edgeworth independence does not lead to exchange-rate policy neutrality. Therefore, Calvo's deterministic results on temporary stabilization are not preserved in this environment. Finally, in section 4, we give conclusions, acknowledge limitations, and make suggestions for further research. Three appendices contain some technical details.

2. A diffusion-jump process for the exchange rate

In order to generate solutions which are analytically tractable, the structure of the economy will be kept as simple as possible. Most of the assumptions have been borrowed from Calvo (1986). The main difference is that the consumer now perceives that, inside an adjustable band, the expected rate of devaluation follows a mixed diffusion-jump process within an adjustable band.

2.1 Prices and assets

Let us consider a small open economy with a single infinitely-lived consumer in a world with a single perishable consumption good. Suppose that the good is freely traded, and its domestic price level, P_t , is determined by the purchasing power parity condition, namely $P_t = P_t^* E_t$, where P_t^* is the foreign-currency price of the good in the rest of world, and E_t is the nominal exchange rate. Throughout the paper, we will assume, for the sake of simplicity, that P_t^* is equal to 1, which implies that the price level, P_t , is equal to the exchange rate, E_t . We also assume that the exchange-rate initial value, E_0 , is known and equal to 1.

We suppose that the number of expected devaluations (*i. e.*, jumps in the exchange rate) per unit of time follows a Poisson process q_t with intensity λ , so $\Pr\{\text{one unit jump during } dt\} = \Pr\{dq_t = 1\} = \lambda dt + o(dt)$, whereas³ $\Pr\{\text{no jump during } dt\} = \Pr\{dq_t = 0\} = 1 - \lambda dt + o(dt)$. Thus, $E[dq_t] = \lambda dt$.

Moreover, it is assumed that the exchange rate, E_t , is restricted to an adjustable band:

$$(1 + \nu)^{q_t} A_t \leq E_t \leq (1 + \nu)^{q_t} B_t, \quad (1)$$

where $1 + \nu$ is the mean expected size of upward jumps in the exchange rate, $q_0 = 0$, and A_t and B_t are the adjustment stochastic processes, linked with monetary policy, which satisfy $A_t < B_t$ with probability one.

Inside the band, the ongoing uncertainty in the dynamics of the expected rate of devaluation is generated by a mixed diffusion-jump process. We suppose that the consumer perceives that the expected inflation rate, dP_t/P_t , and thus the expected rate of the devaluation, dE_t/E_t , follows a geometric Brownian motion with Poisson jumps, more specifically

$$\frac{dP_t}{P_t} = \frac{dE_t}{E_t} = \epsilon dt + \sigma dz_t + \nu dq_t, \quad (2)$$

where the drift ϵ is the mean expected rate of devaluation conditional on no jumps, σ is the standard deviation or volatility of the expected rate of devaluation, and z_t is a standard Wiener process, that is, dz_t is a temporally independent normally distributed random variable with $E[dz_t] = 0$ and $\text{Var}[dz_t] = dt$. The processes dz_t and dq_t are

³ As customary, $o(h)$ means $o(h)/h \rightarrow 0$ as $h \rightarrow 0$.

assumed to be independent. In what follows, it will be convenient to introduce a new parameter, the inverse of the volatility, $\kappa = 1/\sigma$. Because of the specific interest of this paper on once-and-for-all changes in ϵ , κ , λ and ν , we will assume that all of them are (positive) constants. The normalized exchange rate, $e_t = E_t/(1 + \nu)^{q_t}$, inside the band $A_t \leq e_t \leq B_t$, will obey $de_t/e_t = (\epsilon dt + \sigma dz_t) - \nu dq_t$, from where

$$\frac{de_t}{e_t} = \epsilon dt + \sigma dz_t. \quad (3)$$

The consumer holds two real assets: real cash balances, $m_t = M_t/P_t$, where M_t is the nominal stock of money, and an international bond, k_t . The bond pays a constant real interest rate r (*i.e.*, pays r units of the consumption good per unit of time). Thus, the consumer's real wealth, a_t , is defined by

$$a_t = m_t + k_t, \quad (4)$$

where a_0 is exogenously determined. Furthermore, we suppose that the rest of the world does not hold domestic currency.

If capital is perfectly mobile, and dR_t/R_t is the expected nominal interest rate, then the real domestic interest rate, defined as $(dR_t/R_t) - (dE_t/E_t)$, must equal $r dt$ at any instant dt . Consequently,

$$\frac{dR_t}{R_t} = i dt + \sigma dz_t + \nu dq_t,$$

where

$$i = r + \epsilon \quad (5)$$

is the mean expected nominal interest rate conditional on no jumps.

Consider a cash-in-advance constraint of the Clower-Lucas-Feenstra form, $m_t \geq \alpha c_t$, where c_t is consumption, and $\alpha > 0$ is the time that money must be held to finance consumption. Given that $i > 0$, the cash-in-advance constraint will hold with equality,

$$m_t = \alpha c_t. \quad (6)$$

Condition (6) is critical in linking exchange-rate policy and consumption. In such a case, devaluation acts as a consumption tax.

Inside the band, the stochastic rate of return of holding real cash balances, dr_m , is simply the percentage change in the inverse of the price level. By applying the generalized Itô's lemma for diffusion-jump processes to the inverse of the price level, with (2) as the underlying process (see Appendix C, formula (C.1)), we obtain

$$dr_m = P_t d\left(\frac{1}{P_t}\right) = (-\epsilon + \sigma^2)dt - \sigma dz_t + \left(\frac{1}{1 + \nu} - 1\right)dq_t. \quad (7)$$

In the rest of the paper, we will assume that⁴ $\epsilon \geq \sigma^2$.

2.2 The consumer's choice problem

The stochastic consumer's wealth accumulation in terms of the portfolio shares, $w_t = m_t/a_t$, $1 - w_t = k_t/a_t$, and consumption, c_t , is given by⁵

$$da_t = a_t w_t dR_m + a_t(1 - w_t)r dt - c_t dt, \quad (8)$$

with a_0 exogenously determined. By substituting (5), (6) and (7) into (8), we get

$$da_t = a_t \left[(r - \rho w_t) dt - w_t \sigma dz_t + \left(\frac{1 + \nu(1 - w_t)}{1 + \nu} - 1 \right) dq_t \right], \quad (9)$$

where $\rho = \alpha^{-1} + i - \sigma^2$.

