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CIDE

NÚMERO 139

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TRADING IN NAMES UNDER MORAL HAZARD

Abstract

Does the possibility of selling one's reputation (name) improve the average quality sold in an economy by reducing incentives to cheat towards the end of an interaction? It is shown in the context of various economies with finite number of overlapping generations and imperfectly informed buyers, that, in the rare instances in which names' markets are active, not only does trade in names not lead to improved trade outcomes, it can even worsen them.

1 Introduction

When an owner-operator sells his or her firm, the buyer often continues operating the business under the original name. Moreover, consumers are seldom in a position to keep track of the change in ownership. In such cases, the new owner has purchased not only the physical assets that make up the firm but, in effect, also its reputation. This even though the new firm lacks one ingredient of the old firm, namely, the services of the original owner-operator. In so far as these services represent an essential determinant of the quality of the firm's product, the firm's name or reputation is no longer necessarily a good predictor of its future performance. On the other hand, provided sold reputations do not completely loose value, the possibility of selling a firm's name can counter eventual incentives to run down a business' 'good will' prior to its sale. It is not a priori clear then whether the feasibility of selling a business' name (arising out of consumers' limited ability to track changes in ownership) is a good thing or not. The purpose of this paper is to try and throw some light on this question[1].

In order to do that, this paper studies repeated, simultaneous sales under moral hazard (the seller chooses product quality after the buyer has decided whether to purchase or not, and sellers cannot commit ex-ante to supply a certain quality), interrupted at regular intervals by 'name'-trading sessions among sellers.

Buyers are assumed to have imperfect information regarding sellers' types (i.e., they will be assumed not to be able to directly observe whether the seller confronting him or her is honest -always provides high quality, or rational -only does so when it is in his or her best interest). Hence, buyers will try to infer a given seller's type from his or her past actions, introducing a backward-looking 'reputational' element in the story (as in the classic references on reputations, Kreps-Wilson 1982 and Milgrom-Roberts 1982).

A'name' (the only traded asset here) is identified in this work with the actual sequence of quality decisions undertaken by its bearers; a sequence that will be taken to be directly observable by buyers. This notion of 'name' is taken from Tadelis 1998 who studies trade in such 'names' in a pure adverse selection environment. Obviously, it is very special. One could instead take a 'name' to correspond not to the sequence itself but just to the beliefs such sequence would induce (as in Mailath and Samuelson 1998 who study trade in this alternative class of names under moral hazard), and the results will most probably change as a consequence. And, just as evidently, this notion of 'name', as well as the alternative one just mentioned, should be taken as some sort of reduced-form representations of what actual names achieve in practice[2].

Finite sellers' lives (2 periods) are introduced in order to motivate 'name' sales. Again following Tadelis' work, sellers' generations are assumed to overlap for one period. For simplicity, it will be assumed that buyers live only for one period. Further, it will be assumed that the economy itself has a finite horizon, that is, that trading of any sort eventually stops. This in

[1] It is perhaps interesting to note the affinity of this problem with the wider class of problems concerning incentives for efficient use of long-lived assets owned by shorter-lived agents. The market economy offers a solution to this problem by allowing short-lived asset holders to sell those assets at the end of their lives. As is well known, in the absence of asymmetric information and other frictions, this leads to efficient exploitation of those resources.

[2] In fact, it is in my opinion an interesting research programm to study how names, now in a literal sense, can give rise to 'names' in either of the forms above.

order to exclude from consideration norm-type equilibria which arise when trading goes on forever.

As hinted in the opening paragraph, a key feature of the model will be the inability of buyers to keep track of 'names' transfers. On the other hand, buyers will be aware that 'names' can be sold. The key assumptions in this regard will be, first, that buyers cannot observe sellers' ages, and, secondly, that they cannot observe who trades in the market for names.

One key insight from this work is that the exact overlap pattern plus the exact timing of 'names' transactions within a seller's life are crucial in determining whether the market for names is active or not, and, hence, whether trade in names can lead to improved outcomes. By the way it clarifies how the specific demographic structure and pattern of participation in the market for names assumed in Tadelis 1998 represents a 'best case scenario' in inducing trade in names. Moreover, it will be shown that name-trading, even when feasible, is unlikely to lead to actual improvements. More specifically, it will be shown that name-trading will not result in improved trade outcomes in the various scenarios considered. In some, it might even be counterproductive by leading to what I call 'erasing of ones' tracks', that is , cheating and then buying a good name in order to continue trading, though for this 'mixed strategies' (or equivalent constructions) are required.

The intuition behind the importance of the overlap pattern and timing of names' trades can be summarized as follows: In general, trade in names is possible if there is scope for what I want to call 'blending'. 'Blending' takes place if a buyer cannot for sure tell whether the name of the seller confronting him or her has at least partially been generated by the actions of that seller. This requires that in equilibrium some sellers should bear names which they have generated themselves (at least partially). If, for example, sellers live for two periods, only two-actions names can be sold by the old and they are sold only to one year old sellers, after the second period clearly buyers will be able to infer that any bearer of a two-action name must have bought it, and, moreover, cannot possibly have generated any portion of the name him- or herself.

Besides this issue of 'blending', there is the question of the opportunity cost of buying a name. This also will depend on the overlap pattern and timing of names' trades. Someone who already has a good name will not have an incentive to buy one at a positive price (or even at a zero price, if there is some cost of participating in the names market)..

It will become apparent that the set-up in Tadelis 1998, unlike other similarly plausible environments, by specifying that only new borns can buy names and only olds can sell them while at the same time starting the economy off with an exceptional generation of sellers that lives only for one period, necessarily gives rise to blending while at the same time making sure buyers of names have very low opportunity costs of buying names (since being new born they cannot possible own a good name when entering the names' market).

The intuition behind the difficulty in obtaining improved trade outcomes has already been hinted at in the opening paragraph of this introduction: While the possibility of trading in names might generate incentives not to cheat towards the end of one's life, it also somehow devalues names as indicators of future performance. More precisely: A first dimension of this trade-off comes into play because in order for trade in names to provide incentives for a seller not to cheat in the period prior to his death, good names must command a positive price. But good names can only command a positive price if some sellers are actually cheating. If all sellers provide good quality, a good name is just not informative in any way. In this type of situation the only way to make a name valuable is by having a very favorable composition of the pool of buyers but in the absence of any natural separating structure between honest and dishonests, such favorable pools cannot be implemented here. In a sense, this lack of the 'right' separating structure is the crucial difference between this type of setup and the adverse selection model of Tadelis 1998 or the noisy outcomes setup of Mailath and Samuelson 1998.

Moreover, as 'blending' is necessary for there to be an active name market, name trading will tend to water down the value of a good name and hence lower prices that can be commanded in the future by providing good quality today. A third dimension arises because trade in names allows cheaters to dissimulate their bad records by purchasing good names (this presumes that he can capture at least part of the surplus associated with that name, see following paragraphs). Note that this effect again depends crucially on the timing of name transactions: If only newborns can buy names, it will obviously not operate.

In addition to these 'quantity' effects, one should mention certain 'price' effects that tend to reinforce the quantity effects, even though in the analysis below pricing will be very much ad-hoc. The point is that one can expect that in most market structures the greater the supply of names the lower the price names will command, and this will tend to dampen the incentives to participate in the names market and hence not to cheat before one's retirement. Similarly, the more cheaters there are, the higher the demand for good names and presumably (modulo market structure) the higher the price those names command. Thus here again incentives for sellers not to cheat in their last period will be positively correlated with cheating behavior by younger sellers.

Again the comparison with Tadelis 1998 is instructive: In that model it is assumed that all the surplus always goes to the sellers of names. In this way, the case for active names' market is doubly clinched: First, because olds can invariably expect to get a positive payoff from entering the names' market. Second, because even if middle aged sellers are allowed to enter the names' market, it would not pay for them to cheat hoping to then buy a good name as they cannot expect to capture any of the surplus associated with that name.

