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CIDE

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Rodolfo Cermeño

PERFORMANCE OF VARIOUS ESTIMATORS IN DYNAMIC PANEL DATA MODELS

Abstract

This paper investigates the performance of various estimators in dynamic panel data models, in small samples and highly persistent AR processes. It is found that the different estimators of the AR parameter are either downward biased or imprecise. The IV estimator is nearly unbiased, but it becomes extremely imprecise as the true AR parameter approaches one. On the other hand, the GMM and LSDV estimators are relatively efficient, but they are downward biased. Even though they can compete in terms of a mean squared error criterion, the LSDV is by far more efficient. Kiviet's LSDVc estimator, as implemented in this study, performs quite poorly for AR parameter values close to one. The results of this investigation suggest that a natural way to asses the bias/efficiency problems in a highly persistent AR context, would be to use bias-corrected LSDV estimators. This approach is implemented in Cermeño (1997).

Key words: Dynamic Panel Data Models; Instrumental Variables; Generalized Method of Moments; Least Squares Dummy Variables; Monte Carlo simulations.

JEL classification: C15, C23.

1. Introducción

This paper investigates the performance, namely bias and precision, of various dynamic panel data estimators in contexts in which the time and/or cross-sectional dimensions are small and the AR parameter is high, say between 0.8 and 1. These contexts are likely to be found in convergence studies based on macro panels.[1] This investigation seems to be relevant since most dynamic panel data estimators have been motivated by micro applications, where N is large relative to T and the AR parameter values of interest are usually lower than 0.8.[2]

It is well known that the logarithm of the AR parameter, in absolute value, measures the speed of (conditional) convergence of the underlying dynamic process.[3] Thus, estimators that are appreciably downward biased are likely to produce downward biased estimates in the actual samples and, therefore, to over estimate convergence rates. On the other hand, estimators that are extremely inefficient would be unreliable even if they are unbiased.

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[1] See Maddala (1997a, 1997b) for an extensive discussion on the use of panel data models to investigate convergence.

[2] See Hsiao (1986), Ch. 4 for a detailed treatment of several dynamic panel data estimators. See also Baltagi (1995), Ch. 8 for an extensive study of more recent developments.

[3] A Linearization the Ramsey-Cass-Koopmans model gives an AR(1) process for per capita output with an AR parameter equal to $\exp\{-\lambda\}$ where $\lambda>0$ is the speed of convergence, i.e. see Barro and Sala-i-Martin (1995).

Kiviet (1995) presents an extensive study of the performance of various dynamic panel data estimators in small samples. He also develops an asymptotic approximation to the bias of the LSDV estimator in finite samples. The study, however, is limited to cases where N is large relative to T, and the AR parameter is at most 0.8. Judson and Owen (1996) consider samples of sizes similar to those of macro panels. They limit their attention, though, to AR parameter values of 0.2 and 0.8 only.

The rest of the paper is organized as follows. Section 2 presents the experimental design of the study, section 3 reports the main findings, and the final section concludes.

2. Experimental Design

The estimators evaluated in this study are the LSDV, Anderson-Hsiao IV, Arellano-Bond GMM1 and GMM2, and Kiviet's LSDVc. The data generation process is given by the following dynamic specification with no exogenous regressors:

$$y_{it} = \mu_i + \theta t + \beta y_{it-1} + \nu_{it}\ ,\ \ i = 1,\dots,N; t = 1,\dots,T \qquad (1)$$

The error term, v_{it}, is assumed to be independently and identically distributed and is drawn from a standard normal distribution. All coefficients are treated as constants.[4]

For simplicity, all estimators are implemented after removing the time trend by subtracting cross-sectional means from each observation. The LSDV estimator is simply the OLS estimator of the AR parameter once the individual-specific effects are modeled with dummy variables. The Anderson-Hsiao IV estimator is the OLS estimator of the AR paramcter once the individual effects have been removed by differencing the model and the 2-period lag of the dependent variable is used as an instrument. The Arellano-Bond GMM1 estimator is the GLS estimator applied to the first differences of the model after they are pre multiplied by a matrix of instrumental variables. This estimator is, in fact, a generalization of Anderson-Hsiao's IV method since it considers all other lagged dependent variables (besides the 2-period lag) which are uncorrelated with the error term. The GMM2 estimator applies also to the same transformed first difference version of the model, but uses estimated differenced residuals from the GMM1 results to approximate the covariance matrix of disturbances.

[4] Regarding the convergence issue previously mentioned, a trend stationary process (i.e. $\theta \neq 0$ and $\beta < 1$) would be consistent with conditional convergence in the sense that deviations of y (per capita output of economies) around a common trend are stationary.