The overall discounted expected utility at time $t = 0$, V_0 , of the competitive risk-averse consumer is assumed to have the time-separable form:

$$V_0 = E_0 \int_0^\infty \log(c_t) e^{-rt} dt, \quad (10)$$

where E_0 is the conditional expectation on all available information at $t = 0$. Notice that the agent's time-preference rate has been set equal to the constant real international rate of interest, r . In order to generate closed-form solutions and make the analysis manageable, we have chosen the logarithmic utility function.

In maximizing (10), with $\log(c_t) = \log(\alpha^{-1} a_t w_t)$, subject to the wealth constraint as given in (9), the first-order condition for an interior solution is (see Appendix A)

$$\frac{r}{w} - \frac{\lambda \nu}{1 + \nu(1 - w)} = \rho + w \sigma^2. \quad (11)$$

which is a cubic equation with one negative and two positive roots, and only one root satisfying $0 < w^* < 1$. To see this graphically, let us define the left-hand side of (11) by $\varphi(w)$. The function $\varphi(w)$ has the following properties: $\varphi(0^+) = +\infty$, $\varphi(0^-) = -\infty$, $\varphi(1) = r - \lambda \nu$,

$$\varphi\left(\frac{r}{r + \lambda} \left(1 + \frac{1}{\nu}\right)\right) = 0, \quad \varphi\left(\left(1 + \frac{1}{\nu}\right)^+\right) = -\infty, \quad \varphi\left(\left(1 + \frac{1}{\nu}\right)^-\right) = \infty, \quad \varphi(+\infty) = 0,$$

and $\varphi(-\infty) = 0$. With this information on hand we sketch the graph of $\varphi(w)$ in Fig. 1. From the definition of ρ , it is straightforward to see that $\rho > r - \lambda \nu$, as shown in

⁴ This study is only concerned with small values of σ , as observed in the Mexican case of 1989-1994.

⁵ To avoid unnecessary technical complications, the real wage is not included in (8), however, it will be considered in the extension developed in section 3.

Fig. 1. The straight line $\rho + w\sigma^2$, defined by the right-hand side of (11), intersects the graph of $\varphi(w)$ three times. The middle intersection defines the steady-state share of wealth set aside for consumption $w^* \in (0, 1)$. Figure 1 depicts the determination of the optimal⁶ w^* for the case $r - \lambda\nu > 0$. The reader can infer from Fig. 1 that if $0 > r - \lambda\nu = \varphi(1)$, w^* will still be in $(0, 1)$.

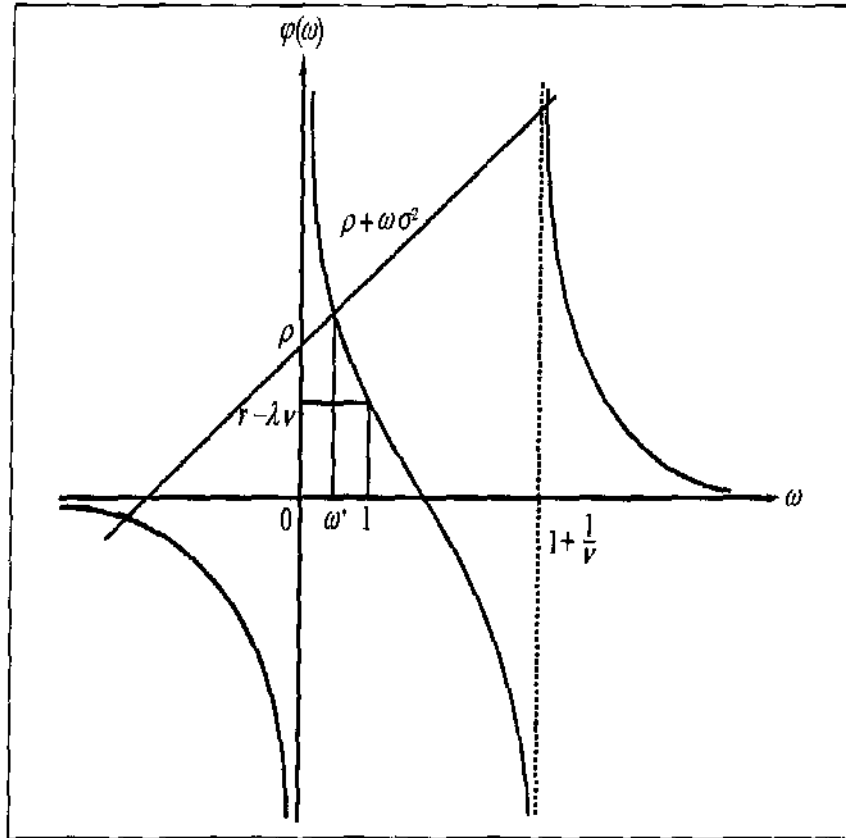


Fig. 1. Determination of optimal w^* .

