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RESEARCH ARTICLE

Privatization in the Natural Gas Sector: A General Equilibrium Analysis*

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Abstract

A broad literature highlights efficiency gains due to cost reduction after privatizations in the energy sector. However, to the best of our knowledge, this literature does not develop general equilibrium models, which are fundamental to account for post-privatization gains from a regional perspective. This paper evaluates the increase in efficiency necessary to make the privatization of a natural gas local distribution company (LDC) worthwhile in a state-level fiscal sense. We propose a general equilibrium model representing a regional economy supplied by a monopolistic LDC, whose ownership is shared between the private sector and federal and state governments and calibrate it for 13 of the major Brazilian LDCs. We find that the necessary unit cost reduction varies substantially across LDCs and depends on the level of underpricing when the asset is sold. The necessary unitary cost reductions range from 1.6% to 64% when we consider the median level of underpricing found in the literature.

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1. Introduction

Since the 1990s many developing countries have been implementing structural reforms involving a larger participation of the private sector into the energy sector. One of the traditional arguments for why privatization might be beneficial is the efficiency gains obtained from profit-maximizing firms (see Megginson and Netter, 2001; Walheer and He, 2020). For example, Brown et al. (2006) find large productivity gains in Romania, Hungary, and Ukraine after the collapse of the Communist Party but significant losses for Russia. More recently, Deutschmann et al. (2023) show that privatizing sewage treatment centers improved efficiency significantly in Senegal and reduced prices paid by households. In this paper, we ask what should the efficiency gains be so that privatization measures are justified? In particular, what is the efficiency gain needed so that state governments ensure their revenues remain constant?

The economic literature has documented a strong correlation between energy consumption per capita and economic growth (Soytas and Sari, 2009; Fei et al., 2011; Mahadevan and Asafu-Adjaye, 2007), which is also true in the case of natural gas consumption (Sasana and Ghozali, 2017; Cheng, 1997; Poveda and Martinez, 2011; Zhixin and Xin, 2011). According to the Latin American Energy Organization (OLADE), in 2020, natural gas comprised 30% of the Latin American and Caribean primary energy supply and 20% of final consumption. For some countries, such as Argentina and Mexico, it comprises more than 50% of the primary energy supply (59% and 58%, respectively). In particular, its consumption is especially relevant in the electricity and industrial sectors, although it is also increasingly consumed by households, commercial establishments, and as automotive fuel in compressed form (CNG). Thus, privatization policies in strategic sectors might be of first-order importance since their effects spread throughout the economy.

We focus on the case of Brazil, which has recently sanctioned a law in 2021 known as the New Gas Law.[1] This law highlights the privatization of local distribution companies (LDCs, henceforth) and increased competition in the oil and gas sectors as one of its goals. In Brazil, however, natural gas represents only 11% of the primary energy supply. This low participation could be partially attributed to the low competition in Brazil's oil and gas sectors, in which Petrobras, a state-owned company, was an effective monopolist. Although in July 2022 Petrobras sold its participation in the LDCs market, 25 out of 27 Brazilian states still have a stake in LDCs. We take the Brazilian New Gas Law as our case study.

We propose a general equilibrium model representing the economy of a certain region supplied by a monopolistic natural gas LDC, whose ownership is shared between the private sector and both federal and state governments. The model is characterized by a representative agent that demands a final consumption good and supplies labor elastically. The production sector is comprised of sub-sectors that require labor and natural gas as production inputs. The LDC is a monopolist that sells natural gas to different sectors at different tariffs, and a producer supplies gas inelastically to the LDC at an exogenous and constant price. Labor and intermediate goods prices are endogenously determined in general equilibrium.

The model is calibrated for 13 major Brazilian LDCs, using financial and operational data from these distributors, as well as economic data from the regions where they operate. With the calibrated model, we calculate the distribution efficiency gains necessary for justifying, from the fiscal point of view, the privatization of the LDCs. In other words, we find the productivity levels for which the increase in state tax revenues covers the loss of dividends. We also study the economic effects of increased competition in gas production, represented here as a reduction in the price of natural gas purchased by LDCs.

The general equilibrium framework we propose is more suitable for counterfactual analysis than reduced form evidence because we can account for how the productive sector adjusts to changes in the ownership structure of LDCs. Given the central role of natural gas as an intermediate input, changes in the efficiency of LDCs result in reduced costs for the other sectors. This, by itself, already improves

[1] Law No. 14134/2021.

the economy's overall efficiency, but it also allows us to quantify the indirect effect on increased tax revenues. In fact, our results suggest that this indirect effect is larger for some states than the direct effect from the contribution of LDCs. This highlights the importance of the general equilibrium approach in studying tax revenue effects in the post-privatization analysis.

In our results, the efficiency gains necessary for privatization vary substantially across LDCs and depend crucially on the degree of underpricing. Evidence shows that governments systematically tend to underprice the asset value in privatization offers (Jones et al., 1999; Megginson, 2010). Jones et al. (1999) estimate that the median level of underpricing for initial share issue privatizations is at 12.4% while the average level is 34.1%. We consider a range of underpricing levels between 0% and 40% and show that the efficiency gains vary substantially across levels of underpricing and states.

When assuming the median underpricing level (12.4%), the necessary efficiency gains range from 1.6% to 64%. For context, Brown et al. (2006) estimate gains of 2% in Ukraine, 8% in Hungary, and 15% in Romania. In the Latin American context, Rossi (2001) estimates a 4% gain in efficiency after ten years due to the privatization of Argentinian LDCs. In our results, five out of eight LDCs require efficiency gains below 13%, which are in the range estimated by the empirical literature. This experiment does not imply that privatization will lead to those efficiency gains, however this allows us to gauge how plausible the efficiency gains need to be in order to make state revenues constant.

The Brazilian natural gas sector is controlled mainly by Petrobras, which held a monopoly over production and had a stake in most LDCs until July 2022 (Mathias and Szklo, 2007). Furthermore, natural gas prices in Brazil have remained consistently high (IEA, 2021) even compared to countries like France, Latvia, or Sweden. These high prices are due to prices being linked to oil products. Thus, enhanced competition is a plausible way of reducing prices. In our model, LDCs are effective monopolies that set tariffs considering the sectoral demand for natural gas.[2] We use our model to simulate an increase in competition in natural gas production by reducing the price exogenously. A drop of 5% in natural gas prices can mildly increase long-run regional GDP by 0.06% and increase substantially the use of natural gas by 6.64%.

There is an ongoing discussion about relevant reforms for the sector. Leal et al. (2019) propose changes in the Brazilian regulatory framework to ensure better long-term incentives to production and distribution companies to invest, also in line with a transition towards a lower emission energy system. In the U.S., deregulation of fuel-fired power plants decreased the price paid for coal but not for natural gas (Cicala, 2015, 2021). For electricity generation costs, Cicala (2022) finds a reduction of 5% in generation costs after liberalization. Our paper contributes to this debate by showing the high degree of heterogeneity across necessary efficiency gains to keep state revenues constant, taking into account underpricing and simulating the effects of a drop in the price of natural gas. Therefore, our model is a tool that can complement public policies in this matter.

A broad literature highlights efficiency gains after privatizations in the energy sector (Plane, 1999; Andrés et al., 2006; Pombo and Taborda, 2006; Pérez-Reyes and Tovar, 2009; Eller et al., 2011; Hartley and Medlock, 2013; Baldassarri et al., 2016; Gakhar and Phukon, 2018). However, these articles cannot account for the general equilibrium effects that emerge from privatization which are fundamental to account for the indirect effects. In particular, we show that the price elasticity of natural gas demand is an endogenous object that will be determined in general equilibrium and varies across sectors. These differences across sectors in both natural gas intensity and the price elasticity of demand are crucial for determining the impacts of productivity increases following privatization.

There is also an extensive literature that considers energy as a production input in general equilibrium contexts (Unalmis et al., 2009; Alpanda and Peralta-Alva, 2010; Bodenstein and Guerrieri, 2011; Antonakakis et al., 2014; Huynh, 2016; Zhao et al., 2016; Fried, 2018; Fried and Lagakos, 2020). We differ from them, modeling energy from natural gas as a relevant input and highlighting the LDC's role

[2]We consider the electricity, industrial, commercial, and residential sectors.

in the natural gas chain. Furthermore, this work also contributes to the literature on regional development and energy transition, aligning the economic benefits that the cost reduction of clean-burning fossil fuels could have for local economies (see for example Le et al. (2019) and Guerrero-Lemus and Shephard (2017)).

The rest of the paper is organized as follows. Section 2 provides a background for the Brazilian natural gas sector. Section 3 presents the general equilibrium model for a regional economy with an LDC. Section 4 presents the data and the calibration strategy. Section 5 presents the main results and counterfactuals. Finally, Section 6 concludes.

