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GROUP REPUTATIONS UNDER ANONYMITY

Abstract

This paper studies incentives to invest in 'group reputations' when the latter result from the inability of certain agents to recognize their partners from past interactions (i.e., anonymity). It does this by embedding the Kreps-Wilson model of reputation and entry deterrence in a random matching game (instead of just looking at isolated bilateral interactions). The paper shows how the presence of this type of 'reputational externality' modifies the development of reputations under varying assumptions about how information is shared among agents, with and without discounting. In particular, it shows how anonymity might completely prevent the development of reputations, regardless of the trading horizon, if payoffs are discounted, and contemporaneous entrants share information while incumbents do not.

1 Introduction

In deciding which cab to take, one is rarely concerned about the identity of the driver, but one often minds whether the taxi is, say, yellow rather than checkered. This is remarkable, as the quality of the service will depend more on who is driving than on the color the vehicle is painted. A simple explanation is that it is hard to keep track of individual taxi drivers, while it is easy to keep track of the color of their cars. As a consequence of this 'limited memory', if a driver' service is less than satisfactory, the only way to punish the surly driver is to avoid using the services of his cab company altogether. Similarly, when one deals only sporadically with big organizations (say, a big bank) and one of its employees turns out to be less than helpful, one reacts not by trying to avoid dealing with that particular employee again, whose identity one can seldom remember, but by seeking the services of another firm altogether. In such situations, beliefs about unobservable characteristics ('kindness', 'honesty',etc.), i.e., reputations, can be said to attach to groups, rather than individuals.

Under these circumstances, one asks whether individuals will have incentives to invest in the group's reputation (a sort of public good), i.e., to engage in costly activities in the short run in order to influence other agents' beliefs about the unobservable characteristics of the group in the longer run. Or to put it in yet another way, one asks to what extent, if at all, does the presence of such 'externalities' lead agents to free-ride on each other, and to what extent does eventual free riding interfere with the formation of group reputations of this sort?

This paper models such situations by introducing imperfect information a la Kreps-Wilson (1982) and Milgrom-Roberts (1982) in random matching games in which agents are not able to recognize their partners from past interactions ('anonymity'), thus giving rise to 'reputational externalities'. A 'reputational externality' arises in any situation where the actions of an agent (an individual yellow cab driver) affect another agent's beliefs (the taxi's customer) regarding the unobservable characteristics ('politeness') of a third agent (another yellow cab driver), even though the first and third agents are independent entities[1].

The main lesson from this analysis is that the exact pattern of information sharing will be crucial in molding incentives to invest in the group's reputation. In particular, it is not the case that the mere presence of a 'reputational externality' will result in 'free riding' (*contemporaneous* information sharing will be necessary for that), nor is it the case that the existence of 'free riding' will necessarily neutralize incentives to invest in reputations[2]. Another insight from this work concerns the key role played by discounting in this kind of environment, in stark contrast with what happens in similar models when agents are 'named'.

Besides the class of situations illustrated by the examples above, where 'anonymity' results either from 'forgetfulness' or from high costs of record keeping, one can identify literal examples of anonymity, such as generic products, or transactions over the phone in which it is not possible to verify the identity of the person at the other end. The model in this paper also yields insights into a host of hybrid cases which do not map straightforwardly into the pure anonymity scenario: Firms in a modern economy posses the ability to change and/or hide their identities by manifold means ranging from straightforward name changes to 'fly by night' schemes, and even mergers[3]. This behavior generates a degree of anonymity in the economy in the sense here relevant, in so far as it prevents agents from recognizing those they have

[1]It might be objected that the individual members of the groups in the above examples are not completely independent entities, but while this might be so, it is also the case that they posses a degree of autonomy in their objectives and hence their decisions, and this is all that is required here.

[2]The reasoning underlying the previous conclusions would seem to trascend the specific form of 'group reputation' studied in this paper. If so, the literature on franchising, which also deals with 'group reputations', although due to the shared brandname rather than anonymity, is wrong in pressuming that the mere presence of a 'reputational externality' (the shared brandname) plus the fact that franchises share customers over time suffices to generate 'free riding', and that this 'free riding' is necessarily going to prevent efficient levels of investment in the common reputation. See Brickley and Dark (1987), Brickley (1987), Rubin (1978), and Mathewson and Winter (1985).

[3]In this regard, one should also not forget that individual identities of physical persons were a strictly local phenomenon well into the second half of the nineteenth century. Prior to the development of national identity registers, photography, and all the other underpinnings of what today we understand as identities, (long distance) relationships were plagued by the problem of clearly establishing a person's credentials (see the remarks on the subject in Corbin 1990).

traded with in the past. Of course, in such an environment there might be incentives for some agents to keep their identities, and to the extent that such incentives are present, it is highly unlikely that strategic manipulation of identities will result in complete anonymity. Nevertheless, it seems reasonable (and customary) to begin by analyzing extreme cases[4] before attacking more complex constellations.

And last but not least, one should point out that 'anonymity' is a common assumption in both the theoretical and experimental literature. It is often made in random matching models to preclude reputation building, and more generally, any kind of repeated game effects. The results in this paper constitute further evidence that this might not always work[5] (see Ellison 1994).

Some comments about modelling, a brief survey of the relevant literature and an outline of the remaining sections complete this introductory section.

1.1 Modelling Approach

The model used here is an extension of the Kreps-Wilson (1982) model of entry deterrence through reputation. In the present extension, two (one-period lived) entrants are matched each period one-on-one and randomly with two (long-lived) incumbents -instead of one short-lived entrant being matched with the same long-lived incumbent each period, as in Kreps-Wilson.

As in those authors' model, the matching goes on for a finite number of periods, entrants share information across periods, and there are two types of incumbents : Toughs and Rationals. Entrants cannot directly tell the type of an incumbent. If an entrant is matched with a rational incumbent, the pair proceeds to play exactly the same stage-game as in Kreps-Wilson (see diagram 1). A tough incumbent is here assumed to be a fighting automaton, i.e., is assumed to have no choice but to fight entry (see diagram 2). In the Kreps-Wilson set-up, in contrast, 'tough' incumbents are assumed to actually enjoy fighting.

[4]Note that the conventional scenario with all agents perfectly recognizable is also an extreme case.

[5]The analysis suggests that there will be scope for reputation building even in very large anonymous populations, as long as the horizon is sufficiently long. This will be so at least in the absence of discounting. I do not explore this tradeoff in this paper.

Despite these differences, incentives to invest in reputations arise here in pretty much the same way as in Kreps-Wilson[6].

A few words about the choice of approach: Much more general versions of the results in the Kreps-Wilson paper just mentioned have been obtained by Fudenberg and Levine (1989,1992) using an alternative, rather intuitive and, in a sense, more elegant approach. There are mainly two reasons why I stick to the Kreps-Wilson treatment: The first reason is that the analysis in Fudenberg and Levine's approach, with its focus on deriving a lower bound for the long-lived player's utility in any Nash equilibrium, tends to leave the exact structure of the underlying behavior unclear. The other reason has to do with the nature of the task at hand: It is not at all clear to me whether their methods can be applied in this environment, as here a long-lived agent by playing his Stackelberg action over and over again cannot hope to eventually convince entrants that this action will be played for sure in the future. At best, he can lead them to expect that this action will be played with probability one half, which equals the probability that an entrant is matched with any given incumbent. This simply because he cannot control the actions of the other long-lived player, and entrants cannot distinguish between incumbents.

1.2 Related Literature

This work draws mainly on the seminal contributions of Kreps and Wilson (1982), and Milgrom and Roberts (1982)[7]. It also draws on the literature on matching games and bargaining surveyed in Rubinstein and Osborne (1990), and on the somewhat narrower literature on imperfect information flows in matching games originating in the contributions of Rosenthal (1979), Kandori (1992), and Okuno-Fujiwara and Postlewaite (1990)[8].

Even though this paper follows the alternative conceptualization originat-

[6]The results obtained in this paper carry over to a seller/buyer interaction with 'fixed prices' in which a seller supplies an item whose quality cannot be ascertained ex-ante by the buyer.

[7]For an overview of the literature originating in these contributions, see Fudenberg and Tirole (1992). As far as I know, none of those follow-up papers have looked at simultaneous interactions.

[8]More recent contributions in this last line are Ellison (1994), and Ghosh and Ray (1996).

ing in the work of Kreps et al., I should also mention here the literature that conceptualizes reputation as a 'norm' in the context of an infinitely repeated game. Papers following this 'norm' approach are Dybvig and Spatt (1980), Klein and Leffler (1981), and Telser (1980). This latter view of reputation is fully forward-looking, and would not seem to capture the backward looking element in the intuitive, everyday idea of reputation.