Finally, Kiviet's LSDVc estimator subtracts an estimated bias from the actual LSDV estimate (uncorrected). Kiviet (1995) derives a formula to estimate the bias of the LSDV in finite samples. It should be pointed out that implementation of Kiviet's LSDV estimator requires estimated residuals from a preliminary consistent estimator. Also it requires N to be large, so that all second order terms in the formula can be omitted.

This investigation is limited to samples of sizes similar to those of actual samples. The following cases are considered: (N = 48, T+1 = 63), (N = 13, T+1 = 120), (N = 57, T+1 = 41) and (N = 100, T+1 = 31).[5] For each of the previous sample sizes the following AR parameters are considered: 0.5, 0.85, 0.95 and 0.99. As pointed out before, the error term is drawn from a standard normal distribution and all coefficients are treated as constants. The number of replications in each simulation is set to 2000. To make the computation of GMM estimators feasible, the number of instruments is limited to 5 and only 1000 replications are implemented. In the case (N = 13 , T+1 = 120), the number of replications was set to 200 since the large time dimension of this sample results in a very large number of moment restrictions which increase dramatically the computation time. All simulations have been implemented using GAUSS programs.

[5] The first sample size correspond to a USA states panel of per capita income. The second one corresponds to an OECD panel of per capita GDP . The third and fourth sample sizes correspond to per capita GPD panels of countries with complete information between 1950-1990 and between 1960-1990 respectively, from Summers and Heston's Penn World Tables, version 5.6.

3. Finite Sample Performance of various Estimators

Tables 1 through 4 summarize the results of the Monte Carlo simulations designed in the previous section. A number of results are worth mentioning. The LSDV estimator is downward biased which is not surprising since this estimator is inconsistent for a given T. The absolute value of the bias is higher the higher the AR parameter and the smaller the time dimension of the samples considered. In terms of dispersion and mean squared error, though, this estimator performs uniformly better than all other estimators. In contrast with results from experiments where T is small and N large, in the cases investigated here the GMM1 and GMM2 estimators are found to be numerically equivalent. This result confirms the well established result by Arellano and Bond (1991) that both estimators are asymptotically equivalent under i.i.d. disturbances.[6]

The best results concerning bias are given by Anderson-Hsiao IV estimator. However, in terms of precision this estimator behaves quite poorly as the AR parameter gets closer to one. This investigation also finds that, as in the case of the LSDV estimator, the GMM estimators are downward biased. On the basis of a mean

[6] In a number of experiments I found that for T less than 10, the GMM1 and GMM2 become closer to each other as N increases but they are not equal even if N=1000. For a given N (i.e. 50) and T higher than 20 they become equal.

Table 1: Performance of various dynamic panel data estimators (AR=0.5)

Sample/		AR=0.5	
Estimator	**bias**	**std. dev.**	**mse**
N=48, T+1=63			
LSDV	-0.0254	0.0167	0.0009
IV	0.0001	0.0328	0.0011
GMM	-0.0174	0.0407	0.0020
LSDVc	-0.0035	0.0168	0.0003
N=13, T+1=120			
LSDV	-0.0121	0.0235	0.0007
IV	0.0018	0.0455	0.0021
GMM	-0.0138	0.0347	0.0014
LSDVc	-0.0010	0.0239	0.0006
N=57, T+1=41			
LSDV	-0.0395	0.0191	0.0019
IV	0.0004	0.0379	0.0014
GMM	-0.0208	0.0446	0.0024
LSDVc	-0.0062	0.0197	0.0004
N=100, T+1=31			
LSDV	-0.0537	0.0172	0.0032
IV	0.0003	0.0336	0.0011
GMM	-0.0163	0.0388	0.0018
LSDVc	-0.0089	0.0177	0.0004

Table 2: Performance of various dynamic panel data estimators (AR=0.85)

Sample/		AR=0.85	
Estimator	**bias**	**std. dev.**	**mse**
N=48, T+1=63			
LSDV	-0.0346	0.0112	0.0013
IV	0.0006	0.0387	0.0015
GMM	-0.0250	0.0274	0.0014
LSDVc	-0.1529	0.0312	0.0244
N=13, T+1=120			
LSDV	-0.0178	0.0157	0.0006
IV	-0.0010	0.0524	0.0028
GMM	-0.0174	0.0254	0.0009
LSDVc	-0.0690	0.0263	0.0055
N=57, T+1=41			
LSDV	-0.0568	0.0148	0.0034
IV	-0.0006	0.0460	0.0021
GMM	-0.0306	0.0344	0.0021
LSDVc	-0.2162	0.0405	0.0484
N=100, T+1=31			
LSDV	-0.0803	0.0139	0.0066
IV	0.0000	0.0423	0.0018
GMM	-0.0270	0.0304	0.0017
LSDVc	-0.2470	0.0342	0.0622

Table 3: Performance of various dynamic panel data estimators (AR=0.95)