In summary, the analysis in this paper suggests that trade in names in this type of overlapping generations set-up with moral hazard cannot be relied upon to improve trade outcomes in a substantial way. In fact, it suggests that very special constellations are required to activate this type of names' markets at all.

The paper is organized as follows: After a discussion of related literature, the benchmark model is presented. Then some notation is introduced. In the next section, the strategies of the different types of agents acting in the various scenarios considered in what follows are described. In section ? the equilibrium for the 'basic' best case setup is described in detail (unlike in the following sections) in order to provide a reference point for readers, and results concerning whether or not name trading is feasible and if so whether it will lead to improved trade outcomes (higher average quality traded). In section ? the issue of 'blending' is highlighted as a necessary condition for trade in names. Afterwards, the feasibility of 'erasing ones' tracks' is evaluated and its effects on trade outcomes are considered. The paper closes with a brief concluding section.

1.1 Literature Overview

The key reference for the present paper is the work by Tadelis 1998, which, as already mentioned, deals with the pure adverse selection case in an overlapping generations environment similar to the various scenarios considered here. The emphasis in Tadelis' work is on obtaining an active names' market and on the make up of the pool of names' buyers as between good and bad types. It shows, in particular, that good agents will not be able to fully separate themselves by buying good names. This due to the interplay of what he calls the 'Reputation Maintenance Effect'(goods can maintain a reputation more easily -makes buying a name relatively attractive for a good type) versus the 'Reputation Start-up Effect' (goods can build up a reputation more easily -makes it relatively unattractive for a good type to buy a name). The intuition is that if only good types buy names then they will be hard to depreciate in the eyes of buyers. Consequently, maintaining the name will be relatively easy, making it very attractive to bad types but not so attractive to good types who can more easily build up their own good name.

Another recent contribution on the subject of names' trading is the one by Mailath and Samuelson 1998 in which they deal with the moral hazard case but in a model with noisy product signals, compulsive cheaters instead of compulsive do-gooders, and long lived agents. Also, they work with the different concept of name mentioned in the previous section, instead of the name as history they take names to correspond to beliefs. First thing to note is that such models display much richer reputational dynamics (protracted reputation buildup and rundown) than the class of models used in this paper, which follow Kreps-Wilson 1982 and Milgrom-Roberts 1982. Mailath and Samuelson emphasize the types of names bought by each type of agent and show that good agents will tend to buy moderately good reputations while bad ones will prefer very good ones. Interestingly, seemingly because of the noisy signals and compulsive cheating by some agents[3], a substantial part of the intuition in Tadelis 1998 seems to carry over to Mailath and Samuelson's environment with moral hazard. Good types will prefer to buy moderate reputations because they will be able to build them up more easily while bad types will buy very good reputations since they are hard to depreciate.

In the set-up of this paper, these two effects do not operate at all, as both good and bad types are just as good at building a good name. In fact, in the analysis that follows the absence of such a structure inducing differential incentives between types (even though never full separation) will play a very important role. This is perhaps the main difference between the two papers

[3]It is much more difficult for a compulsive cheater than for normal agents to build up a good reputation. If instead of compulsive cheaters one works with compulsive do-gooders, then building up and maintaining a reputation for rationals is just as easy as for the automaton types. By the way, note that, in this infinite horizon model, the issue of commitment does not play a role. The only problem is for normal agents to separate themselves from the cheaters.

just quoted and the work presented in this paper. Else, the modelling here follows Tadelis 1998 in adopting the concept of track record as name and an overlapping generations structure.

The above are the papers closest to the work presented here but there are, of course, other (but not many) papers that bear on the issues this paper deals with. There is Kreps 1990 that reinterprets norm equilibria in infinitely repeated games in terms of name trading. Salant 1991 studies norm equilibria in overlapping generations set-ups. Further, there is Aoyagi 1996 who studies how firms' sales under asymmetric information affects incentives to behave aggressively in an infinitely repeated entry deterrence game. In this work, though, what is sold is the firm rather than just its reputation.

2 The Model

It is useful to think of the game as consisting of a sequence of rounds each made up of two stages: A products-sale stage and a names-sale stage. In the product sales stage, a continuum of two-period lived sellers is matched with a continuum of one-period lived buyers, each of unit measure. Each buyer-seller pair proceeds then to play the following extensive form stage-game:

A price is exogenously set equal to the max of the expected value of the good in the eyes of the buyer and the cost of producing a low quality unit. The buyer decides whether to purchase the item or not. If the buyer makes a purchase and the seller is a rational type, the seller has to decide whether to produce high or low quality. If the seller is an automaton, he invariably supplies high quality. Producing an item of high quality costs c_H, while producing one item of low quality costs c_L. Of course, $c_H > c_L$. The remuneration to the seller in this stage is then the price minus the cost. The remuneration to the buyer is her reservation value for the item of the quality supplied minus the price. Evidently, $v_H > v_L$. The relationship between all these parameters is given by $v_H > c_H > c_L > v_L$. This implies that buyers would never knowingly acquire a low quality product at a price that covers its cost of production. In other words, it is not efficient to supply low quality in this economy.

This product sale stage is then followed by a name-selling stage along the following lines: There is a name market for each possible track record with the exception of the empty history. Sellers after having supplied buyers in the preceding product sale stage, decide whether and which name market to enter. The suppliers of names are then matched randomly with the demanders of names within each market. In each match a nonnegative price will be set according to some exogenous rule (which is common knowledge among all players in the game and which might or might not depend on the type of the name-buyer involved), and then the buyer has to decide whether to buy or not. If a purchase takes place then the name is transferred and a new product sale stage starts. The remuneration to a buyer from this stage will be negative and equal to the purchase price, while that to the seller will simply be the price.

In order to exclude some equilibrium outcomes that hinge on very extreme specifications of out of equilibrium beliefs, it will be assumed that there is always an $\varepsilon > 0$ subsample of each generation (with the same composition as the overall sample) that cannot participate in the names' market.

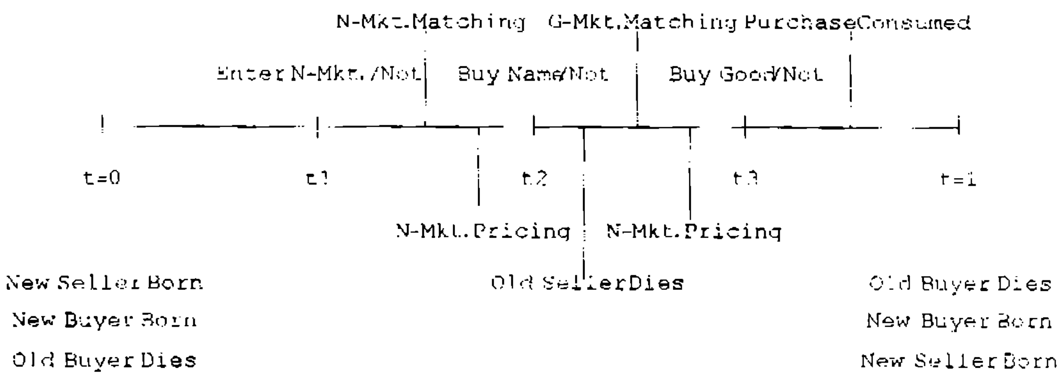
The remuneration of a 2-period lived seller will then be the discounted sum of the payoffs in each stage of the game, where the discounting will take place only across product sale stages. That is, a period for discounting purposes will include the product-sale stage and the subsequent name trading session.

The demographic structure of the economy is given by some pattern of overlapping generations of two-period lived sellers, starting from some initial constellation. The overlap will be somewhat peculiar in order to allow newborns to trade with exiting agents, with non-final agents living for slightly more than two periods in order to allow for this. More precisely, each period will be subdivided in four stages with names' market activity taking place in the first two rounds, product purchases in the third round while quality decisions are taken in the fourth round. So, each non-final seller (i.e., sellers who get to make a name-sale) lives for 8 rounds, with the last two rounds dedicated to name trading. See the time line of the game below.