A recent paper that claims to study collective reputations is Tirole (1995) (in fact, the only recent contribution on the subject that I am aware of). That paper works with a rather peculiar definition of reputation that does not seem to correspond to either of the two mentioned previously: In Tirole's paper there is neither learning nor repeated strategic interaction (as it studies a random matching game with a continuum of agents). The Tirole contribution strikes me as a dynamic version of Akerlof (1970), in which the dynamics are driven not by strategic considerations, but by the evolution of a physical state (namely, the probability that an agent might be identified as a cheater).

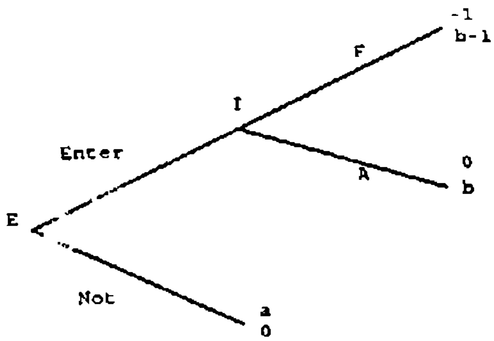
1.3 Paper Outline

The paper proceeds as follows: First, the basic matching game is outlined (Section 2). Then, after briefly dealing with a benchmark case with named incumbents, I analyze the case with anonymous incumbents but no information sharing (Section 3). In Section 4.1, the case with information sharing among entrants but not among incumbents (in the sense that incumbents cannot observe each others' types) is studied. In Section 4.2, it is assumed that incumbents can observe each others' types, and an equilibrium of the two-period game under these conditions is presented. The paper closes with a summary of results and an enumeration of the experiments not performed here (Section 5,6).

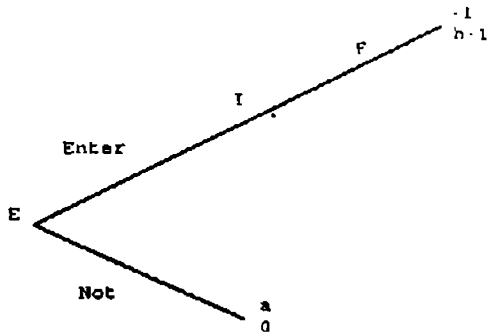
2 The Model

The game lasts for T periods. Following Kreps-Wilson, I will measure time backwards (i.e., T stands for the first period, and 1 for the last). There are $N(1+T)$ players. At any given date, there are $2N$ players: N long-lived incumbents, and N short-lived (one-period-lived) entrants. The entrants are matched randomly with the incumbents each period, and matched agents proceed to play stage-game 1 or 2, as explained immediately (see the diagrams below).

Stage-Game 1



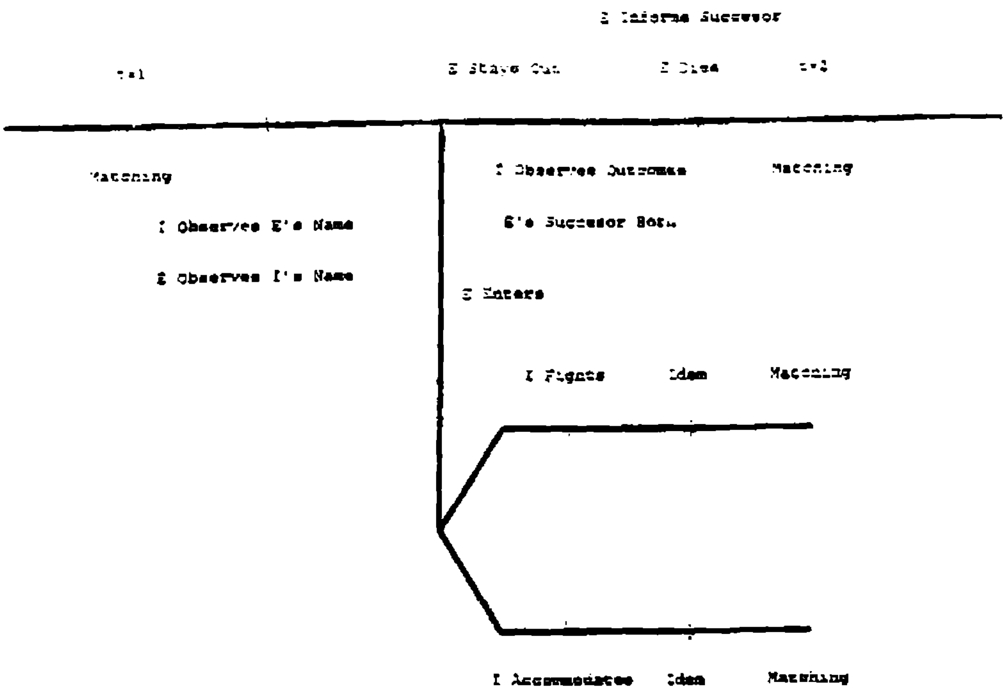
Stage-Game 2



Stage Game

Incumbents might be of one of two types: 'Rational' or 'Tough'. A 'rational' incumbent's plays stage-game 1, while a 'hard-wired' incumbent plays stage-game 2. Note that $0 < b < 1$, $a > 1$. There are N cohorts of short-lived entrants, and each period the current generation of short-lived entrants contains one (and only one) member from each of these. Entrants belonging to the same cohort share their experiences. An individual entrant's 'experience' consists of the outcome of the stage-game that entrant played, as well as the name of the incumbent the entrant faced. A 'Name' in the present context is simply a device that allows a player to associate a specific history (more precisely, a record of the outcomes of past interactions between that incumbent and members of the entrant's cohort) with the player he or she is matched with. I will assume here that it is common knowledge among the players that no two players share the same name, and that players cannot change their names, so that, given the structure of the game being considered, a player's name in effect suffices to identify him or her (i.e., to distinguish his or her record from that of other players). An entrant's payoffs are as in the diagrams above. An incumbent's overall payoffs are given by the undiscounted or discounted sum of the payoffs in the stage-games he or she takes part in. Incumbents know which payoff structure obtains (i.e., they know their type), while entrants do not: Each 'cohort' initially believes that an incumbent is 'rational' with probability $1-p_T$. I assume incumbents' types are drawn independently from identical distributions. I will, most of the time, assume that incumbents cannot observe each others' types. The exception is in Section 4.2. Also, I will invariably assume that incumbents can observe each others' actions ex-post; in other words, that at every moment they know the full history of the economy. Moreover, I will always assume that all entrants share the same initial beliefs, and that these do not vary across incumbents.

The following diagram illustrates the time-line of the game:



Time Line

The solution concept I employ is Kreps and Wilson's notion of sequential equilibrium. Finally, let me emphasize that this paper will look only at symmetric equilibria.

3 Anonymity Without Free-Riding

In this part of the paper, I look at two scenarios: The first has all agents named. This is the case 'nearest' to Kreps-Wilson, and, as such, a useful benchmark. The other scenario assumes that incumbents are anonymous, and there is no information sharing among contemporaneous entrants. The analysis of this case constitutes the core of this paper.

3.1 'Pure' Random Matching: A Benchmark

This section looks at the case with all agents named and no information sharing among contemporaneous agents. No 'information sharing' means that agents are only aware of the outcomes of those matches in which either they themselves, or, in the case of entrants, members of their cohort, have taken part.

Not surprisingly, it turns out that the equilibrium of this game strongly resembles that of Kreps and Wilson:

Proposition 1 *The equilibrium strategies and beliefs are exactly as in Kreps and Wilson, except that the condition guaranteeing reputation formation (from the period-before-last onward) is now $(1/N)a > 1$ (rather than $a > 1$), and entrants will mix with probability $1/Na$ (rather than $1/a$).(For a detailed description, see Appendix).*

Proof. The reasoning is, mostly, analogous to Kreps-Wilson's. Two remarks should suffice to show why this is so: 1) The condition $(1/N)a > 1$ just states that the maximum expected gain at $t = 2$ (letting $t = 1$ be the last period) from deterring entry by fighting, $(1/N)a$, exceeds the cost of doing so, 1. If this is so at $t = 2$, this must be so as well for any $t > 2$ (no discounting); 2) Since cohorts are 'identifiable' here, the value function of an incumbent can be written as the sum of two independent parts, each giving the expected payoff of interacting with a specific cohort. Moreover, since the incumbents are named, their value functions will not depend on each others' actions. This implies that one can think of this game as 4 independent games, one for each cohort/incumbent pairing, and each with payoffs scaled by $1/N$. These 'subgames' can be analyzed as simple 1:1 games[9]. Applying to each such 'subgame' exactly the same arguments as in Kreps-Wilson, and then putting together the resulting 'subequilibria', yields the overall equilibrium described above. ■

Perhaps the most interesting difference between this environment and Kreps-Wilson is contained in the following result:

Proposition 2 *The equilibrium described in the previous proposition is unique.*

Proof. The on-the-equilibrium-path strategies are unique for the same reasons as in Kreps-Wilson (see Appendix). Overall uniqueness follows from the consistency requirement in the definition of sequential equilibrium[10], as it can be shown that this requirement fully determines beliefs off-equilibrium-path. The fact that consistency suffices to pin down beliefs off-equilibrium-path is essentially a consequence of working with automata incumbents, instead of incumbents who 'like' to fight. ■

Of course, the Kreps-Wilson punchline carries over:

Corollary 3 *As $T \to \infty$, even a very small initial assessment that an incumbent is 'hard-wired' will lead to reputation building.*

In conclusion: Modifying Kreps-Wilson in this way leaves their results practically unchanged, except for the strengthened uniqueness, and the two minor differences pointed out in Proposition 1. As these latter features are straightforward consequences of assuming that two cohorts of entrants are each being matched randomly with a different incumbent every period , it seems appropriate to refer to them as 'pure random matching effects'.

