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The Dynamics of Bargaining Power in a Principal-Agent Model

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Abstract

We propose a dynamic principal-agent model where the agent's initial bargaining power is the state variable and a law of motion that governs their bargaining power's behavior. Our numerical results indicate that agents with the same relative risk aversion might show different paths of their bargaining powers, and that more powerful the incentives ensue higher variability in the agent's salary. We implement an empirical equation to identify CEOs' bargaining power and find a set of values of the state variable for which the proposed dynamics explains well the relationship between firm performance and CEO compensation. Finally, by analyzing a panel sample of annual observations for 9,084 CEOs in the U.S., we conclude that our estimates are consistent with empirical findings of a slow yearly growth in CEOs' compensation

Keywords: Dynamic Analysis, Contract Theory

JEL Codes: C61, D86

Resumen

Se propone un modelo dinámico de agente-principal en el cual el nivel inicial del poder de negociación del agente es la variable de estado, al igual que una ley de movimiento que gobierna el comportamiento de dicho poder de negociación. Nuestros resultados numéricos indican que agentes cuyos coeficientes relativos de aversión al riesgo son similares podrían mostrar diferentes trayectorias de su poder de negociación y que incentivos más fuertes podrían traer como consecuencia mayor variabilidad en el salario de los agentes. Se implementa una ecuación empírica para identificar el poder de negociación de gerentes y también encontrar el conjunto de valores de parámetros para los cuales la dinámica propuesta explica adecuadamente la relación entre el desempeño de la firma y la compensación gerencial. Finalmente, al analizar una muestra panel con observaciones anuales de 9,084 gerentes de compañías estadounidenses se concluye que los resultados obtenidos son consistentes con hallazgos empíricos que señalan un crecimiento anual paulatino en la compensación gerencial.

Palabras Clave: Análisis Dinámico, Teoría de Contratos
Clasificación JEL: C61, D86

The Dynamics of Bargaining Power in a Principal-Agent Model

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Abstract

We propose a dynamic principal-agent model where the agent's initial bargaining power is the state variable and a law of motion that governs their bargaining power's behavior. Our numerical results indicate that agents with the same relative risk aversion might show different paths of their bargaining powers, and that more powerful the incentives ensue higher variability in the agent's salary. We implement an empirical equation to identify CEOs' bargaining power and find a set of values of the state variable for which the proposed dynamics explains well the relationship between firm performance and CEO compensation. Finally, by analyzing a panel sample of annual observations for 9,084 CEOs in the U.S., we conclude that our estimates are consistent with empirical findings of a slow yearly growth in CEOs' compensation.

Keywords: Dynamic Analysis, Contract Theory.
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1 Introduction

The level and composition of the CEO pay have been greatly discussed both in the academic and in the popular literature. There is extensive evidence of the heterogeneity of real-life CEO pay packages (Tervio (2008); Gabaix and Landier (2008); Jenter and Kanaan (2015)). Firm size has gained support from theoretical and empirical articles as a determinant of the CEO pay's heterogeneity. For instance, the "size of stakes" theory establishes a positive relationship between the direction and magnitude of the change in the size of all bigger firms and the direction and magnitude of the change in CEOs' compensation, given those firms' higher willingness to pay for managerial talent (Gabaix and Landier (2008); Edmans and Gabaix (2016)). While analyzing the causes of the increase of agency costs in U.S. since 1944, Gayle and Miller (2009) find empirical evidence that links this phenomenon with exogenous growth in firm size; but they find no reason to think that this increase is due to changes in managers' risk preferences since they have remained stable.

Apart from firm size, two additional hypotheses have been analyzed to explain the important increase of CEO compensation (Jenter and Kanaan (2015)). First, there is evidence that CEOs have acquired a higher ability to either extract rent from shareholders or have more influence in the composition of board of directors (Shivdasani and Yermack (1999); Murphy and Sandino (2010); Graham et al. (2020)). Limits to the shareholders' bargaining power have been modeled by introducing limited liability constraints and/or of bargaining settings in principal-agent models. Albeit those are interesting approaches, their predictions regarding CEO's effort and total surplus are mixed (Pitchford (1998); Yao (2012); Li et al. (2013)). Secondly, a highly competitive market for managerial talent has driven this dramatic increase since the 1990's (Jenter and Kanaan (2015)). Both hypotheses contribute to the discussion of determinants of CEO pay's behavior, but none offer a complete picture and more theoretical and empirical work is required to gain better insights into this phenomenon (Jenter and Kanaan (2015)).

In this article we intend to contribute to the analysis of CEOs' power by proposing a dynamic moral hazard model that includes the CEO's initial level of bargaining power as the state variable, and analyzes its evolution throughout the infinitely repeated relationship between the shareholders and the CEO. Given this model's structure, obtaining closed-form solutions proves to be elusive, and a good strategy to gain insight into the optimal contracts that result from this model is through the design and implementation of a numerical algorithm, as in Wang (1997) and Clementi et al. (2010). Our numerical results indicate that it is plausible that CEOs with even the same relative risk aversion parameter might show different stationary paths of their bargaining powers and salaries. This diversity is only explained by the values of the model's state variable and its evolution. From our simulation results, we observe that there is a positive relationship between the agent's salary and the initial value of their bargaining power; and that the more powerful the incentives, the higher the variability of the CEO's salary. Moreover, by combining our theoretical and numerical contributions, we propose an empirical equation that was implemented in an econometric exercise that includes using model-generated data to validate our selection of parameter values, and real-life CEO compensation data. Our results indicate that the validity of our empirical identification depends on the initial value of the CEO's bargaining power, and that the proposed empirical equation provides a good approximation for the variation of CEOs' bargaining power provided that their initial bargaining power is not too high.

Agency models provide a formal environment to analyze the design of incentive based compensation in the presence of asymmetric information between a company's shareholders (principal) and CEO (agent) (Hölmstrom (1979); Grossman and Hart (1983)). When the owner of a company delegates the task of managing it to a CEO, a problem of asymmetric information between the two parties often arises because shareholders are, in general, unable to perfectly observe whether the CEO is really working or shirking. The fact that shareholders are unable to observe and/or perfectly monitor the effort that the CEO exerts in performing the assigned task generates an inefficient allocation of resources. The dynamic principal-agent problem is usually analyzed as the maximization of the discounted expected utility of shareholders subject to two main constraints (Spear and Srivastava (1987)). First, the rationality constraint that ensures that the CEO accepts the contractual arrangement given that the CEO's lifetime discounted expected utility will be equal to their reservation utility. Second, the incentive compatibility constraint that warrants that the CEO chooses a path of optimal effort levels that corresponds to the effort path that shareholders want to implement. Providing optimal incentives to the CEO is achieved through two devices: present and future compensation.

Given the conflict of interests between shareholders and the CEO that emanates from the separation of ownership and control, the structure of the dynamic principal-agent problem makes it possible to envision it as a multi-objective optimization problem (Goldberg (1989)). The advantage of using this approach is that we can consider several incentive-provision arrangements between the principal and the agent in which their utilities have several levels of priority. That is, we consider that the bargaining power of the principal and the agent define those levels of priority. This means that we work with the assumption that the principal does not necessarily impose a contractual arrangement to the agent, as it is assumed by Hölmstrom (1979), Grossman and Hart (1983), and Spear and Srivastava (1987). Many situations in the real world that involve optimizing conflicting objectives between two or more parts can be thought as multi-objective optimization problems (Ehrgott (2005)). Multi-objective optimization problems are characterized by a set of alternative and equivalent solutions because of the lack of information about the relevance of one objective with respect to the others. The set of optimal infinite solutions is called the Pareto Optimal Frontier.

Our multi-objective approach to the dynamic principal-agent model relates to the question of what the objective function of a firm should be. According to Milton Friedman (Friedman (1970)), managers are employees of their shareholders; so, they should simply comply with the shareholders' objectives if they do not imply breaking basic societal rules. A prolific discussion has emerged from this article that has included topics from which Friedman (1970) abstracted from; such as firms' responsibilities towards society and towards internalizing externalities they generate. For instance, Hart and Zingales (2017) propose a model in which there is no separability from the business side of firms and the ethical side of firms, and conclude that a way that a firm maximizes shareholders' welfare is to allow them participate in choosing corporate policies by voting. Like Friedman (1970), we do not tackle ethical issues related to firms; however, the results of our strategy of explicitly modeling the CEO's bargaining power and its evolution allow us to say that efficiency loss is observed when the distance between CEO's pay packages and their performance in maximizing shareholders' welfare, defined by their discounted expected utility, increase. This result is corroborated by our empirical identification of the CEOs' bargaining power.

The remaining of this paper is organized as follows: in Section 2 the multi-objective dynamic principal-agent model is presented. In Section 3 we explain the computational strategy used to numerically approximate the model's solutions. The stationary and simulation results we obtain by implementing the computational strategy, described in the previous section, are discussed in Section 4. In Section 5, we propose an empirical equation to identify the CEO's bargaining power and perform an econometric exercise to estimate it. Finally, we offer our concluding remarks.

2 The Model

In this section, we develop a dynamic principal-agent model based on the standard repeated moral hazard model of Spear and Srivastava (1987). We assume that time is discrete and that it goes on until infinity: $t = 0, 1, 2, \ldots$. There are two individuals: a risk neutral principal and a risk averse agent, who are both discounted expected utility maximizers with a common discount rate $\beta \in (0, 1)$.

Suppose that the agent has a continuous utility function represented by: $v(w_t, a_t)$, which is assumed to be bounded, strictly increasing, and strictly concave with respect to w_t; and strictly decreasing and convex with respect to a_t. The variable $w_t \geq 0$ is the agent's salary or present compensation at the end of every period. The variable a_t is the agent's effort choice made at the beginning of every period, drawn from a compact set $A = [\underline{a}, \overline{a}]$, and it is unobservable to the principal. We also assume that v is either additively or multiplicatively separable in its two arguments, w_t and a_t.

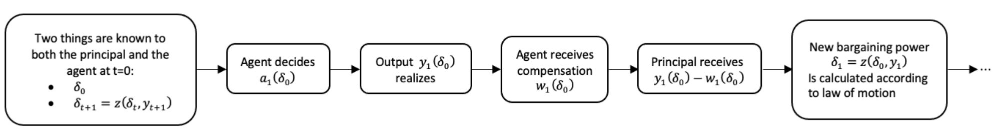
Every period $t \geq 1$ a realization of the output y_t, drawn from the compact set Y, is observed by the principal and the agent. The stochastic relationship between the output realization and the agent's effort choice is given by the time-invariant distribution $F(y_t \mid a_t) > 0$ for all $y_t \in Y$ and for all $a_t \in A$. Also, we assume that this distribution has a density f and that the distribution of outputs is *i.i.d.* from period to period, for a given action.

Now we introduce the agent's bargaining power in our environment. Assume that at the beginning of the principal-agent relationship, at $t = 0$, the agent has an exogenously given bargaining power level, $\delta_0 \in [0, 1]$; while the principal has a bargaining power of $(1 - \delta_0)$. We see the agent's bargaining power constitutes as his ability to extract part of the surplus generated by their productive activity. Moreover, we assume that, at the moment of signing their contract at $t = 0$, the principal and the agent would have already reached an agreement about the evolution of the agent's bargaining power which will be dictated by a law of movement. That is, we are not modelling a situation where the principal and the agent would engage a bargaining process at each period to determine their respective bargaining power at (Binmore et al. (1986)), but a situation where both the agent's initial bargaining power and its law of movement are already determined at $t = 0$.

