CIDE

NÚMERO 154

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PRICES AND REPUTATION

Resumen

Este trabajo estudia el rol de los precios en los mercados reputacionales. Presenta una versión modificada del modelo de Milgrom y Roberts 1982, que sustituye un juego de señalización en cada episodio por el juego de entrada diferida de los autores mencionados. El análisis demuestra que los precios no pueden funcionar como mecanismos de compromiso en este entorno. Además se caracteriza la dinámica de los precios en el (cuasi) único equilibrio secuencial del juego.

Abstract

This paper is concerned with the role or price signals in trading processes that rely on reputations to enforce quality. It presents a modified version of the model in Milgrom and Roberts 1982 that substitutes a non-standard price singnaling stage game for the entry-derretence game those authors worked with. The paper shows that the 'bonding' intuition of other related models does not carry over to this environment. Moreover, it characterizes the price dynamics associated with the (almost) unique equilibrium of the game.

1 Introduction

Most consumer goods are traded in 'reputational' markets, that is, in markets where the quality of the item being traded, while known to the seller, cannot be costlessly ascertained prior to consumption by the buyer. In such markets buyers have an incentive to rely on the seller's fear of loosing repeat business to ensure delivery of the desired quality. In other words, they might rely on a seller's concern about his 'reputation' to ensure the quality of their purchase.

This paper is concerned, generally, with role of prices in this type of trade process and, more particularly, with characterizing the resulting price dynamics.

Prices represent a form of 'pre-trade communication', and might conceivably help in persuading a sceptical buyer to trade. In fact, the literature that models reputations as a norm in an infinitely repeated trade has shown how a particular intertemporal pricing pattern can operate as a quality commitment device under these circumstances -a line of argument often referred to as 'bonding' (see Klein and Leffler[6]). Even if prices should play only a passive role (as will turn out to be the case in this model), I think it is important to characterize price paths associated with the formation of reputations. This because prices are directly and easily observable, unlike beliefs or information flows, and bringing them into the picture gives us a way of tracking the development of reputations empirically.

The major literature strand dealing with the issue of reputation, the one originating in the contributions of Kreps and Wilson[8] and Milgrom and Roberts[10], has, as far as I am aware, pretty much ignored these issues, and concentrated instead on analyzing entry-deterrence stories in which there are no prices.

The present paper aims to make a start in filling this gap by modifying the model presented in Milgrom and Roberts[10], substituting at each stage a 'price signalling' game for the entry deterrence one that those authors worked with. This 'price signalling' game is not quite a signalling game in the usual sense as it lacks the 'single crossing' structure that those games normally have.

Consequently, all the equilibria of the game studied in this paper will be pooling equilibria and, so, the present work will have nothing to say on the interesting question of to what extent can prices serve to separate types (for contributions of that sort, see Milgrom and Roberts[12] and Hertzendorf[5])[1].

Actually, there is a sense in which reputation and separation (but not 'bonding') are opposites: If separation is possible to start with, then there is no role for reputation in the sense of Milgrom and Roberts[10] and Kreps and Wilson[8]. Moreover, characterizing the price dynamic is immediate: Potential cheaters will no sell, while 'honest' sellers will sell at the highest possible price (under take-it-or-leave-it price offers by the seller).

On the other hand, I think the exercise in this paper does throw light on the question of whether something analogous to the 'bonding' story of infinitely repeated games can emerge in this alternative set-up. The answer is negative. The equilibrium displays the recursive structure of the equilibrium in Milgrom and Roberts[10], and, hence, will not allow for the intertemporal linkage of decisions implicit in the 'bonding' reasoning.

The main contribution of this paper is to make a start in characterizing the time paths of prices in such a 'reputational' environment.

Here again the comparison with work concerned with separation is enlightening: The issue of separation is often studied in models of pure adverse selection (Milgrom and Roberts[12] and Hertzendorf[5] work with such pure models)[2], while the model in this paper is a moral hazard/adverse selection hybrid with a finite horizon. The dynamics of these two types of models are quite different: In a pure adverse selection environment, price dynamics will be driven by learning alone and will be, hence, totally backward looking. In the type of model I deal with here, besides this backward looking force (which here tends to raise prices over time as sales continue only if high quality is provided), there is a forward looking force originating in the inability of sellers to commit to supply high quality in the last period of the game (which tends to lower prices as the end of the game is approached). In fact, this paper will mainly be concerned with these two forces and their net effect on the evolution of prices.

Perhaps the most interesting result in this regard is the monotonicity of the path in all but the initial stage of the game: It is shown that prices must fall from the second period on, though they might rise initially depending on parameters. Note that this implies that the dynamics from the first to the second period are qualitatively different. This last feature, by the way, does

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[1] I thank a referee for these references.

[2] Though neither of those two contributions are concerned with characterizing the evolution of prices over time. They concentrate on the pricing decision in the first period.

not disappear as the horizon goes to infinity.

This price pattern is rather surprising on various counts: First, the possible non-monotonicity at the start runs counter to the expectation that prices will track reputations, and hence, fall throughout. The intuition behind this potential non-monotonicity is that, while increasing incentives to cheat as the end of the game is approached will tend to lower prices, backward looking learning will tend to raise prices as time goes on (continuing high quality sales are cumulative evidence that the seller is honest). In this light, what is really remarkable, and in my view, not intuitive, is that from the second period on the former force should prevail over the latter. Actually, this behavior is easily explained far away from the game's conclusion: In such a region, potential cheaters will not be cheating, and, hence, high quality sales will not be informative, so that only the forward looking force mentioned will operate. The puzzling feature is that prices should continue to fall even as the end of the game is very close, when high quality sales become informative (as fewer and fewer rationals will be planning to supply high quality).

Less unexpected is the asymptotic behavior of prices: As the horizon extends to infinity, prices will remain constant at their highest possible level (after the initial period), only to fall from that level towards the end, as just pointed out.