We are now in a position to derive the first important result: A once-and-for-all increase in the rate of devaluation, which results in an increase in the future opportunity cost of purchasing goods, leads to a permanent decrease in the proportion of wealth devoted to future consumption. Indeed, a once-and-for-all increase in ϵ , from ϵ_1 to ϵ_2 , will lead to $w_1^* > w_2^*$. To see this, it is enough to observe that $\epsilon_1 < \epsilon_2$ implies $\rho_1 < \rho_2$, which shifts the line $\rho + w\sigma^2$ upward, reducing the equilibrium value of w^* , as depicted in Figure 2. Alternatively, we may differentiate (11) to find that

$$\frac{\partial w^*}{\partial \epsilon} = -\Lambda,$$

⁶ A negative w implies negative consumption which has no meaning in the context of this problem. A w greater than one is ruled out since the consumer has incentive to minimize his holdings of real money balances.

where

$$\Lambda = \left[\frac{r^2}{(w^*)^2} + \frac{\lambda\nu^2}{[1 + \nu(1 - w^*)]^2} + \sigma^2 \right]^{-1} > 0.$$

A once-and-for-all increase in the inverse of the volatility, κ , will produce a similar effect to that of ϵ on w^* since

$$\frac{\partial w^*}{\partial \sigma^2} = (1 - w^*)\Lambda > 0$$

and

$$\frac{\partial w^*}{\partial \kappa} = -\frac{\partial w^*}{\partial \sigma^2} \frac{2\sigma}{\kappa^2} < 0.$$

In this case, the increase in ρ , due to the decrease in σ^2 , shifts the line $\rho + w\sigma^2$ upward, and at the same time $\rho + w\sigma^2$ becomes flatter since σ^2 also affects its slope. Nonetheless, w^* decreases.

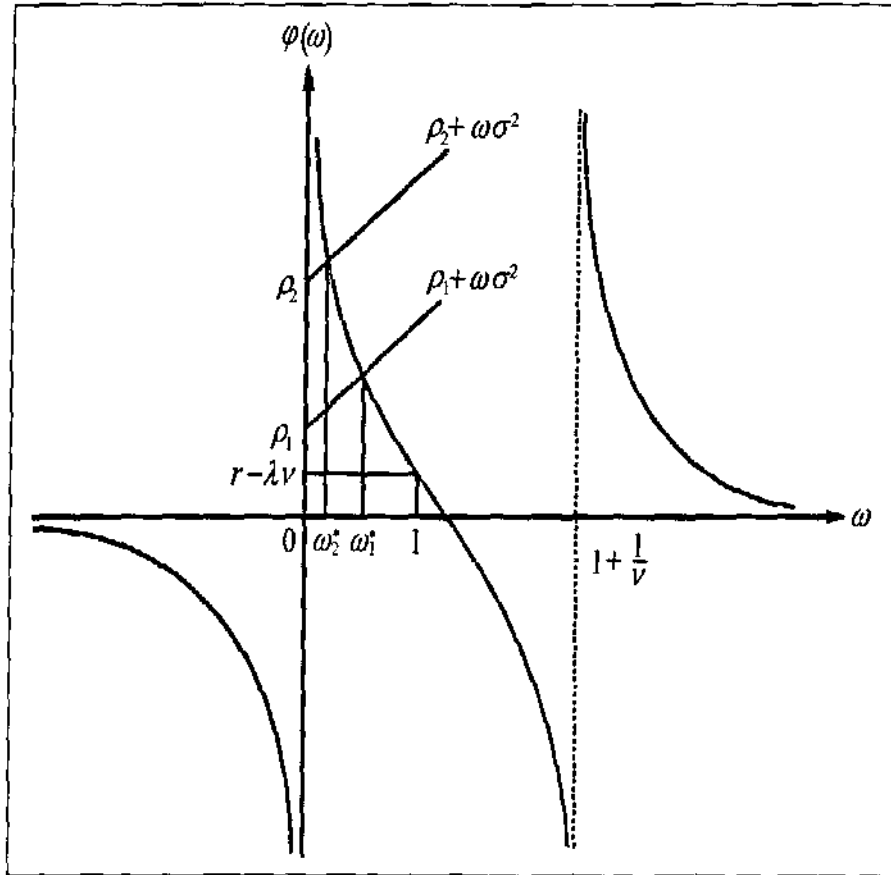


Fig. 2. Response of w^* to once-and-for-all changes in ϵ .

Another relevant result is the response of the equilibrium share of real monetary balances, w^* , to once-and-for-all changes in the intensity parameter, λ : A once-and-for-all increase in the expected number of devaluations per unit of time causes an

increase in the future opportunity cost of purchasing goods. This in turn decreases permanently the proportion of wealth set aside for future consumption. In other words, an increase in λ , from λ_1 to λ_2 , will shift $\varphi(w)$ downward, decreasing the value of w^* , as depicted in Fig. 3. Similarly, a once-and-for-all increase in the expected size of upward jumps in the exchange rate, ν , will also shift downward $\varphi(w)$, resulting in a reduction of w^* . Alternatively, differentiating (11), we get

$$\frac{\partial w^*}{\partial \lambda} = -\frac{\nu \Lambda}{1 + \nu(1 - w^*)} < 0$$

and

$$\frac{\partial w^*}{\partial \nu} = -\frac{\lambda \Lambda}{[1 + \nu(1 - w^*)]^2} < 0.$$

Moreover, under logarithmic utility the corresponding elasticities maintain the same magnitude,

$$\frac{\lambda}{w^*} \frac{\partial w^*}{\partial \lambda} = \frac{\nu}{w^*} \frac{\partial w^*}{\partial \nu}.$$