Furthermore, we assume that the agent's bargaining power's law of movement is given by the function $z(\delta_{t-1}, y_t)$, which maps from $[0, 1]$ into itself. We assume that z is bounded, continuous, and increasing with respect to both its arguments. We would like to emphasize that this law of movement is intended to showcase rewards and punishments for good versus bad performances of the firm. Assuming, as we do, that after signing the contract at $t = 0$ no more bargaining will happen between the principal and the agent is simplistic, and a justification for this assumption can be found in the argument that there are mobility costs for a worker while changing jobs; see, for instance, Baily (1974). But, on the other hand, it has been documented that CEOs face important turnover rates after bad industry performance, and, to a lesser extent, bad market performance Jenter and Kanaan (2015). Hence, this is a caveat of the present model, but one that will allow us to introduce a framework to study how the ability of the CEO to extract surplus from this relationship changes through time.

The contract that defines the infinite relationship of the principal and the agent follows this timeline: At $t = 1$, given a value of $\delta_0 \in [0, 1]$, the agent decides $a_1(\delta_0) \in A$, output $y_1(\delta_0) = y_1(a_1(\delta_0))$ is drawn from the distribution $F(y \mid a_1(\delta_0))$, and the agent receives a compensation $w_1(y_1(\delta_0))$. As we have already mentioned, the principal observes $y_1(\delta_0)$ but not $a_1(\delta_0)$, hence $w_1(y_1(\delta_0))$ only depends on $y_1(\delta_0)$. Consequently, we can write the agent's compensation at $t = 1$ as $w_1(y_1(\delta_0))$. The principal receives $y_1(\delta_0) - w_1(y_1(\delta_0))$, and the agent's bargaining power for $t = 2$, $\delta_1(y_1(\delta_0), \delta_0) \in [0, 1]$ is defined by the agreed-upon law of motion. See Figure 1 for a graphical representation of the principal-agent relationship at $t = 0$ and $t = 1$.

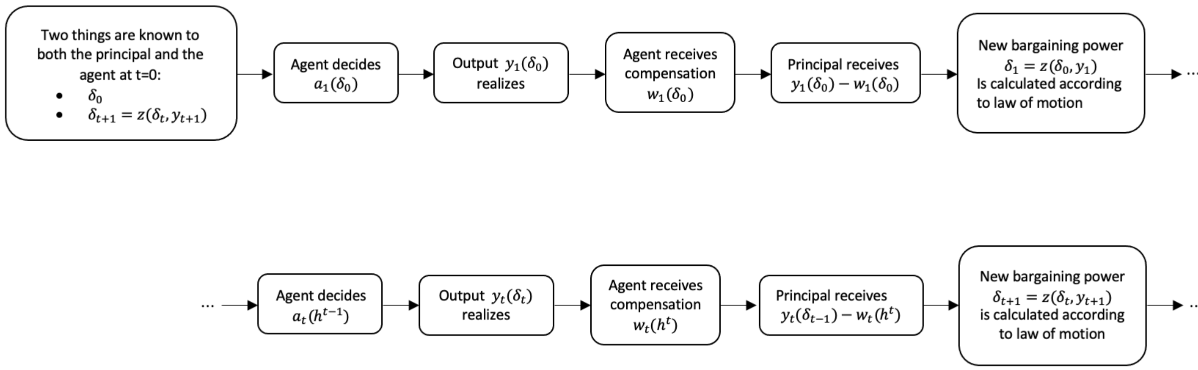
Figure 1: The timeline of the principal and agent relationship at $t = 0$ and $t = 1$.



Now assume that the principal and the agent employ history-dependent pure strategies. At $t = 2$ and given δ_1, the agent decides $a_2(\delta_1(y_1(\delta_0), \delta_0))$; then, output $y_2(\delta_1(y_1(\delta_0), \delta_0))$ is drawn from the distribution $F(y \mid a_2(\delta_1(y_1(\delta_0), \delta_0)))$. The agent receives a compensation $w_2(y_2(\delta_1(y_1(\delta_0), \delta_0)))$ and the principal receives $y_2(\delta_1(y_1(\delta_0), \delta_0)) - w_2(y_2(\delta_1(y_1(\delta_0), \delta_0)))$. The new agent's bargaining power is defined by the aforementioned law of movement and it is equal to: $\delta_2(\delta_1(y_1(\delta_0), \delta_0), y_2(\delta_1(y_1(\delta_0)))) \in [0, 1]$. And, the game is repeated from $t = 3, \ldots$ See Figure 2 for the timeline of the contract at time t.

Therefore, at any time t there is a history of output realizations $h^t = \{(\delta_s, y_{s+1}(\delta_s))\}_{s=0}^{t-1}$; with $h^0 = \delta_0$, such that $y_{s+1}(\delta_s) \in Y$ and $\delta_s \in [0, 1]$ for all $s = 1, 2, \ldots, t-1$. The principal's decision is $w_t(h^t)$, and the agent's decision is $a_t(h^{t-1})$, because the effort decision has to be made before y_t has been realized and given the value of $\delta_t(h^{t-1})$ at the beginning of the period. Let $\pi(h^{t+\tau}; h^t, a_t)$ be the probability distribution of $h^{t+\tau}$ conditional on h^t and a_t. This

Figure 2: The timeline of the principal and agent relationship at time t.



distribution is recursively expressed in the following way:

$$d\pi(h^{t+\tau} \mid h^t, a_t) = f(y_{t+\tau} \mid a(h^{t+\tau-1}))d\pi(h^{t+\tau-1} \mid h^t, a_t)$$

with

$$d\pi(h^{t+1} \mid h^t, a_t) = f(y_{t+1} \mid a(h^t)).$$

The value functions that the principal and the agent, respectively, derive from the sub-game starting from h^t are given by:

$$\begin{aligned} U(h^t, w, a) &= \sum_{\tau=0}^{\infty} \beta^\tau \int_Y \left[y_{t+\tau} - w(h^{t+\tau})\right] d\pi(h^{t+\tau} \mid h^t, a), \\ V(h^t, w, a) &= \sum_{\tau=0}^{\infty} \beta^\tau \int_Y v(w(h^{t+\tau}), a(h^{t+\tau-1}))d\pi(h^{t+\tau} \mid h^t, a). \end{aligned}$$

Given sequences $\delta_t = \{\delta_t(h^{t-1})\}$ and $w_t = \{w_t(h^t)\}$, the sequence $a_t = \{a_t(h^{t-1})\}$ is incentive compatible at h^t if:

$$V(h^t, w, a) \geq V(h^t, w, \overline{a}) = \sum_{\tau=0}^{\infty} \beta^\tau \int_Y v(w_t(h^{t+\tau}), \overline{a}(h^{t+\tau-1}))d\overline{\pi}(h^{t+\tau}; h^t, \overline{a}),$$

for any other sequence $\overline{a}_t = \{\overline{a}_t(h^{t-1})\}$, and $\overline{\pi}$ is the distribution in the future histories induced by δ_t, y_t, w_t and $\overline{a}_t$.

A contract $\sigma_t^{\delta_0}$ is defined by a history-dependent agent's effort recommendation $a_t(h^{t-1})$, and a history-dependent agent's compensation plan $w_t(h^t)$. The agent's history-dependent bargaining power values $\delta_{t+1}(h^t)$ are determined by the agreed-upon law of movement. That is, a contract is given by:

$$\sigma_t^{\delta_0} = \left\{ a_t(h^{t-1}), w_t(h^t) \right\}.$$

We say that a contract $\sigma_t^{\delta_0}$ is feasible if:

$$a_t(h^{t-1}) \in A; \qquad \forall h^{t-1} \in ([0,1] \times Y)^{t-1} \quad \forall t \geq 1, \tag{1}$$

$$0 \leq w_t(h^t) \leq y_t; \qquad \forall h^t \in ([0,1] \times Y)^t \quad \forall t \geq 1, \tag{2}$$

and also the agreed-upon law of motion of the agent's bargaining power must hold:

$$\delta_{t+1}(h^t) = z(h^t) \in [0,1]; \qquad \forall h^t \in ([0,1] \times Y)^t \quad \forall t \geq 0. \tag{3}$$

Condition (1) ensures that the agent's efforts belong to the set of admissible effort values. Condition (2) requires that the agent's salary be non-negative and not greater than the current output. Condition (3) requires that any value of the agent's bargaining power to be contained in the interval $[0, 1]$.

In this environment, for any given δ_t, two conflicting objective functions are simultaneously maximized: the *ex-ante* principal's discounted expected utility, and the *ex-ante* agent's discounted expected utility, subject to incentive compatibility and feasibility. The expected solution is not a unique contract but a unique series of contracts that satisfies Pareto optimality.

A contract $\sigma_t^{\delta_0}$ is Pareto optimal if there is no other feasible and incentive compatible contract $\varphi_t^{\delta_0}$ such that $(U(h^t, \varphi_t^{\delta_0}), V(h^t, \varphi_t^{\delta_0})) \succeq (U(h^t, \sigma_t^{\delta_0}), V(h^t, \sigma_t^{\delta_0}))$, for all h^t. Each Pareto optimal contract $\sigma_t^{\delta_0}$ maximizes both $U(h^t, \sigma_t^{\delta_0})$ and $V(h^t, \sigma_t^{\delta_0})$ subject to feasibility, and incentive compatibility: $V(h^t, w_t, a_t) \geq V(h^t, w_t, \overline{a}_t)$, for all h^t and for all $\overline{a}_t$.

As stated in the previous paragraph, we formulate the dynamic relationship between the principal and the agent that as a multi-objective optimization problem in which both the discounted expected utility of the principal and of the agent are simultaneously optimized subject to feasibility and incentive constraints. Notice that we omit the participation constraint since we do not include in this model the agent's reservation utility, and, this modelling decision could prove to be advantageous because it circumvents the potential time inconsistency problem that might arise from having a forward-looking constraint (the participation constraint in principal-agent environments), as analyzed by Marcet and Marimon (2019). Therefore, we can proceed to transform this problem into a static variational one as in Spear and Srivastava (1987).

The continuation profile from time $t + 1$ onwards for contract σ_t^{δ} at any t, where δ is the initial bargaining power of the agent, given h^t, is determined by $\sigma_t^{\delta} \mid h^t$. This implies a continuation value from time $t + 1$ onwards of $U(\sigma_t^{\delta} \mid h^t)$ for the principal, and of $V(\sigma_t^{\delta} \mid h^t)$ for the agent.

A contract σ_t^{δ} is temporary incentive compatible if, for all t and for all h^t:

$$a_t(h^{t-1}) \in \arg\max_{a \in A} \int_Y [v(w_t(h^t), a) + \beta V(\sigma_t^\delta \mid h^t)] f(y_t; a) dy_t. \quad (4)$$

This constraint ensures that there will be no deviations in the optimal path of the agent's effort decisions, for any δ_t. Furthermore, in order to ensure the validity of the first order approach to this incentive compatibility constraint, we assume that the Monotone Likelihood Ratio Property and the Convexity of the Conditional Distribution Condition are satisfied, following Rogerson (1985).

