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### Número 112

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ENDOGENOUS GROWTH, MONEY, TAXES

AND FOREIGN DEBT

#### Abstract

This paper uses a model of endogenous non-scale growth to examine the trade-off between raising public forcign debt and sustaining growth. It is shown that a tight monetary policy accompanied by an increase in the tax rate on income from capital may be inconsistent with economic growth. In such a case, the economy could instead suffer a recession. The paper also analyzes the effects of distortionary taxes on the perfect foresight equilibrium. The model provides, for the Mexican economy, some insights into the failed trade-off between raising public foreign debt and sustaining growth in 1994, and the sharp fall in growth in 1995 when fiscal and monetary policies, obeying a program of economic adjustment, were inconsistent with a macroeconomic equilibrium with positive growth.

JEL Classification Numbers: F43, E13.

Keywords: Endogenous growth; Fiscal Policy; Monetary Policy.

## 1. Introduction

The real gross domestic product of Mexico grew<sup>1</sup> 3.2 (percent) in 1989; 4.5 in 1990; 3.5 in 1991; 2.8 in 1992; 0.6 in 1993; 3.6 in 1994; and -6.9 in 1995. Why such a sharp fall in 1995? Perhaps, the worst record through the century. There are several papers offering possible answers to this question<sup>2</sup>. Some of them give an explanation in terms of the climate of political risk generated by violence in Chiapas, assassinations, elections and devaluation during 1994, which caused uncertainty on growth projections trigeering foreign debt withdrawal. For other authors the fall in growth is attributable to an overvalued currency and insufficient domestic savings. For some more it was the aftermath of a banking crisis. However, all of the above approaches are based on a partial equilibrium framework. This paper provides an explanation in terms of key economic variables such as capital accumulation, monetary growth, taxes, and private and public foreign debt in a macroeconomic equilibrium context.

The impact of government policies on growth has been a topic of great interest for a long time. Policymakers have been specially concerned with finding trade-offs between reducing taxes and stimulating growth, or between expanding the rate of monetary growth and economic growth; see, for instance, Feldstein (1976), Eaton (1981), Hartman (1988), Turnovsky (1993) and (1996), and Eicher and Turnovsky (1997). In this paper, we utilize an endogenous growth model to examine the trade-off between raising public foreign debt and sustaining growth. Unlike typical endogenous growth models, our model does not lead to balanced growth (all sectors growth at the same rate), which is according to empirical evidence from OECD data<sup>3</sup>. The issue of determining the optimal rate of monetary growth has been also extensively studied. See Friedman (1969), Calvo (1978), Turnovsky and Brock (1980), Lucas and Stokey (1983), Kimbrough (1986), and Abel (1987). This paper determines the optimal quantity of money consistent with a perfect foresight macroeconomic equilibrium in our model. The effects of a tight monetary policy on macroeconomic equilibrium have been analyzed by Sargent and Wallace (1981), Liviatan (1984), and Den Haan (1990). In this paper, the impacts of a tight monetary policy in the resulting perfect foresight equilibrium are examined. Our analysis also considers the effects of various forms of distortionary taxes on the trade-off, and revises the impact of tax policy on economic welfare.

The organization of the paper is as follows. In section 2, we present the model and preliminary results. Here, we set up the decision problems for consumers and firms,

<sup>&</sup>lt;sup>1</sup> Data Source: INEGI.

<sup>&</sup>lt;sup>2</sup> See, for instance, Sachs, Tornell and Velasco (1996), Gil-Díaz and Carstens (1996), Calvo and Mendoza (1996a) and (1996b), Edwards (1996), Dornbusch and Werner (1996), Solís (1996), and Venegas-Martínez (1998).