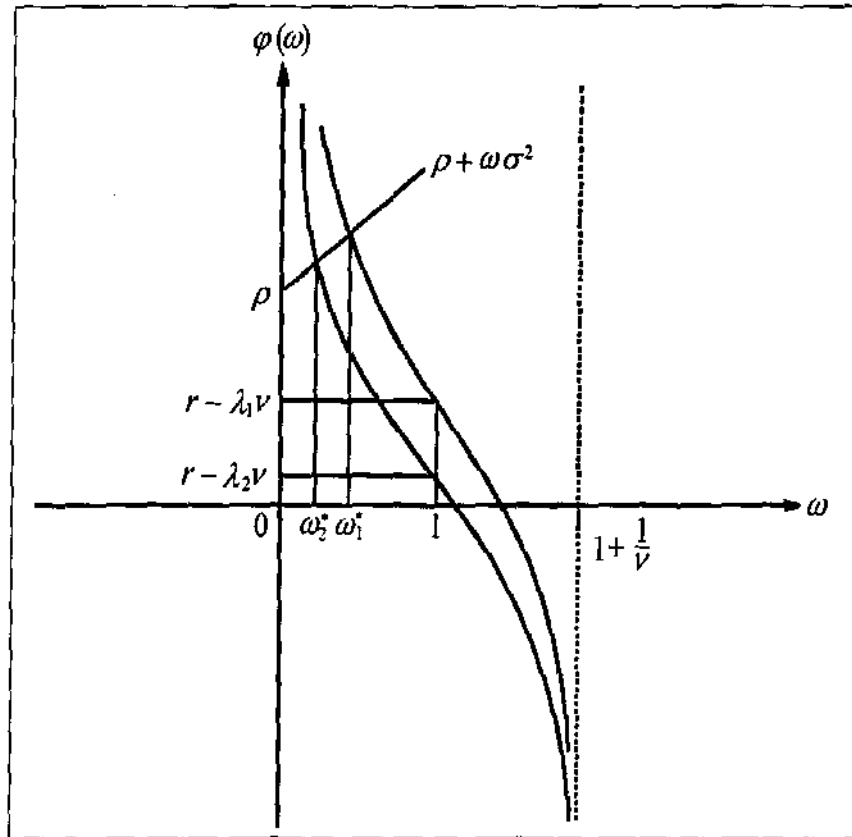


Fig. 3. Response of w^* to once-and-for-all changes in λ .

2.3 Welfare implications

The effects of exogenous shocks on economic welfare will be assessed now. As usual, the welfare criterion, W , of the representative individual is the maximized utility starting from the initial real wealth, a_0 . Therefore, from (A.3), welfare is given by

$$W(\epsilon, \kappa, \lambda, \nu; a_0) = \frac{1}{r} [1 + \log(a_0) + \log(\alpha^{-1}w^*)] - \frac{1}{r^2} \left[\rho w^* + \frac{1}{2}(w^*\sigma)^2 - \lambda \log\left(\frac{1 + \nu(1 + w^*)}{1 + \nu}\right) \right].$$

Table 1 shows the magnitudes of the impacts on welfare of oncc-and-for-all changes in the mean expected rate of devaluation, the inverse of volatility, the probability of devaluation, and the expected size of a devaluation. As it might be expected, welfare behaves as a decreasing function of all the important parameters in our stochastic approach. As before, the critical assumption of logarithmic utility accounts for the results.

increase in	effect on welfare
ϵ	$-\frac{w}{r^2} + \frac{\lambda}{r} [w - 1] < 0$
κ	$-\frac{1}{r^2} [w(1 - \frac{1}{2}w)] \frac{2\sigma}{\kappa^2} < 0$
λ	$\frac{1}{r^2} \log\left(\frac{1 + \nu(1 - w)}{1 + \nu}\right) < 0$
ν	$-\frac{1}{r^2} \left[\frac{\lambda w}{(1 + \nu)(1 + \nu(1 - w))} \right] < 0$

Table 1. Effects of policy changes on economic welfare.

2.4 Wealth and consumption

The stochastic process that generates wealth when the optimal rule is applied will be now derived. After substituting the optimal share w^* into (9), we get

$$da_t = a_t \left[\left(\frac{\lambda \nu w^*}{1 + \nu(1 - w^*)} + (w^*\sigma)^2 \right) dt - w^*\sigma dz_t + \left(\frac{1 + \nu(1 - w^*)}{1 + \nu} - 1 \right) dq_t \right]. \quad (12)$$

The solution to the above stochastic differential equation, conditional on a_0 , is (see Appendix C, formula (C.2))

$$a_t = a_0 e^{\delta_t}, \quad (13)$$

where⁷

$$\delta_t = \eta_t + \gamma_t, \quad \eta_t \sim \mathcal{N}[F(w^*)t, G(w^*)t], \quad \gamma_t = H(w^*)q_t, \quad \text{and} \quad q_t \sim \mathcal{P}(\lambda t), \quad (14)$$

and the stationary components of the parameters of the above distributions are:

$$F(w^*) = \frac{\lambda \nu w^*}{1 + \nu(1 - w^*)} + \frac{(w^* \sigma)^2}{2}, \quad G(w^*) = (w^* \sigma)^2, \quad \text{and}$$

$$H(w^*) = \log\left(\frac{1 + \nu(1 - w^*)}{1 + \nu}\right).$$

Notice that $F(w^*) > 0$ and $H(w^*) < 0$ always hold. From (13), the stochastic process for consumption can be written as

$$c_t^* = \alpha^{-1} w^* a_0 e^{\delta_t}. \quad (15)$$

In the stochastic framework, we cannot determine the level of consumption; only the probability that, at a given time, certain level of consumption occurs. It is also important to note that the assumption that the agent's time-preference rate is equal to the world's interest rate does not ensure a steady-state level of consumption. However, we do have a steady-state share of wealth set apart for consumption.

It is worth revisiting the deterministic case (*i.e.*, $\sigma = \lambda = 0$) with logarithmic utility. In such a case, it can be shown that

$$c^{**} = \frac{r a_0}{1 + \alpha(r + \epsilon)}. \quad (16)$$

In comparing (15) and (16), we observe that c^{**} is the steady-state level of consumption in the deterministic case, whereas c_t^* shows a dynamic behavior in the stochastic case, even if the rate of devaluation were expected to remain fixed forever.