2. The Natural Gas Industry in Brazil

The path that natural gas takes from the natural deposit to the final consumer can be divided into production, transportation, and distribution activities. Production consists of finding, extracting, and processing natural gas from underground reservoirs. Transportation is usually done through high-pressure pipelines over long distances. Finally, distribution is performed at smaller distances but to a more diverse range of final consumers.

Even though the Brazilian Federal Constitution of 1988 establishes that the States are responsible for the activities of distribution, up until 1995, the state-owned Brazilian oil company Petrobras held a monopoly over all activities related to the natural gas industry, such as production and distribution. Since then, some reforms have been aimed at increasing competition in the sector, but they haven't necessarily fulfilled this goal.[3]

The gas market in Brazil is supplied by imports from Bolivia and Argentina, mainly through pipelines and by imports of Liquefied Natural Gas (LNG) from other locations. In the last two decades, domestic production has more than tripled and currently represents 60% of the total supply, with 82% concentrated offshore (Diaz, 2021; Agência Nacional do Petróleo, 2022). According to Agência Nacional do Petróleo (2022), in June 2022, Petrobras was responsible for approximately 90% of natural gas production in Brazil.

The Law 9.478 (Oil Law) from 1997 was aimed at breaking Petrobras' monopoly over the activities of research, exploration, production, and refining of oil and natural gas. In the end, even as other companies were allowed to participate in these activities, they were obliged to work with Petrobras, who held a stake in every activity step. Law 11.909 (Natural Gas Law) from 2009 targeted the natural gas sector more directly, abolishing the state monopoly. Even with the privatization of some companies, Petrobras held a stake in most of the LDCs. The exceptions were the LDCs from Rio de Janeiro and São Paulo, which were privatized in 1998 and 1999, respectively.

In July 2019, Petrobras and the Brazilian competition authority (CADE) signed a Cessation Commitment Term (TCC). Both parties agreed that Petrobras would provide third-party access to infrastructure to increase the number of players in gas commercialization and sell its transportation and distribution assets (Diaz, 2021). In July 2022, Petrobras sold 51% of the Petrobras Gas S.A. (Gaspetro) to Compass Gás e Energia S.A. (Compass) in line with the TCC.

According to Petrobras' release, Gaspetro is a holding company with equity interests in 18 out of the 27 total LDCs in Brazil. Its distribution networks add up to approximately 10 thousand km, serving more than 500 thousand customers, with a distributed volume of around 29 million m^3/day.[4] Although Petrobras sold its stake in Gaspetro, state governments hold, in most cases, 51% of the voting shares,

[3] A comprehensive outline of the Brazilian natural gas industry, from a historical point of view, can be found in Junior and de Almeida (2007) and Diaz (2021).

[4] Its corporate structure, which had Petrobras as the main shareholder with 51% of the shares, becomes 51% of the shares of Compass and 49% of the shares of Mitsui Gás e Energia do Brasil Ltda.

having a pivotal vote for the privatization of LDCs. Therefore, analyzing the benefits for state governments is critical for LDCs' privatization.

3. Model

Our model represents a regional economy that consists of sectors $\{1, 2, \ldots, J\}$ that produce intermediate goods, a final good producer that uses the intermediate goods and natural gas to produce a final consumption good, a household that provides labor and consumes the final good, and the government. Furthermore, there is production and distribution of natural gas. Distribution activities are carried out by the LDC, which behaves as a monopolist that takes into account the sectoral demand to determine the tariff that will charge to each sector. To distribute natural gas, the LDC has to purchase it from the producer that sells it at an exogenous price. Each intermediate good producer $j \in J$ uses natural gas and labor supplied elastically by a representative agent that consumes the final good. We describe the model in full detail in the following sections.

3.1. Natural Gas Production

Natural gas is produced and sold to the LDC in our model at an exogenous price p.[5] We assume the marginal cost of production is constant and equal to z_p expressed in units of the final consumption good, our numeraire. The net profits of the natural gas producer are given by $(p - z_p)\, g$, where g is the amount of gas sold to the LDC. We denote the total production cost by $K_p \equiv z_p g$.

3.2. Natural Gas Distribution

The distribution activities of the LDC consist of buying a quantity g of natural gas from the natural gas producer at a fixed exogenous price p and distributing it to each sector of the economy $i \in \{0, 1, 2, \ldots, J\}$ charging a sector-specific tariff t_i.[6] A tax rate τ is imposed by the state government on the LDC's value added. The net profits of the LDC are given by

$$\Pi_d = \max_{\{t_i\}_{i=0}^{J}} \sum_{i=0}^{J} \left[(1-\tau)(t_i - p) - z_i\right] g_i(t_i)$$

where z_i denotes the distribution cost (in units of final consumption good) per unit of natural gas for sector i, and $g_i(t_i)$ is the sectoral demand curve. The first-order conditions of the maximization problem yield an expression for the tariffs of each sector

$$t_i = \mu_i(t_i)\left(p + \frac{z_i}{1-\tau}\right) \quad (1)$$

$$\mu_i(t_i) = \frac{\varepsilon_i(t_i)}{\varepsilon_i(t_i) - 1} \quad (2)$$

[5] The exogenous price could be micro-founded by the fact that natural gas prices are driven by other energy prices (Nick and Thoenes, 2014), which are determined globally and hardly affected by the actions of a local economy.

[6] The sectors that demand gas from the LDC are the J intermediate good producers and the final good producer, which is indexed with 0.

$$\varepsilon_i\left(t_i\right)=-t_i\frac{g_i'\left(t_i\right)}{g_i\left(t_i\right)} \tag{3}$$

where $\mu_i\left(t_i\right)$ is the mark-up and $\varepsilon_i\left(t_i\right)$ the price elasticity of natural gas demand for sector i.[7] In equilibrium, $\varepsilon_i(t_i)$ will be constant and determined by the elasticity of substitution between natural gas and the other production inputs. Net profits are given by $\Pi_d=\sum_{i=0}^{J}\left[\mu_i\left(t_i\right)-1\right]\left[\left(1-\tau\right)p+z_i\right]g_i\left(t_i\right)$ and total costs, net of natural gas purchase expenses are denoted by $K_d\equiv\sum_{i=0}^{J}z_i g_i\left(t_i\right)$.

3.3. Intermediate Goods Producers

In the production side of the economy, there are J intermediate sectors, each represented by a representative competitive firm. Firm j produces a quantity y_j of the intermediate good j that is sold at price p_j, using natural gas (g_j) and labor (l_j) as inputs, with respective prices t_j and w.[8] A value-added tax is levied by the state government, where the tax rate is denoted by τ. Note that, in this model, the value added of the sector j is given by the expenditure on labor. The production technology is a constant elasticity of substitution (CES) production function given by

$$y_j=F_j\left(g_j,l_j\right)=\left[\alpha_j g_j^{\frac{\rho_j-1}{\rho_j}}+\left(1-\alpha_j\right)l_j^{\frac{\rho_j-1}{\rho_j}}\right]^{\frac{\rho_j}{\rho_j-1}} \tag{4}$$

where α_j denotes the relative weight of each input and ρ_j is the elasticity of substitution between natural gas and labor inputs. The CES production function nests three limiting cases depending on the elasticity of substitution. When $\rho_j=0$, the production function becomes Leontief, and natural gas and labor are perfect complements; if $\rho_j=1$, it becomes Cobb–Douglas; and in the case when $\rho_j\to\infty$ it becomes linear, and the inputs are perfect substitutes.[9]

Profit maximization then implies:

$$t_j=p_j\alpha_j\left(\frac{y_j}{g_j}\right)^{\frac{1}{\rho_j}},\quad j=1,\ldots,J \tag{5}$$

$$w=\left(1-\tau\right)p_j\left(1-\alpha_j\right)\left(\frac{y_j}{l_j}\right)^{\frac{1}{\rho_j}},\quad j=1,\ldots,J \tag{6}$$

Under this CES production function, the elasticity of substitution between labor and natural gas in sector j is constant and equal to ρ_j. In the model, we can express the demand for natural gas of sector j as

$$g_j=\left(\frac{\alpha_j}{t_j}\right)^{\rho_j}\frac{\mathbb{M}_j}{\alpha_j^{\rho_j}t_j^{1-\rho_j}+(1-\alpha_j)^{\rho_j}\left(\frac{w}{1-\tau}\right)^{1-\rho_j}}$$

where $\mathbb{M}=\frac{w}{1-\tau}l_j+t_jg_j$ is the total expenditure on inputs. We can then define the price elasticity of demand as

$$\varepsilon_j^D\equiv\frac{d\log(g_j)}{d\log(t_j)}=-\rho_j+(\rho_j-1)\frac{\alpha_j^{\rho_j}t_j^{1-\rho_j}}{\alpha_j^{\rho_j}t_j^{1-\rho_j}+(1-\alpha_j)^{\rho_j}\left(\frac{w}{1-\tau}\right)^{1-\rho_j}} \tag{7}$$

[7] This assumes that the LDC cannot change the level of production of the intermediate goods producers directly.