Sample/ Estimator	bias	std. dev.	mse
		AR=0.95	
N=48, T+1=63			
LSDV	-0.0446	0.0095	0.0021
IV	0.0010	0.0485	0.0024
GMM	-0.0327	0.0250	0.0017
LSDVc	-2.1123	0.4876	4.6996
N=13, T+1=120			
LSDV	-0.0214	0.0111	0.0006
IV	0.0010	0.0590	0.0035
GMM	-0.0208	0.0170	0.0007
LSDVc	-1.7524	0.8579	3.8071
N=57, T+1=41			
LSDV	-0.0729	0.0126	0.0055
IV	-0.0007	0.0618	0.0038
GMM	-0.0476	0.0329	0.0033
LSDVc	-1.7092	0.3478	3.0423
N=100, T+1=31			
LSDV	-0.0983	0.0116	0.0098
IV	0.0017	0.0620	0.0038
GMM	-0.0497	0.0384	0.0039
LSDVc	-1.3321	0.1865	1.8094

Table 4: Performance of various dynamic panel data estimators (AR=0.99)

Sample/ Estimator	AR=0.99 bias	std. dev.	mse
N=48, T+1=63			
LSDV	-0.0506	0.0080	0.0026
IV	-0.0119	0.4968	0.2469
GMM	-0.0506	0.0251	0.0032
LSDVc	-10.0797	1.9233	105.2994
N=13, T+1=120			
LSDV	-0.0275	0.0090	0.0008
IV	0.0087	0.1959	0.0385
GMM	-0.0263	0.0147	0.0009
LSDVc	-22.8899	9.6570	617.2074
N=57, T+1=41			
LSDV	-0.0772	0.0108	0.0061
IV	0.0278	0.5004	0.2512
GMM	-0.0803	0.0396	0.0080
LSDVc	-4.8732	0.7900	24.3721
N=100, T+1=31			
LSDV	-0.1003	0.0104	0.0102
IV	0.0407	1.5371	2.3642
GMM	-0.1082	0.0498	0.0142
LSDVc	-2.9434	0.3272	8.7709

squared error criterion only, the GMM estimator performs as well as or even better than the LSDV estimator in some cases. However, looking at the magnitudes of their downward biases and standard errors separately, a trade-off between them is found when the AR parameter is the range of 0.5 to 0.95. In that range, the magnitude of the bias of the GMM is smaller than in the case of the LSDV. However for a value of the AR parameter equal to 0.99, the GMM estimator produces similar or even higher downward biases than the LSDV estimator. Overall, even though the downward bias of the LSDV estimator is generally higher than in the case of the GMM estimator, the former has a much lower standard error. In terms of efficiency, the GMM estimator clearly outperforms the IV estimator but not the LSDV estimator.

Kiviet's LSDVc, as implemented here, performs quite well in the case of AR=0.5. However, in the case of AR=0.85, it is highly downward biased, and for values of the AR parameter between 0.95 and 0.99 it performs quite poorly on any criterion considered. There are at least two reasons for this last result. First, Kiviet's bias correction method requires N to be large relative to T which is not the case for the samples considered here. This argument is confirmed by the observation that in the second sample investigated, where N is very small relative to T, the worst results are obtained no matter what the value of the AR parameter is. Second, Kiviet's method requires estimated residuals from a preliminary stage that uses a consistent estimator, i.e. IV. Unfortunately, this estimator becomes quite inefficient in such a context and, therefore, yields quite imprecise and biased residuals. Using residuals from the LSDV estimator does not solve this problem, though, since this estimator is biased. The results reported here use residuals from the last one since they give better results when the AR coefficient is high.

The results of this study suggest that a reliable alternative way to estimate the AR parameter in highly persistent dynamic processes and finite sample sizes, would be to correct the LSDV for its downward bias, thus exploiting the fact this estimator is relatively efficient. This could be achieved with Median-Unbiased estimation, which is based on the exact finite sample distribution of the LSDV estimator.[7]

4. Conclusion

In the contexts investigated here, the estimators of the AR parameter are either downward biased or imprecise and more so the higher the true AR parameter values. The IV estimator is essentially unbiased but becomes extremely imprecise. On the other hand the GMM and LSDV are relatively efficient but they are downward biased. Even though they can compete in terms of mean squared errors, the LSDV is more efficient. Kiviet's LSDVc behaves quite poorly as the true AR

[7] This approach has been pursued by Andrews (1993) for univariate time series, and is implemented, via Monte Carlo simulations, to the LSDV estimator in dynamic panel data models in Cermeño (1997).

parameter approaches one. A natural way to asses the bias/efficiency problems of estimation in finite samples and highly persistent dynamic processes would be to correct the LSDV for its downward bias on the basis of its finite sample distribution.

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