For every $\delta \in [0, 1]$, define $\mathcal{W}(\delta)$ as the set of the principal's and the agent's discounted expected utility values that are generated by contracts that are feasible, incentive compatible, and characterized by the agent's initial bargaining power given by δ and the agreed-upon law of motion z, as follows:

$$\mathcal{W}(\delta) = \{(U(\sigma^\delta \mid h^0), V(\sigma^\delta \mid h^0)) \mid \exists \ \sigma^\delta \ \ s.t. \ (1), (2), (3), \ \ and \ \ (4)\}.$$

Proposition 1 *$\mathcal{W}(\delta)$ is compact for all δ.*[2]

Given that a unique series of Pareto optimal contracts exists, we will characterize it in a Bellman equation From Proposition 1, we define $(U^*(\delta), V^*(\delta))$ as the Pareto optimal values of the principal's and the agent's discounted expected utilities, respectively, that belong to $\mathcal{W}(\delta)$. Now, let Γ be an operator that maps from the space of the Cartesian product of two spaces of continuous and bounded functions, one for the principal and one for the agent, into itself with the $\sup^*$ norm, defined as $\sup^* = \sup(\sup, \sup)$. Given that the functions $(U(\delta), V(\delta))$ defined on a compact set are bounded and continuous, we can express the operator $\sup^*$ as follows: $\sup^* = \max(\sup, \sup)$. The function $U(\delta) : \mathcal{W}(\delta) \to \mathbb{R}$ is bounded because the principal's rewards are bounded, and the function $V(\delta) : \mathcal{W}(\delta) \to \mathbb{R}$ is also bounded because the agent is risk averse and his compensations are bounded. Also, the problem Γ should be understood as a multi-objective optimization problem, and its solutions, in the non-negative orthant, are Pareto optimal or non-dominated, Sawaragi et al. (1985). Hence, for all $(U(\delta), V(\delta)) \in \mathcal{W}(\delta)$:

$$\Gamma(U, V)(\delta) = \max_{w(\delta,y), \overline{V}(\delta,y), \overline{U}(\delta,y)} \{U(\delta), V(\delta)\}$$

where:

$$U(\delta) = \int_Y [y - w(\delta, y) + \beta \overline{U}(\delta, y)] f(y; a^*(\delta)) dy,$$

$$V(\delta) = \int_Y [v(w(\delta, y), a^*(\delta)) + \beta \overline{V}(\delta, y)] f(y; a^*(\delta)) dy;$$

subject to

$$a^*(\delta) \in \arg\max_{a(\delta) \in A} \int_Y [v(w(\delta, y), a(\delta)) + \beta \overline{V}(\delta, y)] f(y; a(\delta)) dy, \quad (5)$$

[2] The proof is in Appendix 7.1.

$$0 \leq w(\delta, y) \leq y \quad \forall \ y \in Y, \tag{6}$$

$$\delta' = z(\delta, y) \in [0,1], \tag{7}$$

$$(\overline{U}(\delta, y), \overline{V}(\delta, y)) \in \mathcal{W}(\delta') \quad \forall \ y \in Y, \tag{8}$$

where (5) is the incentive compatibility constraint; (6) indicates the agent's temporary inability to borrow; (7) guarantees that the future vale of the agent's bargaining power belongs to the interval $[0,1]$, and (8) ensures that the principal's and the agent's future utility plans are feasible. We now establish that $(U^*(\delta), V^*(\delta))$ is a fixed point of Γ.

Proposition 2 $(U^*(\delta), V^*(\delta)) = \Gamma(U^*, V^*)(\delta), \qquad \forall \delta \in [0,1]$.[3]

The operator Γ satisfies Blackwell's sufficient conditions for a contraction, and the contraction mapping theorem ensures that the fixed point $(U^*(\delta), V^*(\delta))$ is unique for all $(U, V) \in \mathcal{W}(\delta)$. This means that along the resulting Pareto Frontier, PF^*, there exists only one pair if maximal values of the principal's and the agent's discounted expected utilities, given a value of δ, for all $(U, V) \in \mathcal{W}(\delta)$; and vice-versa. Now, PF^* must be non-increasing because otherwise either the principal or the agent can achieve a higher level of discounted expected utility and the other individual would be better off (Spear and Srivastava (1987)).

According to Hernández-Lerma and Romera (2004), our multi-objective dynamic optimization problem admits the following Pareto Weights representation with δ as the state variable:

$$\max_{w(\delta,y)} [\delta V(\delta) + (1-\delta)U(\delta)]$$

subject to constraints (5), (6), (7), and (8). Notice that each objective function has a level of priority associated; that is, δ is the priority assigned to the *ex-ante* agent's discounted expected utility, and $1-\delta$ is the priority assigned to the *ex-ante* principal's discounted expected utility.

In the next section we propose a methodology to numerically approximate the Pareto Frontier derived from this model.

3 Computational Strategy

In this section, we present the computational strategy we devised to numerically approximate the optimal solutions of a parameterized version of the multi-objective dynamic principal-agent model proposed in the previous section. First, we will present the algorithms that outline our computational strategy, then we will specify the functional forms and parameter values we used in our computational program.

[3] The proof is in Appendix 7.2.

3.1 The Computational Algorithm

In this sub-section we present two algorithms that outline the computational program we designed to obtain a numerical solution of the model we propose in this article. [4] Algorithm 1 refers to the process of finding the admissible values of the state variable δ. That is, we find the minimal admissible values of the state variable, δ_{min} , and the maximal admissible value of this variable, δ_{max}. Then, we discretize the space of the admissible values that δ can take. In the second part of our computational strategy we proceed with the recursive process of finding the stationary solution of the Bellman equation for the Pareto Weights representation of our multi-objective dynamic principal-agent model. That is, we iterate the Bellman Equation until convergence is achieved, see Algorithm 2.

3.2 Functional Forms and Parameter Values

In this sub-section, we enlist a series of assumptions regarding functional forms and parameter values we made with the purpose of implementing the computational strategy presented in the previous sub-section.

The principal's temporary utility function is: $u(y, w(y, \delta)) = y - w(y, \delta)$. The agent's temporary utility function is: $v(w(y, \delta), a(\delta)) = \frac{w^{1-h}(y,\delta)}{1-h} - a^2(\delta)$, where $0 < h < 1$. Notice that the agent's temporary utility function is of the CRRA type with respect to current compensation, and that the coefficient of relative risk aversion is h, where higher degrees of relative risk aversion are observed with higher values of h. The agent's feasible effort choices are discrete and belong to the set $A = \{a_L, a_H\}$, where a_L is the low effort choice and a_H is the high effort choice.

Also, there are two levels of output: low (L) or high (H), described by the set $Y = \{y_L, y_H\}$. The probability function that formalizes the stochastic relationship between effort and output is:

$$f(y_L; a_L) = f(y_H; a_H) = \frac{2}{3},$$
$$f(y_H; a_L) = f(y_L; a_H) = \frac{1}{3},$$

and these probabilities capture the idea that the higher the agent's effort level choice is, the greater the likelihood of the realization of the high output level.

The law of motion that we propose for the agent's bargaining power is:

$$\delta' = z(\delta, y) = \begin{cases} \min\{1, \delta + \varepsilon \cdot \frac{y}{y_H}\} & if \quad y = y_H, \\ \max\{0, \delta - \varepsilon \cdot \frac{y}{y_H}\} & if \quad y = y_L. \end{cases}$$

where ε is an arbitrarily small and positive number. This law of motion complies with our

[4] For more details, see Appendix 7.3.

Algorithm 1 Admissible Values of State Variable δ

Require: $\delta_{min} > 0$ ▷ Find δ_{min}

$\mathbf{w} \leftarrow (w_H, w_L)$

$EV_i(\mathbf{w}) \leftarrow \left[f(y_H; a_i)\left(\frac{w_H^{1-h}}{1-h} - a_i^2\right) + f(y_L; a_i)\left(\frac{w_L^{1-h}}{1-h} - a_i^2\right)\right], \quad i = H, L$

$EU_i(\mathbf{w}) \leftarrow [f(y_H; a_i)(y_H - w_H) + f(y_L; a_i)(y_L - w_L)], \quad i = H, L$

$\delta_0 \leftarrow 0$

$EV_i^*(0) \leftarrow \max_{0 \leq \mathbf{w} \leq y} \{\delta_0 EV_i(\mathbf{w}) + (1-\delta_0) EU_i(\mathbf{w})\}, \quad i = H, L$ ▷ s.t. (5) and (6)

$EV^* \leftarrow \max\{EV_i^*(0)\}, \quad i = H, L$

$EV^{**} \leftarrow EV^*$

$t \leftarrow 0$

while $EV^* = EV^{**}$ **do**

  $t \leftarrow t + 1$

  $\delta_t \leftarrow \delta_{t-1} + \Delta$ for $t = 1, 2, ...;$ where $\Delta > 0$ is arbitrarily small

  $EV_i^*(t) \leftarrow \max_{0 \leq \mathbf{w} \leq y} \{\delta_t EV_i(\mathbf{w}) + (1-\delta_t) EU_i(\mathbf{w})\}, \quad i = H, L$ ▷ s.t. (5) and (6)

  $EV^{**} \leftarrow \max\{EV_i^*(t)\}, \quad i = H, L$

end while

$\delta_{min} \leftarrow \delta_t$

Require: $0 < \delta_{max} < 1$ ▷ Find δ_{max}

$\delta_0 \leftarrow 1$

$EV_i^*(0) \leftarrow \max_{0 \leq \mathbf{w} \leq y} \{\delta_0 EV_i(\mathbf{w}) + (1-\delta_0) EU_i(\mathbf{w})\}, \quad i = H, L$ ▷ s.t. (5) and (6)

$EV^* \leftarrow \max\{EV_i^*(0)\}, \quad i = H, L$

$EV^{**} \leftarrow EV^*$

$t \leftarrow 0$

while $EV^* = EV^{**}$ **do**

  $t \leftarrow t + 1$

  $\delta_t \leftarrow \delta_{t-1} + \Delta$ for $t = 1, 2, ...;$ where $\Delta > 0$ is arbitrarily small

  $EV_i^*(t) \leftarrow \max_{0 \leq \mathbf{w} \leq y} \{\delta_t EV_i(\mathbf{w}) + (1-\delta_t) EU_i(\mathbf{w})\}, \quad i = H, L$ ▷ s.t. (5) and (6)

  $EV^{**} \leftarrow \max\{EV_i^*(t)\}, \quad i = H, L$

end while

$\delta_{max} \leftarrow \delta_t$

$N \leftarrow \frac{(\delta_{max} - \delta_{min}) \times 2}{\varepsilon + 1};$ where $\varepsilon > 0$ is arbitrarily small ▷ Discretize δ

$K \leftarrow 0$

while $K \leq N$ **do**

  $K \leftarrow K + 1$

  $D(K) \leftarrow \delta_{min} + \frac{K-1}{N-1}[\delta_{max} - \delta_{min}]$

end while

Algorithm 2 Stationary solution of Bellman Equation

$\mathbf{w} \leftarrow (w_H, w_L)$
$K \leftarrow 0$
while $K \leq N$ **do**
  $K \leftarrow K + 1$
  $S_0(K) \leftarrow 0$ ▷ Initialize value function at zero
  $U_0(K) \leftarrow 0$ ▷ Initialize principal's future utility at zero
  $V_0(K) \leftarrow 0$ ▷ Initialize agent's future utility at zero
end while
$t \leftarrow 0$
while $S_t \neq S_{t-1}$ **do**
  $K \leftarrow 0$
  $P \leftarrow 1$
  $Q \leftarrow 1$
  while $K \leq N$ **do**
    $K \leftarrow K + 1$
    $P \leftarrow \min(K + 2, N)$ ▷ Index of δ' at y_H
    $Q \leftarrow \max(K - 1, 0)$ ▷ Index of δ' at y_L
    $EV_t^i(K; \mathbf{w}) \leftarrow \left[f(y_H; a_i)\left(\frac{w_H^{1-h}}{1-h} - a_i^2 + \beta V_{t-1}(P)\right) + f(y_L; a_i)\left(\frac{w_L^{1-h}}{1-h} - a_i^2 + \beta V_{t-1}(Q)\right)\right],$
$i = H, L$
    $EU_t^i(K; \mathbf{w}) \leftarrow [f(y_H; a_i)(y_H - w_H + \beta U_{t-1}(P)) + f(y_L; a_i)(y_L - w_L + \beta U_{t-1}(Q))],$
$i = H, L$
    $S_t^i(K) \leftarrow \max_{0 \leq \mathbf{w} \leq y} \left\{D(K) EV_t^i(K; \mathbf{w}) + (1 - D(K)) EU_t^i(K; \mathbf{w})\right\}, i = H, L$ ▷ s.t.
(5) and (6)
    $S_t(K) \leftarrow \max\{S_t^i(K); i = H, L\}$
  end while
  $t \leftarrow t + 1$
end while

assumptions about the function $z(\delta, y)$, and it provides incentives in the form of a greater next-period bargaining power if output y_H is observed, and a punishment in the opposite direction if output y_L occurs at the current period. We interpret the parameter ε as a measure of how closely future values of the CEO's bargaining power represent rewards or punishments for good versus bad performances of the firm. An additional assumption behind this proposed law of motion is that the principal is more generous with the rewards than astringent with the punishments for increments in the next-period agent's bargaining power are equal to ε whereas reductions in the next-period agent's bargaining power are lower than ε. This assumption is based on asymmetrical responses that have been observed in CEO pay in the face of good versus bad outcomes, see for instance Gopalan et al. (2010) and Bell et al. (2021).