2.5 Stabilization programs

Next, we will analyze a stabilization policy of the form:

$$(\epsilon_t, \kappa_t, \lambda_t, \nu_t) = \begin{cases} (\epsilon_1, \kappa_1, \lambda_1, \nu_1) & \text{for } 0 \leq t \leq T, \\ (\epsilon_2, \kappa_2, \lambda_2, \nu_2) & \text{for } t > T, \end{cases} \quad (17)$$

⁷ As customary, $x \sim \mathcal{N}(a, b)$ denotes a random variable x normally distributed with mean a and variance b , and $x \sim \mathcal{P}(a)$ denotes a Poisson random variable x with mean a .

where T is exogenously determined, $\epsilon_1 < \epsilon_2$, $\kappa_1 < \kappa_2$, $\lambda_1 < \lambda_2$, and $\nu_1 < \nu_2$. That is, we are concerned with inflation stabilization plans in which, initially, not only ϵ is set at a low level, as in Calvo (1986), but also the other key parameters, in the stochastic framework, are small. We now examine the response of consumption to such a stabilization policy. From (15), (17) and the separability of the utility function, the consumption process satisfies:

$$c_t^* = \begin{cases} \alpha^{-1} w_1^* a_0 e^{\delta_t^{(1)}}, & \text{for } 0 \leq t \leq T, \\ \alpha^{-1} w_2^* a_T e^{\delta_{t-T}^{(2)}}, & \text{for } t > T, \end{cases} \quad (18)$$

$$(19)$$

where the superindex “(j)” is related to the optimal portfolio share w_j^* associated to the vector of parameters $(\epsilon_j, \kappa_j, \lambda_j, \nu_j)$, $j = 1, 2$, and

$$a_T = a_0 e^{\delta_T^{(1)}}. \quad (20)$$

It is also important to point out, from (14), that a once-and-for-all increase in $(\epsilon, \kappa, \lambda, \nu)$ modifies the stationary components of the parameters of the distributions of η_τ and γ_τ , and consequently those of δ_τ .

In virtue of (18), consumption at time T satisfies:

$$c_T^* = \alpha^{-1} w_1^* a_T. \quad (21)$$

Substituting (21) into (19) with $t = T + \Delta$, $\Delta > 0$, we obtain

$$c_{T+\Delta}^* = c_T^* \frac{w_2^*}{w_1^*} e^{\delta_\Delta^{(2)}}.$$

Notice that $e^{\delta_\Delta^{(2)}}$ tends to 1 as $\Delta \rightarrow 0$ a.s.(almost surely), thus

$$\lim_{\Delta \rightarrow 0^+} c_{T+\Delta}^* = c_T^* w_2^* / w_1^* \quad \text{a.s.} \quad (22)$$

Though the stationary components of the parameters of η_τ and γ_τ are different before and after time T , such a distinction becomes negligible when $\Delta \rightarrow 0$. We also notice that $w_2^* / w_1^* < 1$, together with (22), imply $c_T^* > \lim_{\Delta \rightarrow 0^+} c_{T+\Delta}^*$ a.s., indicating a jump (boom) in consumption at time T .

If all the parameters were to be constant forever, *i.e.*, if $(\epsilon_t, \kappa_t, \lambda_t, \nu_t) = (\epsilon_2, \kappa_2, \lambda_2, \nu_2)$ for all $t \geq 0$, then we would have

$$c_{t+\Delta}^* = c_t^* e^{\delta_\Delta^{(2)}}. \quad (23)$$

On the right-hand side of (23), the factor $e^{\delta_\Delta^{(2)}} \rightarrow 1$ as $\Delta \rightarrow 0$ a.s., thus consumption would be continuous a.s. for all t . In other words, if the plan is expected to be

temporary, then, as we have shown above, $c_T^* > \lim_{\Delta \rightarrow 0^+} c_{T+\Delta}^*$ a.s., indicating a jump in consumption at T . Therefore, Calvo's (1986) deterministic result on the response of consumption to temporary stabilization is locally maintained (around T) in the stochastic model.

2.6 Simulation exercise

The following experiment is intended to simulate the response of consumption to permanent increases 1) in the mean expected rate of devaluation of 10%, 2) in the inverse of volatility of the exchange rate of 90%, 3) in the probability of devaluation of 10%, and 4) in the expected size of a devaluation of 10%. Table 2 presents a couple of feasible⁸ vectors $(\epsilon_j, \kappa_j, \lambda_j, \nu_j)$, $j = 1, 2$, and, for each case, the optimal share of wealth set apart for consumption and the stationary components of the distribution of δ_t when $r = \alpha = 1$ are computed.

$(\epsilon_1, \kappa_1, \lambda_1, \nu_1) = (0.2, 10, 0.01, 0.2)$	$(\epsilon_2, \kappa_2, \lambda_2, \nu_2) = (0.3, 100, 0.1, 0.3)$
$w_1^* = 0.455299$	$w_2^* = 0.430004$
$F(w_1^*) = 0.001858, G(w_1^*) = 0.002073$	$F(w_2^*) = 0.011026, G(w_2^*) = 0.000019$
$H(w_1^*) = -0.000343$	$H(w_2^*) = -0.004539$

Table 2. Optimal consumption shares in a temporary stabilization plan.

In Figure 4, the solid line represents the simulated⁹ consumption with suitable choices of T and u_0 , and the dashed line corresponds to the observed Mexico's current account deficit. The simulation reproduces the observed positive correlation between the trends of consumption and the current account deficit. Also, the simulated consumption mimics the jump in the current account deficit as observed at the beginning of 1993.

⁸ In order to choose a pair of vectors capable of replicating stylized facts, we tried about 800 different plausible combinations of parameter values.

⁹ For simulation purposes, we have used a discrete-time version of (15) with an appropriate unit of time, and generated independent random numbers drawn from both the normal and Poisson distributions defined by (14).