[9] An intuitive way of seeing this equivalence is to think of each of these games as one where every period an entrant is matched with an incumbent with probability $1/N$ or not at all.

[10] That requirement states that an equilibrium system of beliefs must correspond to the limit of a sequence of beliefs' systems generated via Bayes' Rule from a sequence of completely mixed strategies which themselves converge to the equilibrium strategies.

3.2 Reputations with Beliefs' Averaging: Anonymity and No Information Sharing

This section takes a look at what happens when incumbents are anonymous and there are two cohorts of entrants. In other words, what happens when entrants cannot recognize those incumbents they have interacted with in the past, and contemporaneous entrants cannot observe the outcomes of matches other than their own.

Assumptions about Incumbents' Information With anonymous incumbents, if mixed strategies are used in equilibrium (as they will be in this set-up for some range of initial beliefs), and/or incumbents are not aware of each others' types, the question of whether incumbents can directly observe the actions taken in matches other than their own becomes important because the 'reputational externality' generated by the anonymity of incumbents will give incumbents an incentive to monitor each others' behavior in order to keep track of cohorts' beliefs.

I will just duck the problem, and assume that incumbents can directly observe the outcomes of matches other than their own. Note the resulting asymmetry: I am assuming that entrants are only able to observe the outcome of their own matches, while I assume that incumbents know the history of the whole economy.

I will also assume that incumbents cannot observe each others' types, even though it would not make a substantial difference if I made the alternative assumption[11].

Discounting This section focuses on characterizing the equilibrium of this game when incumbents' stage-game payoffs are not discounted, though it does include a proposition showing that the results of the no-discounting case are not robust to the introduction of discounting.

3.2.1 Equilibrium

The key to understanding the equilibrium of this game is the fact that, in the absence of information sharing of any kind, the anonymity feature affects

[11] It must be emphasized that this is only so under the previous assumption that incumbents know the history of the whole economy.

equilibrium behavior exclusively by modifying the beliefs' updating rule of entrants. In order to form beliefs regarding a 'generic' incumbent, entrants see themselves forced to average out their (updated) beliefs about the incumbent they have just met (and whose behavior they have just observed), with their prior beliefs (in effect, their beliefs regarding the rest of the incumbent population whose current actions they have not observed). Hence, anonymity 'dampens' beliefs' fluctuations. In particular, it will no longer be the case that after observing accommodation, an entrant will set the probability that a (generic) incumbent be an automaton to zero.

Under the thus modified beliefs' updating rule, the unique equilibrium of this game is the following:

Proposition 4 *If $b > \frac{1}{3}$, $\frac{1}{2}a > 1$, then the following is a sequential equilibrium of the game with anonymous incumbents:*

Beliefs for a given cohort are defined recursively, starting from given beliefs p_T (where the last period is $t = 1$):

a) If there is no entry at stage $t + 1$, $p_t = p_{t+1}$.

b) If there is entry at stage $t + 1$, and entry is fought, and $p_{t+1} > 0$, then:

i) If $p_{t+1} > 2\,\overline{p}_1$, then $p_t = \frac{1}{2} + \frac{1}{2}p_{t+1}$.

ii) If $2\overline{p}_1 \geq p_{t+1} > \overline{p}_t$, then $p_t = p_{t+1}$.

iii) If $\underline{p}_{t+1} < p_{t+1} \leq \overline{p}_t$, then $p_t = \overline{p}_t$.

iv) If $p_{t+1} \leq \underline{p}_{t+1}$, then $p_t = \frac{1}{2} + \frac{1}{2}p_{t+1}$.

with $\overline{p}_t \equiv (2b/b+1)^{t-1}\, b$; $\underline{p}_{t+1} \equiv 2\left(\overline{p}_t - \frac{1}{2}\right)$

c) If there is entry at stage $t+1$, and entry is accommodated or $p_{t+1} = 0$, then $p_t = \frac{1}{2}p_{t+1}$.

Strategy of the Entrant:

If $p_t > \overline{p}_t$, entrant stays out. If $p_t < \overline{p}_t$, entrant enters. If $p_t = \overline{p}_t$, then entrant randomizes, staying out with probability $1/2a$.

Strategy of 'rational' incumbent:

If $t = 1$, then incumbent accommodates. If $t > 1$ and if $p_t \leq \underline{p}_t$, then accommodate. If $t > 1$ and $\underline{p}_t < p_t \leq \overline{p}_{t-1}$, then fight with probability,

$$\left(\left[1 - 2\left(\overline{p}_{t-1} - \frac{1}{2}p_t\right)\right] p_t\right) / \left[2\left(\overline{p}_{t-1} - \frac{1}{2}p_t\right)(1 - p_t)\right]$$

Accommodate with the complementary probability. If $t > 1$ and $2\overline{p}_1 \geq p_t > \overline{p}_{t-1}$, then fight. If $t > 1$ and $p_t > 2\overline{p}_1$, then accommodate. If $t > 1$ and $p_t \leq \underline{p}_t$, then accommodate.

For the argument verifying that this assessment is a sequential equilibrium, the reader is referred to the Appendix.

In the next few sections, I look at various features of this equilibrium.

3.2.2 Reputation as the Horizon Expands

Corollary 5 *As $T \to \infty$, without discounting, even a very small initial assessment that an incumbent is 'hard-wired' will lead to reputation building.*

The intuition for this is not quite as obvious as that for the analogous result in the previous section: Here there is a 'reputational externality'-the current actions of an incumbent do affect the entrants' beliefs about the other incumbent. The following subsection aims to clarify why the Kreps-Wilson punchline is nevertheless preserved.

No Free-Riding Despite 'Reputational Externalities' The proposition below might help to understand why the Kreps-Wilson punchline is preserved by making clear that there is no free riding in this equilibrium:

Proposition 6 *An incumbent's choice of action at any given time will be independent of the contemporaneous choice of the other incumbent.*

Proof. In order to prove this proposition, it suffices to show that the value function of an incumbent j matched with an entrant i at any time t, can be written in the following form:

$$V_t\left(i, p_t^i, p_t^k\right) = E_t f\left({}_I a_t^{j(i)}, {}_E a_t^i\right) + \frac{1}{2}V_t\left(p_{t-1}^i\right) + \frac{1}{2}V_t\left(p_{t-1}^k\right), \; i \neq k \; ; \; i, j, k = A, B \qquad (1)$$

In this expression $E_t f\left({}_I a_t^{j(i)}, {}_E a_t^i\right)$ stands for the expected payoff in the stage game to the actions taken at date t by the entrant i and the incumbent j matched with i. $V_t\left(p_{t+1}^i\right)$, $i = A, B$, stands for the value (as of date t) that this incumbent can expect to earn in the future from being matched with members of i's cohort.

In order to see why this should suffice, note that the beliefs of contemporaneous entrants will be independent since they do not share information. Using the notation just introduced, one can express this by writing $p_{t+1}^i\left(p_t^i, {}_I a_t^{j(i)}, {}_E a_t^i\right)$. Since, moreover, entrants are assumed to live only one period and move first, their choice of action will depend only on the beliefs they currently hold. It follows that the incumbent's current choice of action will not depend at all on the action being taken contemporaneously by the other incumbent.