Our set of parameters are the following:

$$h = \frac{1}{2}; \beta = 0.96; Y = \{y_L = 0.4, y_H = 0.8\}; A = \{a_L = 0.1, a_H = 0.2\}; \varepsilon = 0.001.$$

Finally, we must say that our numerical exercise should not be taken as a calibration exercise.

4 Results

In the present section, we show some of the stationary and simulation results of our numerical implementation.[5] Both sets of results provide a different but complementary visions of the model proposed in this article. As a robustness test, we also obtain numerical results of related models, considered as benchmark or reference models: the standard dynamic principal-agent model (SDPA), the multi-objective static principal-agent model (MOSPA), the multi-objective dynamic principal-agent model with no dynamics for the agent's bargaining power (MODPA1), and the full-fledged multi-objective dynamic principal-agent model with dynamics for the agent's bargaining power (MODPA2). [6]

4.1 Stationary Results

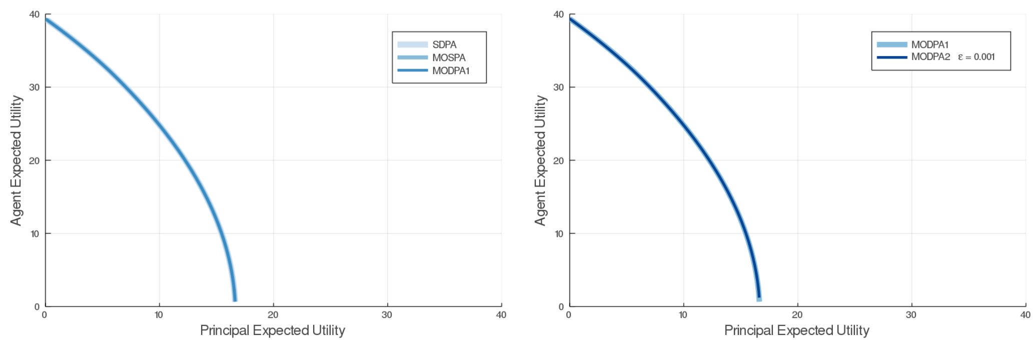
First, in Figure 3 we present our Pareto Frontier results considering the four models that we have already mentioned. In part (a) of this figure, we can observe that the Pareto Frontiers of the three benchmark models (SDPA, MOSPA, MODPA1) coincide. Also, in part (b) we show the Pareto Frontiers of the multi-objective dynamic models without and with dynamics of the agent's bargaining power (MODPA 1 and MODPA2) for a small value of the parameter $\varepsilon = 0.001$. This figure shows that the Pareto Frontiers associated with all the models considered here are identical for this value of ε, which establishes a baseline case from which to analyze further cases. This figure also presents us with an evidence that each initial value of the agent's bargaining power has an associated agent's reservation utility level, see Curiel et al. (2022) for a static analysis.

Figure 4 depicts the Pareto Frontiers we obtained from model MODPA2 by changing some

[5]Further information is available here: https://github.com/genarobasulto/Project-Dynamics-of-Bargaining-Power

[6]See Appendix 7.4 for more details about these models.

Figure 3: Pareto Frontier: Model Comparison

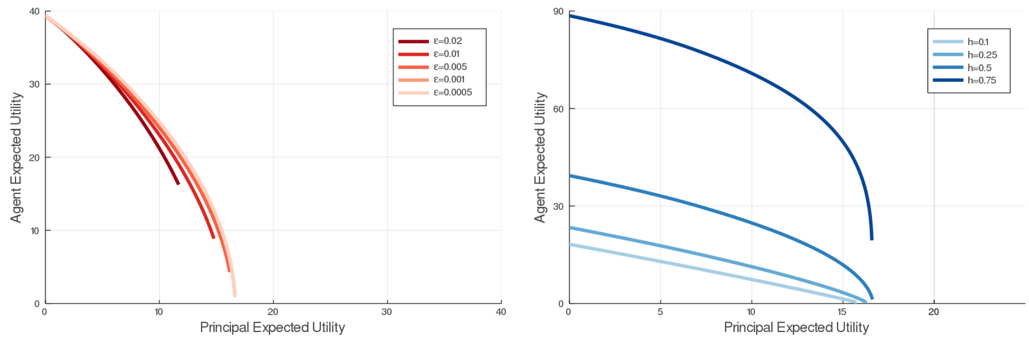


(a) Single objective and multi-objective approach (b) Bargaining dynamics and no dynamics

parameter values. In panel (a), we show results obtained from varying the values of the parameter ε, which in the baseline case has a value of ε is 0.001. We consider the following additional values for this parameter $\{0.02, 0.01, 0.005, 0.0005\}$. Our objective is to understand the impact of values of ε that are higher and lower than 0.001 on the Pareto Frontier, while keeping the agent's relative risk aversion parameter constant at $h = 0.5$. Our results indicate that as ε increases, the results reveal inferior discounted expected utility values for both the principal and the agent, and the maximal discounted expected utility that the principal can achieve during the contractual relationship diminishes. That is, the Pareto Frontier tends to rotate towards the origin staying fixed at the same maximal value the agent can obtain situated at the extreme in the y-axis and its extension decreases. In panel (b), the results of an exercise performed for several values of the agent's relative risk aversion parameter h are presented. Notice that, in this exercise, we have considered the following values of h given by $\{0.1, 0.25, 0.75\}$ as well as the value of 0.5 of the baseline case. Our results show that as the agent's risk aversion parameter decreases; that is, the agent is less risk averse, the Pareto Frontier shifts towards the origin producing combinations of discounted expected utility values for both the principal and the agent that are lower.

Figure 5 renders our results concerning the agent's promised discounted expected utilities, or future compensation, for the models we have considered in this analysis. In all the models, the agent's promised discounted expected utility (for high and low outputs) has a positive and linear relationship with the agent's current utility, and for the values of the parameters considered here ($\varepsilon = 0.001$ and $h = 0.5$), there is no difference in the promised utilities for the different output levels. So, for this set of parameter values, future compensation is not the incentive tool the principal uses. In addition, Figure 6 depicts the same results as in the previous figure but having as the independent variable the state variable of our dynamic model. The results show that there is an increasing and non-linear relationship between the agent's bargaining power and their next-period promised discounted utility. Moreover, the relationship between the two variables is observed to follow a bell-shaped curve; in particular, for lower values of the parameters ε in the first panel and h in the second panel. On the other hand, the distance in this policy rule

Figure 4: Pareto Frontier: Parameter Comparison

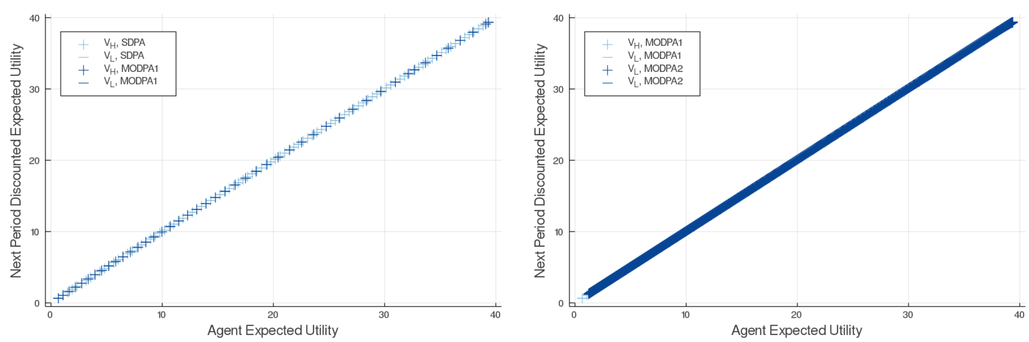


(a) Different Values for ε

(b) Different Values for h

for high and low output realizations decreases with ε, while there is no observed distance when considering different values of the relative risk aversion parameter.

Figure 5: Agent's Promised Discounted Expected Utilities: Model Comparison

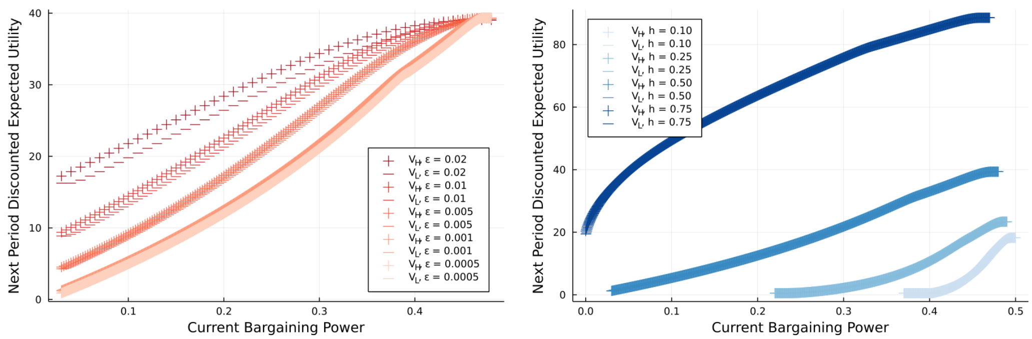


(a) Single objective and multi-objective approach

(b) Bargaining dynamics and no dynamics

Figure 7 illustrates the agent's promised bargaining power results for high and low output realizations, given different parameter values. In panel (a), we show the results when the parameter ε changes. The baseline value for ε is 0.001, and the other values considered here are: $\{0.02, 0.01, 0.005, 0.0005\}$. We observe that as ε increases, the distance between the agent's promised bargaining power results for high and low output realizations also increase. That is, the higher the value of ε, the lower the insurance level the agent receives. In panel (b), we present similar results when varying the agent's relative risk aversion parameter, which seems not to influence the difference between the agent's promised bargaining power results for high and low output realizations.