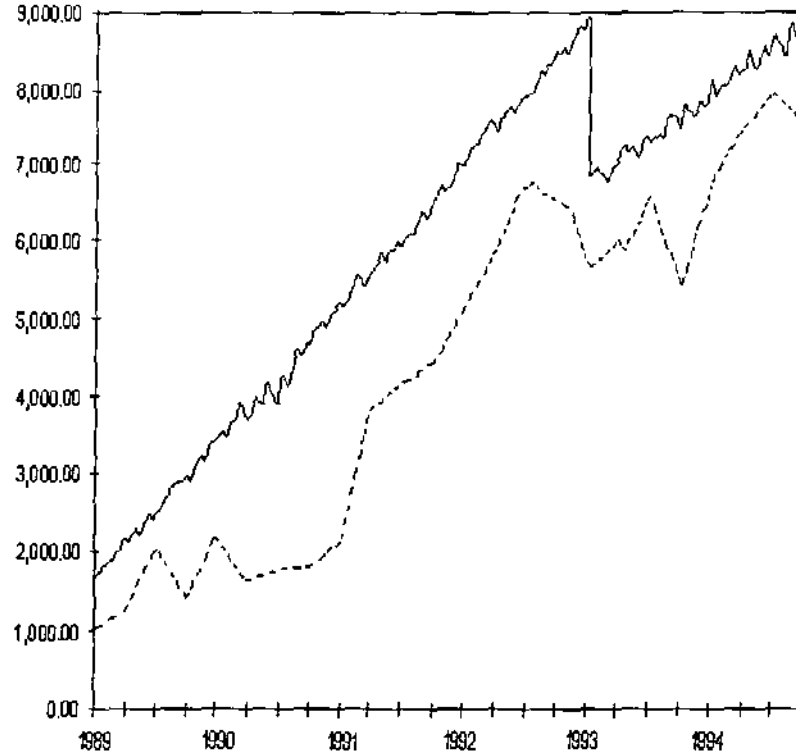


Fig. 4. Simulated consumption and observed current account deficit.
(millions of dollars of 1980)

2.7 Stochastic stabilization horizons

Let T be an exponentially distributed random variable with parameter \bar{T}^{-1} . In such a case, a higher parameter value will increase the probability of shortening the horizon of the stabilization program. Suppose that T is independent of dq_t and dz_t . Then, from (14) and (15), we find that

$$E[\log(c_T)] = E[E[\log(c_T)|T]] = \log(\alpha^{-1}w_1^*a_0) + [F(w_1^*) + H(w_1^*)\lambda]\bar{T}.$$

Therefore,

$$\frac{\partial E[\log(c_T)]}{\partial \bar{T}} = F(w_1^*) + H(w_1^*)\lambda.$$

The above effect is unambiguously positive. Indeed, notice that $x - 1 - \log(x) \geq 0$ holds for all $x > 0$, then

$$\frac{\nu w_1^*}{1 + \nu(1 - w_1^*)} - \log\left(\frac{1 + \nu}{1 + \nu(1 - w_1^*)}\right) \geq 0,$$

and so $F(w_1^*) + H(w_1^*)\lambda > 0$. Consequently, a lower mean stabilization horizon will imply a lower mean growth rate of consumption.

While Calvo' (1986) deterministic study analyzes the response of the level of consumption to a shortening in the stabilization horizon, our analysis has focused on the response of the mean growth rate of consumption to an increase in the probability of shortening the horizon of the stabilization program.

3. Money in the utility function

We now include currency directly in the utility function. In such a case, the stochastic wealth accumulation in terms of the portfolio shares and consumption becomes:

$$da_t = a_t w_t dR_m + a_t(1 - w_t)r dt - c_t dt + y_t dt, \quad (24)$$

where y_t is the real wage. By substituting (7) into (24), we obtain

$$da_t = a_t \left[(r - \phi w_t) dt - w_t \sigma dz_t + \left(\frac{1 + \nu(1 - w_t)}{1 + \nu} - 1 \right) dq_t \right] - c_t dt + y_t dt, \quad (25)$$

where $\phi = i - \sigma^2$.

The expected utility at time $t = 0$, V_0 , now takes the form:

$$V_0 = E_0 \int_0^\infty [\theta \log(c_t) + \log(m_t)] e^{-rt} dt. \quad (26)$$

We have chosen again the logarithmic utility function to generate closed-form solutions. Notice that consumption and real money balances are Pareto-Edgeworth independent. The first-order conditions for an interior solution in maximizing (26), subject to (25), are given by (see Appendix B)

$$c_t = \frac{\theta r}{1 + \theta} a_t \quad \text{and} \quad \frac{r}{(1 + \theta)w} - \frac{\lambda \nu}{1 + \nu(1 - w)} = \phi + w \sigma^2. \quad (27)$$

The second equation above is similar to that in (11), except for the factor $1/(1 + \theta)$ that now appears in the first term of the left-hand side of (27). It is also trivially true that $\phi > r(1 + \theta)^{-1} - \lambda \nu$. Even though w has now a different meaning, the same analysis of the previous section for optimal w^* can be entirely applied.

3.1 Wealth and consumption

To obtain the stochastic process that generates wealth, we substitute the optimal $w^* \in (0, 1)$ and c_t^* into (25), finding

$$da_t = a_t \left[\left(\frac{\lambda \nu w^*}{1 + \nu(1 - w^*)} + (w^* \sigma)^2 \right) dt - w^* \sigma dz_t + \left(\frac{1 + \nu(1 - w^*)}{1 + \nu} - 1 \right) dq_t \right] + y_t dt. \quad (28)$$

The solution to (28) conditional on a_0 is (see Appendix C, formula (C.2))

$$a_t = a_0 e^{\delta t} + \int_0^t y_s e^{\delta(t-s)} ds, \quad (29)$$

where $\delta_{t-s} = \eta_{t-s} + \gamma_{t-s}$,

$$\begin{aligned} \eta_{t-s} &\sim \mathcal{N}[F(w^*)(t-s), G(w^*)(t-s)], \quad 0 \leq s \leq t, \\ \gamma_{t-s} &= H(w^*)q_{t-s}, \quad q_{t-s} \sim \mathcal{P}[\lambda(t-s)], \quad 0 \leq s \leq t, \end{aligned}$$

and the stationary components of the parameters of the distributions are taken as in (14). Hence, the consumption stochastic process is given by

$$c_t^* = \frac{\theta r}{1 + \theta} \left(a_0 e^{\delta t} + \int_0^t y_s e^{\delta(t-s)} ds \right). \quad (30)$$

We also find in this extension that assuming that the agent's time-preference rate equals the world's interest rate does not ensure a steady-state level of consumption. Moreover, we see that Pareto-Edgeworth independence does not lead to exchange-rate policy neutrality since c_t^* depends on a_t , which, in turn, depends explicitly on the exchange rate, ϵ .