Each generation will include the same proportion of rationals and honests or automatons. The initial constellation will vary and might involve an exceptional one period lived generation of sellers as in Tadelis. Also, various exogenously set participation patterns in names-markets will be studied.

The information in the economy is as follows: Buyers will be aware of the proportion of honests and automata in each generation but will not be able to tell directly whether a seller is of one or the other type, neither will they be able to tell whether a good is of good or low quality prior to actually consuming it. They will be able to see only the track record or name carried by each seller. In the name trading session, sellers will sometimes be assumed to be able to recognize each others' types.

The following diagram illustrates the time line of the game:



A word about pricing: Taking pricing as exogenous is, on the one hand, unsatisfactory, as it obviously contributes to blur the equilibrium predictions of the model. On the other hand, though, it allows one to focus on the conditions for trade in names more generally, that is, independently of particular bargaining procedures. Also, it considerably simplifies the analysis.

Another key modelling choice is to work with continua of agents in each generation. This leads to deterministic outcomes (as a result of a -casual- application of the law of large numbers).

2.1 Equilibrium and Notation

The solution concept will be the notion of sequential equilibrium (Kreps and Wilson 1982). Let b_o be the prior probability that a seller of any generation is honest. If H denotes a high quality sale while L denotes a low quality one, let N denote the set of all track records or names, i.e., the set of all sequences $(H, L)^t$ with $t \leq T$ where T is the number of periods in the game (remember that 'period' here refers to discounting periods, not rounds of play). Further, let $G_t(\tau)((H))$ designate the generation of sellers born at period t of type $\tau \in (A, R)$ (where A stands for automaton or honest, and R for rational or dishonest), and who bear the name (H). The measure of sellers in generation $G_{t'}$ of type τ who supply high quality at period t is $\lambda_t(G_{t'}(\tau), H)$. Similarly, $\lambda_t(G_{t'}(\tau), (H))$ refers to the measure of sellers of type τ in generation $G_{t'}$ who carry name (H) at time t; $\lambda_t^{EB}(G_{t'}(\tau),(H))$ designates the measure of sellers in generation $G_{t'}$ of type τ who enter the market for (H)-names at time t on the demand side; $\lambda_t^B(G_{t'}(\tau),(H))$ denotes the measure of sellers in that generation who actually buy a name (H); while $\lambda_t^{ES}(G_{t'}(\tau),(H))$ and $\lambda_t^S(G_{t'}(\tau),(H))$ denote the corresponding magnitudes on the supply side. The probability a buyer assigns to the event that a seller bearing the name (H) is an automaton is given by $\Pr_t(A|(H))$, and $p_t(\Pr_t(A|(H)))$ stands for the price of a good sold by a seller bearing name (H) at time t. The price of a name (H) at time t is expressed by $p_t^N((H))$. It is also important to keep track of the set of all names available for trade at any given time t, N_t, letting n_t^i designate an element of that set, with $i = 1, ..., \#(N_t)$.

The strategy of a buyer alive at time t is simply a mapping from the history up to that point (in particular, price offers and the name of the seller the buyer was matched with) to actions (B, NB), where B stands for 'buy', NB for 'not buy'. Formally,

$$\sigma_{t3}^B : H_{t3} \rightarrow \Delta\ (B, NB)$$

2.2 Strategies in a Benchmark Case

In order to efficiently describe sellers' strategies, I think it best to specialize and then to modify the formulation as alternative scenarios are considered. Accordingly, in what follows I describe the strategies for a benchmark case in which sellers can only sell names at the end of their lives and buy them when they are born, and in which names are traded at the same prices independently of the type of the seller-demander. Moreover, it will be useful to describe strategies for a 'normal' two-period lived seller (a seller who starts and closes his or her life by entering the names' market), exceptional 'initial' two- or one-period lived sellers (who do not start out trading in names), and exceptional 'final' two- or one period sellers (who do not trade names at the end of their lives), as they will come up in the various demographic constellations analyzed below.

The strategy of an agent is then a sequence of mappings , one for each round of play in which he or she has the move, from the history of play up to that point[4] to mixtures over the set of actions available in that round.

[4]The formulation of strategies below implies that agents know everything that has

Hence the strategy of a normal two-period lived rational seller will consist of six mappings denoted by $\sigma_{kr}^{N2_t}$, where kr stands for a date-round. In round 1, the seller has to decide whether to enter the names' market as a demander, and which names' market out of all those open, so this mapping is given by

$$\sigma_{t1}^{N2_t} : H_{t1} \rightarrow \Delta(EB\left(n_{t1}^1\right), ..., EB\left(n_{t1}^{\#(N_{t1})}\right), NE)$$

where $EB\left(n_{t1}^1\right)$, for example, stands for the decision to enter as a demander the market for names n_{t1}^1, and H_{t1} stands for the history preceding time $t1$.

In the second round, if he entered a names' market, that seller has to decide whether to buy the name offered to him, in case there is a positive match. This latter eventuality will be designated by M, with no match denoted by NM.

$$\sigma_{t2}^{N2_t} : H_{t2}^{M/EB(N2_t)} \rightarrow \Delta\left(B, NB\right)$$

where $H_{t2}^{M/EB(N2_t)}$ stands for all histories up to time-round $t2$ such that a seller $N2_t$ has entered a names market and has been matched with a seller offering a name for sale.

In the fourth stage of a normal seller's life, the product sale takes place, and the corresponding mapping is given by

$$\sigma_{t4}^{N2_t}(R) : H_{t4}^{B_{t3}(N2_t)} \rightarrow \Delta\left(H, L\right)$$

with $H_{t4}^{B_{t3}(N2_t)}$ denoting the set of all histories such that the buyer matched with a seller $N2_t$ has made a purchase at date-round $t3$.

The fourth mapping is

$$\sigma_{(t+1)4}^{N2_t}(R) : H_{(t+1)4}^{B_{(t+1)4}(N2_t)} \rightarrow \Delta\ (H, L)$$

The fifth mapping is somewhat different as now the seller enters a market as a supplier,

$$\sigma_{(t+2)1}^{2N_t} : H_{(t+2)1}^{B(N2_t)} \rightarrow \Delta\left(ES(n_{(t+2)1}(N2_t)), NE\right)$$

happened in the economy. Note that in this type of model, with each generation being made up of a continuum of agents, this assumption is not restrictive. One could just as well assume that agents only know what they themselves have experienced directly, and the results would not be affected. This because, first, agents will never care about the exact match outcomes (who exactly was matched with whom) since all agents of a certain type and generation must behave identically; and, second, because agents will always be able to infer exactly the actual course of the game from equilibrium, even if it should include mixed strategies, as the law of large number (casually applied) will leave no doubt about aggregate outcomes.

where $H^{B(N2_t)}_{(t+2)1}$ refers to the set of all histories such that a this seller has made a sale.

Finally, the sixth and last round mapping is given by

$$\sigma^{N2_t}_{(t+2)2} : H^{B/M/ES(N2_t)}_{(t+2)2} \rightarrow \triangle (S, NS)$$

where S stands for 'sell', NS for 'not sell', and $H^{B/M/ES(N2_t)}_{(t+2)2}$ describes the set of all histories such that this seller has made a sale, entered the market for names as a seller and been matched positively.

The representation of a strategy for a normal two period lived automaton is basically the same, except that the mappings corresponding to quality decisions fall off.

The strategy of an initial two-period lived seller of generation $G^{I2}_{t_0}$ is again a sequence of six mappings as below with the relevant experience in the first such mapping given by the price offer and the action undertaken by the buyer with whom the seller was matched when young:

$$\sigma^{I2}_{t_0 4}(R) \quad : \quad H^{B_{t_0 3}(I2)}_{t_0 4} \rightarrow \triangle (H, L)$$

$$\sigma^{I2}_{(t_0+1)4}(R) \quad : \quad H^{B_{(t_0+1)3}(I2)}_{(t_0+1)4} \rightarrow \triangle (H, L)$$

$$\sigma^{I2}_{(t_0+2)1} \qquad H^{B(I2)}_{(t_0+2)1} \rightarrow \triangle [ES \left(n_{(t_0+2)1}(I2)\right), NE]$$

$$\sigma^{I2}_{(t_0+2)2} \quad : \quad H^{B/M/ES(I2)}_{(t_0+2)2} \rightarrow \triangle (S, NS)$$

The representation of a strategy for an initial two period lived automaton is basically the same, except that the first and second mappings fall off as the automaton always provides high quality.