Paper Overview In the next section, the game is outlined and the solution concept used is discussed. The paper then characterizes the equilibria of the stage game. After this, the two period case is analyzed to illustrate why prices need not always fall. In the next section, a more general formulation along the lines of Milgrom and Roberts[10] is developed, and the asymptotic behavior of prices is characterized. Finally, existence and uniqueness are discussed, and conclusions are drawn.

2 The Game

In the stage game a T-period lived seller confronts a one period lived buyer (who shares information across generations). The seller can produce either high (H) or low (L) quality at a unit cost (disutility per unit) of c_H and c_L, respectively, with $c_H > c_L$. The buyer is endowed with v_H units of a non-produced good ('money'), which is assumed to enter linearly both buyers' and sellers' objectives. Buyers have unit demands for the good produced by the seller, with reservation values v_H for a high quality unit and v_L for a low quality unit. The following inequality relates buyers' reservation values and sellers' unit costs of production, $v_H > c_H > c_L > v_L$. Note that buyers will only pay a price above costs if they expect to be supplied with a high quality unit. Buyers will be assumed not to be able to tell apart a high quality unit from a low quality one ex-ante, that is, before consuming the good. The seller might be of one of two basic types, Honest or Rational. The honest seller will always supply high quality (that is, as long as it is individually rational to do so)[3]; the rational one might or might not, depending on the circumstances. Buyers assess prior probability δ that the seller is rational.

In order to avoid indeterminacy along the path of play[4] (as opposed to the familiar multiplicity that results from the freedom to choose out of equilibrium beliefs, which I shall handle via a refinement -more on this below), I will assume that a rational seller can be of one of a continuum of sub-types each with a different unit cost of producing the high quality good. These unit costs are assumed to range from c_L to c_H. More precisely, let each such rational type be indexed by $s \in [0, 1]$, and let $c_H(s) : [0, 1] \to [c_L, c_H]$ be a strictly decreasing, continuous function[5].

In the stage game, the seller moves first offering to supply the buyer with one unit of the produced good in exchange for a certain amount of money. The buyer accepts or rejects the offer. If the buyer accepts, the buyer pays the price and the seller chooses (if he is rational) whether to produce high or low quality, after which he delivers the good to the buyer. The buyer consumes, and by doing so, finds out if the good supplied to him was of high or low quality. This signalling game is depicted in the diagram below:

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[3]Note that the seller will always be able to make sure the buyer rejects an offer by demanding a sufficiently high price. Hence, in order to keep things simple, I will not explicitly allow sellers to refuse to trade.

[4]The exact nature of the multiplicity I am referring to here is the following: In the two-period case, for initial beliefs above $\overline{\delta}$, sellers have to mix between providing high and low quality (for similar reasons as in Kreps and Wilson[7]), but the mixture is not unique. This because now it is possible to make buyers indifferent between buying or not (a necessary condition in order for sellers to be indifferent between supplying high or low quality) for a whole range of values of posterior beliefs by setting the price equal to the expected value of a purchase given those beliefs.

This results, for any given initial beliefs, in a continuum of equilibria indexed by the probability that the seller provides high quality.

[5]This modification is in the same class as those introduced by Milgrom and Roberts[11] into the Kreps and Wilson framework in order to generate unique pure strategy equilibria.

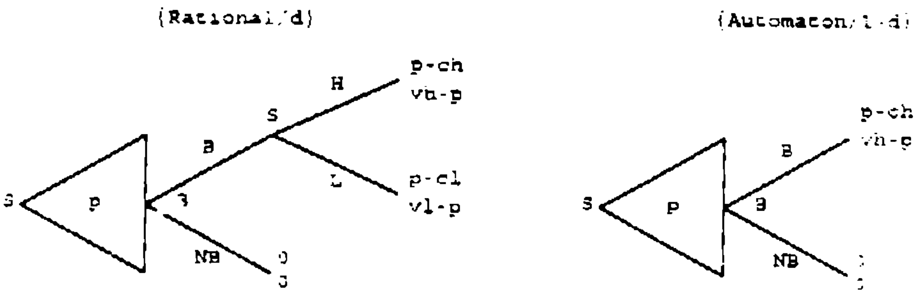

Fig. 1: Stage Game

2.0.1 Solution Concept

The solution concept I shall use is the notion of sequential equilibrium (Kreps and Wilson 1982b). In order to deal with the multiplicity of equilibria resulting from the freedom to choose out of equilibrium beliefs, it is only natural to introduce a refinement. Unfortunately, the sui generis nature of this 'signalling' game makes the conventional refinements of little use (i.e., Cho and Kreps[3] intuitive criterium and extensions thereof)[6]. Instead I use an argument in the spirit of the 'divinity' refinement introduced by Banks and Sobel[1]. The idea is the following: Since whenever an honest type gains from deviating, it is also profitable for a rational type to deviate, it seems reasonable to require that buyers stick to their prior beliefs when confronted with a deviation[7]. In a similar vein, if the deviation could not possibly lead to a gain for either type then beliefs remain unchanged.

In a repeated signalling game this refinement amounts to the statement that, under whatever 'theory' a deviator of a given type entertains concerning

[6] All selling equilibria satisfy the intuitive criterium. To see this, note that all price deviations downwards are not profitable, regardless of sellers' type and of beliefs held by buyers. Price deviations upwards (to a price at or below v_H) could be profitable if buyers faced with such an offer conclude that the deviator is honest. In other words, condition 1 in Cho[2] (p.1373) is not satisfied for either type here.

Cho's forward induction criterium represents, for signalling games, a strenghtening of the intuitive criterium, so it cannot be of help either (for there are no BAD deviations here). See Cho[2].