[8] In principle, it could be the case that natural gas could be substituted by liquefied petroleum gas or other energy sources. In this case, it would be necessary to compare the price of natural gas relative to that of other energy sources. In that context, increased efficiency in using natural gas can induce energy transitions away from other energy sources. To evaluate what would be the fiscal benefits, we would need first to assess how the efficiency improvement would change the relative prices and the relative demands. Therefore, an increase in natural gas efficiency could drive consumers of other energy sources towards natural gas, thus increasing consumption of natural gas which would revert in fiscal revenues.

[9] See Klump et al. (2012) for an in-depth survey on the CES production function and its properties.

which crucially depends on the elasticity of substitution ρ_j.

The use of a CES production function serves three main purposes. First, it is analytically tractable and allows us to find the price elasticity of demand for natural gas in closed form, which is a crucial object in this context. Second, it is flexible enough to incorporate the three aforementioned limiting cases (Leontief, Cobb–Douglas, and linear). Third, it introduces a key driving force of economic growth, substituting scarce factors by abundant factors. This flexibility allows us to calibrate the elasticity of demand for natural gas according to empirical evidence. In our context, an increase in the efficiency of the LDC will induce changes in the tariffs, which will change the demand for natural gas. The extent to which the demand for natural gas changes will be at the core of evaluating the benefits of privatization. Furthermore, it follows the canonical model of monopolistic competition of Dixit and Stiglitz (1977).[10] This implies that the profits made by the LDC will also depend on the elasticity of substitution between natural gas and labor.

3.4. Final Consumption Good Producer

A single final consumption good is produced by a representative competitive firm using intermediate goods and residential natural gas as inputs. Its production technology is given by

$$y = F_0\left(g_0, \{c_j\}_{j=1}^{J}\right) = A\left[\left(1-\sum_{j=1}^{J}\beta_j\right)g_0^{\frac{\rho_0-1}{\rho_0}} + \sum_{j=1}^{J}\beta_j c_j^{\frac{\rho_0-1}{\rho_0}}\right]^{\frac{\rho_0}{\rho_0-1}} \tag{8}$$

where y is the quantity of final good produced, g_0 is the residential natural gas demand and c_j is the intermediate good j demand. We normalize the price of the final consumption good to 1. The first order conditions with respect to g_0 and c_j yield:

$$t_0 = A^{\frac{\rho_0-1}{\rho_0}}\left(1-\sum_{j=1}^{J}\beta_j\right)\left(\frac{y}{g_0}\right)^{\frac{1}{\rho_0}} \tag{9}$$

$$p_j = A^{\frac{\rho_0-1}{\rho_0}}\beta_j\left(\frac{y}{c_j}\right)^{\frac{1}{\rho_0}} \qquad j=1,\ldots,J \tag{10}$$

As for the intermediate goods producers, we can show that the price elasticity of residential natural gas demand is also constant and equal to ρ_0. Note that a similar expression to equation (7) for the price elasticity of demand can be obtained for the final consumption good producer.

3.5. Households

There is a representative household that consumes the final good (c), supplies labor (l), and receives a wage of w for each unit of labor. We follow Greenwood et al. (1988) and assume that preferences take the form

$$U(c,l) = \left(\left(c-\psi\frac{l^{1+\theta}}{1+\theta}\right)^{1-\sigma} - 1\right)\frac{1}{1-\sigma} \tag{11}$$

where $\theta > 0$ controls the Frisch elasticity of labor supply, ψ is a scaling constant controlling the disutility of labor, and $\sigma > 0$.[11] Apart from wages, the consumer also receives a lump sum transfer T from the

[10] Related to our paper, Accinelli and Tenorio (2012) argue that natural monopolies can be perpetuated if efficiency improvements allow for the incumbent to outcompete entrants.

[11] The Greenwood-Hercowitz-Huffman (GHH) functional form for preferences is mathematically convenient to work with, as closed-form expressions for consumption good demand and labor supply are easily obtained. For our purposes, we do not need to calibrate σ since the model is static and a welfare analysis is out of the scope of this paper.

government, as well as part of the net profits from the natural gas producer and the LDC, Π_p and Π_d. We denote the participation of the federal government in the production sector as π_p^f, its participation in the LDC as π_d^f, and the participation of the state government in the LDC as π_d^e. The consumer optimization problem consists of maximizing the utility (11) subject to the budget constraint (12)

$$c = wl + T + \left(1 - \pi_p^f\right)\Pi_p + \left(1 - \pi_d^e - \pi_d^f\right)\Pi_d. \tag{12}$$

The optimal choices for the representative agent are given by:

$$l = \left(\frac{w}{\psi}\right)^{\frac{1}{\theta}} \tag{13}$$

$$c = w\left(\frac{w}{\psi}\right)^{\frac{1}{\theta}} + T + \left(1 - \pi_p^f\right)\Pi_p + \left(1 - \pi_d^e - \pi_d^f\right)\Pi_d. \tag{14}$$

3.6. Government

We model the government in two levels; the federal and state. The federal government receives a share π_p^f of the natural gas producer's net income and a portion π_d^f of the net profit from the LDC. These revenues are used for government purchases denoted c^f. The state government revenues come from collecting taxes from the LDC's value-added, from the intermediate goods producers' value-added, and from the share of profits of the LDC (π_d^e). These revenues are then rebated lump-sum to the households as transfers (T). Thus, the total state government revenue (R_e) in this economy is given by:

$$R_e = \underbrace{\pi_d^e \Pi_d}_{\substack{\text{LDC's}\\ \text{profit}}} + \underbrace{\tau \sum_{i=0}^{J} (t_i - p)\, g_i}_{\substack{\text{LDC's}\\ \text{taxes}}} + \underbrace{\tau \sum_{j=1}^{J} (p_j y_j - t_j g_j)}_{\substack{\text{Intermediate sectors'}\\ \text{taxes}}} \tag{15}$$

while the federal government's revenue (R_f) is given by:

$$R_f = \underbrace{\pi_p^f \Pi_p}_{\substack{\text{NG production firm's}\\ \text{profit}}} + \underbrace{\pi_d^f \Pi_d}_{\substack{\text{LDC's}\\ \text{profit}}}. \tag{16}$$

3.7. Equilibrium

Given the natural gas price p, the equilibrium is characterized by a set of tariffs $\{t_i\}_{i=0}^J$, intermediate goods prices $\{p_j\}_{j=1}^J$, a wage rate w, a set of aggregate allocations $\{c, l, y, g\}$, sectoral allocations $\left\{\{c_j, y_j, l_j\}_{j=1}^J, \{g_i\}_{i=0}^J\right\}$, transfers T, net profits Π_p and Π_d, total costs K_p and K_d, and federal government purchases c^f such that:

1. Given $\{w, T, \Pi_p, \Pi_d\}$, $\{c, l\}$ maximize the utility of the representative consumer, satisfying (12), (13) and (14).
2. Given $\{p_j\}_{j=1}^J$ and t_0, $\left\{y, g_0, \{c_j\}_{j=1}^J\right\}$ maximize the profits of the final consumption good producer, satisfying (8), (9) and (10).

3. Given w and $\{t_j, p_j\}_{j=1}^{J}$, $\{y_j, g_j, l_j\}_{j=1}^{J}$ maximize the profits of the intermediate goods producers, satisfying (4), (5) and (6).
4. Tariffs $\{t_i\}_{i=0}^{J}$ satisfy the optimality conditions for the monopolistic LDC described by (1).
5. Final consumption good market clears: $K_p + K_d + c_f + c = y$.
6. Intermediate goods markets clear: $c_j = y_j, \quad j = 1, \ldots, J$.
7. Natural gas market clears: $\sum_{i=0}^{J} g_i = g$.
8. Labor market clears: $\sum_{j=1}^{J} l_j = l$.
9. Federal government budget is balanced: $R_f = c^f$.
10. State government budget is balanced: $R_e = T$.
11. Profits of the natural gas producer satisfy: $\Pi_p = (p - z_p)\, g$.
12. Profits of the LDC satisfy: $\Pi_d = \sum_{i=0}^{J} [\mu_i(t_i) - 1] [(1 - \tau)\, p + z_i]\, g_i$.
13. Total natural gas production costs are given by: $K_p = z_p g$.
14. Total natural gas distribution costs are given by: $K_d = \sum_{i=0}^{J} z_i g_i$.