The formulation of the strategies for the other types mentioned (one period lived initial sellers and one period lived final sellers) is similar to that of the strategies just described and I shall not be writing them out in detail.

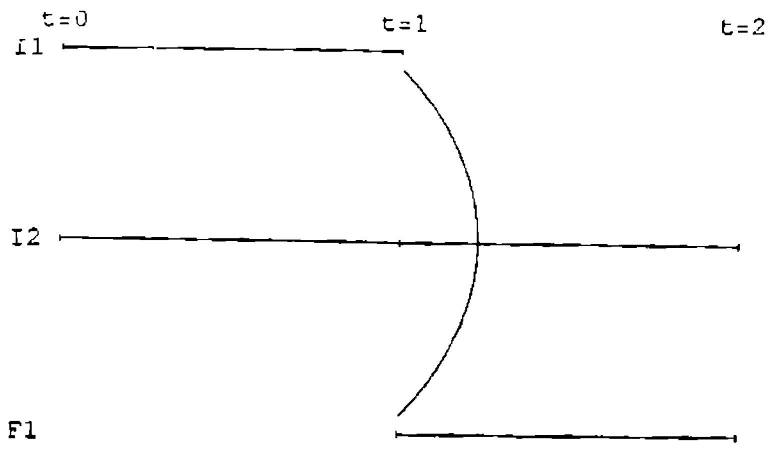
In order to be able to define formally a sequential equilibrium for this game, I introduce a probability space Ω whose elements correspond to a particular realization of nature moves and a complete history $h_T \in H_T \subseteq H$ (the set of all terminal histories as a subset of the set of all histories). Then a strategy profile induces a probability distribution P over Ω. A beliefs' system of buyers is then a mapping $\phi_B : H_B \rightarrow \triangle\Gamma$, where $\triangle\Gamma$ denotes the set of all probability distributions over the space $N\times\ (A, R)$,with N the space of all names, and H_B denotes the set of all histories after which a buyer has the move. It should be that for any history $h \in H_B$ occurring with positive probability, the posterior belief that a seller with a given name n be of type τ is given by Bayes' Rule, i.e., $\phi_B(h)(\tau,n) = P(\tau,n|h)$ whenever $P(h) > 0$. The consistency requirement in the definition of a sequential equilibrium also imposes the restriction that $\phi_B(h.s)(\tau,n) = 0$ whenever $\phi_B(h)(\tau,n) = 0$ (where $h.s$ stands for the concatenation of a continuation history and its pre-history). Moreover, names longer than the length of the history (more precisely, the number of sale episodes in a given history) must clearly be assigned 0 probability [5]. Finally, let $V_{tr}\left(n_t^j\left(G_{t'}(\tau)\right), h_t\left(G_{t'}(\tau)\right); \sigma, \phi_B|\tau\right)$ express the value of a name $n_t^j \in N_t$ as round r at date t after history h_t, being held by a member of generation t' of type τ, given a profile of strategies σ, and a system of beliefs of buyers ϕ_B.

In order to define an equilibrium precisely the exact configuration of the population of sellers must be specified, so I go straight into consideration of the first environment studied in this paper, namely the basic case in Tadelis 1998, p.12.

3 A Best Scenario for Name Trading

The diagram below illustrates the demographic composition of the population:

[5] In principle, one could specify a system of beliefs for sellers as well concerning the types of fellow sellers with whom they are matched in the names' market. However, with exogenous pricing such beliefs are not relevant.



There is one initial one-period lived, one initial two-period lived and one final normal one-period lived generation. Moreover, only olds are allowed to sell names, while only new-borns are allowed to buy them. In other words, the middle-aged do not take part in the names' market. In order to complete the specification of the economy, one must define the pricing rule in the market for names. It will be assumed that name-buyers are charged a proportion α of the value of a given name to an automaton demander, i.e., in terms of the notation above[6],

$$p_{t2}^{N}(n) = \alpha[V_{t2}\left(n, h_{t2}^{EB/M/B}; \sigma, \phi_B | A\right) - V_{t2}\left(\{\emptyset\}, h_{t2}^{EB/M/NB}; \sigma, \phi_B | A\right)]$$

An equilibrium of this game can then be defined as follows:

Definition 1 *A sequential equilibrium of this game is a profile of strategies σ and a system of beliefs for buyers ϕ_B such that:*

$$\sigma_{t_04}^{I2}(R)\left(h_{t_04}^{B_{t_0 3}(I2)}\right) \in \arg \max_{\beta_{t_04}^{I2} \in \Delta(H,L)} \sum_{a \in (H,L)} \beta_{t_04}^{I2}(a)\left\{[p_{t_03}(b_0) - c_a] + \right.$$

[6]Note that it does not make a difference if instead of A one writes R, as the surplus is the same for either type. In fact, in order for there to be trade in names it must be that

$$\begin{aligned} p_{t2}^{N}(n) \leq \; & V_{t2}\left(n, h_{t2}^{EB/M/B}; \sigma, \phi_B | \tau\right) - \\ & V_{t2}\left(\{\emptyset\}, h_{t2}^{EB/M/NB}; \sigma, \phi_B | \tau\right) \end{aligned}$$

$$\beta V^{I2}_{t_04}\left((a), h^{B_{t_0 3}(I2)}_{t_04}.a; \sigma, \phi_B|R\right)\}$$

$$for\ all\ h^{B_{t_0 3}(I2)}_{t_04}$$

$$\sigma^{I2}_{(t_0+1)4}(R)\left(h^{B_{(t_0+1)3}(I2)}_{(t_0+1)4}\right) \in \arg\max_{\beta^{I2}_{(t_0+1)4} \subset \Delta(H,L)} \sum_{a\in(H,L)} \beta^{I2}_{(t_0+1)4}(a)$$

$$[p_{(t_0+1)3}\left(\phi_B\left(h^{B_{(t_0+1)3}(I2)}_{(t_0+1)4}\right)A|n^{I2}_{t_0+1}\right) - c_a]$$

$$for\ all\ h^{B_{(t_0+1)3}(I2)}_{(t_0+1)4}$$

$$\sigma^{I1}_{t_04}(R)\left(h^{B_{t_0 3}(I1)}_{t_04}\right) \in \arg\max_{\beta^{I1}_{t_04} \in \Delta(H,L)} \sum_{a\in(H,L)} \beta^{I1}_{t_04}(a)\{[p_{t_03}(b_0) - c_a] +$$

$$V^{I1}_{t_04}((a), h^{B_{t_0 3}(I1)}_{t_04}.a, ; \sigma, \phi_B|R)\}$$

$$for\ all\ h^{B_{t_0 3}(I1)}_{t_04}$$

$$\sigma^{I1}_{(t_0+1)1}(\tau)\left(h^{B(I1)}_{(t_0+1)1}\right) \in \arg\max_{\beta^{I1}_{(t_0+1)1} \in \Delta[ES\left(n^{I1}_{(t_0+1)1}\right), NE]} \{\beta^{I1}_{(t_0+1)1}(NE)$$

$$V^{I1}_{(t_0+1)1}\left(n^{I1}_{(t_0+1)1}, h^{B(I1)}_{(t_0+1)1}.NE; \sigma, \phi_B|\tau\right)$$

$$+\beta^{I1}_{(t_0+1)1}\left(ES\left(n^{I1}_{(t_0+1)1}\right)\right)\max[\frac{\lambda^{EB}_{(t_0+1)1}\left(n^{I1}_{(t_0+1)1}\right)}{\lambda^{ES}_{(t_0+1)1}\left(n^{I1}_{(t_0+1)1}\right)}, 1]$$