[7] In the notation of Cho and Kreps[3] I require that whenever
$D_A \cup D_A^o = D_R \cup D_R^o$, then $\mu(A, m') / \mu(R, m') = \pi(A) / \pi(R)$.

the further development of the game after the deviation, if he expects to gain from deviating, so should the other type

3 Equilibrium

In characterizing the equilibrium, I will proceed first by looking at the equilibrium of the stage game, then at the equilibrium in the two-period case, and, finally, at the arbitrary horizon case (including the infinite horizon scenario). Moreover, I will first characterize the form a selling equilibrium would take if it existed, and only afterward will existence and uniqueness be discussed. I will focus attention on equilibria involving sales, which will turn out to be unique and pooling such that a sale takes place every period until the game is over or low quality is supplied. Separating equilibria do exist, but no sales take place in them.

The reason for looking at the two-period case separately is, in part, to emphasize the qualitatively distinct behavior exhibited in the initial period of any interaction. In part, to anchor the discussion of existence (more specifically, the discussion concerning the range of initial beliefs for which a selling equilibrium exists).

3.1 Stage Game Equilibrium

The equilibria of the stage game are illustrated in the diagram below:

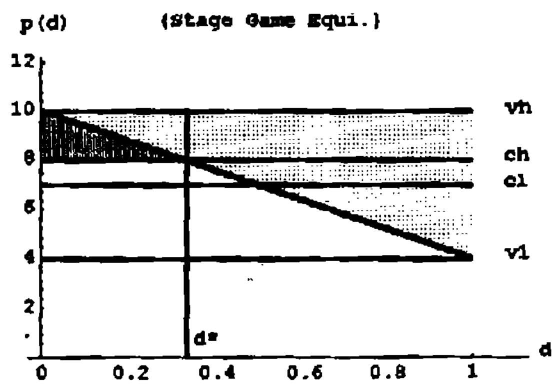

Fig. 2: Equilibria of Stage Game

Clearly, in a one-shot situation, rational types will never supply high quality. The only equilibria involving a sale are pooling equilibria in the region $\left[0, \bar{\delta}\right]$, with prices in the shaded area marked (I). The reason for this is straightforward: In the complementary region, any price at or below the diagonal line representing the expected value of the good to a buyer lies below the cost of supplying high quality. Hence, a seller who demands such a price will be expected to supply low quality, and, consequently, no sales will take place.

It is easy to see that, under the refinement, there is in the original stage game a unique selling price configuration for each value of δ in the region $\left[0, \bar{\delta}\right]$, given by $p_A^E = p_R^E = p(\delta)$, where $p(\delta)$ stands for the expected value of the good given initial beliefs δ; p_A^E for the price charged by the honest type (or automaton), and p_R^E for the price charged by the rational type.

The introduction of a continuum of rational types in the above fashion has no effect on the refined equilibrium of the one-stage game. In the two (or more) periods case, though, it will be shown to induce a unique path of actual sales' prices.

4 Characterizing the Equilibrium for the Two-Period Case

I start by introducing some additional notation: First of all, note that I will be counting time backwards: Period t will precede period $t-1$. Let T represent the horizon of the game (here $T = 2$), and let δ_2 represent the initial beliefs of the buyer (that is, the probability that the buyer alive at $t = 2$ assigns to the event that the seller is rational). Let $\widetilde{p}_t$ be the price charged at period t. $\delta_1(H)$ will designate the posterior probability assessment that the seller is rational, given that high quality was supplied at $t = 2$. Of course, $\delta_1(L) = 1$. Finally, $\rho_2(H)$ denote the share of rational types aiming to supply high quality at $t = 2$, $\widetilde{p}_t(\delta)$ refers to the price charged at t as a function of initial beliefs, and $\delta_1(\delta)$ refers to the posterior value generated in the equilibrium corresponding to initial beliefs δ. The following proposition describes the unique (along the equilibrium path) sale equilibrium of the game as a function of initial beliefs:

Proposition 1 *The following beliefs and strategies form a sequential equilibrium of the two-period game. The equilibrium outcome is unique under the refinement introduced in the previous section.*

For all $\delta_2 \leq \min\left[1, \widetilde{\delta}, \widehat{\delta}, \overline{\overline{\delta}}\right]$, *with* $\widehat{\delta}$ *given by the solution to* $\delta_1(\delta) = \overline{\delta}$ *(which is equivalent to* $\widetilde{p}_1(\delta) = c_H$*);* $\widetilde{\delta}$ *given by the solution to* $\widetilde{p}_2(\delta) = c_H - \beta(\widetilde{p}_1(\delta) - c_H)$, *and* $\overline{\overline{\delta}}$ *is given by the solution to* $\widetilde{p}_2(\delta) = c_L$:

a) Beliefs:

Price deviations at any time leave beliefs unchanged; $\delta_1(L) = 1$ *always; beliefs after purchasing a high quality unit are given by*

$$\delta_1(H) = \frac{\rho_2(H)\,\delta_2}{\rho_2(H)\,\delta_2 + (1-\delta_2)} \tag{1}$$

b) Strategies for sellers:

i) Pricing:

At $t = 2$, *both, rational and honest types charge*

$$\widetilde{p}_2 = \delta_2\left[\rho_2(H)\,v_H + (1-\rho_2(H))\,v_L\right] + (1-\delta_2)\,v_H \tag{2}$$

At $t = 1$, *after supplying high quality, a seller charges*

$$\widetilde{p}_1 = \delta_1(H)\,v_L + (1-\delta_1(H))\,v_H \tag{3}$$

At $t = 1$, *a rational seller who supplied low quality charges any price above* v_L.

ii) Qualities:

At $t = 2$, *there exists a rational subtype indexed by* s^* *such that all subtypes with* $s < s^*$ *provide low quality, while the rest provides high quality, with* s^* *given by the fixed point of* (1), (2), (3) *and* (4) *below.*

$$\rho_2(H) = 1 - c_H^{-1}\left[\beta(\widetilde{p}_1 - c_L) + c_L\right] = 1 - s^*. \tag{4}$$

At $t = 1$, *rational types always supply low quality.*

c) Strategies for Buyers:

At $t = 2$, for $\delta_2 \leq \min\left[1, \widetilde{\delta}, \widehat{\delta}, \overline{\overline{\delta}}\right]$, buy only if

$$\widetilde{p}_2 \leq \delta_2\left[\rho_2(H)\, v_H + (1 - \rho_2(H))\, v_L\right] + (1 - \delta_2)\, v_H$$

At $t = 1$, for $\delta_1 \leq \overline{\delta}$, buy only if high quality was supplied the previous period and

$$\widetilde{p}_1 \leq \delta_1(H)\, v_L + (1 - \delta_1(H))\, v_H$$

Else, don't buy except if $\widetilde{p}_1 \leq v_L$.