<sup>&</sup>lt;sup>3</sup> For surveys on OECD countries we direct the reader to the references contained in Eicher and Turnovsky (1997).

and establish the government budget constraint. In section 3, we characterize the perfect foresight equilibrium, and investigate the conditions that allow us to exploit a trade-off between raising public foreign debt and sustaining growth. In section 4, we provide some insights into the Mexican case of 1989-1995 in terms of our model. Finally, in section 5 results are summarized and conclusions are drawn. Appendix A contains the first-order conditions for the consumer's decision problem, and Appendix B provides complete proofs of a number of propositions in the paper.

# 2. Structure of the Economy

We consider an economy that has access to an international debt market. The determinants of growth in the model are endogenous. Our formulation is rich enough to examine the trade-off between raising public foreign debt and sustaining growth and to study the effects of fiscal and monetary policy. Through this section we describe the behavior of consumers, firms and government. We also derive the consumer's and firm's optimal decisions.

### 2.1 Consumers

We normalize the number of consumers at time t=0 to unity, and suppose that the consumer population size, at any time t>0, is given by  $N(t)=e^{nt}$ , where n is taken as exogenous and constant. Each consumer provides  $\ell$  hours of labor services. Thus, the total labor input, L, is given by  $L=\ell N$ . We assume that the economy produces and consumes a single good. Each consumer has foreign debt d and holds two real assets: capital k and real cash balances m=M/P, where M is the nominal stock of money and P is the price level. Consumers have perfect foresight of the inflation rate so  $\dot{P}/P=\pi^e=\pi>0$ , that is, consumers accurately perceive the rate at which inflation is proceeding. The value P(0) is assumed to be known.

Consumers obtain utility from a consumption good and assets (money), and derive disutility from labor and liabilities (debt). For literature regarding debt in the utility function see, for instance, Bardhan (1967) and Intriligator (1971). At time t=0 (at the present) the consumer seeks to maximize her/his overall discounted utility, U, given by:

$$U = \int_0^\infty u(c, m, \ell, d)e^{-(\rho - n)t}dt, \qquad (1)$$

where c is per capita consumption of a perishable good, and  $\rho$  is the rate of time preference. We have included moncy directly in the utility function because of its liquidity services. We also suppose that the consumer is selfish, that is,  $\rho > n$ . In order to make the analysis tractable, we propose the following specific functional form for u:

$$u(c, m, \ell, d) = \frac{(c^{\beta} m^{1-\beta})^{1-\theta}}{1-\theta} - \frac{(\ell^{\nu} d^{1-\nu})^{1+\sigma}}{1+\sigma},$$

where the substitution parameters  $\theta$ ,  $\beta$ ,  $\nu$ , and  $\sigma$  satisfy:  $\theta > 0$ ,  $\theta \neq 1$ ,  $0 < \beta < 1$ ,  $0 < \nu < 1$ , and

$$\sigma > \max\left\{\frac{\nu}{1-\nu}, \frac{1-\nu}{\nu}\right\}.$$

Observe that  $\sigma > 1$  always holds. With the above specification of u, it can be readily shown that

$$u_c, u_m, -u_\ell, -u_d > 0, \qquad u_{cc}, u_{mm}, u_{\ell\ell}, u_{dd} < 0,$$

and

$$-u_{cc}\frac{c}{u_c}-u_{mm}\frac{m}{u_m}=1+\theta, \quad u_{\ell\ell}\frac{\ell}{u_\ell}+u_{dd}\frac{d}{u_d}=\sigma-1>0.$$

Moreover, it can be shown that the following limit conditions hold:

$$u_c(0, m, \ell, d) = u_m(c, 0, \ell, d) = \infty, \quad u_\ell(c, m, 0, d) = u_d(c, m, \ell, 0) = 0,$$

$$u_c(\infty, m, \ell, d) = u_m(c, \infty, \ell, d) = 0$$
, and  $u_\ell(c, m, \infty, d) = u_d(c, m, \ell, \infty) = -\infty$ .

We summarize the values of the elasticities of marginal utilities and disutilities in Table 1.