A simple induction argument shows that an incumbent's value function can be written in this additive form at any t:

At date $t = 0$, by initializing $V_0\left(p_1^i\right) = 0$, $i = A, B$, this is trivially true. Assume that this is true up to and including date t. The standard value function at date $t + 1$ is given by

$$V_{t+1}\left(A, p_{t+1}^A, p_{t+1}^B\right) = E_{t+1} f\left({}_I a_{t+1}^A, {}_E a_{t+1}^A\right) + \frac{1}{2} V_t\left(A, p_t^A, p_t^B\right) + \frac{1}{2} V_t\left(B, p_t^A, p_t^B\right)$$

Substituting for $V_t\left(i, p_t^i, p_t^k\right)$ the expression (1) above, one gets

$$V_{t+1}\left(A, p_{t+1}^A, p_{t+1}^B\right) = E_{t+1} f\left({}_I a_{t+1}^A, {}_E a_{t+1}^A\right) +$$

$$\tfrac{1}{2}\left(E_t f\left({}_I a_t^A, {}_E a_t^A\right) + \tfrac{1}{2} V_t\left(p_{t-1}^A\right) + \tfrac{1}{2} V_t\left(p_{t-1}^B\right)\right)$$

$$+\tfrac{1}{2}\left(E_t f\left({}_I a_t^B, {}_E a_t^B\right) + \tfrac{1}{2} V_t\left(p_{t-1}^A\right) + \tfrac{1}{2} V_t\left(p_{t-1}^B\right)\right)$$

By the induction hypothesis, it then follows that one can set

$$V_{t+1}\left(p_t^i\right) = E_t f\left({}_I a_t^{j(i)}, {}_E a_t^i\right) + V_t\left(p_{t-1}^i\right)$$

Thus the induction goes through. ■

This suggests that it is the absence of information sharing among contemporaneous entrants that is crucial in determining whether or not there is free riding. The sole presence of 'reputational externalities' will not suffice to generate that type of behavior.

A Note on Discounting The fact that in this environment after an episode of accommodation entrant's beliefs do not collapse to zero, turns out to be of great importance if incumbents' payoffs are discounted. As the proposition below shows, the Kreps-Wilson punchline does not survive the introduction of discounting because of this feature of the beliefs' updating rule[12]:

Proposition 7 *With discounting, anonymous incumbents and no information sharing among either entrants or incumbents, the previous equilibrium breaks down in the limit, i.e., as $T \to \infty$. Furthermore, for any given set of parameters b, β, a and T, a necessary and sufficient condition for there to exist an equilibrium of the general form of Proposition 4, is*

$$\frac{1}{2}\beta^{\left[T-k\left(\frac{1}{2}b\right)+1\right]} \sum_{j=0}^{\left[k\left(\frac{1}{2}b\right)-k(b)-1\right]} \beta^{j} > 1$$

with $k(p) \equiv \inf\{t : \bar{p}_t < p\}$ for $p > 0$, and $k(0) = \infty$.

Proof. The reader is referred to the Appendix.

The intuition for this result is actually straightforward: As $T \to \infty$, the cost of accommodating in the region where incumbents are supposed to fight for sure (namely, the parameter a times the number of periods at which entry takes place after a deviation, minus that same parameter times the number of periods at which entry would have taken place if the incumbent had not deviated), vanishes due to discounting as, for given initial beliefs, the additional entry episodes will invariably take place in an endgame whose duration will remain constant as the horizon stretches to infinity. The gain from such a deviation (the avoided cost of fighting), on the other hand, will accrue immediately, and, hence, will not vary with the horizon of the game. It is important to emphasize that this breakdown has nothing to do with free riding in the usual sense[13].

[12] A question left open here is whether or not there is an equilibrium as $T \to \infty$. My guess is that there is not, but I have not been able to come up with a definitive argument.

[13] The formula in the proposition reveals a further, rather subtle difference between the current set-up and the one with named incumbents: For $b = \frac{1}{3}$, and $\beta = 1$, this formula takes the form $\frac{a}{2} > 1$. In other words, this previous condition is now necessary and sufficient, instead of just being sufficient as in the game with identifiable incumbents.

3.2.3 Pure Anonymity Effects

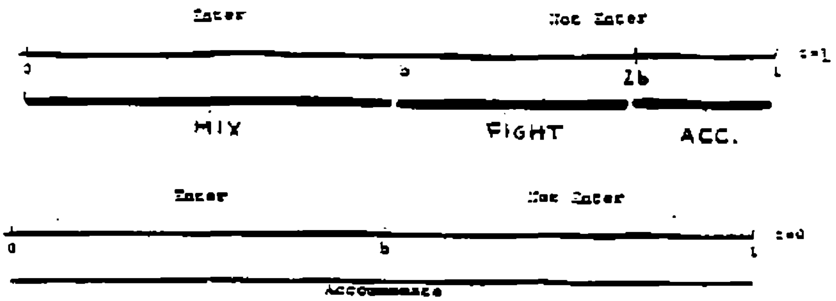
While the 'pure random matching' effects identified in the previous section are still present in this equilibrium, now there appear three additional 'pure anonymity' effects :'Tail' effects concerning behavior at extreme beliefs; a restriction on the relative profitability for an entrant of being fought versus accommodated, i.e., a restriction on the parameter b; and a modification of the range of values at which incumbents play mixed strategies and of the probabilities with which they mix (hence, of the critical beliefs separating the entry from the no-entry regions).

Modified Mixing and Critical Beliefs Roughly speaking, at $t = 2$ (letting $t = 1$ be the last period), and for any given beliefs below b, in order for an observation of fighting to lead to posterior beliefs exactly equal to b under the new averaging rule for updating beliefs, fighting must take place with a lower probability than was the case when incumbents were named. As a consequence, the critical beliefs separating the entry from the no-entry regions, $(2b/b+1)^{t-1}\,b$, will be higher than in the set-up with named incumbents (where the corresponding value was b^t). This will, in turn, feed back into the probabilities of fighting for previous periods. The probabilities have to be lower in order to overcome both the averaging of beliefs and the higher critical value prevailing one period ahead.

The implication is that reputation formation, understood as the gradual enlargement of the no-entry region as the horizon lengthens, is slowed down relative to the case where incumbents are assumed identifiable.

Tail Effects Another consequence of beliefs' averaging is to make threats of fighting non-credible in the regions at the extremes of the beliefs' line.

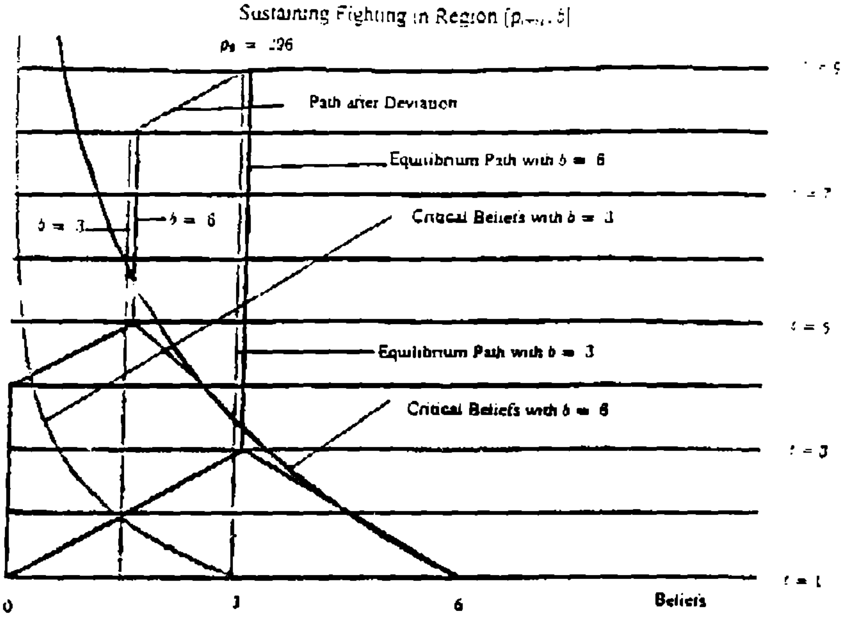
Take the case with $b < \frac{1}{2}$. The following diagram illustrates the resulting equilibrium (for $T = 2$):



Tail Effects

In the region $[2b, 1]$, it is no longer possible for an incumbent to credibly threaten to fight entry. The reason being that deviating from the threatened action and accommodating instead will not lower entrants' beliefs sufficiently to induce entry the following period.

Restriction on the Relative Profitability of Entry The key to understanding this restriction (that $b > \frac{1}{3}$) is to note that the critical beliefs separating the entry from the no-entry regions $((2b/b+1)^{t-1}b)$ vary with the value of the parameter b. The requirement that this parameter exceeds $\frac{1}{3}$ amounts to limiting the speed at which the critical beliefs increase as the game approaches its conclusion. The role of this restriction is to guarantee that fighting with probability 1 can be supported where prescribed in the equilibrium. The case with $b = 0.3$ is presented in the diagram below:



Evolution of Critical Values and Path of Play

4 Free Riding with Information Sharing

This part of the paper looks at what happens when incumbents are anonymous and there is information sharing among contemporaneous entrants. That is, what happens when all entrants alive at any given time belong to one and the same cohort.

The first section looks at the case where incumbents cannot observe each others' types and discount their payoffs.

The second section then looks at what happens when incumbents know each others' types. The analysis is, at best, exploratory in that it deals only with the case $T = 2$.

4.1 No Reputation Building Regardless of Horizon

Assuming that incumbents cannot observe each others types and discount their payoffs at a sufficiently high rate allows one to calculate an equilibrium where there are no incentives to invest in reputation regardless of the length of the game. More precisely, in this equilibrium the critical beliefs separating the entry from the no-entry regions remain constant as $T \rightarrow \infty$.