It is worthwhile to revisit again the deterministic case (*i.e.*, $\sigma = \lambda = 0$). In such a case, it is found that

$$c^{**} = \frac{\theta r}{1 + \theta} \left(a_0 + \int_0^\infty y_s e^{-rs} ds \right). \quad (31)$$

In comparing (30) and (31), we observe that c^{**} is the steady-state consumption in the deterministic case, and does not depend on the rate of devaluation ϵ ; however, c_t^* is dynamic and depends on ϵ . We summarize the contrasting differences in Table 3.

Deterministic	Stochastic
c^{**} is the steady state	c_t^* is dynamic
exchange-rate policy is neutral	exchange-rate policy is not neutral

Table 3. Significant deviations from the deterministic case.

3.2 Stabilization programs

Now let us analyze a stabilization policy of the form given in (17). From (30) and (17), the consumption process satisfies

$$c_t^* = \begin{cases} \frac{\theta r}{(1 + \theta)} \left(a_0 e^{\delta t^{(1)}} + \int_0^t y_s e^{\delta(t-s)} ds \right), & \text{for } 0 \leq t \leq T, \\ \frac{\theta r}{(1 + \theta)} \left(a_T e^{\delta(t-T)} + \int_T^t y_s e^{\delta(t-s)} ds \right), & \text{for } t > T, \end{cases} \quad (32)$$

$$\quad (33)$$

where, as before, the superindex “(j)”, $j = 1, 2$, is associated with the optimal portfolio share w_j^* when the rate of devaluation takes the value ϵ_j , and

$$a_T = a_0 e^{\delta_t^{(1)}} + \int_0^T y_s e^{\delta_{t-s}^{(1)}} ds. \quad (34)$$

We notice that a once-and-for-all increase in $(\epsilon, \kappa, \lambda, \nu)$ affects the stationary components of the parameters of the distributions of η_τ and γ_τ , as in previous results.

By means of (32), consumption at time T is

$$c_T^* = \frac{\theta r}{(1 + \theta)} a_T. \quad (35)$$

Substituting (35) into (33), with $t = T + \Delta$, $\Delta > 0$, we get

$$c_{T+\Delta}^* = c_T^* \left(\frac{a_T e^{\delta_\Delta^{(2)}} + \int_T^{T+\Delta} y_s e^{\delta_{\Delta-s}^{(2)}} ds}{a_T} \right). \quad (36)$$

The term inside the parentheses in (36) goes to 1 as $\Delta \rightarrow 0$ a.s. Thus, $\lim_{\Delta \rightarrow 0^+} c_{T+\Delta}^* = c_T^*$ a.s., that is, consumption is continuous a.s. at time T .

If all the parameters were to be constant forever, *i.e.*, if $(\epsilon_t, \kappa_t, \lambda_t, \nu_t) = (\epsilon_2, \kappa_2, \lambda_2, \nu_2)$ for $t \geq 0$, then we would get

$$c_{t+\Delta}^* = c_t^* e^{\delta_\Delta^{(2)}} + \frac{\theta r}{(1 + \theta)} \int_t^{t+\Delta} y_s e^{\delta_{\Delta-s}^{(2)}} ds. \quad (37)$$

Since the second term of the right-hand side of (37) goes to 0 as $\Delta \rightarrow 0$ a.s., consumption would be continuous a.s. for all t . Hence, if the plan is expected to be temporary, then c_t^* is also continuous a.s. for all t , but with different stationary parts of the distributional parameters before and after time T . Although there is no jump, the exchange-rate policy is by no means neutral regarding consumption.

4. Summary and concluding remarks

We have studied a small open stochastic economy in order to examine the response of consumption to temporal stabilization policies in which, initially, not only the rate of devaluation is set at a low level, but also the other key parameters are small. We have also considered the impact of temporary stabilization on economic welfare, carried out simulation experiments to reproduce stylized facts, and examined the effects of stochastic stabilization horizons on consumption.

The paper provides an explanation of the response of consumption to stabilization plans under uncertainty, as observed in the Mexican case of 1989-1994 when the

public was unsure about any government commitment to defend the exchange-rate policy. The broad message of this paper, although only demonstrated for a specific form of utility index, is that the use of the rate of devaluation as the nominal anchor in inflation stabilization programs which are expected to be temporary due to uncertainty, may lead to unsustainable current account deficits.

Using a logarithmic utility function, and a mixed diffusion-jump process for the expected exchange-rate dynamics, we have seen significant qualitative and quantitative differences between Calvo's (1986) deterministic results and our stochastic modelling: 1) If money is needed to finance consumption, equality between the agent's time-preference rate and the world's interest rate did not lead to steady-state behavior of domestic consumption, and 2) If consumption and money are Pareto-Edgeworth independent, then exchange-rate neutrality is not maintained. However, an important aspect of the paper was to show that Calvo's (1986) deterministic result on the response of consumption to temporary stabilization in a cash-in-advance economy, is locally preserved despite stochastic considerations.

It is important to point out that the obtained results depend strongly on the assumption of logarithmic utility; a limit case of the family of constant relative risk aversion utility functions. The extension of our stochastic analysis to such a family it is not straightforward because of technical difficulties introduced by the jump component in the exchange-rate process. In such a case, results might only be obtained via numerical methods. We have left steady-state distributions out of our analysis since they involve considerable computational problems because of the jump component. Needless to say, further work is needed in the above aspects.

Finally, we believe that more research should be undertaken in this stochastic framework to include government transfers and a stochastic budget constraint for the government (in a full general equilibrium), and to extend the analysis to both nontradable and durable goods.