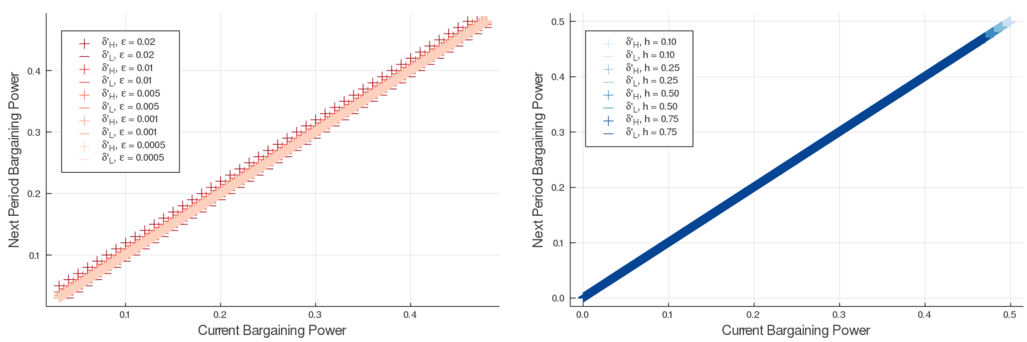
Figure 6: Agent's Promised Expected Utility on Current Bargaining Power: Parameter Comparison



(a) Different Values for ε

(b) Different Values for h

Figure 7: Agent's Promised Bargaining Power: Parameter Comparison

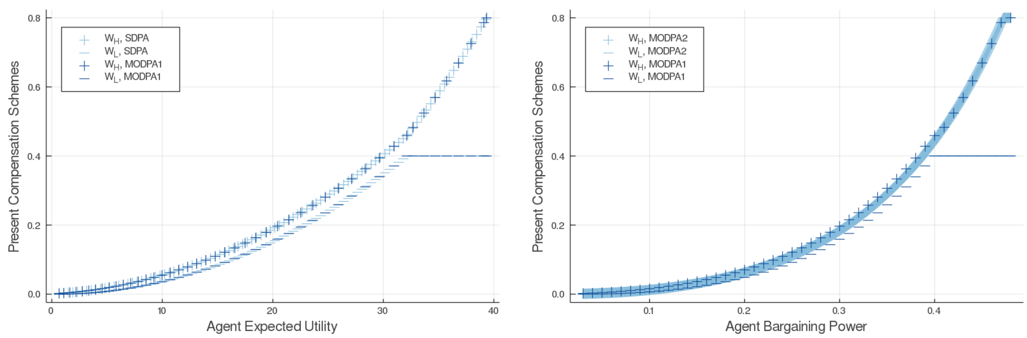


(a) Different Values for ε

(b) Different Values for h

Figure 8 presents the results we obtained for the agent's present compensation (salary) for high and low output realizations, for the different models we include in our analysis. In panel (a), we show the results for the following models: SDPA, MOSPA, and MODPA1. The three models have the same results and the agent consistently gets a higher salary when the low output realization is obtained when compared to the salary for the low output realization. However, when comparing the full-fledged multi-objective model with dynamics for the agent's bargaining power versus the multi-objective model without such a dynamics, we observe that in the full-fledged model, salaries are equal regardless the output realization. That is, in the full-fledged model, the preferred incentive tool is the agent's promised bargaining power and neither future nor present compensations provide incentives in this model.

Figure 8: Current Compensation: Model Comparison



(a) Single objective and multi-objective approach

(b) Bargaining dynamics and no dynamics

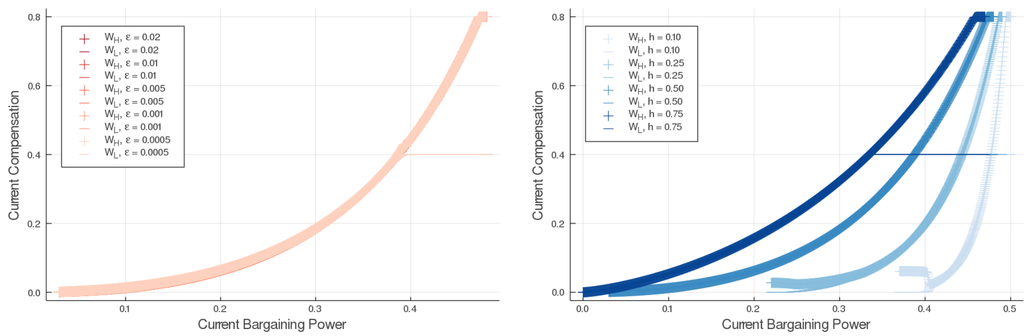
Finally, in Figure 9 we show the results we obtained for the agent's present compensation for high and low output realizations, for the full-fledged multi-objective model but considering different values for the parameters. In panel (a), the results we obtain when we vary ε are depicted, and we conclude that variations in this parameter value do not affect the agent's salary. In panel (b), we present the results we get when the parameter h changes, and we can state that as the agent becomes more risk averse, their salary increases, for a given level of the agent's bargaining power. From the results illustrated in Figures 7 and 9, we conclude that it is plausible that CEOs with even the same relative risk aversion parameter might show different stationary paths of their bargaining powers and salaries that can only be explained by the agreed-upon law of motion for their bargaining power at the beginning of the principal-agent relationship. That is, the initial value of the CEO's bargaining power and the value of ε in the law of motion determine the evolution of the optimal contractual arrangements between the CEO and the shareholders.

4.2 Simulation Results

In this sub-section, we present and discuss the results of simulation exercises we performed by allowing the CEO's initial bargaining power, δ_0, to take four values: $\{0.1, 0.2, 0.3, 0.4\}$. The idea is to analyze the impact of each of those initial conditions on the evolution of the contractual arrangements between the principal and the agent. We also consider a set of several values of ε, apart from 0.001 which constitutes our baseline value, given by: $\{0.02, 0.01, 0.005, 0.0005\}$. We allow for 100 contracting periods, and we assume in the exercises that the agent chooses the high effort level, which implies a higher probability of obtaining the higher firm performance.

In Figure 10 we show the results of the agent's bargaining power, which has a positive trend with respect to the contractual periods. This positive trend is explained by the higher probability of the higher output, given that the agent chooses the high effort level. The agent's bargaining power increments are directly proportional to the values of ε; that is, the higher ε, the faster the

Figure 9: Current Compensation: Parameter Comparison



(a) Different Values for ε

(b) Different Values for h

agent's bargaining power reaches its maximal value, even in the case when the initial value of the bargaining power is the lowest considered in our simulation exercise, see panel (a).

The simulation results regarding the agent's salary are presented in Figure 11. We observe that for lower values of ε and/or δ_0, there is a positive relationship between the agent's salary and the contractual periods. However, for higher values of ε and/or δ_0 we observe greater variability of salary with respect to the previous contractual period. This behavior is related to the agent's bargaining power behavior when they reach values close to their maximal values which show wiggles. So, when the agent's reaches faster the maximal values of their bargaining power, they tend to bring as a consequence a higher variability in their salaries. Finally, the data generated by these simulation exercises will be used in the next section to perform an econometric analysis with the purpose of validating some of our parameter values.

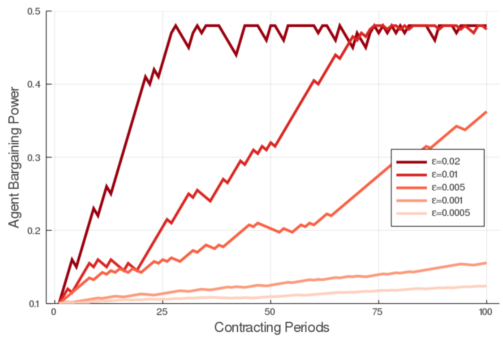
5 Econometric Exercise

The numerical results of our model provide a way to estimate the parameter ε that appears in the proposed functional form for the law of movement $z(\delta, y)$, described in subsection 3.2. We interpret this parameter as a measure of of the power of incentives in our model because the higher the value of ε, the higher the CEO's level of insurance and the lower the power of incentives.

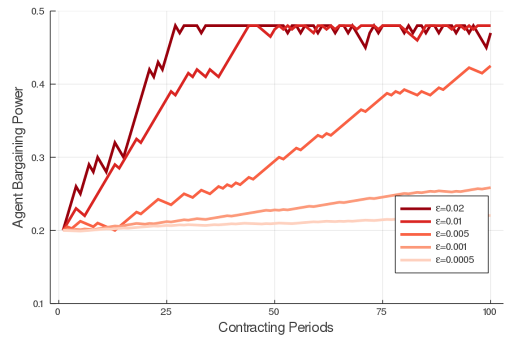
We begin by considering the results depicted in Figure 9, where we plot the agent's current compensation by corresponding values of the bargaining power parameter. The plot shows that the agent bargaining power can be represented as a concave function of their current compensation. We assume that both the agent compensation and his bargaining power are stochastic at any period of time, and that this relationship takes the following form:

$$\delta_t = a + b\sqrt{w_t} + e_t \tag{9}$$

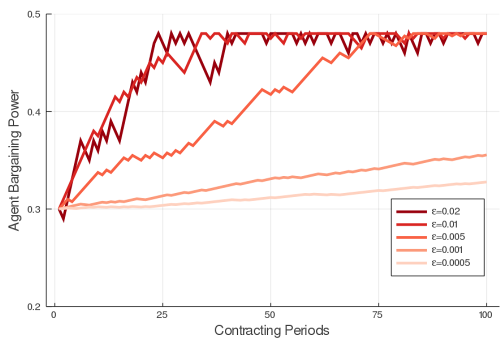
Figure 10: Simulation: Agent's Bargaining Power



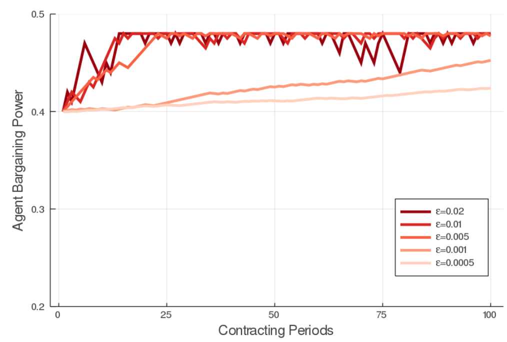
(a) $\delta_0 = 0.10$



(b) $\delta_0 = 0.20$



(c) $\delta_0 = 0.30$



(d) $\delta_0 = 0.40$

where e_t are independent and normally distributed errors, with mean 0 and variance σ^2 for all t.

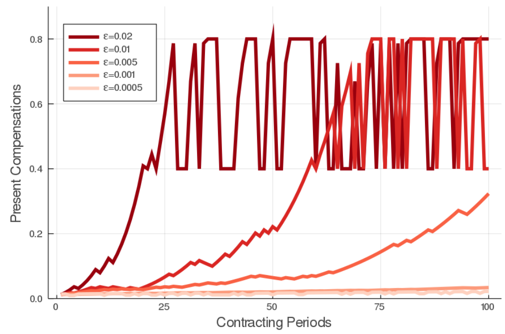
Furthermore, our numerical results allow us to conclude that when the principal holds all of the bargaining power, the agent compensation will be the lowest possible, which can be normalized to 0 and the parameter a in (9) takes the value of zero. Similarly, the agent will receive the maximal compensation level when they hold all the bargaining power, and the parameter b in (9) takes the value of $1/\sqrt{y_H}$. So, from (9) and the results for a and b, we attain the following equation:

$$\delta_t = \frac{1}{\sqrt{y_H}}\sqrt{w_t} + e_t. \tag{10}$$

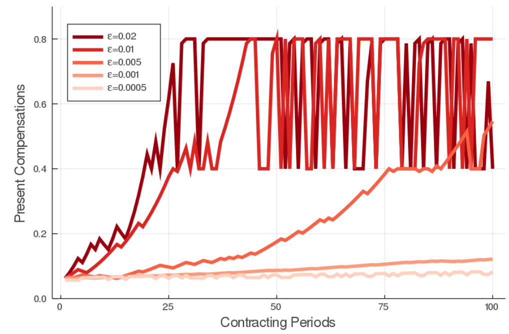
From the law of motion for the agent's bargaining power, proposed in sub-section 3.2, we obtain:

$$\delta_{t+1} = z(\delta_t, y) = \begin{cases} \min\{1, \delta_t + \varepsilon \cdot \frac{y_t}{y_H}\} & if \quad y_t = y_H, \\ \max\{0, \delta_t - \varepsilon \cdot \frac{y_t}{y_H}\} & if \quad y_t = y_L. \end{cases}$$

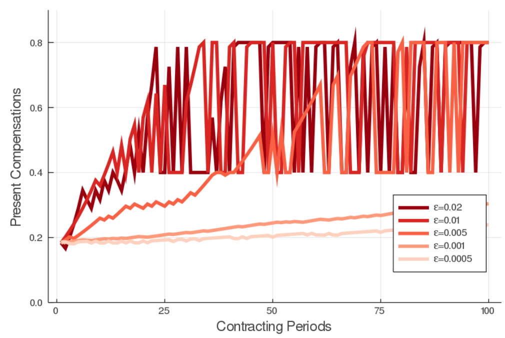
Figure 11: Simulation: Agent's Compensation



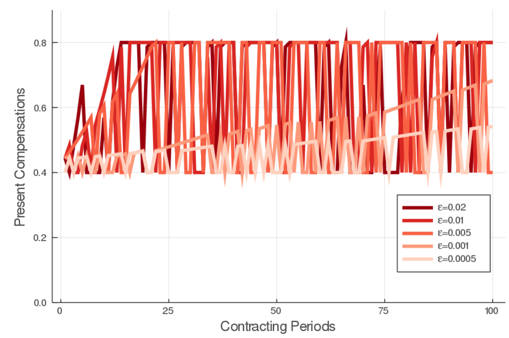
(a) $\delta_0 = 0.10$



(b) $\delta_0 = 0.20$



(c) $\delta_0 = 0.30$



(d) $\delta_0 = 0.40$

This is equivalent to:

$$\delta_{t+1} - \delta_t = \varepsilon \left((-1)^{I_t^-} \frac{y_t}{y_H} \right),$$

where I_t^- is a dichotomous variable that takes the value 0 when $y_t = y_H$, and the value 1 otherwise.