Proposition 8 *If incumbents cannot observe each others' types, and if $b > \frac{1}{2}$, $\beta a > 1$, and $\frac{\beta}{1-\beta}a < \frac{1}{b}$, then there is no reputation formation even as the horizon goes to infinity.*

This is the equilibrium (with the last period corresponding to $t = 0$):

Beliefs:

a) If no entry takes place or $p_{t+1} = 0$, then $p_t = p_{t+1}$.

b) If $t \geq 1$, $0 < p_{t+1} \leq 1/\frac{1-\beta^t}{1-\beta}\beta a$ and observed action profile is:

i) (F, F) , then $p_t = 1$.

ii) (F, AC) , then $p_t = \frac{1}{2}$.

iii) (AC, AC) ,then $p_t = 0$.

c) If $t \geq 1$, $1/\frac{1-\beta^t}{1-\beta}\beta a \leq p_{t+1} \leq 1$ and observed action profile is:

i) (F, F) , then $p_t = p_{t+1}$.

ii) (F, AC) , then $p_t = \frac{1}{2}p_{t+1}$.

iii) (AC, AC) , then $p_t = 0$

Strategies:

Incumbents :

a) If $t \geq 1$ and $p_{t+1} < 1/\frac{1-\beta^t}{1-\beta}\beta a$, then accommodate.

b) If $t \geq 1$ and $p_{t+1} \geq 1/\frac{1-\beta^t}{1-\beta}\beta a$, then fight.

If $t = 0$: Accommodate always.

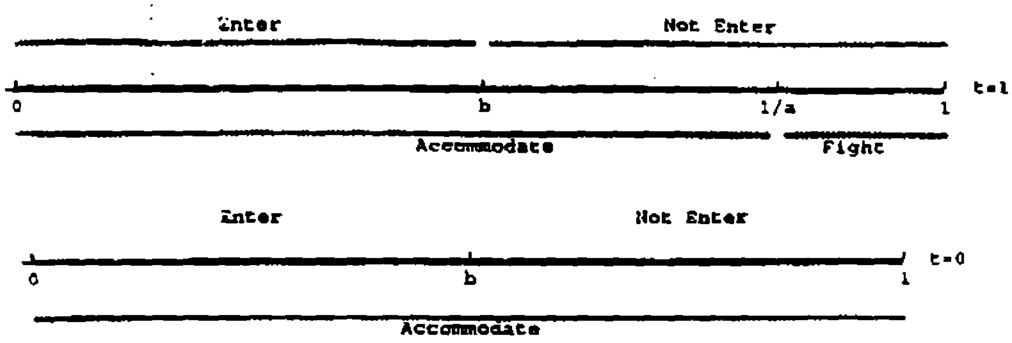
Entrants:

If $p_{t+1} < b$, then enter; if $p_{t+1} \geq b$, stay out.

Proof. See Appendix.

Note that this equilibrium is not unique. There is another equilibrium that has incumbents fighting with probability 1 for beliefs above b, for all t. The path of play, though, is the same in both equilibria.

To understand what is going on here, it seems best to start by looking at a two period, no-discounting version of this game. The diagram below illustrates the discussion:



No Reputation in Two Periods

In the last period, here, as in all the scenarios in this paper, nothing changes. In the period before last, starting off from initial beliefs somewhere to the left of $\frac{1}{a}$, the equilibrium prescribes accommodation with probability 1. To see that accommodation is a best response here, note that if an incumbent deviates from this action and fights instead, her expected payoff will be given by

$$p_2 a + (1 - p_2)\, 0$$

For all initial beliefs below $\frac{1}{a}$, this expected payoff will not match the additional cost of fighting, namely 1, making the deviation under consideration unprofitable. On the other hand, for initial beliefs in $\left[0, \frac{1}{a}\right]$, the net payoff from accommodating is zero. Hence, it is clearly best to do so.

The role of discounting is the following: If incumbents were not to discount, the reward from convincing entrants that incumbents are tough[14] would grow arithmetically to infinity as the horizon expands, eventually pushing the expected payoff from deviating and fighting above 1. To prevent this, it is necessary that incumbents discount their payoffs at a sufficiently high rate, namely, a rate β such that

$$\frac{\beta}{1-\beta} a < \frac{1}{b}$$

[14]Note that, by the very nature of the result this section is aiming at, it must always be feasible to convince entrants that incumbents are tough in the range of beliefs $[0, b]$. For, if the critical beliefs are not to shift leftward as the horizon expands, accommodation with probability one must be the prescribed action in equilibrium in this region. But then, by deviating and fighting in this range, there is always a positive probability that an incumbent can convince an entrant that he is tough.

The right hand-side of this expression gives the reward from discouraging entry forever, from next period on (hence the additional β). The condition guarantees that the beliefs making an incumbent indifferent between deviating-and-fighting or conforming-and-accommodating (the inverse of the right hand-side expression), always (i.e., for any horizon) lie to the right of b. In this way, since the expected gain from deviating varies monotonically with initial beliefs, no matter how long the games goes on, it will always remain a best response to accommodate in the region $[0, b]$[15].

Note that there is free riding here because what the other incumbent is doing contemporaneously is modifying the way this incumbent's current actions influence the beliefs of entrants.

Finally, note that under these same parameter values, reputations would develop in the 1:1 Kreps-Wilson environment.

4.2 Reputation Despite Free Riding

When incumbents are aware of each others' types, the computation of an equilibrium becomes more involved: Incumbents' strategies depend now on profiles of types, in addition to beliefs and time. Moreover, the formulas for updating beliefs become considerably more complicated, as entrants now have to 'average out' all states (profiles of types) under which an observed profile of actions might have arisen under the equilibrium being considered.

For these reasons, I study here only the $T = 2$ case. Even the equilibrium of this truncated game is considerably less transparent than those encountered before (and this, despite the fact that it is, as were those other equilibria, recursive in beliefs).

Proposition 9 *If incumbents can observe each others' types and actions, $a > 1, T = 2$ and $b \geq \frac{1}{2}$, then, depending on the exact value of b, there might exist a sequential equilibrium with reputation formation exactly as in the Kreps and Wilson model (i.e., the probability of entry remains unchanged each period).*

The equilibrium assessment is as follows:

[15]The case $b < \frac{1}{2}$ is problematic. Let $T = 2$. Under these conditions, for initial beliefs in a neighbourhood to the right of $2b$, there would not exist an equilibrium.

Beliefs:

a) If no entry takes place or $p_{t+1} = 0$, then $p_t = p_{t+1}$.

b) If $0 < p_{t+1} \leq \tilde{\delta}(b)$ and observed action profile is:

i) (F, F), then $p_t = b$.
ii) (F, AC), then $p_t = \frac{1}{2}$.
iii) (AC, AC), then $p_t = 0$.

c) If $\tilde{\delta}(b) < p_{t+1} \leq b$ and observed action profile is:

i) (F, F), then $p_t \geq b$.
ii) (F, AC), then $p_t \leq \frac{1}{2}$.
iii) (AC, AC), then $p_t = 0$

d) If $p_{t+1} \geq b$ and observed action profile is:

i) (F, F), then $p_t = p_{t+1}$.
ii) (F, AC), then $p_t = \frac{1}{2} p_{t+1}$.
iii) (AC, AC), then $p_t = 0$.

Strategies:

Incumbents :

If realized types are (R, R) and $t > 0$:

a) If $p_{t+1} < b$, then accommodate.
b) If $p_{t+1} \geq b$, then fight.

If realized types are (R, A) and $t > 0$:

a) If $p_{t+1} < \tilde{\delta}(b)$, then mix, fighting with probability $\frac{(1-b)\bar{\delta}}{(2b-1)(1-\bar{\delta})}$, (where $\bar{\delta}$ stands for initial beliefs).
b) If $p_{t+1} \geq \tilde{\delta}(b)$, then fight.

If $t = 0$: Accommodate always.

Entrants:

At $t \neq 0$, if $p_{t+1} < \delta(b)$, then enter; if $p_{t+1} \geq \delta(b)$, stay out. At $t = 1$, if $p_{t+1} < b$, enter; if $p_{t+1} > b$, stay out; if $p_{t+1} = b$ then mix, staying out with probability $\frac{1}{a}$.