4. Calibration – Taking the Model to the Data

In this section, we take the model to the data by calibrating its parameters to match certain empirical regularities. We have two sets of parameters depending on whether they have a direct counterpart in the data or not. For those parameters that we can pin down directly from a moment of the data, we do so. For the rest of the parameters that do not have this direct counterpart, we jointly calibrate them so that the model matches a set of data moments.

We calibrate our model to 13 LDCs that provided sufficient data and that had a significant volume of natural gas sold from ten different Brazilian states.[12] Furthermore, we include the industrial, electricity, commercial, and residential sectors as intermediate goods producers. We aggregate all other remaining sectors that purchase natural gas into a single sector labeled *"others"*. Thus, the model has $J = 4$ intermediate good producers (electricity, industrial, commercial, and others), and we assign the residential sector to the final consumption good aggregator.

In the following sections we detail the calibration procedure. First, we calibrate those parameters that have a direct counterpart in the data or have readily available estimates. Second, we show how we internally calibrate the rest of the parameters without a direct counterpart by matching a set of data moments.

[12] Based on data provided by the Brazilian Association of Piped Gas Distribution Companies (Abegás), the 13 largest natural gas distribution companies (LDCs) account for approximately 83% of Brazil's total natural gas distribution. Of the remaining 17%, Gasmar and Cigás, two distributors located in Brazil's northern and northeastern regions, manage 13% of natural gas distribution. These distributors were not included in this paper since Gasmar and Cigás sold 100% and 98% of their gas to thermoelectric plants, respectively. This concentration of distribution to a single sector could have additional economic implications that are not currently considered by our model. Specifically, a distribution company that caters solely to one industry needs to be modelled separately to factor in the monopsony power that the sector may possess. This is currently out of the scope of the model.

4.1. Natural Gas Production and Distribution Parameters

We calibrate the parameters related to the production and distribution of natural gas using financial information of each LDC, volumes of natural gas, tariffs, and other relevant information. Instead of calibrating the model to a particular year, we focus on averages from 2014 to 2019, avoiding irregularities in revenue and costs from non-recurrent events.

Some of the information was directly available in the financial and operational reports of the LDCs, as well as in some tables made available by the Brazilian Association of Piped Natural Gas Distribution Companies (Abegás). Other information, however, was not directly available and had to be estimated. Table 1 presents the calibrated values for the parameters of production and distribution costs. Below we detail the choice for each parameter.[13]

Table 1: *Natural gas production and distribution costs by LDC.*

LDC	Natural gas sector cost parameters (R$/m^3)						
	p	z_p	Residential	Electricity	Industrial	Commercial	Others
Sulgás	0.84	0.74	1.91	0.00	0.22	1.12	0.14
SCGás	0.89	0.79	1.21	0.00	0.40	0.78	0.37
Compagás	1.05	0.93	0.60	0.42	0.41	0.51	0.39
Comgás	0.73	0.64	2.69	0.40	0.54	1.49	0.41
Gás Brasiliano	1.00	0.88	1.74	0.00	0.48	0.90	0.42
Naturgy São Paulo	0.99	0.88	1.52	0.00	0.49	1.15	0.37
Naturgy Rio Capital	0.81	0.71	3.37	0.18	0.59	1.51	0.82
Naturgy Rio Interior	0.77	0.68	0.99	0.13	0.16	0.65	0.15
Gasmig	1.17	1.04	1.02	0.37	0.33	0.69	0.48
ESGás	0.99	0.88	1.66	0.22	0.22	0.59	0.20
MSGás	0.45	0.40	0.43	0.06	0.19	0.36	0.12
Bahiagás	1.01	0.89	0.78	0.25	0.30	0.38	0.30
Copergás	0.53	0.47	0.52	0.14	0.19	0.47	0.17

Natural gas price (p) We use the average value for the volume (in m^3) of natural gas purchased by each LDC. The price is defined as total purchases over total volume.

Unit cost of natural gas production (z_p) Since Petrobras is the effective monopolist that sells natural gas to LDCs in each state, we use the assumption of Petrobras' profit margin being the same across all states. The net profit to gross revenue ratio gives this profit margin, but only relative to Petrobras' natural gas operation. In the model, this ratio is given by $x = (p - z_p)/z_p$. We can then use the calibrated price p and set $z_p = p/(1 + x)$.

Unit cost of natural gas distribution for each sector i (z_i) We decompose the unit distribution cost into $z_i = z^d + z_i^{nd}$, where z^d represents the unit direct cost and z_i^{nd} the unit indirect cost. The difference between net revenue and gross profit is the direct cost. The net revenue is defined as the gross revenue minus the purchase of gas net of taxes. Dividing the direct cost by the total volume of natural gas pins down z^d. This implies the unitary direct cost is the same for all sectors. The indirect cost is defined as

[13] In 2018, CEG, CEG Rio, and Gás Natural Fenosa became controlled by the same holding company and were renamed Naturgy. To refer to each one of these companies, we use the names Naturgy Rio Capital, Naturgy Rio Interior, and Naturgy São Paulo, respectively. In 2020, the distribution of natural gas in Espírito Santo started to be carried out by ESGás, no longer BR Distribuidora. Still, we use information from BR Distribuidora to calibrate the model for ESGás.

the difference between gross profit and net profit, representing non-operating costs and expenses. The assumption here is that the indirect cost is not proportional to the volume of natural gas sold to sector i, but proportional to the gross profit from that sector.[14] After sharing the indirect cost among different sectors, z_i^{nd} is given by the ratio between indirect costs and volume sold to sector i.

For all LDCs, the largest unit distribution costs are attributed to the residential and commercial sectors, in line with what was expected. These segments have many individual customers, with an average natural gas consumption much lower than the electricity and industrial sectors, implying a smaller scale gain.[15]

Value-added tax (τ) We use the state value-added tax rate (ICMS) as the value for τ. Table 2 lists these rates for each state.

Table 2: Value-added tax (ICMS) rates for each state.

State	Tax Rate (%)
RS	18
SC	17
PR	18
SP	17
SP	18
SP	18
RJ	20
RJ	18
MG	17
ES	18

Government shareholdings in gas production and distribution (π_p^f,π_d^f,π_d^e) We use the federal government's share of Petrobras' total capital for the natural gas producer shares (π_p^f). In the case of LDCs, federal participation (π_d^f) is through Petrobras, so it is necessary to multiply Petrobras' participation in the company by federal participation in Petrobras to obtain federal involvement in the LDC. State participation in LDCs (π_d^e) occurs both directly and through state mixed capital companies. Table 3 summarizes the ownership distribution for each LDC.

Frisch elasticity of labor supply Parameter θ controls the inverse of the Frisch elasticity of labor supply, estimated for Brazil by Moura (2015). The point estimate is 0.246, which is the value we choose for θ.

[14] Assuming that the indirect cost is proportional to the volume of gas would give us z_i equal for all sectors. However, for some LDCs, this would lead to negative net profit for sectors with lower profit margins, such as the electricity and industrial sectors. For Naturgy Rio Capital we assume that the indirect cost for the electricity sector is zero due to net profits exceeding gross profits. This occurs when indirect income (e.g., rebates or rental income) is larger than indirect expenses.

[15] Between 2014 and 2019, Sulgás, SCGás, Gás Brasiliano, and Naturgy São Paulo did not sell to the electricity sector. In the case of these LDCs, we assumed that sales to the electricity sector remained zero in the experiments.

Table 3: *Ownership of natural gas production and distribution companies.*

LDC	Shareholders participation (%)		
	Federal	State	Others
Sulgás	22.7	51.0	26.3
SCGás	19.0	3.4	77.6
Compagás	11.3	15.8	72.8
Comgás	0.0	0.0	100.0
Gás Brasiliano	23.6	0.0	76.4
Naturgy São Paulo	0.0	0.0	100.0
Naturgy Rio Capital	0.0	0.0	100.0
Naturgy Rio Interior	0.0	0.0	100.0
Gasmig	0.0	17.0	83.0
ESGás	22.7	51.0	26.3
MSGás	22.7	51.0	26.3
Bahiagás	19.2	17.0	63.8
Copergás	19.2	17.0	63.8

4.2. Internally Calibrated Parameters

There are still 15 remaining parameters to calibrate. Since we do not have a direct counterpart in the data, we calibrate these parameters so that the model is consistent with certain empirical regularities. We use natural gas volumes distributed for each sector as targets to calibrate parameters $\{A, \alpha_1, \alpha_2, \alpha_3, \alpha_4\}$. We use the relative participation of each sector in GDP obtained from the Regional Accounts System (SCR) of the Brazilian statistical office (IBGE) as targets for $\{\beta_1, \beta_2, \beta_3, \beta_4\}$. We calibrate the constant that controls the disutility of labor (ψ) to match GDP.[16] We still have to calibrate the five elasticities of substitution $\{\rho_0, \rho_1, \rho_2, \rho_3, \rho_4\}$. We assume that all parameters $\rho_i = \bar{\rho}\ \forall i \in J$, and we calibrate them so that the model matches the aggregate price elasticity of demand for natural gas.