In the region $[\min\left[1, \widetilde{\delta}, \widehat{\delta}, \overline{\overline{\delta}}\right], 1]$, there are no equilibria involving a sale, but there are a multiplicity of no-sale equilibria, which I will not describe in detail.

Proof. See Appendix.

Proposition 2 *A necessary and sufficient condition for the existence of selling equilibria (potential sales in both periods), for initial beliefs $\delta_2 \in \left[0, \min\left(1, \widetilde{\delta}, \widehat{\delta}, \overline{\overline{\delta}}\right)\right]$, is*

$$c_H(1) - c_L < \beta(v_H - c_L)$$

Proof. Rewrite equation (1-4) above as follows

$$[(c_H(s) - c_L) - \beta(v_L - c_L)]\, \delta_2(1 - s) + (1 - \delta_2)(c_H(s) - c_L)$$

$$= \beta(v_H - c_L)(1 - \delta_2)$$

If one sets $s = 1$, the left-hand side expression will be everywhere falling in $\delta_2 \in [0, 1]$, starting at $c_H(1) - c_L$ and ending at 0. If now one sets $s = 0$, this same expression will be rising everywhere in $\delta_2 \in [0, 1]$, starting at $c_H(0) - c_L$ $(> c_H(1) - c_L)$. For all $s \in (0, 1)$, the expression will take values in the area enclosed by the two schedules just outlined. So, if $c_H(1) - c_L > \beta(v_H - c_L)$, the only point of intersection between the schedule defined by the right-hand side expression and the family of schedules defined by the left-hand side expression will be at $\delta_2 = 1$ and $s = 1$ (see diagram below), in which case no buyer will be prepared to pay a price above $v_L (< c_L)$. On the other hand, it is easy to see that $\widetilde{p}_2$ cannot be below c_L in equilibrium.

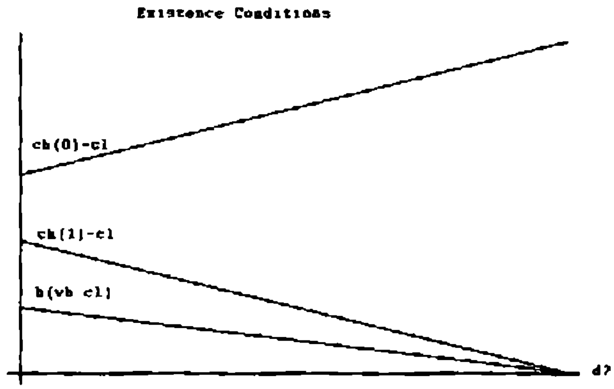

Fig. 3: Existence of Fix Point

If $c_H(1) - c_L = \beta\,(v_H - c_L)$, any candidate equilibrium will have $s = 1$. But in an equilibrium it cannot be that all rational types supply low quality: In such scenario, it would pay for a rational type to deviate and supply high quality as this would convince the buyer that he is honest. (Only mixed strategies could solve this problem, but here it is not possible to make a positive measure of rationals indifferent). In conclusion: If the condition of the proposition is violated only no-sale equilibria can exist. ■

4.1 Characterizing Price Paths

4.1.1 An Example of Falling Prices

The diagram below illustrates a parametric example, with $c_H\,(s) = sc_L + (1-s)\,c_H$ and $v_H = 10; c_H = 8; c_L = 7; v_L = 4$, and $b = \frac{1}{2}$:

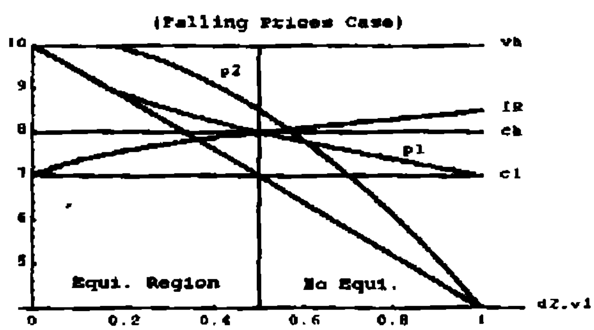

Figure 4a: Falling Prices

In the figure the schedule IR plots the individual rationality constraint for the honest seller,

$$c_H - \beta\left(\widetilde{p}_1(\delta) - c_H\right)$$

(if individual rationality of honest sellers is satisfied, so must be individual rationality of rationals). The x-axis represents initial beliefs, i.e., δ_2. The vertical line corresponds to the min $\left[1, \widetilde{\delta}, \widehat{\delta}, \overline{\overline{\delta}}\right]$. Note that in this example, min $\left[1, \widetilde{\delta}, \widehat{\delta}, \overline{\overline{\delta}}\right] = \widehat{\delta}$, and prices are falling over time.