$1-\beta+\beta\theta$	elasticity of marginal utility of consumption
$\beta + \theta(1-\beta)$	elasticity of marginal utility of real cash balances
$ u\sigma - (1- u) $	elasticity of marginal disutility of labor
$\sigma(1-\nu)-\nu$	elasticity of marginal disutility of debt

Table 1. Elasticities of marginal utilities.

The consumer's flow budget constraint is given by

$$\dot{k} + \dot{m} - \dot{d} = (1 - \tau_w)w\ell + (1 - \tau_r)rk - (\pi + n)m - rd - (1 + \tau_c)c + g - nk + nd,$$
 (2)

where w is the wage rate, r is the rental payment for physical capital (i.e., capital pays r unit of the consumption good per unit of time),  $\pi m$  stands for depreciated real monetary balances, g are government lump-sum transfers,  $\tau_w$  is the tax rate on labor income,  $\tau_r$  is the tax rate on income from capital, and  $\tau_c$  is the tax rate on consumption. The initial values k(0), M(0) and d(0) are all supposed to be known.

If we denote the consumer's real wealth net of foreign debt by

$$a = k + m - d, (3)$$

then we may rewrite (2) in the following form:

$$\dot{a} = (1 - \tau_w)w\ell + [(1 - \tau_r)r - n]a - \tau_r rd + (1 + \tau_c)c - (i - \tau_r r)m + g, \qquad (4)$$

where  $i = r + \pi$  is the nominal interest rate. We assume that consumers do not want to have any real wealth left over at the end. Then, there is a transversality condition to be satisfied:

$$\lim_{t \to \infty} ae^{-(r-n)t} = 0. \tag{5}$$

The first-order conditions for the maximization of (1) subject to (4), after routine computations, lead to the following behavioral relations of substitution between consumption and real cash balances, and between debt and labor effort respectively (see Appendix A):

$$\left(\frac{\beta}{1-\beta}\right)\frac{m}{c} = \frac{1-\tau_c}{i-\tau_r}, \qquad \left(\frac{\nu}{1-\nu}\right)\frac{d}{\ell} = -\frac{(1-\tau_w)w}{\tau_r r}, \tag{6}$$

along with the growth rates

$$\frac{\dot{c}}{c} = \frac{1}{\theta} [(1 - \tau_r)r - \rho], \quad \text{and} \quad \frac{\dot{d}}{d} = \frac{1}{\sigma} [(1 - \tau_r)r - \rho]. \tag{7}$$

In order to make easier the notation, let us denote growth rates generically by  $\gamma_x = \dot{x}/x$ . Combining (6) and (7) we get

$$\gamma_c = \gamma_m = \frac{1}{\theta} [(1 - \tau_r)r - \rho], \quad \text{and} \quad \gamma_d = \gamma_\ell = \frac{1}{\sigma} [(1 - \tau_r)r - \rho]. \quad (8)$$

#### 2.2 Firms

There is a representative firm producing goods, and making rental payments for both capital and labor inputs. We will assume that the firm has access to the "y = Ak" technology (cf. Barro and Sala-i-Martin 1992). The firm maximizes the present value of its discounted net cash flow. If we assume no adjustment costs, the firm's decision problem reduces to the maximization of profit. At a point in time in a competitive equilibrium, with positive physical capital, profit is given by

$$\Pi = Ak - (r + \delta)k - w\ell, \tag{9}$$

where  $\delta$  is the constant rate of depreciation. Profit maximization requires that marginal products of capital and labor equal the interest rate and the wage rate respectively, that is,

$$A = r + \delta$$
, and  $w = 0$ . (10)

We may think of w = 0 as the wage rate to raw labor without being augmented by human capital. See, for instance Barro (1995, ch.4) where each individual supplies inelastically one unit of labor per unit time at zero wage rate.