The price elasticity of demand is a key parameter in the model. Since we want to find the efficiency gains that would leave government revenues unchanged, an increase in efficiency will imply some reduction in the tariff. Therefore, how much the demand for natural gas reacts to the decline in the tariff is controlled by the price elasticity of demand. As equation (7) shows, parameter ρ_j is crucial to control this elasticity. Note, however, that assuming $\rho_i = \bar{\rho}$ for all $i \in J$ does not imply that the price elasticity of demand for each sector will be the same since it depends on the tariff and parameters α_j and β_j. The elasticity of the demand to the price of natural gas p is given by:

$$\bar{\nu}_i^D \equiv \frac{d\log(g_i)}{d\log(p)} = \frac{d\log(g_i)}{d\log(t_i)} \times \frac{d\log(t_i)}{d\log(p)} = \varepsilon_i^D \times \frac{(1-\tau)p}{(1-\tau)p + z_i} \tag{17}$$

which induces further heterogeneity in the sectoral price elasticities of demand through z_i.

The evidence on the price elasticity of natural gas is abundant, although there is no clear consensus on its value. This is partly due to the fact that this price elasticity of demand will depend on various factors such as the stage of development (see Shahbaz et al., 2014), the sector, the country, the time horizon (long or short run), or the model specification. We follow Burke and Yang (2016) who estimate the price and income elasticities of natural gas demand using data from multiple countries, including Brazil. Their estimate for the long-run price elasticity of natural gas demand is -1.25, which is the target we use.[17]

[16] In Appendix A, Tables 11 and 12 show daily sectoral volumes of natural gas by sector and LDC and sectoral participation in GDP by state, respectively. Table 13 shows the GDP of each region supplied by each LDC.

[17] Labandeira et al. (2017) find the long-run price elasticity of energy demand to be in the range $[-1.16, -0.31]$, while

Table 4: *Calibrated parameters by LDC.*

LDC	Calibrated Parameters										
	A	α_1	α_2	α_3	α_4	β_1	β_2	β_3	β_4	ψ	$\bar{\rho}$
Sulgás	253	-	0.012	0.004	0.004	0.066	0.337	0.170	0.427	0.008	1.64
SCGás	162	-	0.023	0.005	0.008	0.086	0.328	0.194	0.392	0.005	1.93
Compagás	186	0.038	0.014	0.003	0.008	0.100	0.324	0.184	0.391	0.001	1.85
Comgás	548	0.015	0.012	0.009	0.003	0.096	0.309	0.202	0.389	0.001	2.45
Gás Brasiliano	64	-	0.047	0.011	0.007	0.074	0.322	0.186	0.416	0.002	2.00
Naturgy São Paulo	174	-	0.017	0.006	0.002	0.088	0.328	0.188	0.395	0.001	2.04
Naturgy Rio Capital	156	0.090	0.027	0.028	0.021	0.093	0.243	0.172	0.483	0.001	2.01
Naturgy Rio Interior	94	0.287	0.037	0.003	0.007	0.060	0.230	0.142	0.567	0.001	1.52
Gasmig	217	0.049	0.022	0.004	0.003	0.079	0.304	0.162	0.454	0.001	1.71
ESGás	89	0.189	0.071	0.005	0.006	0.064	0.250	0.174	0.512	0.010	1.56
MSGás	66	0.090	0.016	0.002	0.001	0.088	0.275	0.152	0.484	0.004	1.53
Bahiagás	119	0.020	0.043	0.007	0.007	0.080	0.325	0.153	0.440	0.001	1.70
Copergás	82	0.115	0.023	0.004	0.005	0.094	0.288	0.172	0.446	0.001	1.69

Table 5: *Average and maximum relative deviations by LDC.*

LDC	Relative Deviation (%)	
	Average	Maximum
Sulgás	0.029	0.152
SCGás	0.053	0.260
Compagás	0.037	0.199
Comgás	0.049	0.302
Gás Brasiliano	0.066	0.366
Naturgy São Paulo	0.023	0.122
Naturgy Rio Capital	0.086	0.669
Naturgy Rio Interior	0.113	0.876
Gasmig	0.047	0.304
ESGás	0.129	0.925
MSGás	0.048	0.318
Bahiagás	0.092	0.564
Copergás	0.099	0.623

4.3. Numerical Calibration Procedure

Let $\Theta = (A, \alpha_1, \ldots, \alpha_4, \beta_1, \ldots, \beta_4, \psi, \bar{\rho})$ be the vector of parameters to be numerically calibrated. Let m_d be the vector of target statistics and $m(\Theta)$ the vector of model statistics. We choose Θ^* so that we minimize the distance between m_d and $m(\Theta^*)$. In particular, we solve the problem

$$\Theta^* = \arg\min_{\Theta} \left(m(\Theta) - m_d\right)^T \mathbf{W} \left(m(\Theta) - m_d\right)$$

where $\mathbf{W}$ is a diagonal weighting matrix defined as $\mathbf{W} = \operatorname{diag}\left(1/m_{d,k}^2\right)$ where $m_{d,k}$ denotes data moment k.

Table 4 shows the values obtained with the calibration procedure for each LDC, while Table 5 summarizes deviations between model and data moments. The model fits the data very precisely. The maximum deviation across moments and LDCs is 0.925% while the average is below 0.13%. In terms of the elasticity $\bar{\rho}$, the largest value for this elasticity is 2.45 (Comgás) and the lowest is 1.52 (Naturgy Rio Interior).

Table 6: *Price elasticity of demand by sector and LDC.*

LDC	Price Elasticity of Demand				
	Residential	Electricity	Industrial	Commercial	Aggregate
Sulgás	-0.43	-	-1.25	-0.63	-1.25
SCGás	-0.73	-	-1.25	-0.94	-1.25
Compagás	-1.10	-1.24	-1.26	-1.17	-1.25
Comgás	-0.44	-1.45	-1.28	-0.70	-1.25
Gás Brasiliano	-0.64	-	-1.25	-0.95	-1.25
Naturgy São Paulo	-0.71	-	-1.27	-0.84	-1.25
Naturgy Rio Capital	-0.32	-1.42	-1.04	-0.60	-1.25
Naturgy Rio Interior	-0.58	-1.05	-1.19	-0.74	-1.25
Gasmig	-0.83	-1.20	-1.27	-0.99	-1.25
ESGás	-0.52	-1.12	-1.21	-0.91	-1.25
MSGás	-0.71	-1.27	-1.02	-0.78	-1.25
Bahiagás	-0.87	-1.29	-1.24	-1.16	-1.25
Copergás	-0.77	-1.19	-1.18	-0.82	-1.25

5. Numerical Experiments

In this section, we use the calibrated model to perform two counterfactual exercises. The first experiment involves the privatization of 8 LDCs that currently have state participation. The second experiment assesses the impact of a reduction in the price of natural gas purchased by LDCs on all 13 companies.

5.1. Privatization of LDCs

In this counterfactual experiment, we compute the minimum efficiency gain necessary to keep state-level revenues constant after the privatization of each LDC. With this experiment, we do not intend to discuss whether or not privatization does bring efficiency gains or if there are differences in the objective function between private and state-owned companies. Instead, we assess what are the efficiency gains that would justify the privatization.

The process of privatizing a company involves a complex set of negotiations and financial considerations. However, one common observation is that, in many cases, the privatization of a company tends to occur with the asset being sold at a price that is perceived to be under its true value. Jones et al. (1999) find that political factors play a role in offer pricing, share allocation, and other outcomes related to privatization.[18] Furthermore, they find that governments tend to underprice offers with an average level of 34.1% and a median level of 12.4% for initial share issue privatizations.[19] In our experiment, we assume that the government gets revenues from selling the shares after privatization, but these sales are underpriced. We assume this underpricing ranges from 0% to 40%, which contains the median and average underpricing found by Jones et al. (1999). We discretize this interval and find the necessary cost reduction to keep revenues constant for each level of underpricing. That is, we find the constant κ such that κz_i is the distribution cost that leaves state revenues constant. We assume this cost reduction is uniform across sectors (i.e., κ does not change across sectors).

Figure 1 shows the necessary unit cost reduction as a function of the degree of underpricing for each LDC and the average across LDCs. The two dashed vertical lines show the average and median

[18] Although there are other forms of privatization, Jones et al. (1999) study share issue privatizations that involve the government selling a portion or all of its stake through public share offerings.

[19] Ljungqvist (2007) report that underpricing is also a common phenomenon in initial public offerings (IPOs) for private corporations. In particular, this discount averaged 40% between 2000 and 2004. Hoque and Mu (2021) find that the average underpricing level in China from 2004 to 2012 had been 61.26%.