4.1.2 An Example of Rising Prices

With all parameter values as above except for the discount factor (here $\frac{1}{3}$), the situation depicted in the diagram below arises:

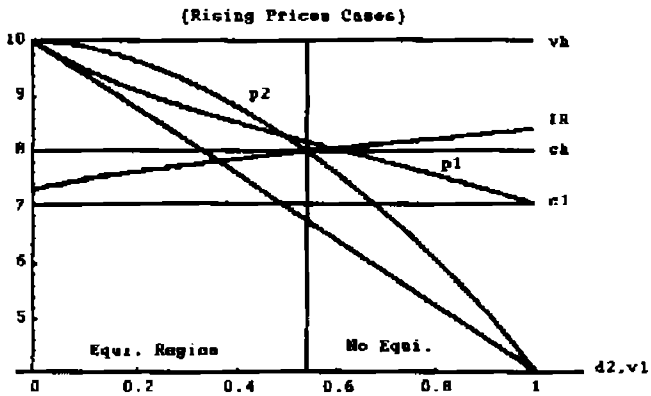

Fig. 4b: Rising Prices

Here again, IR stands for individual rationality (of the honest seller). Note that in a neighborhood to the left of the vertical line (denoting the min $\left[1, \widetilde{\delta}, \widehat{\delta}, \overline{\overline{\delta}}\right] = \widetilde{\delta}$), the $\widetilde{p}_2$-schedule is below the $\widetilde{p}_1$-schedule, i.e., prices rise. (At the vertical line itself, $\widetilde{p}_2$ is actually below c_H, meaning that losses are incurred that period by honest sellers. Also, the IR schedule intersects the $\widetilde{p}_2$-schedule at exactly the vertical line -by construction).

4.1.3 Interpretation

One way of looking at this is just to concentrate on the formal structure of this two-period equilibrium, and note that the decision to supply high quality today is totally independent of the price prevailing today, and that, hence, there is no reason why the price today should be linked in any particular way to the price tomorrow (of course, individual rationality will link both prices but so long as today's price covers the cost of producing high quality it will not require that prices rise or fall over time).

More intuitively, one can note that there two 'forces' operating in opposite directions on prices over time: The first stems from the fact that as the end of the game is approached, incentives for rational types to supply high quality fall,. i.e., the share of rationals who sell high quality falls. This tends to lower the price. On the other hand, every time high quality is supplied, buyers become more 'bullish' on the likelihood of the seller being honest, which tends to raise the price. There is no reason in this two period case why one of these forces should systematically prevail, and so it should not be mysterious that the price tomorrow might end up exceeding the price today.

Note that in the example presented, prices rise over time only for sufficiently pessimistic beliefs. This seems intuitive as well. The stronger the 'reputation' effect, i.e., the higher the measure of rational sellers aiming to supply high quality in equilibrium, the higher the expected value of a purchase at $t = 2$. On the other hand, the higher the measure of rational sellers intending to supply high, the less informative a high quality purchase (for given priors), and, hence, the lower the price obtainable in the last period. As priors worsen, in order to induce rational types to sell high quality, it becomes necessary to lower the measure of rational sellers so as to make a high quality purchase highly informative about the seller's type. This lowers the expected value of the good today, while simultaneously making the price tomorrow correspondingly higher.

Finally, one should make an important caveat regarding the interpretation of the results in this two-period case: It will be shown that it is wrong to presume that these results apply in an end-phase of a longer game, even if the equilibrium of the longer game can be shown to be recursive in beliefs. A formal argument for this is given in Corollary 6, section 5.2.2, and some intuition in the discussion that follows that corollary. Here I just would like to emphasize that the intuition offered in this section in no way contradicts the results obtained in the arbitrary horizon case: The key is to think of the two period analysis not as illustrating the behavior in the last two periods of a longer game, but rather the behavior in the first two periods. The discussion following Corollary 6 elaborates on this.

4.1.4 The 'Bonding' Issue

This two period example suffices to illustrate why the 'bonding' does not apply in this type of set-up, as, evidently, the price charged at $t = 2$ is totally irrelevant in inducing rational sellers to supply high quality at that date[8]. Mathematically, this expresses itself in the fact that one can solve for all the other endogenous variables in the system $(\tilde{p}_1, \rho_2(H), \delta_1(H))$, without using the equation generating $\tilde{p}_2$. In other words, it is the recursivity of the equilibrium, i.e., the fact that actions at any time depend only on the value of the beliefs held at that time, plus the fact that these beliefs depend only on the actual quality provided the previous period, and not in any way on the price charged at that time, that accounts for the absence of 'bonding'.

The nearest this model seems to get to the 'bonding' intuition is when $\tilde{p}_2$ dips below c_H, as in one of the diagrams above. The reason for that temporary loss being, as already pointed out, that in order to make a high quality sale sufficiently informative, it might be necessary for relatively few rational types to supply high quality. This then feeds back into $\tilde{p}_2$. Note that it is not the losses per se that matter, but the 'informativeness' of the signal. The losses incurred at $t = 2$ are merely incidental. What remains true is that in order to satisfy individual rationality, if losses are incurred today, the seller of high quality must be hoping to make a sale tomorrow (though I want to emphasize again that the reverse does not apply). Finally, note in particular that the observation of a price below c_H cannot in any way be taken as a 'signal' that high quality will be provided, for sellers of low quality will not incur losses in equilibrium.

5 Asymptotic Behavior

In order to study longer horizons, I switch to a formalization of the equilibrium along the lines of Milgrom and Roberts[9], in which the 'state variable' at time t is given by the maximum of all the critical index values separating at each previous stage the rational players willing to supply high quality from those not prepared to do so (rather than using buyers' beliefs regarding the likelihood that the seller they are confronting is honest, as I did in the previous section). This magnitude will express at any given time the 'reputation' of a seller given the history of play.

[8] This property is robust to longer horizons.

5.1 Formal Definition of Equilibrium

In order to express more formally the conditions defining a sequential equilibrium for the game being studied here, I introduce some notation: H_n denotes the history of moves up to the start of stage n (not including any action taken in that stage), while H_n^1 refers to the history of moves up to stage n including the price and the purchase decision at that stage. A plus sign superscript, H_n^+, denotes that high quality was supplied in the previous stage (a minus sign denoting low quality). An additional tilde superscript, H_n', denotes the history up to stage n including the price offer made at that stage, and an o superscript, H_n°, denotes that no sale took place in the preceding stage. The set of all possible histories up to a given stage is then designated by calligraphics, $\mathcal{H}$.