$$V^{I1}_{(t_0+1)1}\left(n^{I1}_{(t_0+1)1}, h^{M/B(I1)}_{(t_0+1)1}.ES\left(n^{I1}_{(t_0+1)1}\right); \sigma, \phi_B|\tau\right)$$

$$for\ all\ h^{B(I1)}_{(t_0+1)1}$$

$$\sigma^{I1}_{(t_0+1)2}\left(h^{B/M/ES(I1)}_{(t_0+1)2}\right) \in \arg\max_{\beta^{I1}_{(t_0+1)2}\in\Delta[S,NS]} \beta^{I1}_{(t_0+1)2}(S)\, p^{N}_{(t_0+1)2}\left(n^{I1}_{(t_0+1)1}\right)$$

$$for\ all\ h^{B(I1)}_{(t_0+1)1}$$

$$\sigma^{F1}_{(t_0+1)1}(\tau)\left(h_{(t_0+1)1}\right) \in \arg\max_{\beta^{F1}_{(t_0+1)1}\subset\Delta[\underline{EB},NE]}$$

$$\sum_{a\in\underline{EB}}\{\beta^{F1}_{(t_0+1)1}(a)\,[\max(\frac{\lambda^{ES}_{(t_0+1)1}(a)}{\lambda^{EB}_{(t_0+1)1}(a)},1)V^{F1}_{(t_0+1)1}(\{\emptyset\},h^{M}_{(t_0+1)1}.a;\sigma,\phi_B|\tau)$$

$$+\left(1-\max(\frac{\lambda^{ES}_{(t_0+1)1}(a)}{\lambda^{EB}_{(t_0+1)1}(a)},1)\right)V^{F1}_{(t_0+1)1}(\{\emptyset\},h^{NM}_{(t_0+1)1}.a;\sigma,\phi_B|\tau)]\}$$

$$+\beta^{F1}_{(t_0+1)1}(NE)\,V^{F1}_{(t_0+1)1}(\{\emptyset\},h_{(t_0+1)1}.NE;\sigma,\phi_B|\tau)$$

$$for\ all\ h_{t_0+1}$$

$$\left(EB\left(n^{1}_{(t_0+1)1}\right),...,EB\left(n^{\#\left(N_{(t_0+1)1}(\sigma)\right)}_{(t_0+1)1}\right)\right)\equiv\underline{EB}$$

$$\sigma^{F1}_{(t_0+1)2}(\tau)\left(h^{M/EB(F1)}_{(t_0+1)2}\right) \in \arg\max_{\beta^{F1}_{(t_0+1)2}\in\Delta[B,NB]}\{\beta^{F1}_{(t_0+1)2}(B)\,[(-p^{N}_{(t_0+1)2}\left(EB^{-1}\right)+$$

$$V^{F1}_{(t_0+1)2}(EB^{-1},h^{EB(F1)}_{(t_0+1)2}.B;\sigma,\phi_B|\tau)]+$$

$$\beta^{F1}_{(t_0+1)2}(NB)\,V^{F1}_{(t_0+1)2}(\{\emptyset\},h^{EB(F1)}_{(t_0+1)2}.NB;\sigma,\phi_B|\tau)]\}$$

$$for\ all\ h^{M/EB(F1)}_{(t_0+1)2}$$

$$\sigma^{F1}_{(t_0+1)4}(R)\left(h^{B_{(t_0+1)3}(F1)}_{(t_0+1)4}\right) \in \arg\max_{\beta^{F1}_{(t_0+1)4}\in\Delta(H,L)}\sum_{a\in(H,L)}\beta^{F1}_{(t_0+1)4}(a)$$

$$[p_{(t_0+1)3}\left(\phi_B\left(h^{B_{(t_0+1)3}(F1)}_{(t_0+1)4}\right)\left(A|n^{F1}_{(t_0+1)3}\right)\right)-c_a]$$

$$for\ all\ h^{B_{(t_0+1)3}(F1)}_{(t_0+1)4}$$

This environment is most favorable in three respects: First, it is just not feasible for a rational seller to cheat and then avoid the consequences by buying a name. Second, 'blending' is built into names' trading (that is the role of the two-period lived initial generation). Third, the only participants on the demand side in the market for names being new-borns favors trade for names in as far as new borns can only get a 'good' name by buying it (middle-aged participants would have the option of building their own 'good' name instead of buying it).

The first proposition shows that even in this scenario, which can be considered the most favorable to improving (product) trade outcomes via trade in names, there is no equilibrium in pure strategies that leads to such improvement.

A caveat: Because of the continuum of agents feature, any mixed strategy equilibrium is equivalent to a pure strategy equilibrium in which appropriate proportions of each population behave in a certain way. Hence, when it is said that there is no pure strategy equilibrium what is meant is that there is no equilibrium which leads to an improved outcome when sellers' quality choices represent strictly preferred options.

Proposition 2 *There is no equilibrium in pure strategies in which sellers' quality decisions represent strictly preferred choices that leads to an improvement in product trade outcomes (relative to a situation where there are no names' markets but there is reputation building).*

Proof. Showing that there is no strict pure strategies equilibrium with

$$\lambda_{t_0 4}(I1, H) = 1$$

$$\lambda_{t_0 4}(I2, H) = 1$$

suffices to prove the claim, since the assumption of a continuum of agents of each kind forces all players of a certain category and type to behave in the same way in any equilibrium -if the decision is strict; and, clearly, with or without trade in names, it must be that

$$\lambda_{(t_0+1)4}(I2(R), L) = 1 - b_0$$

$$\lambda_{(t_0+1)4}(F1(R), L) = 1 - b_0$$

Now, in order for $\sigma_{t_0 4}^{I1}(R)\left(h_{t_0 4}^{B_{t_0 3}(I1)}\right)(H) = 1$ to be a best response it must be that

$$\max(\frac{\lambda_{(t_0+1)1}^{EB}(H)}{\lambda_{(t_0+1)1}^{ES}(H)}, 1)p_{(t_0+1)1}^{N}(H) \geq c_H - c_L$$

By the hypothesis of strict quality choices, this inequality must be strict. It then follows that all $I1$'s enter the names' market, so that $\lambda_{(t_0+1)1}^{ES}(H) = 1$.

The previous inequality implies that $p_{(t_0+1)1}^{N}(H)$ must be positive . In order for this to be the case, if all $F1$'s enter the names market, it must be that

$$pr_{(t_0+1)3}(A|(H)) > b_0$$

The above inequality must be satisfied since there is always an ε subsample of the final generation that cannot participate in the names market, and buyers must belief in any equilibrium that a seller bearing a $\{\emptyset\}$-name after the last round of name trading must be an automaton with probability b_0.But this cannot be if all enter, as then

$$pr_{(t_0+1)3}(A|(H)) = b_0$$

It must be then that a subsample of $F1$'s including both rationals and automatons, and such that the proportion of automatons is strictly higher than in the original $F1$-pool, enters the names' market, i.e., it must be that

$$\frac{\lambda_{(t_0+1)1}^{EB}(F1(A),(H))}{\lambda_{(t_0+1)1}^{EB}(F1(R),(H))} > \frac{b_0}{1-b_0}$$

In other words, some $F1$'s of each type must not enter the names market, i.e., given the continuum assumption, those $F1$'s must be indifferent between entering and not entering. But it is just not possible to make both types indifferent: If automatons are indifferent, then rationals strictly prefer to enter, and if rationals are indifferent, then automatons strictly prefer not to enter (and in equilibrium it cannot be that only automatons stay out, for then it pays for rationals to deviate and remain outside the market). Hence, the result follows.

■

First thing to note about this result is that it does not depend on how exactly the surplus is divided. In other words, it does not depend on the exact pricing rule. It just follows from the fact that under these conditions a name can only generate a surplus if a special pattern of entry into the market is implemented, a task that turns out to be impossible. Also note that the key to the impossibility of implementing this specific entry pattern is the absence of the right 'separating structure' between automatons and rationals, unlike what happened in the adverse selection model of Tadelis 1998 or in the moral hazard model with noisy outcomes of Mailath and Samuelson 1998.