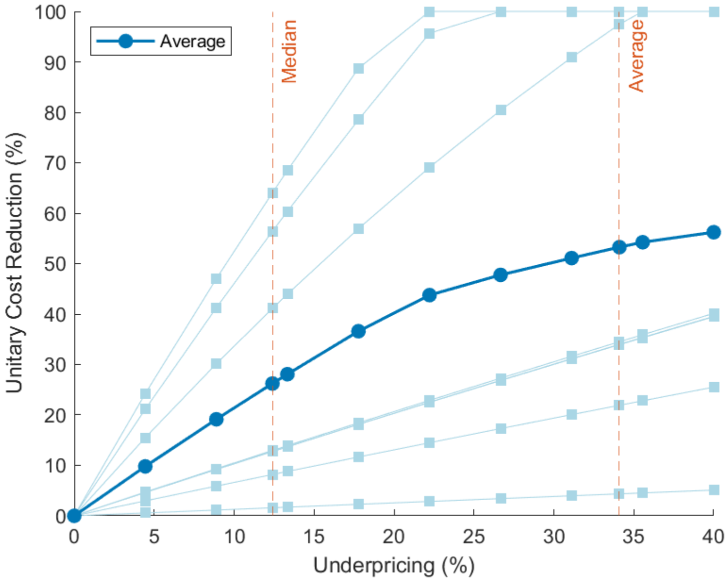

Figure 1: *Unitary Cost Reduction and Privatization*

Note: The vertical lines denote the median and average underpricing found in Jones et al. (1999).

underpricing levels estimated by Jones et al. (1999), respectively, while the squared clear blue lines correspond to each LDC. The first result is that the unitary cost reduction is increasing in the underpricing level for all LDCs. Second, there is substantial heterogeneity in the necessary cost reduction across LDCs. If we take the median underpricing of 12.4%, the necessary efficiency gain ranges from 1.59% (SCGás) to 64.06% (MSGás) with an average across LDCs of 26.23%. We interpret these numbers as a way to measure how easy it is for privatization to be advantageous in a fiscal sense. A lower unitary cost reduction implies that it is relatively easier for the government to keep revenues the same and, therefore, justify from this point of view the privatization of the LDC. For Sulgás, ESGás, and MSGás, if the underpricing exceeds a certain threshold, it is not possible to keep state revenues constant. For Sulgás, this threshold is 35.5%, whereas for ESGás it is 26.7%, and for MSGás it is 22.2%.

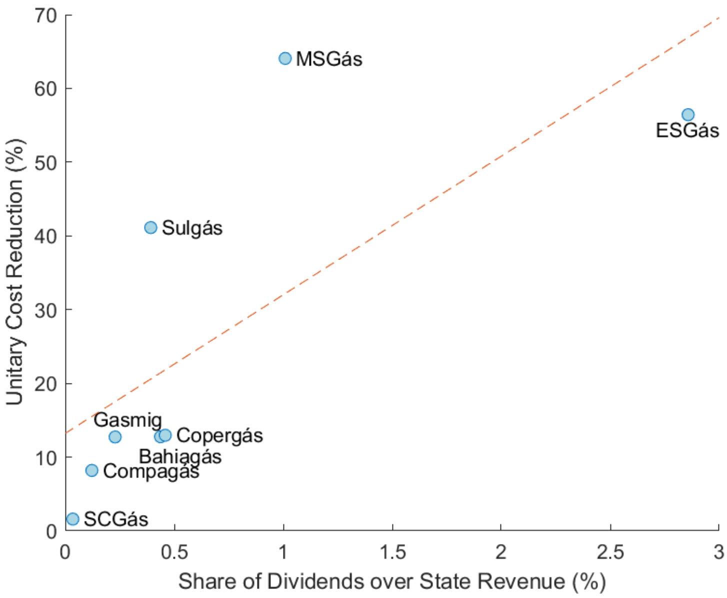

Figure 2: *Share of Dividends on State Revenue and Unitary Cost Reduction*

The magnitude of the necessary efficiency gain is directly related to LDC's profitability and the size of the state's ownership over the company. Figure 2 illustrates this point. On the horizontal axis, we have the share of total state revenues due to dividends $(\pi_f^e \Pi_d / R_e)$, while on the vertical axis, we have the necessary efficiency gain for the median underpricing of 12.4%. The necessary efficiency gain is, on average, around 18.8% for each percentage point of revenue reverted to dividends to the state.

One question that could arise is how efficiency gains in distribution spread throughout the economy and impact tax collection. Figure 3 shows each productive sector's contribution to state tax revenues. For each sector, we compute the change in revenues in the counterfactual relative to the benchmark and divide this by the total revenues. For each state, the sum of sectoral contributions is equal to 100%. There is clear heterogeneity in the magnitude of each contribution, although the contributions are positive in all cases. In all cases, the sector with the largest contribution is the one labeled "others" followed by the industrial sector.

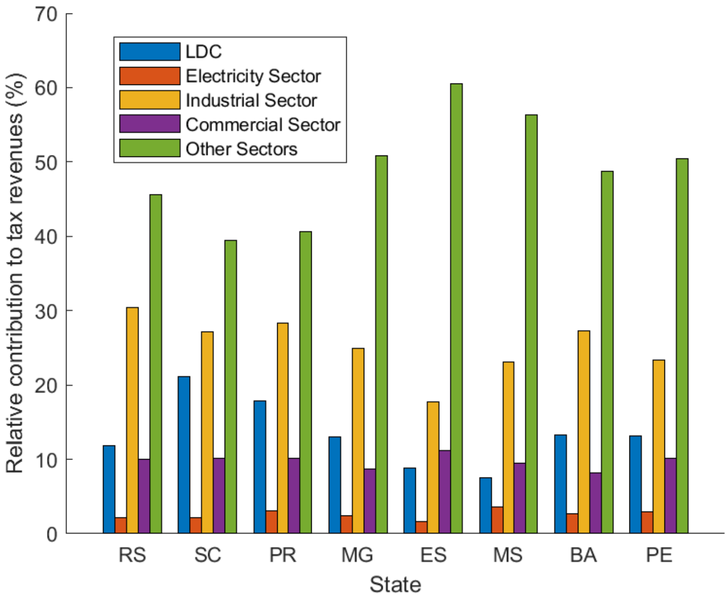

Figure 3: *Relative contribution to state revenues by sector.*

This counterfactual highlights that privatization might be worth it in the fiscal sense for some, though not all, distribution companies. Taking the estimates of Brown et al. (2006), productivity gains after privatization range from 2% to 15%. In the case of a median underpricing, five companies have efficiency gains that are contained in this range; SCGás (1.59%), Compagás (8.19%), Gasmig (12.73%), Bahiagás (12.76%), and Copergás (12.96%). In this sense, these are the companies that are easier to privatize. For the rest, the efficiency gains necessary to keep state revenues constant might be too large compared to what the empirical literature has estimated. This counterfactual also shows that companies requiring lower unitary cost reductions are those with lower state participation. This might indicate a selection into privatization because these companies are more productive or less important for state revenues.

5.2. Increased competition in natural gas production

Within the discussion of the New Gas Law, there is an expectation of increased competition in gas production which will likely induce a reduction in the price of natural gas purchased by the LDCs. We use our model to assess the impact of a reduction in the price of natural gas purchased by LDCs upon the volume consumed and its effects on the regional economies. According to data on natural gas production, currently monopolized by Petrobras, the activity's profit margin is approximately 10%. Therefore, to represent an increase in competition within gas production, we considered an intermediate reduction of 5% in the natural gas price p, which would mean a reduction by half of the profit margin while maintaining the unitary production cost parameter z_p constant. In addition to the LDCs considered in the previous privatization experiment, we also included the already private LDCs from São Paulo (Comgás, Gás Brasiliano, and Naturgy São Paulo) and Rio de Janeiro (Naturgy Rio Capital and Naturgy Rio Interior).