### 2.3 Government

To close the model, we introduce the consolidated government budget constraint. The government has no consumption, generates no utility for consumers, and has no effects on productivity for firms. It collects taxes on income from capital, taxes on income from labor, and taxes on consumption. The government gives back to the consumers the proceeds of the inflation tax in the form of a lump-sum subsidy. Moreover, taxes collected from consumers are redistributed in a lump-sum fashion, and public foreign debt is transferred to the consumer. Hence, the government budget constraint in per capita terms, to be satisfied at any instant, is

$$g = m\gamma_M + \dot{b} - rb + nb + \tau_c c + \tau_r rk, \tag{11}$$

where,  $\gamma_M = M/M$  is the rate of monetary growth to be endogenously determined, and b is public foreign debt, with b(0) given. The growth rate of public foreign debt,  $\gamma_b$ , is taken as exogenous. We rule out chain letter debt finance for the government, so

$$\lim_{t \to \infty} b e^{-(r-n)t} = 0.$$

In addition, the growth rate of real monetary balances always satisfies

$$\gamma_m = \gamma_M - \pi. \tag{12}$$

### 3. Perfect Foresight Equilibrium

Our next task is to combine the rational behavior of consumers and firms with the government actions. In what follows, for the sake of simplicity, we will assume that  $\sigma = \theta$ . This implies  $\theta > 1$ . From (8), (10), and (11) we readily obtain

$$\gamma_c = \gamma_m = \gamma_\ell = \gamma_d = \frac{1}{\theta} [(1 - \tau_r)(A - \delta) - \rho], \tag{13}$$

$$\dot{v} = (A - \delta - n)v - c, \qquad v = k - f, \qquad f = d + b, \tag{14}$$

along with the transversality conditions

$$\lim_{t \to \infty} ae^{-(A-\delta-n)t} = 0, \qquad \lim_{t \to \infty} be^{-(A-\delta-n)t} = 0, \tag{15}$$

where f is total (private and public) per capita foreign debt, and v denotes capital net of total foreign debt.

The assumption on the technology that ensures growth in c, after taxing income from capital, is

$$(1 - \tau_r)(A - \delta) > \rho, \quad \text{for all } \tau_r \in (0, \bar{\tau}). \tag{16}$$

A tax rate greater than the threshold  $\bar{\tau}$  will reverse the inequality in (16). Furthermore, to assure that the indirect utility, V, remains bounded, it is required that

$$\rho - n > \frac{1+\theta}{\theta} [(1-\tau_r)(A-\delta) - \rho]. \tag{17}$$

In such a case, it can be shown that

$$V = \frac{[c(0)^{\beta} m(0)^{1-\beta}]^{1-\theta}}{1-\theta} \left[ \frac{1}{\rho - n - \frac{1-\theta}{\theta} [(1-\tau_r)(A-\delta) - \rho]} \right] - \frac{[\ell(0)^{\nu} d(0)^{1-\nu}]^{1+\theta}}{1+\theta} \left[ \frac{1}{\rho - n - \frac{1+\theta}{\theta} [(1-\tau_r)(A-\delta) - \rho]} \right].$$
(18)

Observe that (17) implies  $\rho - n > \frac{1-\theta}{\theta}[(1-\tau_r)(A-\delta) - \rho]$ , thus V < 0. In virtue of (16), we also find that, as long as  $\tau_r < \bar{\tau}$ ,

$$\gamma_c = \gamma_m = \gamma_d = \gamma_\ell = \frac{1}{\theta} [(1 - \tau_r)(A - \delta) - \rho] > 0.$$
 (19)

For  $\tau_r > \bar{\tau}$  the inequality in (19) is reversed. The model has no transitional dynamics in c, m, d, and  $\ell$ . It is important to point out that in the Ak model the long-run per capita growth rate equals the short-run per capita growth rate. If we substitute the general solution for consumption  $c = c(0) \exp\{(1/\theta)[(1-\tau_r)(A-\delta)-\rho]t\}$  in (14), where c(0) is to be determined, then the solution to the resulting non-homogeneous, first-order differential equation in v is

$$v = \left[v(0) - \frac{c(0)}{B}\right] e^{(A-\delta-n)t} + \frac{c(0)}{B} e^{\frac{1}{\theta}[(1-\tau_r)(A-\delta)-\rho]t}$$
 (20)

where

$$B \equiv (A - \delta)(1 - \tau_r) \left(\frac{\theta - 1}{\theta}\right) + \frac{\rho}{\theta} - n.$$