With

$$\delta(b) = \frac{(1-2b)+\sqrt{-8b^3+16b^2-8b+1}}{2(1-b)} \quad if \;\; b \geq b^*$$

$$\delta(b) = 1-(1-b)^{1/2} \qquad if \;\; b < b^*$$

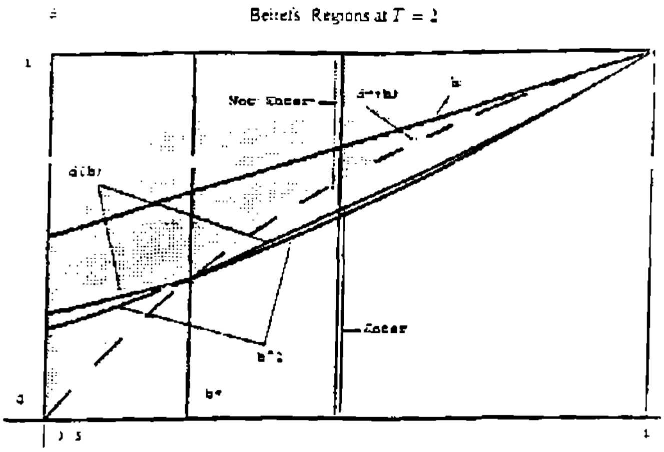
$$where\ b^*\ s.t.\ 1-(1-b)^{1/2} = \tilde{\delta}(b) \;\; and\ \tilde{\delta}(b) = \frac{2b-1}{b}$$

Proof. See Appendix.

The first thing to note is that there is free riding here, as in the equilibrium of the previous section. Take the state (R,R) (i.e., both incumbents are rational), and initial beliefs below b. Under these conditions, it is equilibrium behavior for an incumbent to accommodate for sure. The reason being that unilateral fighting cannot succeed in convincing an entrant that incumbents are tough. More precisely: The incumbent realizes that should she deviate and fight, the entrant will observe the outcome (F,AC) (i.e., one incumbent fights; the other accommodates). According to the equilibrium, this outcome should only arise if the state is (R,A). Hence, after observing (F,AC), the entrant will assign probability $\frac{1}{2}$ to the event that an incumbent is tough. But this belief is not high enough to deter entry the following period (as $b > \frac{1}{2}$).

Note further that in the above equilibrium, when the state is (R,R), incumbents accommodate entry throughout this range of beliefs, while in the Kreps-Wilson game they would have fought entry with positive probability. On the other hand, when the state is (R,A), incumbents will, at any given beliefs in this range, fight with a higher probability than they would in the Kreps-Wilson environment. These two circumstances tend to shift the critical beliefs in opposite directions, so that,without further arguments, is hard to say whether the critical beliefs will be above or below those in Kreps-Wilson.

But the following diagram makes plain that beliefs will never be above the Kreps-Wilson value.



Interestingly, there will be a value of b, namely, b^*, at which there will be no 'loss' in reputation relative to Kreps-Wilson. Also, as this parameter takes values near one, the loss in reputation becomes negligible. For any value of b, the loss in reputation will be relatively small though.

It would seem that the additional information sharing among incumbents allows them to better coordinate their responses, and thus avoid the extreme form of free riding which led to the drastic result of the previous section (fighting can now be sustained when the state is (R,A) precisely because the rational incumbent can now be sure that the other incumbent will not accommodate).

A final remark on the issue of uniqueness: This question is left open in this section. It is not clear to me even whether the equilibrium presented here is unique among the class of equilibria recursive in beliefs.

5 Summary of Results

Here is a summary of the results presented in the paper:

First of all, the paper shows that, generally, there will be scope for reputation even in anonymous populations.

Secondly, the paper shows that the presence of a 'reputational externality' due to the anonymity of incumbents will not suffice to generate 'free riding' if entrants do not know what is going on simultaneously in matches other than their own. This will be so even if incumbents share entrants over time. A rather counterintuitive implication is that more information sharing, if not of the 'right' sort, might actually be detrimental to the formation of reputations.

Thirdly, the paper also suggests that 'free riding' per se might not be enough to prevent the development of reputations. The two period case shows that, if there is information sharing among both entrants and incumbents, while there will be 'free riding', its effects on reputation formation will be small to nil.

Overall, these results emphasize the importance of the exact pattern of information sharing among agents (both among entrants and among incumbents) in molding incentives to invest in reputations when 'reputational externalities' due to anonymity are at work.

Finally, the paper also shows that discounting will play a very important role when incumbents are anonymous. This in stark contrast to what happens with named agents. For example, it can be shown that, with discounting, the equilibrium that results when incumbents are anonymous but there is no information sharing among entrants would eventually breakdown as one lets the horizon extend to infinity[16].

6 Experiments Not Done

I close with a list of a few of the experiments I did not perform even though the model suggests them rather naturally: I did not look at situations with anonymous entrants. In a sense, I have studied only the 'perfect discrimination' case. Neither did I look at the implications of having cohorts start out with different priors. And this paper has concentrated on the 'easy' parameter values; specifically, it has studies only the case $b > \frac{1}{2}$. This restriction considerably eases the analysis, in that it makes unilateral fighting unprofitable (by the way 'biasing' the conclusions towards less reputation building). Furthermore, I have assumed throughout that incumbents know the history

[16]Note that this breakdown has nothing to do with free riding, as there is no such behavior in this environment, with or without discounting.

of the whole economy[17]. And, finally, it bears repeating that I have only considered in the previous section the case $T = 2$.

[17] I feel this is not an implausible assumption: Presumably, businesses (the long-lived players investing in reputation) have the means and the motivation to track closely what their competitors are doing. Customers (the short-lived players) are less likely to engage in similar research about other customers.

A.1 Description of Equilibrium for Prop. 1

For $(1/N)a > 1$, the following strategies and beliefs form a sequential equilibrium of the matching game just described:

Starting from given initial beliefs p_T, define beliefs recursively as follows:

a) If there is no entry at stage $t+1$, then $p_t = p_{t+1}$.

b) If there is entry at stage $t+1$, this entry is fought, and $p_{t+1} > 0$, then $p_t = \max(b^n, p_{t+1})$

c) If there is entry at stage $t+1$, and either this entry is met by accommodation or $p_{t+1} = 0$, then $p_t = 0$.

The rational incumbent's strategy is given by:

If $t = 1$, the incumbent accommodates. If $t > 1$ and $p_t \geq b^{t-1}$, the incumbent fights. If $t > 1$ and $p_t < b^{t-1}$, the incumbent fights with probability $(1 - b^{t-1})\, p_t / (1 - p_t)\, b^{t-1}$, and accommodates with the complementary probability.

The entrant's strategy is given by:

If $p_t > b$, entrant stays out. If $p_t < b$, then entrant enters. If $p_t = b^t$, then entrant stays out with probability $1/Na$.

A.2 Proof of Proposition 4

Proof. I check optimality of strategies and consistency of beliefs:

CONSISTENCY OF BELIEFS:

If no entry takes place at stage t, nothing is learned, and $p_t = p_{t+1}$. If $p_{t+1} > 2\, \overline{p}_1$, and the entrant enters and is fought, since the rational incumbent was not supposed to fight, the entrant must conclude that he has been matched with a 'hard-wired' incumbent. As the entrant does not know with which of the two incumbents he will matched, his best guess as to the type of the incumbent he is going to be matched with next period is given by $\frac{1}{2} + \frac{1}{2}p_{t+1}$: One the incumbents (the one who just fought the entrant) is for sure 'hard-wired', hence the first term. The entrant beliefs about the other incumbent remain unchanged from the previous period, hence the second term. If $2\overline{p}_1 \geq p_{t+1} > \overline{p}_t$, then both types fight, so posteriors equal priors. If $\underline{p}_{t+1} < p_{t+1} \leq \overline{p}_t$, then given the mixing probabilities the rational incumbent is using, the resulting posterior is exactly $\overline{p}_t$. If $p_{t+1} \leq \underline{p}_{t+1}$, then, since the rational incumbent is supposed to accommodate, the fact that entry was fought leads the entrant to conclude that the incumbent he is currently matched with is 'hard-wired', and the posterior is exactly as in case i). Finally, if there is entry, and it is accommodated, then the entrant must conclude that the incumbent he is matched with is rational ($p_t = 0$), and his best guess as to the likelihood of the incumbent he will be matched with next period being rational is given by $\frac{1}{2}p_{t+1}$. Note the out-of-equilibrium path beliefs: If $p_{t+1} = 0$ and the incumbent fights, then $p_t = \frac{1}{2}p_{t+1}$; and if $2\overline{p}_1 \geq p_{t+1} > \overline{p}_t$ and the incumbent accommodates, then $p_t = \frac{1}{2}p_{t+1}$. The interpretation is exactly the same as in Kreps and Wilson: Any accommodation is taken as proof that at least one of the incumbents is rational.