Table 7: *Relative change in sectoral tariffs after a 5% reduction in natural gas price.*

LDC	Tariff changes (%)				
	Residential	Electricity	Industrial	Commercial	Others
Sulgás	-1.3	-	-3.8	-1.9	-4.2
SCGás	-1.9	-	-3.2	-2.4	-3.4
Compagás	-3.0	-3.4	-3.4	-3.1	-3.4
Comgás	-0.9	-3.0	-2.6	-1.4	-3.0
Gás Brasiliano	-1.6	-	-3.2	-2.4	-3.3
Naturgy São Paulo	-1.7	-	-3.1	-2.1	-3.4
Naturgy Rio Capital	-0.8	-3.9	-2.6	-1.5	-2.2
Naturgy Rio Interior	-1.9	-4.1	-4.0	-2.4	-4.0
Gasmig	-2.4	-3.6	-3.7	-2.9	-3.3
ESGás	-1.7	-4.0	-4.0	-2.9	-4.0
MSGás	-2.3	-4.3	-3.3	-2.5	-3.8
Bahiagás	-2.6	-3.8	-3.7	-3.4	-3.7
Copergás	-2.3	-3.8	-3.5	-2.4	-3.6
Average	-1.9	-3.8	-3.4	-2.4	-3.5

Table 7 reports the implied tariff changes after the reduction in the natural gas price. For all sectors and LDCs, the tariff reduction is below 5%. In the model, the change in tariffs depends solely on the change in the natural gas price p since, as shown in equation (1), the rest of the parameters (τ, z_i, and ρ_i) remain constant. However, note that the elasticity of the tariff to the price of natural gas is the second term in equation (17), that is

$$\frac{d\log(t_i)}{d\log(p)} = \frac{(1-\tau)p}{(1-\tau)p + z_i}$$

which shows that for a given reduction in p, the change in the tariff for all sectors i is going to be smaller than the given change in p but always in the same direction. The results show that, in all cases, the reduction in the tariff is less than 5% but with substantial heterogeneity across sectors. The largest average reduction in the tariff is for the electricity sector of 3.8% while the smallest one is for the residential sector of 1.9%.

On average, the volume consumed increased by 6.64% after the price reduction, which was very similar across LDCs. Table 8 shows the effects on total volume, tax revenues, and GDP for all LDCs. In the case of GDP and tax revenue, we found an average increase of 0.08% and 0.06%, respectively. Although small, these are sizeable effects in line with the empirical evidence. Rubaszek et al. (2021) find an insignificant response in economic activity to a positive supply shock in the U.S. natural gas market. Their analysis is based on a Bayesian structural vector autoregression model and find an increase in production and reduced spot prices, similar to our results.

LDC	Total Volume	Tax Revenue	GDP
Sulgás	6.65	0.04	0.02
SCGás	6.63	0.04	0.03
Compagás	6.64	0.02	0.02
Comgás	6.46	0.03	0.02
Gás Brasiliano	6.62	0.05	0.04
Naturgy São Paulo	6.62	0.02	0.01
Naturgy Rio Capital	4.61	0.06	0.04
Naturgy Rio Interior	6.37	0.05	0.04
Gasmig	6.69	0.04	0.03
ESGás	6.60	0.16	0.09
MSGás	5.34	0.02	0.01
Bahiagás	6.64	0.10	0.07
Copergás	6.25	0.03	0.02
Average	6.32	0.05	0.03

Table 8: Relative change in aggregate variables after a 5% reduction in natural gas price without the electricity sector.

Table 9: *Relative change in sectoral tariffs after a 5% reduction in natural gas price.*

LDC	Relative Volume Changes (%)				
	Residential	Electricity	Industrial	Commercial	Others
Sulgás	2.2	-	6.6	3.2	7.2
SCGás	3.8	-	6.6	4.9	6.8
Compagás	5.7	-	6.6	6.1	6.7
Comgás	2.3	-	6.8	3.6	7.7
Gás Brasiliano	3.3	-	6.6	5.0	7.0
Naturgy São Paulo	3.7	-	6.7	4.4	7.4
Naturgy Rio Capital	1.7	-	5.5	3.1	4.6
Naturgy Rio Interior	3.0	-	6.4	3.8	6.5
Gasmig	4.3	-	6.7	5.2	6.0
ESGás	2.7	-	6.6	4.8	6.7
MSGás	3.7	-	5.3	4.0	6.1
Bahiagás	4.6	-	6.6	6.1	6.6
Copergás	4.0	-	6.2	4.2	6.4
Average	3.5	-	6.4	4.5	6.6

The change in volumes consumed by each sector is driven by their price elasticity of demand and is reported in Table 9. The average increase in the residential sector (the lowest average price elasticity of demand of -0.66%) was 3.5%. For the electricity sector (with a larger average elasticity), the average increase was 7.1%. We also perform this counterfactual by removing the electricity sector completely to see the impacts on aggregate volumes, tax revenues, and GDP and find that the effects remain relatively unchanged. On average, the total volume of natural gas increases by 6.3%, tax revenues by 0.05%, and GDP by 0.03%. Table 8 in Appendix A shows the results for all LDCs.

Table 10: *Relative change in sectoral revenues after a 5% reduction in natural gas price.*

LDC	Relative Revenues Changes (%)			
	Electricity	Industrial	Commercial	Others
Sulgás	0.01	0.03	0.01	0.02
SCGás	0.02	0.05	0.02	0.02
Compagás	0.09	0.03	0.01	0.02
Comgás	0.10	0.05	0.02	0.01
Gás Brasiliano	0.02	0.07	0.02	0.02
Naturgy São Paulo	0.01	0.02	0.01	0.01
Naturgy Rio Capital	0.84	0.08	0.06	0.07
Naturgy Rio Interior	1.18	0.15	0.10	0.11
Gasmig	0.15	0.06	0.03	0.02
ESGás	0.70	0.23	0.10	0.10
MSGás	0.29	0.04	0.03	0.03
Bahiagás	0.07	0.12	0.05	0.05
Copergás	0.50	0.07	0.05	0.05
Average	0.31	0.08	0.04	0.04

To better understand the impact of price reduction on GDP, Table 10 reports the change in sectoral revenues. The electricity sector shows the largest increase in revenue, of 0.31% on average. For the other sectors, the change in revenues is more moderate, between 0.04% and 0.08% on average. Therefore, through the lens of our model, a reduction of 5% in the price of natural gas can lead to substantial reductions in tariffs (on average, between 1.9% and 3.8% depending on the sector), increased volume of natural gas, and moderate effects on tax revenues and GDP. These effects suggest that more competition in the natural gas market might be a good policy to induce transitions to more intensive natural gas use.

6. Concluding Remarks

In this article, we developed a general equilibrium model to assess the necessary gains in efficiency to justify privatizing LDCs. In particular, the efficiency gains required to keep state revenues constant after the privatization. We calibrated the model for 13 of the major Brazilian LDCs, taking into account these companies' financial and operational data, as well as economic data from the regions in which they operate. We find that the median gains needed to justify their privatization depend on the level of underpricing of shares and vary substantially across LDCs. For example, in a scenario with median underpricing of state shares, the necessary unitary cost reductions range between 1.6% and 64%. For five companies, the necessary unitary cost reductions are below 13%, which is in the range of values found in the empirical literature (Rossi, 2001; Brown et al., 2006).

We also assess the gains from increased competition in natural gas production, which is another goal of the New Gas Law. We find that a 5% reduction in the price of natural gas purchased by LDCs leads to a moderate average increase in GDP of 0.06%, a significant average increase in the volume of natural gas of 6.6%, and moderate tax revenue increases of 0.08%. Both counterfactual experiments studied show significant heterogeneity in the results for different LDCs and regions, highlighting the importance of assessing both privatization and competition gains on a case-by-case basis.

Our results help assess how big efficiency gains should be so that it is justified to privatize LDCs from a fiscal point of view. Although it might not be the only reason for privatization, keeping fiscal revenues stable is crucial since privatization might incur costs from broader adjustment processes. Furthermore, even if the consensus in the literature is that privatization *"works"* (Megginson and Netter, 2001) our results help us shed light on the potential heterogeneity by showing how much efficiency should increase across 13 different regional economies. In particular, our results suggest that the efficiency gains necessary will depend on the degree of underpricing. Lastly, our results also show that it is of first-order importance to account for general equilibrium effects. In particular, when dealing with policies that affect a key intermediate input like natural gas or, more broadly, energy.