A (refined) sequential equilibrium is then given by a strategy for each firm such that:

1) For all $n = 1, ..., N$, and for all $H \in \mathcal{H}_n'$,

$$b_n(H, p_n) = \begin{cases} 1 \quad (BUY) \qquad if \quad \rho_n(H) v_H + [1 - \rho_n(H) v_L] \geq p_n \\ 0 \quad (NOT\ BUY)\ else \end{cases}$$

2) For all $n = 1, ..., N$, for all $t_r \in [0, 1]$ (where t_r stands for the rational sub-type),

a) for all $H \in \mathcal{H}_n$,

$$\begin{aligned} s_n^{r0}(H_n, t_r) &= p_n \\ if \qquad \beta V_{n-1}(t_r, H^\circ) &< \max\{p_n - c_H(t_r) + \beta V_{n-1}(t_r, H^+), \\ & \qquad p_n - c_L + \beta V_{n-1}(t, H^-)\} \\ & \qquad and \\ \beta \widetilde{V}_{n-1}(\omega, H^\circ) &< p_n - c_H + \beta \widetilde{V}_{n-1}(\omega, H^+) \end{aligned}$$

$$\textit{where } p_n = \rho_n(H)\, v_H + [1 - \rho_n(H)\, v_L]$$

$$\textit{else} \qquad s_n^{r0}\left(H^-, t_r\right) = v_H + \sigma \quad \textit{where} \quad \sigma \in (0, +\infty)$$

where $\widetilde{V}$ stands for the value function of an honest seller, ω stands for honest type, and this additional condition reflects the fact that if honests are not expected to sell, rationals will neither.

b) for all $H \in \mathcal{H}_n^1$,

$$s_n^{r1}(H, t_r) = Q_H$$

$$\textit{if} \quad p_n - c_H(t_r) + \beta V_{n-1}\left(t_r, H^+\right) \geq p_n - c_L + \beta V_{n-1}\left(t_r, H^-\right)$$

$$= Q_L \quad \textit{else}$$

where V is defined recursively, given $V_o \equiv 0$ and $\widetilde{V}_k \geq 0$, for all $k \leq n$,[9] by

$$\begin{aligned} V_n(t_r, H) = {} & \max\{b_n(H, v_H + \rho) \max\{v_H + \sigma - c_H(t_r) + \beta V_{n-1}\left(t_r, H^+\right), \\ & v_H + \sigma - c_L + \beta V_{n-1}\left(t_r, H^-\right)\} + [1 - b_n(H, v_H + \rho)]\, \beta V_{n-1}\left(t_r, H^o\right), \\ & b_n(H, p_n) \max\{ p_n - c_H(t_r) + \beta V_{n-1}\left(t_r, H^+\right), \\ & p_n - c_L + \beta V_{n-1}\left(t_r, H^-\right)\} + [1 - b_n(H, p_n)]\, \beta V_{n-1}\left(t_r, H^o\right)\} \end{aligned}$$

3) For all $n = 1, ..., N$, for all $H \in \mathcal{H}_n$,

[9] If it does not play for an honest type to sell at a given stage, it cannot pay for him to sell at any subsequent stage: If he does not sell, neither can rationals, so that beliefs remain unchanged. The horizon, though, is shorter, so that it is not possible to expect a higher payoff than in the previous period. It follows that if any sales by honest types are to take place in equilibrium, once they start they must continue until the end of the game. Else, all rationals would plan to supply low quality in the period prior to the end of sales, but this cannot be in equilibrium, as a rational would have an incentive to deviate and supply high quality in that period, thus convincing buyers that he is honest.

$$
\begin{aligned}
s_n^h(H,\omega) &= p_n \\
& \qquad if \quad \beta\widetilde{V}_{n-1}(\omega, H^\circ) < p_n - c_H + \beta\widetilde{V}_{n-1}(\omega, H^+) \\
where\ p_n &= \rho_n(H)\, v_H + [1-\rho_n(H)\, v_L] \\
else \qquad &= v_H + \sigma \qquad with\ \sigma \in (0, +\infty)
\end{aligned}
$$

where $\widetilde{V}$ is defined recursively, given $\widetilde{V}_o \equiv 0$, by

$$
\begin{aligned}
\widetilde{V}_n(\omega, H) = & \max\{b_n(H, v_H+\sigma)(v_H + \sigma - c_H + \beta V_{n-1}(\omega, H^+)) + \\
& [1 - b_n(H, v_H+\sigma)]\, \beta\widetilde{V}_{n-1}(\omega, H^\circ), \\
& b_n(H, p_n)\left(p_n + \sigma - c_H + \beta\widetilde{V}_{n-1}(\omega, H^+)\right) + \\
& [1 - b_n(H, p_n)]\, \beta\widetilde{V}_{n-1}(\omega, H^\circ)\}
\end{aligned}
$$

4) For all $n = 1, ..., N$, for all $H \in \mathcal{H}_n$,

$$
\rho_n(H) = \Pr\{s_n(H, \tau_r) = Q_H | H\}
$$

This is not the most general formulation as it incorporates the pricing rule that follows from the refinement of beliefs I am using, namely, that sales will always take place at expected values (if they take place). Also, I am not being specific about out-of-equilibrium beliefs (besides saying that prices will not affect buyers' beliefs), but the fact is that in this type of model out-of-equilibrium beliefs are pinned down by the consistency requirement in the definition of sequential equilibrium, and, hence, are not really a problem.

The additional condition in 2) a) refers to the requirement that honest types be willing to sell. If honests are not willing to sell, neither can rationals as they would have to ask for a price below that being requested by honest (which exceeds the expected value of the good), and in doing so they would reveal their type to buyers.