OPTIMALITY:

Verifying that the entrants are playing optimally is straightforward: If the rational incumbent is supposed to fight, then clearly it can never pay for the entrant to enter. If $p_t > 2\overline{p}_1$, it does not pay for the entrant to enter because $p_t > b = \overline{p}_1$. Verifying that rational incumbents are playing optimally is more involved: As before (Proposition 1), the condition $\frac{1}{2}a > 1$ just ensures that it is worthwhile to build a reputation. The condition $b > \frac{1}{3}$ ensures that it is worthwhile for the incumbent to fight when he is supposed to. To see why this so, it is best to go back to the original Kreps and Wilson model (where the tough types actually like to fight, and hence out of equilibrium beliefs are not as severely restricted as they are when one assumes 'hard-wired' incumbents) . There, in order to support fighting in equilibrium at stage T, it was not really necessary to set $p_{T-1} = 0$ after a deviation. It sufficed to set beliefs so as to ensure that the condition $\overline{p}_{k(p_T)} > p_{T-1}$ was satisfied (where $k(p) \equiv \inf\{t : \overline{p}_t < p\}$ for $p > 0$, and $k(0) = \infty$). The left-hand side expression corresponds to the critical value separating the entry from the no-entry regions at the latest stage where entry is deterred, assuming the incumbent sticks to the equilibrium strategy from the start. This means that accommodating today and sticking to the specified play thereafter will eventually lead to entry at least one period earlier than would have been the case had the incumbent stuck to the equilibrium from the start. Since the gain from deterred entry (a) is assumed bigger than the cost (=1), this means that the incumbent should fight when fighting is the prescribed equilibrium action. The analogous condition in the present context takes the form $\overline{p}_{k(p_T)} > \frac{1}{2}p_T$. Note now that, for a given magnitude of p_T, the left-hand side is invariant to changes in the length of the horizon T. One can show by induction that, for values of p_T between $\overline{p}_t$ and $\overline{p}_{t-1}$(with $T \geq t$), $\overline{p}_{k(p_T)} = \overline{p}_t$. Taking the upper bound of the region, substituting it into the condition above, and using the expression for $\overline{p}_t$, it follows that $b > \frac{1}{3}$ is necessary and sufficient for values of $p_T \in (0, b)$ to satisfy the condition[1]. If $p_t \leq \underline{p}_t$, then even if the incumbent fights when he is supposed to accommodate, entry is not deterred, so there is no need to have recourse to mixed strategies, and accommodation is the optimal action. The rest of the proof closely follows the reasoning in Kreps and Wilson, and involves writing down the corresponding value function and then applying the one-deviation principle . ■

A.3 Proof of Proposition 7

The cost of deviating and accommodating in the region where fighting for sure is prescribed, is given by

$$\frac{1}{2}\beta^{\left[T-k\left(\frac{1}{2}p_T\right)+1\right]}\sum_{j=0}^{\left[k\left(\frac{1}{2}p_T\right)-k(p_T)-1\right]}\beta^j \tag{2}$$

By the argument in the previous proof , for a given magnitude of p_T, $k\left(\frac{1}{2}p_T\right)$ and $k\left(p_T\right)$ remain unchanged as $T \to \infty$. It follows that this cost goes to 0 as $T \to \infty$. This proves the first part of the proposition.

To see that for there to exist an equilibrium of the form of Prop. 4, it must be that

$$\frac{1}{2}\beta^{\left[T-k\left(\frac{1}{2}b\right)+1\right]}\sum_{j=0}^{\left[k\left(\frac{1}{2}b\right)-k(b)-1\right]}\beta^j > 1 \tag{3}$$

it suffices to show that the cost of deviating is falling in initial beliefs, and, hence, if the above condition is satisfied, the analogous condition for arbitrary beliefs will be satisfied for all initial beliefs below b. (On the other

[1]For values above b but below $2b$, the condition is always satisfied. For values above $2b$, it is never satisfied (hence the incumbent has to accommodate in that region).

hand, if it is violated, then, clearly, in a neighborhood to the left of b, fighting for sure cannot be sustained in equilibrium). To show that (2) is falling in initial beliefs, write the time inverse of the critical beliefs as

$$t(p_T) = \frac{\ln p_T - \ln b}{\ln 2b - \ln(b+1)} + 1$$

It follows that $k(p) = [\ t(p) \mid$ *rounded up to next integer*$]$. Now, $t(p)$ is clearly increasing in p, hence, so is $k(p)$. On the other hand,

$$k\left(\frac{1}{2}p_T\right) - k(p_T) = \frac{-\ln 2}{\ln 2b - \ln(b+1)}$$

Hence, the result follows. By the way, note that the RHS of (3) is increasing in b (as $k(b) = 1$ for all b, and $k\left(\frac{1}{2}b\right)$ is increasing in b), increasing in β, and falling in T. Note further that (3) cannot be satisfied for $b < \frac{1}{3}$.

A.4 Proof of Uniqueness of the Equilibrium Prop.4

In the last period, the equilibrium is unique for given beliefs, except when beliefs are exactly b. In the period before last, the equilibrium specified is again unique, except again at the critical beliefs separating the entry from the no-entry region. Note that in this two period scenario, entrants' actions in the last period when beliefs are exactly b are determined. Uniqueness follows since, in the last period, sequential rationality requires that actions taken be optimal responses given current beliefs. So, the only way actions in that period could vary with actions taken in previous periods (or with beliefs prevailing in previous periods) is if agents were indifferent between the prescribed actions. This is never the case for incumbents, who always strictly prefer to accommodate rather than to fight in this last period. For entrants, this is only the case at b, but as already noted, actions at such beliefs are pinned down by the requirement that incumbents be indifferent in the previous period between accommodating and fighting in the region $[0, b]$. Now, go one period further back. Again, the sequential rationality argument applies, now to actions in the period before last. Again, this implies that strategies cannot possibly be non-recursive (in beliefs), and again this leads to a unique equilibrium set of actions at the period preceding the one before last. By induction, this reasoning extends to all periods.

A.5 Proof of Proposition 9

Proof. To prove that the candidate equilibrium is in fact one, I check the optimality of strategies and the consistency of beliefs:

OPTIMALITY OF STRATEGIES:

Incumbents:

I) If state is (R, R):

i) $\delta_2 < b$:

Best to accommodate, since if, instead, incumbent fights, resulting beliefs will be given by $\delta_1 = \frac{1}{2} < b$, and so entry will not be deterred.

ii) $\delta_2 \geq b$:

Best to fight, since if, instead, the incumbent accommodates, the resulting beliefs will be given by $\delta_1 = \frac{1}{2}\delta_2 < \frac{1}{2} < b$ (observed outcome would be (F, AC)). If fight, then beliefs would be given by $\delta_1 = \delta_2 \geq b$, so incumbent must be either indifferent between fighting and accommodating, or strictly prefer the former to the latter.

II) If state is (R, A):

i) $\delta_2 < \widetilde{\delta}(b)$:

Fighting with probability 1 will lead to $\delta_1 < b$ after (F, F) is observed. To see this: Note that the probability of an entrant being currently matched with an incumbent who is of type A, after observing (F, F), is given by

$$p(A|(F,F)) = p((A,A)\,|\,(F,F)) + \tfrac{1}{2}p((A,R)\,|\,(F,F))$$

$$+\tfrac{1}{2}p((R,A)\,|\,(F,F)) + \ 0\,p((R,R)\,|\,(F,F)) =$$

$$\frac{\delta_2^2}{p(F,F)} + \frac{p(F|R,(R,A))\delta_2(1-\delta_2)}{p(F,F)}$$

with $p(F|R,(R,A)) = p((F,F)\,|\,(A,R)) \equiv x$

Given that there is accommodation by R when the state is (R, R), we have

$$p(F, F) = \delta_2^2 + 2x\delta_2(1-\delta_2)$$

$$p(A|(F,F)) = \frac{\delta_2^2 + x\delta_2(1-\delta_2)}{\delta_2^2 + 2x\delta_2(1-\delta_2)}$$

Setting $p(A|(F,F)) = b$, solving for x satisfying this equation, and denoting such an x by $\tilde{x}$, we get

$$\tilde{x}(b) = \frac{(1-b)\delta_2}{(1-\delta_2)(2b-1)}$$

Note that $\tilde{x}(b)$ is increasing in δ_2. Now, define $\tilde{\delta}$ by setting $\frac{(1-b)\tilde{\delta}}{(1-\tilde{\delta})(2b-1)} = 1$, to get

$$\tilde{\delta}(b) = \frac{2b-1}{b}$$

All $\delta_2 > \tilde{\delta}(b)$ will yield $x > 1$; all $\delta_2 \leq \tilde{\delta}(b)$ will result in $x < 1$. For $\delta_2 < \tilde{\delta}(b)$, $\tilde{x} < 1$, and so, if a rational incumbent fights with probability 1, we get $x > \tilde{x}$, and, hence, $p(A|(F,F)) < b$ ((since $p(A|(F,F))$ is decreasing in x). It follows that it does not pay to fight.