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A. Additional Tables

Table 11: *Daily sectoral volumes of natural gas by LDC (thousand m³ per day).*

LDC	Daily volume by sector (thousand m³)				
	Residential	Electricity	Industrial	Commercial	Others
Sulgás	12	0	1,499	31	422
SCGás	4	0	1,410	15	299
Compagás	24	591	846	16	396
Comgás	676	1,476	10,359	386	576
Gás Brasiliano	5	0	704	7	32
Naturgy São Paulo	17	0	1,054	16	38
Naturgy Rio Capital	306	4,706	1,337	224	2,344
Naturgy Rio Interior	13	5,519	1,760	10	613
Gasmig	9	888	2,427	32	100
ESGás	10	992	1,824	8	129
MSGás	2	1,362	370	5	13
Bahiagás	14	90	3,358	36	229
Copergás	7	2,943	1,094	11	202

Table 12: *Sector participation in GDP by state (%).*

State	Sector			
	Electricity	Industrial	Commercial	Others
RS	2.4	34.6	11.3	51.6
SC	2.7	34.6	12.8	49.8
PR	3.8	34.4	12.3	49.3
SP	1.8	31.2	11.1	55.6
RJ	2.7	17.6	8.7	70.2
MG	2.9	28.8	10.0	58.1
ES	2.4	20.2	12.1	64.7
MS	4.3	24.9	10.2	60.3
BA	3.0	31.9	9.3	55.3
PE	4.1	26.8	11.5	57.2

Table 13: *GDP of the regions supplied by each LDC*

LDC	GDP (million R$)
Sulgás	731.4
SCGás	447.1
Compagás	731.4
Comgás	3,049.3
Gás Brasiliano	176.3
Naturgy São Paulo	731.4
Naturgy Rio Capital	680.4
Naturgy Rio Interior	423.6
Gasmig	961.4
ESGás	202.2
MSGás	161.8
Bahiagás	468.7
Copergás	273.1

Table 14: *Relative change in aggregate variables after a 5% reduction in natural gas price.*

LDC	Total Volume	Tax Revenue	GDP
Sulgás	6.65	0.04	0.02
SCGás	6.63	0.04	0.03
Compagás	6.63	0.03	0.02
Comgás	6.61	0.04	0.03
Gás Brasiliano	6.62	0.05	0.04
Naturgy São Paulo	6.62	0.02	0.01
Naturgy Rio Capital	6.65	0.12	0.09
Naturgy Rio Interior	6.66	0.17	0.14
Gasmig	6.64	0.05	0.04
ESGás	6.66	0.23	0.14
MSGás	6.67	0.08	0.05
Bahiagás	6.64	0.10	0.07
Copergás	6.65	0.11	0.08
Average	6.64	0.08	0.06

B. Finding the Equilibrium

Despite, for a general set of parameters, not being possible to determine the equilibrium analytically, it is possible to analytically represent all equilibrium objects as a function of only the salary w, given the parameters. This considerably simplifies the numerical effort as it turns it into an equivalent one-dimensional problem.

B.1. Prices

Given that, for all sectors of the economy, the price elasticity of natural gas demand is constant and independent of the tariff ($\varepsilon(t_i) = \rho_i$), by equations (1) and (2) we have

$$\mu_i = \frac{\rho_i}{\rho_i - 1} \quad i = 0, \dots, J \tag{18}$$

$$t_i = \mu_i \left(p + \frac{z_i}{1 - \tau} \right) \quad i = 0, \dots, J \tag{19}$$

Solving (5) for g_j, (6) for l_j, substituting both expressions in (4), and doing some algebraic manipulations, we get p_j as a function of w

$$p_j(w) = \left(\alpha_j^{\rho_j} t_j^{1-\rho_j} + (1 - \alpha_j)^{\rho_j} \left(\frac{w}{1 - \tau} \right)^{1-\rho_j} \right)^{\frac{1}{1-\rho_j}} \quad j = 1, \dots, J \tag{20}$$

Similarly, for the final consumption good sector, solving for g_0 and c_j in (9) and (10), replacing them in equation (8), gives us the relationship

$$A^{1-\rho_0} = \left(1 - \sum_{j=1}^{J} \beta_j \right)^{\rho_0} t_0^{1-\rho_0} + \sum_{j=1}^{J} \beta_j^{\rho_0} p_j(w)^{1-\rho_0} \tag{21}$$

Notice that the only unknown in equation (21) is w. Although it is not generally possible to analytically solve this equation for w, it can be easily solved numerically.

B.2. Allocations as functions of w

Labor supply, l, is already a function of w by equation (13)

$$l(w) = \left(\frac{w}{\psi} \right)^{\frac{1}{\theta}} \tag{22}$$

By manipulating equation (10), replacing the market clearing conditions for intermediate goods markets, $c_j = y_j$, we get

$$y_j = A^{\rho_0 - 1} \left(\frac{\beta_j}{p_j} \right)^{\rho_0} y \quad j = 1, \dots, J \tag{23}$$

Solving equation (6) for l_j and substituting y_j by the expression in (23)

$$l_j = A^{\rho_0 - 1} (1 - \alpha_j)^{\rho_j} \left(\frac{(1 - \tau) p_j}{w} \right)^{\rho_j} \left(\frac{\beta_j}{p_j} \right)^{\rho_0} y \quad j = 1, \dots, J \tag{24}$$

Substituting (22) and (24) in the market clearing condition for the labor market, $\sum_{j=1}^{J} l_j = l$, we find an explicit expression for $y(w)$, given by

$$y(w) = \frac{l(w)}{A^{\rho_0 - 1} \sum_{j=1}^{J} (1-\alpha_j)^{\rho_j} \left(\frac{(1-\tau) p_j(w)}{w}\right)^{\rho_j} \left(\frac{\beta_j}{p_j(w)}\right)^{\rho_0}} \tag{25}$$

Therefore, y_j and l_j as functions of w become

$$y_j(w) = A^{\rho_0 - 1} \left(\frac{\beta_j}{p_j(w)}\right)^{\rho_0} y(w) \quad j = 1, \ldots, J \tag{26}$$

$$l_j(w) = A^{\rho_0 - 1} (1-\alpha_j)^{\rho_j} \left(\frac{(1-\tau) p_j(w)}{w}\right)^{\rho_j} \left(\frac{\beta_j}{p_j(w)}\right)^{\rho_0} y(w) \quad j = 1, \ldots, J \tag{27}$$

By solving (5) and (9) for g_0 and g_j, we get that the demands and total supply of natural gas are

$$g_0(w) = A^{\rho_0 - 1} \left(\frac{1 - \sum_{j=1}^{J} \beta_j}{t_0}\right)^{\rho_0} y(w) \tag{28}$$

$$g_j(w) = \left(\frac{\alpha_j p_j(w)}{t_j}\right)^{\rho_j} y_j(w) \quad j = 1, \ldots, J \tag{29}$$

$$g(w) = g_0(w) + \sum_{j=1}^{J} g_j(w) \tag{30}$$

The remaining allocations are easily obtained from the remaining equilibrium conditions. Table 15 summarizes all expressions.

$\mu_i = \frac{\rho_i}{\rho_i - 1} \quad i = 0, \ldots, J$
$t_i = \mu_i \left(p + \frac{z_i}{1-\tau} \right) \quad i = 0, \ldots, J$
$p_j(w) = \left(\alpha_j^{\rho_j} t_j^{1-\rho_j} + (1-\alpha_j)^{\rho_j} \left(\frac{w}{1-\tau} \right)^{1-\rho_j} \right)^{\frac{1}{1-\rho_j}} \quad j = 1, \ldots, J$
$l(w) = \left(\frac{w}{\psi} \right)^{\frac{1}{\theta}}$
$y(w) = \frac{l(w)}{A^{\rho_0 - 1} \sum_{j=1}^{J} (1-\alpha_j)^{\rho_j} \left(\frac{(1-\tau) p_j(w)}{w} \right)^{\rho_j} \left(\frac{\beta_j}{p_j(w)} \right)^{\rho_0}}$
$y_j(w) = A^{\rho_0 - 1} \left(\frac{\beta_j}{p_j(w)} \right)^{\rho_0} y(w) \quad j = 1, \ldots, J$
$c_j(w) = y_j(w) \quad j = 1, \ldots, J$
$l_j(w) = A^{\rho_0 - 1} (1-\alpha_j)^{\rho_j} \left(\frac{(1-\tau) p_j(w)}{w} \right)^{\rho_j} \left(\frac{\beta_j}{p_j(w)} \right)^{\rho_0} y(w) \quad j = 1, \ldots, J$
$g_0(w) = A^{\rho_0 - 1} \left(\frac{1 - \sum_{j=1}^{J} \beta_j}{t_0} \right)^{\rho_0} y(w)$
$g_j(w) = \left(\frac{\alpha_j p_j(w)}{t_j} \right)^{\rho_j} y_j(w) \quad j = 1, \ldots, J$
$g(w) = g_0(w) + \sum_{j=1}^{J} g_j(w)$
$\Pi_p(w) = (p - z_p) g(w)$
$\Pi_d(w) = (\mu_0 - 1) [(1-\tau) p + z_d] g_0(w) + \sum_{j=1}^{J} (\mu_j - 1) [(1-\tau) p + z_d] g_j(w)$
$T(w) = \pi_d^e \Pi_d(w) + \tau_e \left\{ (t_0 - p) g_0(w) + \sum_{j=1}^{J} [p_j(w) y_j(w) - p g_j(w)] \right\}$
$c(w) = w l(w) + T(w) + \left(1 - \pi_p^f \right) \Pi_p(w) + \left(1 - \pi_d^e - \pi_d^f \right) \Pi_d(w)$
$K_p(w) = z_p g(w)$
$K_d(w) = \sum_{i=0}^{J} z_i g_i(w)$
$c_f(w) = \pi_p^f \Pi_p(w) + \pi_d^f \Pi_d(w)$

Table 15: Equilibrium prices and allocations as functions of the wage rate w.