Notice that (17) implies B > 0. We may also rewrite real wealth as

$$a = v + m + b$$

therefore

$$0 = \lim_{t \to \infty} ae^{-(A-\delta-n)t} = \lim_{t \to \infty} ve^{-(A-\delta-n)t} + \lim_{t \to \infty} m(0)e^{-Bt}.$$

Hence, from (20) we get

$$0 = \lim_{t \to \infty} \left[ \left( v(0) - \frac{c(0)}{B} \right) + \frac{c(0)}{B} e^{-Bt} \right],$$

which leads to

$$c(0) = [k(0) - (d(0) + b(0))]B.$$

To guarantee positive initial consumption, we need to assume that k(0) > d(0) + b(0). Therefore,

$$v = \frac{c(0)}{B} e^{\left[\frac{1}{\theta}(1-\tau_r)(A-\delta)-\rho\right]t}.$$

We may conclude that

$$\gamma_v = \frac{1}{\theta} [(1 - \tau_r)(A - \delta) - \rho]. \tag{21}$$

Hence, there is no transitional dynamics for v. We are now in a position to derive several important results:

# Proposition 1.

(i) Let us denote the debt-capital ratio, b/k, by  $\alpha(t)$ . Then,

$$\gamma_u = \gamma_k = [1 - \alpha(t)]\gamma_d + \alpha(t)\gamma_b. \tag{22}$$

(ii)  $\gamma_b > \gamma_d > 0$  implies  $\gamma_y > 0$ .

(iii) If 
$$\gamma_b = \gamma_d + \varepsilon$$
, for some  $\varepsilon > 0$ , then  $\gamma_y = \gamma_b + [\alpha(t) - 1]\varepsilon$ .

The proof of the above Proposition is given in Appendix B. Part (i) states a linear combination between the growth rates of private and public foreign debt. Part (ii) gives the condition that guarantees growth. Part (iii) explains a trade-off between raising public foreign debt and sustaining growth, provided that either public foreign debt grows faster than private foreign debt or public foreign debt remains at a higher level than the stock of physical capital.

Corollary 1. If  $\gamma_b > \gamma_d$ , then

$$\frac{y}{A} = k = k(0)e^{\gamma_d t} + b(0)e^{\gamma_d t}[e^{(\gamma_b - \gamma_d)t} - 1] > 0.$$
 (23)

The proof of the above corollary readily follows from (22). Corollary 1 determines the levels of output and capital as long as  $\gamma_b > \gamma_d$ .

Corollary 2. Under the hypothesis of part (ii) in Proposition 1. A once-and-for-all increase in the tax rate on income from capital will lead to a permanent reduction in the growth rate since

$$\frac{\partial \gamma_y}{\partial \tau_r} = -\frac{1}{\theta} (A - \delta) < 0. \tag{24}$$

The above corollary stablishes that an increase in the tax rate on income from capital, which discourages investment, slows down growth, regardless the value of  $\tau_r$ .

**Proposition 2.** The unique rate of expansion of nominal money which is consistent with perfect foresigth, optimal decisions of consumers and firms, and general equilibrium is given by

$$\gamma_M = \pi + \frac{1}{\theta} [(1 - \tau_r)(A - \delta) - \rho].$$

The proof for Proposition 2 follows straightforward from (12) and (19). Notice that if  $\tau_r > \bar{\tau}$ , then  $\gamma_M > 0$ , otherwise  $\gamma_M$  has ambiguous sign.