On the other hand, it is immediate that many of the above expressions simplify. Define

$$\widetilde{p}_n(H) \equiv \rho_n(H) v_H + [1 - \rho_n(H)] v_L$$

Then it is obvious that, in equilibrium, $b_n(H, \widetilde{p}_n(H)) = 1$, while, for any H,

$$b_n(H, v_H + \sigma) = 0$$

So, the recursions defining the long term players' value functions take the simpler form

$$V_n(t_r, H) = \max\{ \quad \beta V_{n-1}(t_r, H^\circ), \max\{ \widetilde{p}_n(H) - c_H(t_r) + \beta V_{n-1}(t_o, H^+),$$
$$\widetilde{p}_n(H) - c_L + \beta V_{n-1}(t_r, H^-)\}$$

$$\widetilde{V}_n(\omega, H) = \max\{\beta \widetilde{V}_{n-1}(\omega, H^\circ), \widetilde{p}_n(H) - c_H + \beta \widetilde{V}_{n-1}(\omega, H^+)\}$$

In fact, since whenever it pays for an honest to sell, it will also pay for a rational to do so, and since the recursive definition for a rational agent value function at stage n presupposes that $\widetilde{V}_k \geq 0$, for all $k \leq n$, one might just as well leave the first max operator in the definition of V_n out of the definition altogether. Note by the way, that honest must willing to sell at all subsequent stages along the equilibrium path in order for a sale to take place. In fact, since if it does not pay to sell at n, it will not pay to sell in the following stages, there are only two possible equilibrium paths of planner sales by honest types: Either they sell always or never.

5.2 Characterizing the Equilibrium

5.2.1 An Equilibrium Recursive in Reputations

In what follows I basically retrace the steps in the argument of Milgrom and Roberts[9] in this environment.

1) Clearly, $V_n(t_r, H_n^-) = 0, \forall n \leq n + 1$ Im

2) If it pays to sell high quality at stage n with type $\tau_r = t$, then it also pays to sell when $\tau_r = t' > t$. It follows that V is increasing in t. This has two implications for the equilibrium:

a) One can restrict attention to pure strategies.

b) In any equilibrium, for any given history and at any given stage if τ_r exceeds some critical value x_n, then high quality is provided (where this value is 1 if low quality was ever supplied). This justifies the following definition:

Definition 3 *The reputation of a seller x is the maximum of all critical values governing sellers' past quality decisions. Let $x = -\infty$ if low quality was ever provided, and -1 if no sales have taken place yet.*

At this point, it is possible to describe more precisely an equilibrium for the game that is recursive in the sense that all firms' decisions at any given stage will depend only on the value of their types and the seller's current reputation. Moreover, a seller's reputation upon entering a new stage will be a function only of its reputation in the previous period and the actions taken then. In this description I will presume that at each stage the rational seller will only sell if the honest type is planning to do so.

Here is a recursive description of the strategies making up such an equilibrium:

Step 0: Initialization

Let $V_0(t, x) \equiv 0$, $\forall t, x$, and let $x_N = 1$.

Step 1: Beliefs' Transitions

a) If $x \neq -\infty$ upon entering stage n, then the following rule applies: If no sale takes place, its reputation upon entering stage $n-1$ is x; if low quality is provided then it is $-\infty$; and, finally, if high quality is provided then it is $x \vee x_n$, where $\vee$ is the max operator and

$$x_n \equiv \inf\{s \in [0,1] \,|\, t > s \Rightarrow p_n - c_H(t) + \beta V_{n-1}(t,t) > p_n - c_L\}$$

b) If $x = -\infty$ upon entering stage n, then upon entering stage $n-1$, $x = -\infty$ regardless of what happens in stage n.

Step 2: Seller's Actions

a) If $\tau_r = t_r \in [0,1]$, and $x \neq -\infty$, seller will charge $\widetilde{p}_n(x)$ (see Step 3 below) if

$$\beta V_{n-1}(t_r, x) < \max\left\{ \widetilde{p}_n(x) - c_H(t_r) + \beta V_{n-1}(t_r, x \vee x_n), \widetilde{p}_n(x) - c_L \right\}$$

Else, the seller will charge $v_H + \sigma$, $\sigma \in (0, -\infty)$.

b) If at the offered price, a purchase takes place, then the seller will provide high quality if

$$-c_H(t_r) + \beta V_{n-1}(t_r, x \vee x_n) > -c_L$$

Step 3: Buyers' Actions

Let now $\rho_n(x)$ stand for the probability that the nth. buyer assigns to high quality being provided, when the seller's reputation is x. Then, if ε gives the initial mass assigned to the event that a seller is honest, $\rho_n(x)$ is given by

$$\rho_n(x) = \begin{cases} \frac{\varepsilon + [1-(x \vee x_n)]}{\varepsilon + (1-x)} & if \;\; x \geq 0 \\ 0 & if \;\; x = -\infty \\ \frac{\varepsilon + (1-x_n)}{1+\varepsilon} & if \;\; x = -1 \end{cases}$$

Buyer's optimal actions are to buy if the offered price does not exceed the expected value of the good, given the seller's reputation[10].

Step 4: Transition,from n to $n-1$

[10] Note that this rather terse formulation of the updating rule is perfectly general in the sense that it takes account of the updating of buyers' beliefs along both relevant dimensions, the probability they assign to the seller being honest and the likelihood that a rational seller be of a given type conditional on high quality having been supplied the previous period.

a) Since $V_0 \equiv 0$, it follows that

$$V_1(x, t_r) = \max\{0, \max\{\widetilde{p}_n(x) - c_H(t_r), \widetilde{p}_n(x) - c_L\}\}$$

For $n > 1$,

$$\begin{aligned} V_n(t_r, x) &= \max\{\beta V_{n-1}(t_r, x), \\ &\quad \max\{\tilde{p}_n(x) - c_H(t_r) + \beta V_{n-1}(t_o, x \vee x_n), \widetilde{p}_n(x) - c_L\}\} \end{aligned}$$

b) Since $\widetilde{V} \equiv 0$, one has that $\widetilde{V}_1(x, t_r) = \widetilde{p}_n(x) - c_H$. For $n < 1$,

$$\widetilde{V}_n(\omega, x) = \max\{\beta \widetilde{V}_{n-1}(\omega, x), \ \widetilde{p}_n(x) - c_H + \beta \widetilde{V}_{n-1}(\omega, x \vee x_n)\}$$

To show that these strategies represent a sequential equilibrium, one has to show that they satisfy sequential rationality and consistency of beliefs. Optimality follows from the way the strategies are specified. The consistency of beliefs follows from the following reasoning:

We have to verify that if the sellers follow the above strategies, the buyers should make the inferences specified.