What kind of strict pure strategies equilibria exist (if any)? The following proposition provides an answer:

Proposition 3 *If in the absence of trade in names, there would have been reputation building, there is a unique strict equilibrium in pure strategies involving no trade in names.*

Proof. That such an equilibrium exists follows obviously from the hypothesis that there is reputation building in the absence of trade in names.

The only other candidate equilibrium of this kind (besides the one considered in the preceding proposition) has all $I2(R)$'s providing low quality in the first period, while all $I1(R)$'s provide high quality. If there are any holders of $(\emptyset)$-names after the names' market closes, again the argument in the proof of the preceding proposition applies: It must be that, first, $F1$'s of both types are holding such names, and, second, both types must be indifferent between entering and not. This is impossible.

If there are no holders of $(\emptyset)$-names, i.e., all $F1$'s entered the names' market, it must be that

$$pr(A|(H)) > b_0$$

This implies however that it cannot be a best response for $I2(R)$'s to choose to supply low quality in the first period, given the hypothesis that in the absence of names' trading there would have been reputation building. ■

An immediate corollary is the following:

Corollary 4 *If there would not have been reputation building in the absence of trade in names, such trade can give rise to an equilibrium in which all $I2(R)$'s provide low quality in the first period, while all $I1(R)$'s provide high quality.*

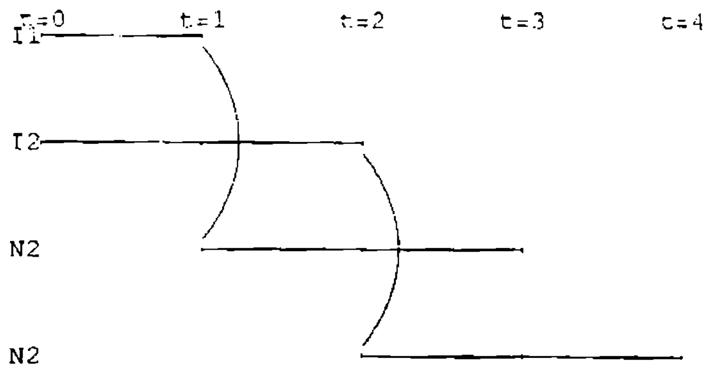
The basic logic underlying the previous result, which hinges on the absence of the 'right' separating forces between rationals and automatons, also provides the key to understanding the nature of mixed strategy equilibria with name trading:

Proposition 5 *If there is reputation building in the absence of name trading, there are no mixed equilibria in which trade in names leads to an improvement in trade outcomes.*

Proof. The argument proceeds by considering two cases: The case where all $I1\,(R)$'s supply high quality, and that in which only some supply high quality. The first case is easily taken care of: If all $I1\,(R)$'s provide high quality, then by exactly the same argument as in proposition 2, trade in names cannot happen. This since that argument applies regardless of whether all $I2\,(R)$'s are providing low quality or just a fraction.

If not all $I1\,(R)$'s provide high quality, then there could be rationing in the names' market. The question is whether this would make any difference, and the answer is no: If not all $F1$'s enter the market, the 'impossibility of simultaneous indifference' still applies. If all enter, then, since matching in the market is random and there is a continuum of agents on both sides, the composition of those matched must correspond to the composition of those not matched, and, hence, to that of the entering population. It follows that, as before, it is better for $I2\,(R)$'s to supply high quality. ■

An interesting question is whether, even if reputation building was feasible in the absence of name trading, name trading can lead to an improvement in trade outcomes as the length of the game increases. There is one case where it is obvious it cannot: If buyers can only observe the last period performance in a sellers' current track record, i.e., if names are of length one. On the other hand, even if buyers can observe longer track records, but there is no trade in names in the first round, there cannot be trade in names thereafter. The following diagram illustrates:



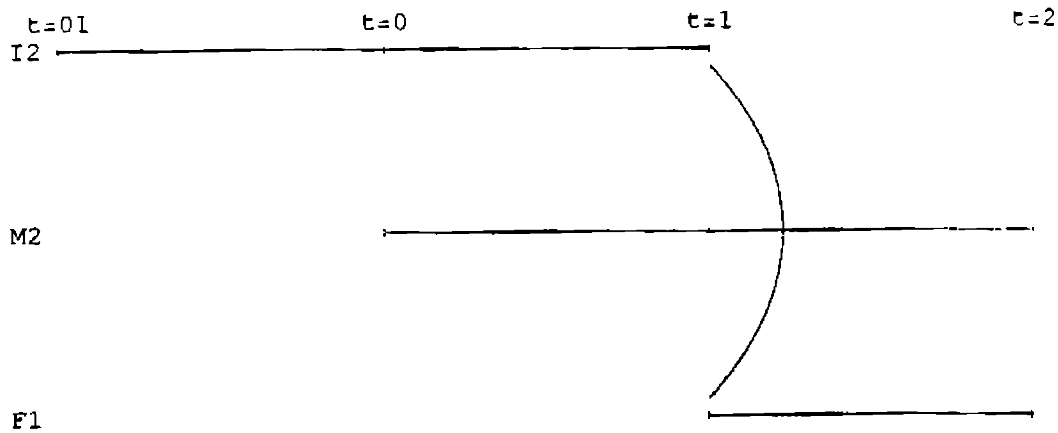
Since there is no trade in names in the first round, in the second round there is no 'blending' in the sense that buyers will know that anyone bearing a name of length two after $t = 2$, must have bought it. As will be shown below, modulo a reasonable refinement, under such circumstances there cannot be trade in names at positive prices.

The interesting case is then that in which there is no reputation building to start with. Under such circumstances, it appears possible that trade in names might deliver an overall improvement in trade outcomes. What is clear, though, is that mixed equilibria (if they exist) will never improve trade outcomes beyond what can be achieved in the 'pure' equilibrium, i.e., that the total measure of rationals of any generation supplying high quality can never exceed unity. This because trade in names will always tend to decrease the value of reputation investments for $I2\,(R)$'s due to the 'devaluing' or 'diluting' effect such trade invariably has on the informativeness of a 'good' name. In fact, this last result carries over literally to games with longer horizons: At most a unit measure of rationals will provide high quality, regardless of the length of the game. This follows straightforwardly from the previously argued fact that there will not be trade in names after the second period.

To summarize this section: While name trading can lead to improved trade-outcomes, this will only happen in the case where reputation is not operating to start with. This result is remarkably negative considering that this scenario represents a best case one for trade in names, as will become apparent in what follows.

4 No Trade in Names in the Absence of 'Blending'

In this section, I consider the scenario illustrated by the following diagram:



The purpose is to show how in the absence of 'blending' there cannot be trade in names at positive prices. As shown, instead of there being a one period lived initial generation besides a two period lived one, as in the setup considered in the previous section, there is only one two period lived initial generation. Otherwise, the environment is as before: Only exiting agents are allowed to sell names, and only new-borns are allowed to purchase them.

Proposition 6 *In this economy there is then no trade in names at positive prices.*

Proof. Take any two-period name. In the second and last round of product trading, any buyer confronted with a buyer bearing such a name will know that it must have been bought. Consequently, this buyer's belief that the seller bearing such a name is an automaton will be given by the ratio

$$\frac{\lambda^{EB}_{(t_0+2)1}\left(F1\left(A\right),\left(H\right)\right)}{\lambda^{EB}_{(t_0+2)1}\left(F1\left(R\right),\left(H\right)\right)}$$

Since all $F1$'s must enter (if any enter)- by the impossibility of making automatons and rationals simultaneously indifferent, this ratio must be equal to the ratio in the original population. Now, since there is always an ε subsample of the final generation that cannot participate in the names market, buyers must belief in any equilibrium that a seller bearing a $\{\emptyset\}$-name after the last round of name trading must be an automaton with probability b_0. This immediately implies that the price of the two period name under consideration must be zero, for otherwise, it would be best not to enter the market. ■

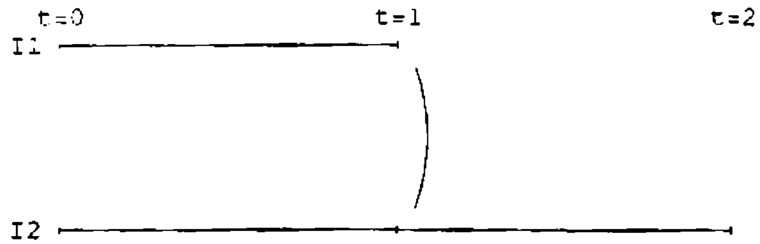
This still allows for trade in names at zero prices. A tiny cost of entering the market would suffice to get rid of those outcome as well. Note that, analogously to what happened in the previous section, this result is robust to lengthening the game[7].