**Proposition 3.** The impact on economic welfare of an increase in the tax rate on income from capital is ambiguous since from (18)

$$\frac{\partial V}{\partial \tau_r} = -\frac{A - \delta}{\theta} \left[ \frac{[([k(0) - d(0) - b(0)]B)^{\beta} m(0)^{1-\beta}]^{1-\theta}}{[\rho - n - \frac{1-\theta}{\theta}((1 - \tau_r)(A - \delta) - \rho)]^2} - \frac{[\ell(0)^{\nu} d(0)^{1-\nu}]^{1+\theta}}{[\rho - n - \frac{1+\theta}{\theta}((1 - \tau_r)(A - \delta) - \rho)]^2} \right],$$

which may be of either sign depending on the initial values of the decision variables k(0), d(0),  $\ell(0)$ , the initial stock of government foreign debt h(0), and the preference and technological parameters.

**Proposition 4.** Tax rates on both labor income or consumption are neutral to welfare and policies directed towards optimal growth compatible with general equilibrium.

The above results follows from the fact that taxes collected from consumers are redistributed in a lump-sum fashion.

**Theorem 1.** All possibilities of equilibria when  $\gamma_b > \gamma_d$  are characterized as follows:

	fiscal policy	monetary policy	growth
I	$ au_r < ar{ au}$	$\gamma_{\scriptscriptstyle M} > 0$	$\gamma_c, \ \gamma_\ell, \ \gamma_d, \ \gamma_k, \ \gamma_y > 0$
II	$ au_r > ar{ au}$	$\gamma_{\scriptscriptstyle M} < 0$	$\gamma_c, \gamma_\ell, \gamma_d, \gamma_k, \gamma_y < 0$
III	$ au_{r} > ar{ au}$	$\gamma_{\scriptscriptstyle M} > 0$	$\gamma_c, \ \gamma_\ell, \ \gamma_d, \ \gamma_k, \ \gamma_y < 0$

#### 4. The Mexican case of 1989-1995

In this section section, we highlight a number of stylized facts about the Mexican economy during 1989-1995. The government build up a large foreign debt on the projection of fast growth. The ratio of public foreign debt and the stock of physical

capital was maintained at a high level through the whole period (with an average ratio greater than 1). In 1994, the government intended to sustain growth by raising the growth rate of public foreign debt above that of private foreign debt, which in turn increased the debt-capital ratio. However, during the first semester of 1995, obeying a program of economic adjustment, monetary policy was tightened up, in fact the rate of monetary growth was negative. Moreover, an increase in the tax rate on income from capital between 1994 and 1995 discouraged investment slowing down economic growth. As a consequence, the desirable results of a trade-off between raising public foreign debt and sustaining economic growth were not met, and a sharp fall on growth was recorded in 1995.

We suppose that the Mexican economy is at its steady-state per capita growth rate. Let us now interprete the above evidence in terms of our model. First, the ratio of public foreign debt and the stock of physical capital<sup>5</sup>,  $\alpha(t)$ ,satisfied  $\alpha(t) > 1$  during 1989-1995. Secondly, in 1994, the government intended to sustain growth by increasing<sup>6</sup>  $\gamma_b$  above  $\gamma_d$ , which in turn increased  $\alpha(t)$ . However, monetary policy was tightened up between December 1994 and June 1995—in fact<sup>7</sup>  $\gamma_M < 0$ . In the same period, an increase in the tax rate on income from capital by more than 20 percent<sup>8</sup> discouraged investment which slowed down growth, which may be expressed as  $\tau_r > \bar{\tau}$  and  $\partial \gamma_y/\partial \tau_r < 0$ . These circumstances were inconsistent with a macroeconomic equilibrium for which  $\gamma_y > 0$ , as stated in the Type I equilibrium of Theorem 1.