Replacing w_{t+1} and w_t from (10) in the last expression, the following result is obtained:

$$\sqrt{w_{t+1}} = \sqrt{w_t} + \varepsilon \left((-1)^{I_t^-} \frac{y_t}{\sqrt{y_H}} \right) + u_t, \tag{11}$$

with $u_t = (e_t - e_{t+1})\sqrt{y_H}$.

Expression (11) could be used to empirically estimate ε using a time series database with CEO compensations and company revenues.

Notice that in (9) a strong assumption is made; that is, the errors e_t are assumed to be independent. In a time series setting it is common for the errors to be serially correlated, so the best way of estimating the regression is to use an auto-regressive moving average (ARMA) model that accounts for the correlation in order to provide consistent estimators for ε. Another possibility is to consider panel data with observations from different companies during the same period of time. Exploiting the structure of panel data could give us more insight on how ε varies from one company to another and how it affects the CEO's contractual packages. In the next two sub-sections, we apply this functional form for an empirical analysis of CEOs' bargaining power. The rest of the present section is structured as follows: First, using data generated by the simulation exercises, we evaluate parameter values for which this functional form might provide a good identification. We conclude this section by empirically identifying CEOs' bargaining power using real-life CEO compensation data, that we describe in detail in the corresponding sub-section.

5.1 Testing with simulation data.

In this sub-section we present an econometric exercise using the simulation data that we presented in the previous section. The objective is to identify sets of values for our parameters for which the functional form given by equation (11). In Table 1 we show the results of running the linear regression: $log(\delta_t) = \beta \log(w_t)$. Given the parameter values of our numerical implementation, we expect to obtain estimates of $\hat{\beta}$ close to 0.5, and our results indicate that that such estimates are close to this value for several values of ϵ.

Table 1: Regression Results on shape of bargaining power on expected compensation.

	Dependent variable:		
	log(Deltas)		
	$\varepsilon = 0.001$	$\varepsilon = 0.005$	$\varepsilon = 0.01$
log(exp_comp)	0.554***	0.554***	0.553***
	(0.003)	(0.007)	(0.010)
Observations	901	181	91
R^2	0.969	0.969	0.969
Adjusted R^2	0.969	0.969	0.969
Residual Std. Error	0.298 (df = 900)	0.299 (df = 180)	0.301 (df = 90)
F Statistic	28,118.090***	5,618.442***	2,812.328***
Note:			*p<0.1; **p<0.05; ***p<0.01

Then we proceed to run a regression for implementing equation (11). In Table 2 we summarize the results we obtain by using model-generated data for 100 contractual periods for different values of ϵ and different values of the state variable, δ_0. The first section corresponds to

$\epsilon = 0.001$; and we observe that when the initial value of the agent's bargaining power is 0.10, the ϵ-estimate is the real value in the model. Furthermore, when those initial values belong to the interval $[0.20, 0.30]$, the ϵ-estimate is close to its real value and statistically significant. However, when $\delta_0 = 0.40$, the estimate of ϵ cannot be considered as a good approximation, it even has a negative sign. These results indicate that when the initial value of the agent's bargaining power is low, our proposed dynamics explains well the relationship between the agent's compensation and the firm's performance. On the other hand, when the initial value of the agent's bargaining power is high, the dynamics seems to change. From Figure 11, we can state that when the initial value of the agent's bargaining power is high, compensation is in a stationary state; that is, it does not grow because it has reached the maximum value possible, and it does not decrease either because the agent is still exerting high effort levels. Therefore, the results we observe for high values of the state variable are expected.

The second and third blocks of the table correspond to the following set of values: $\epsilon = \{0.05, 0.01\}$, respectively. We notice that the lower the value of ϵ, the lower the values of the state variable for which we obtain good estimates of ϵ. So, if we expect that the agent's bargaining power does not change much from one period to the next, we will be able to estimate this change correctly. A hypothesis is that this is due to the linearity of our dynamics because if the real dynamics is a function of a higher order as ϵ grows, so do the region where we are approximating and the errors.

5.2 Empirical Identification

In this subsection, we perform a regression analysis to empirically identify equation (11). To achieve this goal, we use two databases from S&P Global Market Intelligence. The first database is the Compustat's Snapshot database which contains annual financial data from 24,657 companies with headquarters in the United States territory, ranging from the year 1998 to 2022. Secondly, we use the Capital IQ's Compensation Detail database which contains compensation data from 46,256 different CEOs of companies based in the United States from the year 1999 to 2020.

The main variable of interest in the Compustat's Snapshot database is the annual Earnings Before Interest and Tax (EBIT) from each company, which we identify as the companies' output y_t for our econometric implementations. The measure we use for CEO current compensation is reported in the Compensation Detail database, and it is Total Compensation (Salary + Bonus). We also include a measure of CEO's compensation that includes the already mentioned Total Compensation and the present value of two measures of CEO's future compensation: Long Term Incentive Plans (LTIP) and Value of Options Granted. Each one of these variables were adjusted accounting for inflation taking the base year as 1999. We then classified the above measures by director and fiscal year of observation and merge the databases. After data cleaning, our final panel sample consists of annual observations of 8,327 CEOs from the fiscal year 1999 to 2020, and this yields a total of $68,190$ observations. Descriptive statistics from the final sample are shown in Table 3. The average EBIT among all companies and all years is approximately $503.34MM, while the executives' average salary, total compensation and Total Compensation plus LTIP and Value of Options Granted are about $0.41MM, $0.5MM, and $2.15MM, respectively.

Table 2: Regression Results Using simulated data.

	Dependent variable:			
	y			
$\varepsilon = \mathbf{0.001}$	$(\delta_0 = 0.10)$	$(\delta_0 = 0.20)$	$(\delta_0 = 0.30)$	$(\delta_0 = 0.40)$
X	0.001^{***}	0.002^{***}	0.002^{***}	-0.048^{***}
	(0.00002)	(0.00002)	(0.00003)	(0.008)
R^2	0.986	0.985	0.985	0.263
F Statistic (df = 1; 99)	$6,954.480^{***}$	$6,667.812^{***}$	$6,483.073^{***}$	35.355^{***}
$\varepsilon = \mathbf{0.005}$				
X	0.009^{***}	0.008^{***}	-0.035^{***}	-0.116^{***}
	(0.0002)	(0.002)	(0.012)	(0.018)
R^2	0.950	0.202	0.083	0.301
F Statistic (df = 1; 99)	$1,893.272^{***}$	25.006^{***}	9.005^{***}	42.633^{***}
$\varepsilon = \mathbf{0.01}$				
X	-0.037^{***}	-0.059^{***}	-0.069^{***}	-0.099^{***}
	(0.013)	(0.015)	(0.015)	(0.017)
R^2	0.074	0.134	0.177	0.253
F Statistic (df = 1; 99)	7.934^{***}	15.261^{***}	21.302^{***}	33.474^{***}
Observations	100			
Note:				*p<0.1; **p<0.05; ***p<0.01

Table 3: Data Summary

	Variable	Mean	Standard Deviation	Unique Values	Min	Max
1	salary	0.41	0.2	44,427	0.0550	1.3288
2	total comp.	0.5	0.34	48,906	0.0971	3.0984
3	total comp. + LTIP + Value of Options Granted	2.15	2.18	67813	0.1399	17.3610
4	ebit	503.34	885.22	22,266	3.1325	11,066.1107
5	age	54.24	7.4	69	28	96
6	gvkey	-	-	2,060	-	-
7	execid	-	-	8,327	-	-
8	year	-	-	22	1999	2020
9	state	-	-	53	-	-
10	inflation_rate	0.02	0.01	22	-0.003	0.0383
11	Y	0	0.11	54191	-1.1443	1.0907
12	I^-	0.69	0.46	2	0	1
13	I_1^-	0.61	0.49	2	0	1
14	X	-9.78	16.62	22286	-105.1955	51.8344
15	X_1	-7.47	15.42	22286	-86.3376	40.6622

Next, we compute the variables:

$$Y_{it} = \sqrt{w_{i,t+1}} - \sqrt{w_{i,t}}$$

and

$$X_{it} = \left((-1)^{I^-_{it}} y_{it}/\sqrt{y_{iH}}\right),$$

where i indexes the CEO and t indexes time, H refers to the high output realization, I^- is a dichotomous variable, already defined in the previous sub-section, that takes the value 0 when $y_t = y_H$ and the value 1 otherwise. In the data the mean of I^-, measures the proportion of times in which the companies' performance was closer to its highest value than its lowest. In table 3, the variables Y, I^- and X are computed using company EBIT and total CEO compensation, the variables I_1^- and X_1 are computed using total CEO compensation plus LTIP and Value of Options Granted.

We run three regression models where the associated coefficient of X is the parameter ε of our model, using EBIT and Total Current Compensation. The results of these regressions are summarised in Table 4. The first model, OLS1, is a simple linear model adjusted by OLS with only the variable X as regressor. Notice that even in this model the estimate for ε is positive and small, as in our theoretical approach, and it is statistically significant at 1%. However, this first approach does not take into account the panel structure of the data. We take this in consideration and run a fixed effects model with a within estimation. The results are shown in column (3), and we observe that the estimator for ε is still statistically significant at 1% and stays well within the order of magnitude we use for the numerical approach. We run a second OLS model, OLS2, in which we add additional controls, such as the state where the company is based and the CEO age, with the objective of obtaining a better estimation for the effect of ε in the model by limiting the influence of unidentified variables. The results from this regression are shown in column (2) of Table 4. When controlling with factors such as geographical location and company age, we obtain an estimate for ε that is positive and statistically significant at 1%, and the geographical location and the CEO's age are also statistically significant at 1%. Given that the variation of the estimate for ε are of very small numerical magnitude, our results are consistent with what is reported in (Jenter and Kanaan (2015)) with respect to present compensation, that its growth shows an increasing pattern but that it not as dramatic as the one that future compensation measures show.

Following Bell et al. (2021), we run additional regressions using a measure of CEO compensation that not only includes the elements of the measure Total Compensation used in the previous regressions, but it also includes the present value of two instances of future compensation: LTIP and Value of Options Granted. The purpose of this additional exercise is to observe whether the estimate of ϵ significantly changes by including the present value of future CEOs compensation, which is the part of CEO compensation that has been observed to increase dramatically since the decade of the '90s, see Jenter and Kanaan (2015). Table 5 shows the results obtained with this new measure of compensation, and we conclude that the estimates' numerical values are slightly higher but very close to those shown in the previous table. So, our results indicate that even by including in the compensation measure the present value of two instances of future compensation, the bargaining power of the real-life CEOs included in this sample show small variations from period to period.