This latter feature reflects what appears to be a general feature of this type of models: The importance of initial conditions. If one thinks instead in steady state terms, this kind of problem disappears. The robustness to the game horizon is being generated in both instances by the fact that, if there is no trade in names to start with, there cannot be trade in names afterwards. In steady state mode one can just start from the premise that there has always been trade in names, and, by this device, generate 'blending' in each period. Or to put it in somewhat more technical terms, it would seem that in this type of models name-trading steady states are often not stable.

5 Erasing One's Tracks

In this section an environment is considered that in a way represent a worst case world for trading names in the presence of 'blending'.

[7] It might be objected that, in a more flexible model, sellers could induce 'blending' by not selling in the first period. This objection really concerns the extent to which an agent's track record is observable to buyers. In certain environments, the above objection could be met by expanding the observable track record of an agent to include all of his actions at each stage of his or her life (instead of just having the observable track record include solely certain type of decisions, as is done here).



As the diagram above shows, there is an initial one-period lived generation and one initial two- period lived generation overlapping in the first period of their lives.. This allows for 'blending', but it also allows for a strategy of 'erasing one's tracks', that is, of cheating in the first period, then purchasing a 'good' name as one reaches middle age in order to trade again in one's old age (a strategy that was not feasible in the setups studied above). Moreover, it gives potential name buyers (i.e., $I2$'s) the possibility of 'constructing' their own good name as an 'alternative' to purchasing one; a possibility new-born buyers of names did not have. This will exclude honest $I2$'s as buyers of good names (at positive prices).

The question this section tries to answer is whether name trading is feasible under these circumstances, and whether, if feasible, it might worsen trade-outcomes instead of improving them. Since now the exact price level at which names are traded will play a very important role, in order to get definite results I will impose the additional restriction that all surplus in names' trades should go to the short side of the market. Also, in order to get rid of a rather artificial multiplicity of equilibria resulting from irrelevant indifferences characteristic of environments in which middle aged sellers are allowed to participate in the names' market, I will introduce an $\varepsilon-$cost of trading in names.

Proposition 7 *If in the absence of name trading reputations emerged, then with names' markets there will also be such an equilibrium.*

Under the additional restriction that excess demand leads to all surplus going to the sellers, while excess supply leads to the opposite result, the only *equilibria involving 'erasing of tracks' that are robust to an $\varepsilon-$cost of entering the names' market require balance in that market. These equilibria arise if $b_0 \leq \frac{1}{2}$. In all equilibria the trade outcome worsens.*

Proof. The first statement follows straightforwardly since entry into the name market is simultaneous and it cannot pay to enter if no one on the opposite side enters.

As it is clear that in the presence of $\varepsilon-$cost of entering the names' market, no $I2$ holding an $(H)-$name will enter that market, the question is really what kind of equilibria involving $I2\,(R)$ supplying low quality and then purchasing a good names are there, if any. The two relevant conditions in this respect are given by,

$$
\begin{aligned}
c_H - c_L &\lesseqgtr \min(\frac{\lambda^{EB}_{(t_0+1)1}(H)}{\lambda^{ES}_{(t_0+1)1}(H)}, 1)p^N((H)) \\
c_H - c_L &\lesseqgtr \min(\frac{\lambda^{ES}_{(t_0+1)1}(H)}{\lambda^{EB}_{(t_0+1)1}(H)}, 1)p^N((H)) + \\
&\left[1 - \min(\frac{\lambda^{ES}_{(t_0+1)1}(H)}{\lambda^{EB}_{(t_0+1)1}(H)}, 1)\right]\beta\left(p\left(pr\left(A|\left(H\right)\right)\right) - c_L\right)
\end{aligned}
$$

If there 'erasing of tracks' and there is excess supply of names it must be that

$$c_H - c_L \geq p^N((H)) \tag{1}$$

in order for $I2\,(R)$'s to 'erase their tracks' (i.e., supply low quality in the first period, and then purchase a good name). If the inequality is strict, then in such an equilibrium is must be that

$$
\begin{aligned}
\lambda^S((H)) &= b_0 \\
\lambda^B((H)) &= \lambda^B(I2\,(R), (H)) = 1 - b_0
\end{aligned}
$$

Hence, if $b_0 > 1 - b_0$ (i.e., if $b_0 > \frac{1}{2}$), there will be excess supply of names, and their price will tend to be low. In fact, it would be reasonable to assume that it is 0 as there is excess supply (in this model this price is only bounded above by $\beta\left(p\left(pr\left(A|\left(H\right)\right)\right) - c_L\right)$, and, in this equilibrium, $pr\left(A|\left(H\right)\right) = b_0$). If there are residual costs of entering the name market, this class of equilibria would disappear.

If condition 1 should hold with (almost) equality, then there is indifference on both the supply side and the demand side of the names' market so long as there is either excess supply or balance. In the case of excess supply, as said, it seems reasonable to assume a 0 price for names, but this would contradict the condition. In the case of market balance, at any strictly positive price for names, it must be that $\lambda^{S}((H)) \geq b_0$, while the demand for names is always bounded above by $1 - b_0$. So, it follows that market balance with a strictly positive price (as implied by equality in condition 1) can only happen if $b_0 \leq \frac{1}{2}$. There is a continuum of such equilibria, but in all of them the total measure of rationals of either initial generation supplying high quality is $1 - 2b_0$, which is less than the total measure of rationals supplying high quality in the first period ($= 1$) in the absence of trade in names (by the way note that $pr(A|(H)) = b_0$ in any of these equilibria).

On equilibria with excess demand: The relevant conditions are

$$\begin{aligned} c_H - c_L &\lesseqgtr p^N((H)) \\ c_H - c_L &\lesseqgtr \frac{\lambda^{ES}_{(t_0+1)1}(H)}{\lambda^{EB}_{(t_0+1)1}(H)} p^N((H)) + \\ &\quad \left[1 - \frac{\lambda^{ES}_{(t_0+1)1}(H)}{\lambda^{EB}_{(t_0+1)1}(H)}\right] \beta\left(p\left(pr(A|(H))\right) - c_L\right) \end{aligned}$$

Under the additional restriction on pricing postulated, they both reduce to

$$c_H - c_L \lesseqgtr p^N((H))$$

The first case considered has

$$c_H - c_L > p^N((H)) > 0$$

This implies that $\lambda^{ES}((H)) = b_0$. In order for there to be excess demand it must be that $b_0 < \frac{1}{2}$ as $\lambda^{EB}((H)) \leq 1 - b_0$. Moreover, $\lambda^{EB}((H)) = 1 - b_0$. This implies that

$$pr(A|(H)) = \frac{1}{2} > b_0$$

but then

$$\beta\left(p\left(\frac{1}{2}\right) - c_L\right) > c_H - c_L$$

as by the hypothesis of reputation building

$$\beta \left(p \left(b_0 \right) - c_L \right) \geq c_H - c_L$$

Hence, there cannot be an excess demand equilibrium of this kind.