# 5. Summary and Concluding Remarks

Unlike most explanations of the sharp fall in growth, which are based on a partial equilibrium scheme, we have elaborated an analytical framework based on a general equilibrium context. The proposed model of endogenous growth was used to show that as long as either public foreign debt grows at a higher rate than private foreign debt or public foreign debt remains at a higher level than the stock of physical capital, then there was a trade-off between raising public foreign debt and sustaining growth. We have also analyzed the effects of tax policy and monetary policy on the perfect foresight general equilibrium. The developed model has provided some insights into the singular Mexican experience of 1989-1995 in terms of economic key variables such as capital accumulation, monetary growth, taxes, and private and public foreign debt. The broad message of this paper, although only demonstrated for a particular case of utility index, is that growth, money and foreign debt are linked through fragile equilibrium relationships which should be considered with great caution by policymakers when building growth projections. Finally, it is important to point out that unlike typical endogenous growth models, our approach does not lead to balanced

<sup>5</sup> Data Source: SHCP.

<sup>&</sup>lt;sup>6</sup> Data Source: SHCP.

<sup>&</sup>lt;sup>7</sup> Data Source: Banxico.

<sup>&</sup>lt;sup>8</sup> Data Source: Banxico.

growth equilibrium, which deserves much more attention than that we have attempted here. Needless to say, more research should be undertaken in such a respect.

## Appendix A

The present value Hamiltonian of the problem of maximizing (1) subject to (4) is given by

$$H = \left[ \frac{(c^{\beta} m^{1-\beta})^{1-\theta}}{1-\theta} - \frac{(\ell^{\nu} d^{1-\nu})^{1+\sigma}}{1+\sigma} \right] e^{-(\rho-n)t} + \lambda \{ (1-\tau_w)w\ell + [(1-\tau_r)r - n]a - \tau_r rd - (1+\tau_c)c - [i-\tau_r r - n]m + g \}.$$

The first order conditions

$$-\frac{\partial H}{\partial a} = \dot{\lambda}, \quad \frac{\partial H}{\partial c} = 0, \quad \frac{\partial H}{\partial m} = 0, \quad \frac{\partial H}{\partial \ell} = 0, \quad \frac{\partial H}{\partial d} = 0$$

lead to

$$-[(1-\tau_r)r-n]\lambda = \dot{\lambda},\tag{A1}$$

$$\beta c^{\beta(1-\theta)-1} m^{(1-\theta)(1-\beta)} e^{-(\rho-n)t} = \lambda (1-\tau_c), \tag{A2}$$

$$(1 - \beta)e^{\beta(1-\theta)}m^{-\theta(1-\beta)-\beta}e^{-(\rho-n)t} = \lambda(i - \tau_r r - n),$$
 (A3)

$$-\nu \ell^{\nu(\sigma+1)-1} d^{(1+\sigma)(1-\nu)} e^{-(\rho-n)t} = -\lambda (1-\tau_w) w, \tag{A4}$$

$$-(1-\nu)\ell^{\nu(\sigma+1)}d^{\sigma(1-\nu)-\nu}e^{-(\rho-n)t} = \lambda \tau_r r.$$
 (A5)

Dividing (A2) and (A4) by (A3) and (A5) respectively, we get both equations in (6). By differentiating (A2) with respect to t, we obtain

$$\lambda(1-\tau_r)[\beta(1-\theta)-1]\gamma_c + (\beta-1)(\theta-1)\gamma_m - (\rho-n) + [(1-\tau_r)r - n] = 0.$$

Using the fact that  $\gamma_c = \gamma_m$  in the above equation we obtain the first equation in (8). The second equation appearing in (8) is derived by differentiating (A4) and using now  $\gamma_\ell = \gamma_d$ .

# Appendix B

The proof for part (i) of Proposition 1 follows by writing (21) as

$$\gamma_d = \gamma_v = \gamma_k \frac{k}{v} - \gamma_b \frac{b}{v} - \gamma_d \frac{d}{v},\tag{B1}$$

from where

$$\gamma_{y} = \gamma_{k} = \gamma_{d} \left( 1 - \frac{b}{k} \right) + \gamma_{b} \frac{b}{k}. \tag{B2}$$

To show part (ii), it is enough to see that  $\gamma_d/(\gamma_d - \gamma_b) < 0 < \alpha(t)$  always holds. Part (iii) follows from (22).

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