6 Conclusions

This paper contributes to the literature of dynamic moral hazard by proposing a multi-objective dynamic principal-agent model with the CEO's initial bargaining power as the state variable, which contrasts with the agent's reservation utility that has been the usual state variable in repeated principal-agent models. Moreover, we introduce a law of motion that governs the evolution of the agent's bargaining power characterized by awarding measurable increases or decreases in the next-period CEO's bargaining power level according to high or low, respectively, output realizations. Another feature of this law of motion is that it includes an arbitrarily small and positive parameter, ε, that can be interpreted as a measure of the power of the incentives provided by this system of rewards/punishments.

From the results we obtain from the implementation of a numerical algorithm that we devised, we can state that it is plausible that agents with even the same relative risk aversion parameter might show different stationary paths of their bargaining powers and salaries. This diversity is only explained by the values of the model's state variable and its law of motion. Our simulation results indicate that the level and variability of the CEO's salary is positively linked to the initial values of the CEO's bargaining power; whereas the relationship is negative between the

Table 4: Regression Results Using EBIT and Total Current Compensation.

	Dependent variable:		
	Y		
	OLS1	*OLS2*	*panel linear*
	(1)	(2)	(3)
X	0.0001^{***}	0.0001^{***}	0.0002^{***}
	(0.00001)	(0.00001)	(0.00002)
as.factor(state)AL		0.072^{***}	
		(0.003)	
as.factor(state)AR		0.077^{***}	
		(0.003)	
⋮	⋮	⋮	⋮
as.factor(state)WY		0.076^{***}	
		(0.010)	
age		-0.001^{***}	
		(0.00003)	
Observations	68,190	68,190	68,190
R^2	0.0003	0.031	0.003
Adjusted R^2	0.0003	0.030	-0.136
Residual Std. Error	0.058 (df = 68189)	0.057 (df = 68135)	
F Statistic	18.785^{***}	39.139^{***}	162.645^{***}
	(df = 1; 68189)	(df = 55; 68135)	(df = 1; 59862)
Note:			*p<0.1; **p<0.05; ***p<0.01

Table 5: Regression Results Using EBIT and Total Current Compensation + LTIP + Value of Options Granted.

	Dependent variable:		
	Y		
	OLS		*panel linear*
	(1)	(2)	(3)
X_1	0.0003***	0.0002***	0.0004***
	(0.00003)	(0.00003)	(0.00004)
as.factor(state)AL		0.067***	
		(0.006)	
as.factor(state)AR		0.073***	
		(0.006)	
⋮	⋮	⋮	⋮
as.factor(state)WY		0.053**	
		(0.021)	
age		−0.001***	
		(0.0001)	
Observations	68,091	68,091	68,091
R^2	0.002	0.009	0.002
Adjusted R^2	0.002	0.009	−0.137
Residual Std. Error	0.113 (df = 68090)	0.113 (df = 68036)	
F Statistic	151.316***	11.616***	129.902***
	(df = 1; 68090)	(df = 55; 68036)	(df = 1; 59766)
Note:			*p<0.1; **p<0.05; ***p<0.01

variability of the CEO's salary and the value of the parameter ε. That is, the more powerful the incentives, the higher the variability of the CEO's salary.

Finally, we propose an empirical equation to identify the bargaining power values of real-life CEOs based on our theoretical and numerical results. We perform a regression analysis and estimate the parameter ε. By using model-generated data, we arrive to the finding that when the initial value of the agent's bargaining power is low, our proposed dynamics explains well the relationship between the agent's compensation and the firm's performance. But we also observe that for high values of the agent's bargaining power the proposed dynamics does not explain well such relationship. By analyzing a panel sample of annual observations for 9,084 CEOs from the fiscal year 1999 to 2020 constructed from the Compustat's Snapshot database, we observe that the sign and magnitudes of our estimates for ε are positive, of a small numerical magnitude and statistically significant at 1% both for the measure of present compensation (salary + bonus) and the measure that includes present compensation and the present value of two instances of future compensation. Moreover, the estimates for both set of regressions are very similar in term of numerical value, which allows us to state that indicate that even by including in the compensation measure the present value of two instances of future compensation, the bargaining power of the real-life CEOs included in this sample show small variations from period to period.

7 Appendix

7.1 Proof of Proposition 1

Proof. Let δ be arbitrary fixed. $\mathcal{W}(\delta)$ is bounded. We need to prove that $\mathcal{W}(\delta)$ is also closed. Let $\{(U^n(\delta), V^n(\delta))\} \in \mathcal{W}(\delta)$ such that $\underset{n \longrightarrow \infty}{lim} \{(U^n(\delta), V^n(\delta))\} = \{(U^\infty(\delta), V^\infty(\delta))\}$. We have to show that $\{(U^\infty(\delta), V^\infty(\delta))\} \in \mathcal{W}(\delta)$, or that there exists a contract $\sigma^{\delta,\infty}$ that satisfies (1), (2), (3), (4), $U(\sigma^{\delta,\infty} \mid h^0) = U^\infty(\delta)$, and $V(\sigma^{\delta,\infty} \mid h^0) = V^\infty(\delta)$. We construct this optimal contract $\sigma^{\delta,\infty}$. The definition of $\mathcal{W}(\delta)$ allows us to say that there exists a sequence of contracts $\{\sigma^{\delta,n}\} = \{a_t^{\delta,n}(h^{t-1}), w_t^{\delta,n}(h^t)\}$ that satisfies (1), (2), (3), and (4), $\forall n$. Hence,

$$U^\infty(\delta) = \underset{n \longrightarrow \infty}{lim} \sum_{t=0}^{\infty} \beta^t \int_Y (y_t - w_t^{\delta,n}(h^t) f(y_t; a_t^{\delta_0,n}(h^{t-1}))dh^t,$$

$$V^\infty(\delta) = \underset{n \longrightarrow \infty}{lim} \sum_{t=0}^{\infty} \beta^t \int_Y (v(w_t^{\delta,n}(h^t), a_t^{\delta,n}(h^{t-1}))f(y_t; a_t^{\delta,n}(h^{t-1}))dh^t.$$

At $t = 1$, $\sigma_1^\delta = \{a_1^{\delta,n}(h^0), w_1^{\delta,n}(h^1)\}$ is a finite collection of bounded sequences, so there exists a collection of subsequences $\{a_1^{\delta,n_q}(h^0), w_1^{\delta,n_q}(h^1)\}$ that satisfy:

$$\underset{n_q \longrightarrow \infty}{lim} a_1^{\delta,n_q}(h^0) = a_1^{\delta,\infty}(h^0), \quad \text{and}$$

$$\underset{n_q \longrightarrow \infty}{lim} w_1^{\delta,n_q}(h^1) = w_1^{\delta,\infty}(h^1).$$

The following law of motion must hold:

$$\delta_1^{\delta, n_q}(h^0) = z(h^0).$$

Also, $(U^\infty(\delta), V^\infty(\delta))$ must be equal to $(U(\sigma_1^\delta \mid h^0), V(\sigma_1^\delta \mid h^0))$. If $V^\infty(\delta) < V(\sigma_1^\delta \mid h^0)$, the agent would not accept this contract given that the principal is obtaining $U(\sigma_1^\delta \mid h^0)$, and if $V^\infty(\delta) > V(\sigma_1^\delta \mid h^0)$, $V^\infty(\delta)$ would not belong to $\mathcal{W}(\delta)$ because it does violate the principle of Pareto optimality, given that the principal is getting $U(\sigma_1^{\delta_0} \mid h^0)$. A similar argument proves that $U^\infty(\delta) = U(\sigma_1^\delta \mid h^0)$.

Repeating this procedure for $t = 2, ..., \infty$, and letting:

$$\sigma^{\delta,\infty} = \{a_t^{\delta,\infty}(h^{t-1}), w_t^{\delta,\infty}(h^t)\},$$

we obtain the desired contract $\sigma^{\delta,\infty}$. ■

7.2 Proof of Proposition 2

Proof. Let δ be arbitrary fixed. First, we show that $\Gamma(U^*, V^*)(\delta) \leq (U^*(\delta), V^*(\delta))$. This is true if there exists σ^δ that is feasible and incentive compatible such that $(U(\sigma^\delta \mid h^0), V(\sigma^\delta \mid h^0)) = \Gamma(U^*(\delta), V^*(\delta))$. We construct this contract σ^δ by letting $a(\delta)$, $w(\delta, y)$, $\overline{U}(\delta, y)$, and $\overline{V}(\delta, y)$ be the solution to $\Gamma(U^*(\delta), V^*(\delta))$, and by letting:

$$a_1(h^0) = a(\delta), \text{and } \; w_1(h^1) = w(\delta, y), \qquad \forall h^0, h^1.$$

For a given $y_1 \in Y$, there exists $\sigma_{y_1}^\delta$ such that the principal receives $\overline{U}(\delta, y_1)$ and the agent receives $\overline{V}(\delta, y_1)$. Let

$$\sigma^\delta \mid h^1 = \sigma_{y_1}^\delta, \qquad \forall h^1.$$

Notice that $\sigma_{y_1}^\delta$ complies with Pareto optimality, because $\overline{U}(\delta, y_1) = U^*(\sigma_{y_1}^\delta \mid h^1)$, and $\overline{V}(\delta, y_1) = V^*(\sigma_{y_1}^\delta \mid h^1)$. So, there is no other contract $\varphi_{y_1}^\delta$ that is Pareto optimal such that $U^*(\varphi_{y_1}^\delta \mid h^1)$ and $V^*(\varphi_{y_1}^\delta \mid h^1)$ dominate $U^*(\sigma_{y_1}^\delta \mid h^1)$ and $V^*(\sigma_{y_1}^\delta \mid h^1)$; that is, $U^*(\varphi_{y_1}^\delta \mid h^1) \prec U^*(\sigma_{y_1}^\delta \mid h^1)$ and $V^*(\varphi_{y_1}^\delta \mid h^1) \prec V^*(\sigma_{y_1}^\delta \mid h^1)$. So, $\sigma_{y_1}^\delta$ is the contract we need, and $\Gamma(U^*, V^*)(\delta) \leq (U^*(\delta), V^*(\delta))$.

The second part of the proof shows that $(U^*(\delta), V^*(\delta)) \leq \Gamma(U^*, V^*)(\delta)$. Let $\sigma^{\delta *}$ be an optimal contract. Hence,

$$U^*(\delta) = U(\sigma^{\delta *} \mid h^0) = \int_Y [y_1 - w^{\delta *}(y_1) + \beta U(\sigma^{\delta *} \mid h^1)] f(y_1; a^{\delta *}(h^0)) dy_1,$$

$$V^*(\delta) = V(\sigma^{\delta *} \mid h^0) = \int_Y [v(w^{\delta *}(y_1), a^{\delta *}(h^0)) + \beta V(\sigma^{\delta *} \mid h^1)] f(y_1; a^{\delta *}(h^0)) dy_1,$$

and

$$(U^*(\delta), V^*(\delta)) \leq \Gamma(U^*, V^*)(\delta);$$

if we set $a(\delta) = a^{\delta*}(h^0), w(\delta, y_1) = w^{\delta*}(y_1), \overline{U}(\delta, y_1(\delta_1)) = U^*(\sigma^{\delta*} \mid y_1)$, and $\overline{V}(\delta, y_1(\delta_1)) = V^*(\sigma^{\delta*} \mid y_1)$, for $y_1 \in Y$; for (7), (8), and (9) are satisfied. It must be noted that $U^*(\delta) = \overline{U}(\delta, y_1)$ and $V^*(\delta) = \overline{V}(\delta, y_1(\delta_1))$ because, $\sigma^{\delta*}$ is Pareto optimal; and there is no other contract $\varphi^{\delta*}$ that is also Pareto optimal. ■

7.3 Computational Algorithm

The first step of the algorithm is to compute the set of admissible values of the agent's bargaining power. The objective is to discretize the range of the admissible values of the state variable, δ in n steps, such that n is sufficiently large given ε.