If, instead, the incumbent accommodates with probability 1, this would imply that, when $\delta_2 < \tilde{\delta}(b)$, incumbents accommodate across all states ($\tilde{\delta}(b) \leq b$ for $b \in \left[\frac{1}{2}, 1\right]$), and there would be an incentive to fight instead when the state is (R, A), as the observed outcome would be (F, F), and, so, $\delta_1 = 1$. It follows that incumbents must mix. In order for the incumbent to be indifferent between fighting and accommodating, it must be that an entrant, after observing (F, F) when $\delta_2 < \tilde{\delta}(b)$, must himself mix, so that

$$p\left(E|(F,F), \left(\delta_2 < \tilde{\delta}(b)\right)\right)0 +$$

$$\left(1 - p\left(E|(F,F), \left(\delta_2 < \tilde{\delta}(b)\right)\right)\right)a - 1 = 0$$

$$\Rightarrow p\left(E|(F,F), \left(\delta_2 < \tilde{\delta}(b)\right)\right) = 1 - \tfrac{1}{a}$$

In order to make the entrant willing to mix after observing (F, F) in the region $\delta_2 < \widetilde{\delta}(b)$, it must be that $p(A|(F,F)) = b$. This implies that incumbents must fight with probability

$$\widetilde{x}(b) = \frac{(1-b)\,\delta_2}{(1-\delta_2)(2b-1)}$$

ii) $\widetilde{\delta}(b) \leq \delta_2 < b$:

If you accommodate, the observed outcome will be (F, AC), and, hence, $\delta_1 \leq \frac{1}{2}$. Since $b > \frac{1}{2}$, entry takes place. If , instead, the incumbent fights, the observed would be (F,F), and the resulting beliefs $\delta_1 \geq b$. Hence, entry would be deterred. To see this: To obtain $p(A|(F,F)) = b$, incumbent has to fight with probability $\widetilde{x}(b)$. But whenever $\widetilde{\delta}(b) \leq \delta_2$, $\widetilde{x}(b) > 1$. Since $p(A|(F,F))$ is falling in x, it must be that

$$p(A|(F,F))(1) \geq p(A|(F,F))(\widetilde{x}(b)) = b$$

(with equality only if $\delta_2 = \widetilde{\delta}(b)$).

iii) $\delta_2 \geq b$:

Accommodating leads to observation (AC, F), and beliefs $\delta_1 = \frac{1}{2}\delta_2 < \frac{1}{2} < b$. Hence, entry is not deterred. Fighting, on the other hand, leads to observation (F,F), and beliefs $\delta_1 = \delta_2 > b$. Hence deterring entry.

Entrants:

At $T = 2$, in the region $[0, b]$, the probability that an entrant is fought (across all states) is given by the solution to the following equation (which must hold if the expected value of entry is to be 0):

$$\delta_2^2 + 2(1-\delta_2)\,\delta_2\left(\frac{1}{2}x + \frac{1}{2}\right) = b \tag{4}$$

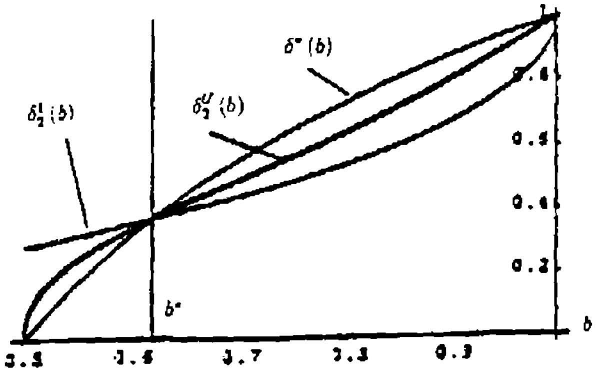
Substituting $\frac{(1-b)\delta}{(1-\delta)(2b-1)}$ for x above, solving for δ_2, and picking the positive root, one obtains

$$\delta_2^U(b) = \frac{(1-2b) + (16b^2 - 8b^3 - 8b + 1)^{\frac{1}{2}}}{2(b-1)}$$

Substituting instead $x = 1$, and solving, one obtains

$$\delta_2^1(b) = 1 - (1-b)^{\frac{1}{2}}$$

The following plot illustrates the properties of these functions:



From the diagram, note that in the region $[\frac{1}{2}, b^*]$, $\delta_2^U(b) > \widetilde{\delta}(b) \Rightarrow \widetilde{x}(b) > 1$ (as $\widetilde{x}(b)$ is increasing in δ). It follows that $\delta_2^U(b)$ overestimates the probability of fighting. Note that the probability that an entrant is fought (from (1) above),

$$\delta^2 + (1-\delta)\,\delta\left(\frac{(1-b)\,\delta}{(1-\delta)\,(2b-1)} + 1\right)$$

is increasing in δ. Also, $\delta^2 + 2(1-\delta)\,\delta$ is increasing in δ. It follows that the true critical value of δ is given by $\delta_2^1(b)$, for $[0.5, b^*]$; and by $\delta_2^U(b)$, for $[b^*, 1]$. Evidently, for $\delta < \delta(b)$, entry should take place; not so for $\delta > \delta(b)$ (this follows from the payoffs to entry being monotonically decreasing as beliefs increase -since the probability of being fought is increasing, as just shown).

CONSISTENCY OF BELIEFS:

I) $\delta_2 < \widetilde{\delta}(b)$:

i) If observation is (F, F) : By construction.

ii) If (F, AC) :

If the state were (R, R) there would be accommodation (note that $\widetilde{\delta}(b) < b$; see plot above). It must be that the state is (R, A) when this observation is made.

iii) If (AC, AC) : Obvious.

II) $\tilde{\delta}(b) \leq \delta_2 < b$:

i) If (F, AC) : At least one incumbent is rational, and so posterior beliefs must be below or at $\frac{1}{2}$.

ii) If (F, F) : Posterior beliefs must above or at b by a previous argument.

iii) If (AC, AC) : Obvious.

III) $\delta_2 > b$:

i) If (F, F) :

Since incumbent fights regardless of type, $\delta_2 = \delta_1$.

ii) If (F, AC) :

Since someone deviated by accommodating, it must be that $\delta_1 \leq \frac{1}{2}$. From the definition of sequential equilibrium (which implies that deviations should be uncoordinated), it follows that $\delta_1 = \frac{1}{2}\delta_2$.

iii) If (AC, AC) : Obvious.

A.6 Proof of Proposition 8

Proof. I present the argument for the two period case. Since the equilibrium is stationary, generalizing the result to longer horizons is straightforward.

OPTIMALITY OF STRATEGIES:

Incumbent:

I) $\delta_2 < \frac{1}{\beta a}$:

If the incumbent fights instead of accommodating, then, with probability δ_2, the other incumbent will be tough. In that case, the entrant will observe outcome (F, F), and, hence, $\delta_1 = 1$, thus deterring entry. With probability $1-\delta_2$, on the other hand, the outcome will be (F, AC), and entry will not be deterred. So, the expected payoff of fighting in this beliefs' region is given by

$$\delta_2 \beta a - 1 \leq 0$$

In words: It is best to accommodate.

II) $\delta_2 \geq \frac{1}{\beta a}$:

Now it is best to fight as accommodation will surely induce entry, while fighting lead to an outcome of (F, F), and, so, to beliefs $\delta_1 = \delta_2 \geq \frac{1}{\beta a} \geq b$ (with equality iff $\delta_2 = \frac{1}{\beta a}$).

Entrant:

Since in the region $[0, \frac{1}{\beta a}]$ at $T = 2$, an incumbent fights only if she is tough, the critical value separating the entry from the no-entry regions, $\delta_2(b)$, is given by the solution to the following equation,

$$p\left(F|\delta_2 \in [0, \frac{1}{a}]\right)(b-1) + \left(1 - p\left(F|\delta_2 \in [0, \frac{1}{a}]\right)\right)b = 0$$

Since $p\left(F|\delta_2 \in [0, \frac{1}{a}]\right) = \delta_2^2 + 2\delta_2(1-\delta_2)\frac{1}{2}$, this yields $\delta_2(b) = b$.

CONSISTENCY OF BELIEFS:

I) $\delta_2 < \frac{1}{\beta a}$:

i) If observed outcome is (F, F) :

Since rational incumbents are supposed to accommodate in that region, the entrant should conclude $\delta_1 = 1$, i.e., that the incumbent is tough for sure.

ii) If (AC, AC), then, evidently, $\delta_1 = 0$.

iii) If (F, AC), then $\delta_1 = \frac{1}{2}$, as only a tough incumbent can be expected to fight in this region.

II) $\delta_2 \geq \frac{1}{\beta a}$:

i) If (F, F), then as both rational and tough types are supposed to fight, priors should equal posteriors.

ii) If (AC, AC), then evidently $\delta_1 = 0$.

iii) If (F, AC) :

It follows that $\delta_1 = \frac{1}{2}\delta_2$ as now both tough and rational incumbents are supposed to fight, and deviations are taken to be uncoordinated. (Note this is an out-of-equilibrium outcome, yet the consistency requirement in the definition of sequential equilibrium suffices to pin down beliefs).

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