Appendix A

The Hamilton-Jacobi-Bellman equation for the stochastic optimal control problem of maximizing (8), with $\log(c_t) = \log(\alpha^{-1}a_t w_t)$, subject to (7) is:

$$\begin{aligned} \max_w H(w_t; a_t, t) \equiv \max_w \left\{ \log(\alpha^{-1}a_t w_t)e^{-rt} + I_a(a_t, t)u_t(r - \rho w_t) + I_t(a_t, t) \right. \\ \left. + \frac{1}{2}I_{aa}(a_t, t)a_t^2 w_t^2 \sigma^2 \right. \\ \left. + \lambda \left[I\left(a_t \left(\frac{1 + \nu(1 - w_t)}{1 + \nu} \right), t\right) - I(a_t, t) \right] \right\} = 0, \end{aligned} \quad (\text{A.1})$$

where

$$I(a_t, t) = \max_w E_t \int_t^\infty \log(\alpha^{-1}a_t w_t) e^{-rs} ds$$

is the agent's indirect utility function—or welfare function—and $I_a(a_t, t)$ is the co-state variable. The first-order condition for an interior solution is:

$$H_w = 0. \quad (\text{A.2})$$

Given the exponential time discounting in (8), we postulate $I(a_t, t)$ in a time-separable form as

$$I(a_t, t) = e^{-rt}[\beta_1 \log(a_t) + \beta_0], \quad (\text{A.3})$$

where β_0 and β_1 are to be determined from (A.1). Substituting (A.3) into (A.1), and then computing the first-order conditions in (A.2), we find that $w_t \equiv w$ is time invariant and

$$\frac{1}{\beta_1 w} - \frac{\lambda \nu}{1 + \nu(1 - w)} = \rho + w\sigma^2. \quad (\text{A.4})$$

The coefficients β_0 and β_1 are determined from (A.1). Thus, $\beta_1 = r^{-1}$ and¹⁰

$$\beta_0 = \frac{1}{r} [1 + \log(\alpha^{-1} w^*)] - \frac{1}{r^2} \left[\rho w^* + \frac{1}{2} (w^* \sigma)^2 - \lambda \log \left(\frac{1 + \nu(1 + w^*)}{1 + \nu} \right) \right].$$

Equation (A.4) is cubic with one negative and two positive roots, and only one root satisfying $0 < w^* < 1$, as sketched in Fig. 1.

Appendix B

The Hamilton-Jacobi-Bellman equation for the stochastic optimal control problem of maximizing (24) subject to (23), and control $\hat{c}_t = c_t - y_t$ is:

$$\begin{aligned} \max_{\hat{c}, w} H(\hat{c}_t, w_t; a_t, t) \equiv \max_{\hat{c}, w} & \left\{ [\theta \log(\hat{c}_t + y_t) + \log(a_t w_t)] e^{-rt} + I_a(a_t, t) a_t (r - \phi w_t) \right. \\ & - I_a(a_t, t) \hat{c}_t + I_t(a_t, t) + \frac{1}{2} I_{aa}(a_t, t) a_t^2 w_t^2 \sigma^2 \\ & \left. + \lambda \left[I \left(a_t \left(\frac{1 + \nu(1 - w_t)}{1 + \nu} \right), t \right) - I(a_t, t) \right] \right\} = 0. \end{aligned} \quad (\text{B.1})$$

The first-order conditions for an interior solution are:

$$H_{\hat{c}} = 0 \quad \text{and} \quad H_w = 0. \quad (\text{B.2})$$

We postulate $I(a_t, t) = e^{-rt}[\beta_1 \log(a_t) + \beta_0]$, where β_0 and β_1 are to be determined from (B.1). Substituting $I(a_t, t)$ into (B.1), and then computing the first-order conditions in (B.2), we find that $w_t \equiv w$ is time invariant,

$$c_t = \beta_1 \theta a_t, \quad \text{and} \quad \frac{1}{\beta_1 w} - \frac{\lambda \nu}{1 + \nu(1 - w)} = \phi + w\sigma^2. \quad (\text{B.3})$$

¹⁰ There is also a transversality condition, $\lim_{t \rightarrow \infty} I(a_t, t) = 0$, that is satisfied.

The coefficients β_0 and β_1 are determined from (B.1). Thus, $\beta_1 = (1 + \theta)r^{-1}$ and

$$\beta_0 = \frac{1}{r} \left[1 + \theta \log\left(\frac{\theta r}{1 + \theta}\right) + \log(w^*) \right] - \frac{1 + \theta}{r^2} \left[\phi w^* + \frac{1}{2}(w^* \sigma)^2 - \lambda \log\left(\frac{1 + \nu(1 + w^*)}{1 + \nu}\right) \right].$$

The second equation in (B.3) has the same properties as those in (A.4). There is a similar transversality condition to that in Appendix I that is met.

Appendix C

In this appendix we state without proof¹¹ two useful results in the development of this paper:

1) The generalized Itô's lemma for mixed diffusion-jump processes, which can be enunciated as follows. Given the homogeneous linear stochastic differential equation

$$dx_t = x_t(\mu dt + \sigma dz_t + \nu dq_t)$$

and $g(x_t)$ twice continuously differentiable, then the "stochastic" differential of $g(x_t)$ is given by

$$dg(x_t) = [g_x(x_t)\mu x_t + \frac{1}{2}g_{xx}(x_t)\sigma^2 x_t^2]dt + g_x(x_t)\sigma x_t dz_t + [g(x_t(1 + \nu)) - g(x_t)]dq_t. \quad (C.1)$$

2) The solution to the nonhomogeneous linear stochastic differential equation

$$dx_t = (\beta_t + \mu x_t)dt + \sigma x_t dz_t + \gamma x_t dq_t$$

is given by

$$x_t = \left[x_0 + \int_0^t \exp \left\{ -\left(\mu - \frac{1}{2}\sigma^2\right)s - \sigma \int_0^s dz_u - \log(1 + \gamma) \int_0^s dq_u \right\} \beta_s ds \right] \times \exp \left\{ \left(\mu - \frac{1}{2}\sigma^2\right)t + \sigma \int_0^t dz_u + \log(1 + \gamma) \int_0^t dq_u \right\}. \quad (C.2)$$

It is also worthwhile to keep in mind, when using (C.2), that for $s \leq t$ the following properties for z_t and q_t hold:

$$E \int_s^t dz_u = 0, \quad E \left(\int_s^t dz_u \right)^2 = E \int_s^t du = t - s, \quad \text{and} \quad E \int_s^t dq_u = \lambda(t - s).$$

¹¹ For the proofs we refer the reader to Gihman and Skorohod (1972, ch. 3).

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