If instead

$$p^N \left((H) \right) = 0$$

then it can be

$$1 - b_0 < \lambda^{ES} \left((H) \right) \leq b_0$$

This since again we must have $\lambda^{EB} \left((H) \right) = 1 - b_0$. Moreover, $b_0 > \frac{1}{2}$. Now,

$$b_0 > pr \left(A | \left(H \right) \right) = \frac{b_0}{\lambda^{ES} \left((H) \right) + b_0} > \frac{1}{2}$$

The first inequality follows since $\lambda^{ES} \left((H) \right) > 1 - b_0$,while the second follows since $\lambda^{ES} \left((H) \right) \leq b_0$. Now, $p^N \left((H) \right) = 0$ iff $p \left(pr \left(A | \left(H \right) \right) \right) < c_H$ (this follows from the requirement that the individual rationality constraint of automatons be satisfied and the pricing rule chosen). This requires

$$p \left(\frac{1}{2} \right) < c_H$$

Even if one assumes that this is so, the additional specification of an $\varepsilon-$cost of entering the names' market destroys this equilibrium. In any case, trade outcomes are clearly worse than in the situation without names' markets.

Finally, consider the case with

$$c_H - c_L = p^N \left((H) \right)$$

Again this implies that $b_0 < \frac{1}{2}$ as $\lambda^{ES} \left((H) \right) \geq b_0$. Also, $\lambda^{EB} \left((H) \right) \leq 1 - b_0$, so that

$$pr \left(A | \left(H \right) \right) = \frac{b_0}{\lambda^{ES} \left((H) \right) + b_0} > b_0$$

where the inequality follows since $\lambda^{ES} \left((H) \right) \leq 1 - b_0$. But by hypothesis of reputation building,

$$\begin{aligned} c_H - c_L &\geq \beta \left(p \left(b_0 \right) - c_L \right) \Rightarrow \\ c_H - c_L &> \beta \left(p \left(pr \left(A | \left(H \right) \right) \right) - c_L \right) \end{aligned}$$

contradicting the condition we started with. ∎

Note that under the additional pricing restriction, if $I2\,(R)$'s indulge in 'erasing their tracks', then of necessity no $I1\,(R)$ will have an incentive in equilibrium to supply high quality. This is the key to absence of gains from name trading in this environment, as it neutralizes the one potential source of gains (the incentives of $I1\,(R)$'s to supply high quality).

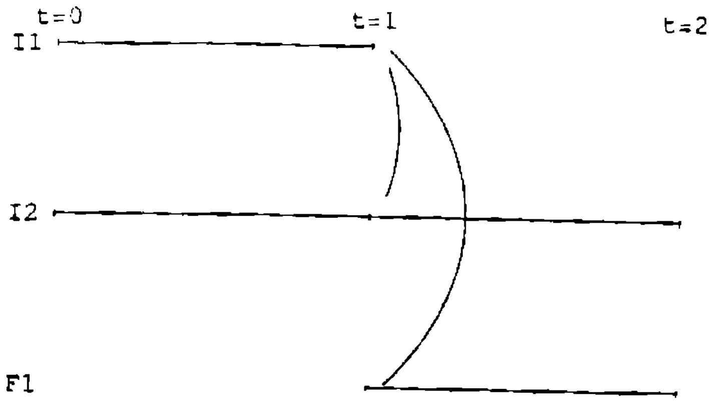
An immediate corollary of the previous proposition is

Corollary 8 *If there is no reputation building to start with, there cannot be 'erasing of tracks' when a market for names is opened.*

Also note that the reputational equilibrium with no trade in names might break down in the presence of names' markets if one would assign more initiative to the participants in that market (for example, allow $I2$ sellers to enter the market and make a price offer to potential $I1$ name sellers).

Of course, the equilibria identified above are rather fragile constructions that rely heavily on indifference conditions, at least under market structures that deliver the pricing behavior postulated. On the other hand, I would conjecture that the fragility of these equilibria might disappear in markets with more elastic demand and supply behavior, one which responds to changes in prices. After all, the mere existence of excess supply or demand in a market need not lead to full surplus transfers, but rather to balance at some intermediate positive price.

As sort of a first step in that direction, in as far as some heterogeneity is introduced on the demand side, consider the following economy:



This is the case considered in the opening section except that now middle-aged sellers are allowed to buy names.

Take the case where there would have been sales in the absence of names' markets. This is a weaker condition than the one postulating reputation building in the absence of names' transactions. It just requires

$$p(b_0) - c_H > 0$$

The pricing rule has to be modified to read as follows: If there is excess demand all surplus of the marginal buyer, i.e., the buyer with the smallest surplus, goes to the short side. If there is excess supply, price of a name still goes to zero.

Proposition 9 *Under the above conditions, there are only two types of equilibria: Equilibria in which $F1$'s do not participate in the names' market, there is market balance, and $I2(R)$'s 'erase their tracks'; and one equilibrium where there is no trade in names.*

Proof. That there is an equilibrium with no trade in names follows trivially. To show that the only other equilibria are those described above, start by noting that there cannot possibly be equilibria with trade in names if there is excess supply, as this would imply that the price of names is zero (now there is an $\varepsilon-$cost of entering the names' market). Hence the only possibility for an equilibrium not involving market balance -but name trading by $F1$'s-is for there to be excess demand. In fact, in any equilibrium in which $F1$'s participate, there must be excess demand if $I1$'s also participate, since if any $F1$ enters the names' market, all must (for exactly the same reasons as in the proof of Proposition ?). Participation in names' trading by $I2$'s implies that these agents must have supplied low quality, as otherwise it can never be best to enter the market (again, due to the $\varepsilon-$cost of entering). So, I concentrate on the case of excess demand with $F1$ participation in name trading. If $F1$'s participate, it must be that

$$pr\left(A|\left(H\right)\right) > b_0$$

since entering the market is costly and the refinement used guarantees that $pr\left(A|\left(\emptyset\right)\right) = b_0$. But if $I2\left(R\right)$'s are 'erasing their tracks' and there is random matching in the names market, this can never be, for $I2\left(R\right)$'s will always 'taint' the pool of entrants (due to random matching the composition of the pool of entrants is the same as that of the pool of final holders of (H)-names). Finally, note that the equilibrium with market balance and 'track erasing' described in the previous proposition implies always

$$pr\left(A|\left(H\right)\right) = b_0$$

so that, in fact, $F1$'s do not have an incentive to enter the market for names. ■

There is one feature about this game that deserves to be highlighted, namely the fact that the new pricing rule breaks the inverse relationship between incentives of $I1\left(R\right)$'s to sell good names and those of $I2\left(R\right)$'s to 'erase their tracks'. Under excess demand, if participation by $F1$'s could be sustained in equilibrium, it would be possible for $I1\left(R\right)$'s to supply good names while $I2\left(R\right)$'s 'erase their tracks'. From the basic formulas, it is clear that there is a link between this type of incentives and pricing. The above feature highlights an additional link between heterogeneity of demand for names and those incentives (the previous pricing rule being a special case of the one used here for the case of homogenous demand).

Finally, note that these results are again robust to lengthening the horizon of the economy. If only middle aged are allowed to buy names, then after the first round there won't be blending, and, hence, no trade in names. If also new-borns are allowed to buy names, then there can, in principle, be blending after the first period, but, as the last proposition shows, there won't be trade in names in the first period, and so no 'blending' in the second, and no further trade in names.

6 Conclusions

Obviously, there are plenty of variations which have not been explored here (e.g., to mention only one that seems particularly interesting, what if names can be sold by the middle aged and not only the old?). But, at least in the scenarios explored here, what is striking is the apparent difficulty of getting names' markets to operate in this type of environment, and moreover, the impossibility of improving trade outcomes by trading names. And this quite independently of the particular pricing behavior one might choose to work with.

In fact, the results of the last section suggest that name trading can easily be counterproductive (though the precarious nature of the equilibria identified in that section makes one reluctant to actually claim this). Of course, the ad-hoc nature of pricing considered here (pricing being crucial for this type of behavior, unlike for the other issues studied in this paper) can only add to that reluctance.

Also, some of the issues identified here would seem to carry over to pure adverse selection environments, namely, the need for 'blending' to get active trade in names, the instability of steady state outcomes, and the importance of initial conditions.

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