For this, we define the current expected utilities of the agent and the principal, respectively, given the the exertion of the high and low effort levels on the part of the agent, respectively:

$$EV_h(w_H, w_L) = \left[f(y_H; a_H) \left(\frac{w_H^{1-h}}{1-h} - a_H^2 \right) + f(y_L; a_H) \left(\frac{w_L^{1-h}}{1-h} - a_H^2 \right) \right],$$

$$EV_l(w_H, w_L) = \left[f(y_H; a_L) \left(\frac{w_H^{1-h}}{1-h} - a_L^2 \right) + f(y_L; a_L) \left(\frac{w_L^{1-h}}{1-h} - a_L^2 \right) \right],$$

$$EU_h(w_H, w_L) = \left[f(y_H; a_H) (y_H - w_H) + f(y_L; a_H) (y_L - w_L) \right],$$

$$EU_l(w_H, w_L) = \left[f(y_H; a_L) (y_H - w_H) + f(y_L; a_L) (y_L - w_L) \right].$$

Then, for the minimum bargaining power δ_{min} that guarantees interior solutions, we solve two optimization problems, one to incentivize the agent to exert low effort and one for high effort.

First, we set $\delta_0 = 0$ and solve for the high effort model, as follows:

$$\max_{w_H, w_L} \left\{ \delta_0 EV_h(w_H, w_L) + (1-\delta_0) EU_h(w_H, w_L) \right\} \tag{HE}$$

subject to:

$$EV_h(w_H, w_L) \geq EV_l(w_H, w_L),$$

$$0 \leq w_H, w_L \leq y \qquad \text{for all } y.$$

Now, we do the same for the case of the low effort model:

$$\max_{w_H, w_L} \left\{ \delta_0 EV_l(w_H, w_L) + (1-\delta_0) EU_l(w_H, w_L) \right\} \tag{LE}$$

subject to:

$$EV_l(w_H, w_L) \geq EV_h(w_H, w_L),$$

$$0 \leq w_H, w_L \leq y \qquad \text{for all } y.$$

From solving these two problems, we obtain the corner solution: $EV^{**} = \{EV_i | \delta_0 EV_i^* + (1-\delta_0)EU_i^* \geq \delta_0 EV_j^* + (1-\delta_0)EU_j^*; i, j = H, L\}$.

At this point, we iterate $\delta_t = \delta_{t-1} + \Delta$, for $t = 1, 2, ...$; where Δ is an arbitrarily small and positive number, and solve the same two problems that we solved for δ_0, (HE) and (LE), just varying δ_t.

If the solution: $EV^* = \{EV_i | \delta_t EV_i^* + (1-\delta_t)EU_i^* \geq \delta_t EV_j^* + (1-\delta_t)EU_j^*; i, j = H, L\}$, when $EV^* \neq EV^{**}$ is satisfied, the iteration stops, and we set $\delta_{min} = \delta_t$.

For the maximum admissible bargaining power, we do something similar, this time we set $\delta_0 = 1$, solve (HE) and (LE), and define EV^{**} as before. Afterwards, we iterate $\delta_t = \delta_{t-1} - \Delta$, $t = 1, 2, ...$, then solve (HE) and (LE). Again, the iteration stops when $EV^* \neq EV^{**}$, then $\delta_{max} = \delta_t$.

To finish the first step, we compute the set of admissible values of the agent's bargaining power: $\mathbb{D} = \{D(K), K = 1, ..., N\}$; where $N = \frac{(\delta_{max} - \delta_{min}) \times 2}{\varepsilon + 1}$. Notice that this characterization of N ensures the step between two consecutive bargaining powers is equal to $\frac{\varepsilon}{2}$, and:

$$D(K) = \delta_{min} + \frac{K-1}{N-1}[\delta_{max} - \delta_{min}].$$

The second step is to find the stationary solution of the Bellman Equation. We start with an all-zero guess for the value function $S_0(K) = 0, \forall K = 1, ..., N$; and initialize all-zero vectors for the agent and principal expected discounted utilities $U_0(K) = 0, V_0(K) = 0, \forall K = 1, ..., N$. Define:

$$EV_t^H(K; w_H, w_L) = \left[f(y_H; a_H) \left(\frac{w_H^{1-h}}{1-h} - a_H^2 + \beta V_{t-1}(P) \right) + f(y_L; a_H) \left(\frac{w_L^{1-h}}{1-h} - a_H^2 + \beta V_{t-1}(Q) \right) \right],$$

$$EV_t^L(K; w_H, w_L) = \left[f(y_H; a_L) \left(\frac{w_H^{1-h}}{1-h} - a_L^2 + \beta V_{t-1}(P) \right) + f(y_L; a_L) \left(\frac{w_L^{1-h}}{1-h} - a_L^2 + \beta V_{t-1}(Q) \right) \right],$$

$$EU_t^H(K; w_H, w_L) = [f(y_H; a_H)(y_H - w_H + \beta U_{t-1}(P)) + f(y_L; a_H)(y_L - w_L + \beta U_{t-1}(Q))],$$

$$EU_t^L(K; w_H, w_L) = [f(y_H; a_L)(y_H - w_H + \beta U_{t-1}(P)) + f(y_L; a_L)(y_L - w_L + \beta U_{t-1}(Q))];$$

where $P = \min(K+2, N)$, $Q = \max(K-1, 0)$. Notice that the value for P (Q) gives us the index for δ' in $\mathbb{D}$ when output y_H or y_L is observed.

Suppose that we are now at the t-th iteration, $t \geq 1$:

$$S_t(K) = \max\{S_t^i(K); i = H, L\},$$

where

$$S_t^H(K) = \max_{w_H, w_L} \left\{D(K)EV_t^H(K; w_H, w_L) + (1 - D(K))EU_t^H(K; w_H, w_L)\right\};$$

subject to:

$$EV_t^H(w_H, w_L) \geq EV_t^L(w_H, w_L),$$

$$0 \leq w_H, w_L \leq y \qquad \text{for all } y,$$

and

$$S_t^L(K) = \max_{w_H, w_L} \left\{D(K)EV_t^L(K; w_H, w_L) + (1 - D(K))EU_t^L(K; w_H, w_L)\right\};$$

subject to:

$$EV_t^L(w_H, w_L) \geq EV_t^H(w_H, w_L),$$

$$0 \leq w_H, w_L \leq y \qquad \text{for all } y.$$

The algorithm stops once we find the stationary solution; that is, $S_t(K) = S_{t-1}(K)$, for all $K = 1, ..., N$.

7.4 Benchmark Models

We use three benchmark or reference models in our numerical implementation. Here we present them:

7.4.1 The Standard Dynamic Principal-Agent Model (SDPA)

A first reference model is the standard dynamic principal-agent model. In particular, we adapt the model of Wang (1997), which is a model formulated as the maximization of the expected discounted utility of the principal subject to the participation constraint, the incentive compatibility constraint, and feasibility constraints. The value function is the principal's expected discounted utility is given by:

$$U(\hat{V}) = \mathbb{E}[y - w(y, \hat{V}) + \beta\overline{U}(y, \hat{V})],$$

where y is the observed output, $w(y, \hat{V})$ is the compensation given the observed output y and β is the discount factor of the principal and the agent. The agent´s lifetime discounted expected utility is given by :

$$V(\hat{V}) = \mathbb{E}[v(w(y, \hat{V}), a(\hat{V})) + \beta\overline{V}(y, \hat{V})],$$

where $v(w(y, \hat{V}))$ is the temporary utility function of the agent, and $a \in A$ is the effort exerted by the agent. In addition, $\overline{V}(y, \hat{V})$ is the agent future discounted utility, which is the promised expected utility from tomorrow on. $\hat{V} \in \mathbf{V}$ is the model's state variable and it is the agent's reservation utility. This is an important difference with respect to our multi-objective dynamic models given that our models' state variable is the agent's initial bargaining power.

The dynamic maximization program is:

$$\max_{w(y,\hat{V}),\overline{V}(y,\hat{V})} \mathbb{E}[y - w(y, \hat{V}) + \beta\overline{U}(y, \hat{V})]$$

subject to

$a(\hat{V}) \in argmax_{a'} \quad \hat{V}(a', \hat{V}),$ Incentive Compatible;

$V(\hat{V}, a(\hat{V})) = \hat{V},$ Individual Rationality;

$a(\hat{V}) \in A,$ Feasible Effort;

$0 \leq w(y, \hat{V}) \leq y$ for all y, Limited Liability;

$\overline{V}(y, \hat{V}) \in \mathcal{V}$ for all y, Feasible and Incentive compatible $\overline{V}$.

Let $\mathcal{V}(\hat{V})$ and $\mathcal{U}(\hat{V})$ be the set of feasible and incentive compatible expected discounted utilities of the agent and principal, respectively. Wang (1997) demonstrates that $\mathcal{U}(\hat{V})$ is compact. Therefore, by virtue of the Bellman equation, there exists a principal's maximal expected discounted utility that is feasible and incentive compatible.

7.4.2 The Multi-Objective Static Model (MOSPA)

A second reference model is the multi-objective static principal-agent model. In this setting, the static contracting problem is to choose an action $a \in A$ and a compensation scheme $w(y, \delta) \in [0, y]$, $\forall y \in Y$, to maximize the Pareto Weights function of the expected utility of the principal and that of the agent; that is:

$$\max_{a(\delta),w(y,\delta)} [\delta v(w(y, \delta), a(\delta)) + (1 - \delta)u(y, w(y, \delta))],$$

subject to

$$\int_Y v(w(y,\delta), a(\delta)) f(y;a) dy \geq \int_Y v(w(y,\delta), a'(\delta)) f(y;a'(\delta)) dy \quad \forall a'(\delta) \in A, \tag{12}$$

$$0 \leq w(y,\delta) \leq y \quad \forall y \in Y. \tag{13}$$

7.4.3 The Multi-Objective Dynamic Model With No Bargaining Power Dynamics (MODPA1)

A third reference model is the multi-objective dynamic principal-agent model with no dynamics of bargaining power. In particular, we analyze the optimal contractual arrangements having the agent's bargaining power, δ, as the model's state variable, but without the implications of explicitly including a law of motion for δ. That is, in this version of the model we analyze the maximization of the Pareto Weights function of the expected discounted utility of the principal and that of the agent, subject to the feasibility and incentive compatible constraints. This problem is formulated as follows:

$$\max_{a(\delta), w^\delta(y^\delta, W), \overline{V}^\delta(y^\delta, W), \overline{U}^\delta(y^\delta, W)} [\delta V^\delta + (1-\delta) U^\delta],$$

where:

$$U(\delta) = \int_Y [y - w(y,\delta) + \beta \overline{U}(y,\delta)] f(y; a(\delta)) dy, \tag{14}$$

$$V(\delta) = \int_Y [v(w(y,\delta), a(\delta)) + \beta \overline{V}(y,\delta)] f(y; a(\delta)) dy; \tag{15}$$

subject to

$$a(V) \in argmax_{\,a'} \quad \hat{V}(a', V), \tag{16}$$

$$0 \leq w(y,\delta) \leq y \quad \forall y \in Y, \tag{17}$$

$$\delta \in (0,1), \tag{18}$$

$$(\overline{U}(y,\delta), \overline{V}(y,\delta)) \in \mathcal{W}(\delta') \quad \forall y \in Y. \tag{19}$$

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