So, if a seller entered stage n with a reputation of $x \neq -\infty$, buyers will know that the seller is of type $t > x$. Now, there are two cases:

Case 1: $x > x_n$

So, $x = x \vee x_n$, and

$$-c_H(t) + \beta V_{n-1}(t, x) \geq -c_H(x) + \beta V_{n-1}(x, x) > 0 \quad for\ t \geq x$$

The first inequality results from $c_H(.)$ being strictly decreasing and V_{n-1} being increasing in t_r. The second inequality follows from the definition of x_n and the hypothesis that $x > x_n$. Conclusion: A seller that has demonstrated that he or she is of type $t_r > x_n$, will supply high quality at that stage.

Case 2: $x \leq x_n < 1$ (i.e., $x_n = x \vee x_n$)

Since c_H is decreasing in t, and V_{n-1} is increasing in this variable and continuous (since V_0 is and so are all functions entering the recursion defining V_{n-1}), for $x_n > 0$ one gets

$$0 = c_L - c_H(x_n) + \beta V_{n-1}(x_n, x_n) \gtreqless c_L - c_H(t) + \beta V_{n-1}(t, x_n) \quad as\ \ x_n \gtreqless t$$

This implies that the seller will provide high quality iff $t > x_n$. If $x_n = 0$, then LHS ≥ 0 and Q_H will be provided. If $x_n = 1$, LHS ≤ 0, $t \leq x_n$ and Q_L will be provided. This proves the consistency of the beliefs specified.

5.2.2 Monotonic Critical Values and Reputation in the Limit

In establishing that reputations will eventually emerge as the horizon of the game extends into infinity, it is necessary to make the assumption below:

$$c_H(1) - c_L < \frac{\beta}{1-\beta}(v_H - c_H(1))$$

This is a necessary condition for the emergence of reputations: It states that the loss incurred in providing high quality by the rational type with the lowest cost of producing that quality is smaller than the revenues that same agent would obtain from selling high quality from the next period on forever.

The central result of this section is then:

Proposition 4 *For any n, $x_{n+1} \leq x_n$, with strict inequality if $x_n \neq 0$ or 1. Further, $x^* \equiv \lim_{n\to\infty} x_n = \max[0, \mathbf{x}]$, where $\mathbf{x}$ is given by the solution to the equation below*

$$c_H(\mathbf{x}) - c_L = \frac{\beta}{1-\beta}(v_H - c_H(\mathbf{x}))$$

Proof. See Appendix.

A straightforward corollary of this result is the following:

Corollary 5 *If $x^* = 0$, then $\lim_{n\to\infty} \rho_n = 1$, and, hence, $\lim_{n\to\infty} \widetilde{p}_n = v_H$. Moreover, the convergence to this value will take place in finite time. If $x^* > 0$, the convergence will take place only asymptotically. Moreover, if $\widetilde{p}_n(\mathbf{x}) \geq \frac{c_H - \beta v_H}{1-\beta}$, and initially high quality is provided, $\widetilde{p}_\infty = v_H$. If, on the other hand, $\widetilde{p}_n(\mathbf{x}) < \min[c_L, \frac{c_H - \beta v_H}{1-\beta}]$, then $\widetilde{p}_\infty = v_H + \sigma$.*

In words: The monotonicity of critical values implies that, if it pays to supply high quality when N periods remain, it will also pay to supply that quality when more than N periods remain. Also, in the limit, the price will not differ substantially from v_H, if sales are feasible. Moreover, if $x^* = 0$, the sale price will remain at v_H most of the time except for a period of finite length towards the end of the game when it will fall from that level. In this case, the length of this end-game will be independent of the length of the horizon (for sufficiently large horizons). This latter feature follows from the recursivity of the equilibrium. It is important to note that, in this case, none of the asymptotic properties depend in any way on the value of $\varepsilon > 0$, the probability that the seller be honest.

Moreover, the analysis of the two-period case suggests that the length of the end-game will vary directly with initial beliefs, and that for sufficiently pessimistic initial beliefs (i.e., for sufficiently high initial ε) increasingly longer periods of no sales (always immediately preceded by a low quality sale) might result towards the end of the game.

On the other hand, if $x^* > 0$ and $\widetilde{p}_n(\mathbf{x}) \geq \frac{c_H - \beta v_H}{1-\beta}$, the asymptotic properties will depend on initial beliefs via $\widetilde{p}_n(\mathbf{x})$. This is a remarkable difference with the results in Kreps-Wilson (but not so with the model of Milgrom and Roberts that has an analogous property[11]). What the condition here represents is the trade-off between facing an honest seller for whom it might not pay to incur high losses today (if initial beliefs are too pessimistic) in order to earn profits in the future (just as in the Milgrom-Roberts environment it might not pay for an entrant to enter if he or she is sufficiently convinced that the incumbent is tough).

Also, in this case, there is no end-game as in Kreps-Wilson, as the length of the game will affect the beliefs with which a buyer enters any given set of periods.

The asymptotic results closely track those in the Milgrom and Roberts paper, though there is at least one important difference in the overall dynamics: Note that unlike what happened in the Milgrom and Roberts environment where entry (corresponding to no sales episodes in this game) might take place sporadically, here sales will take place continuously, if at all (though, as pointed out, they might stop before the end of the game via a low quality sale). This is a consequence of the fact that sellers in choosing the price in effect are able to control whether or not a sale takes place.

The following additional corollary completes the characterization of the price dynamics:

[11]Even though in the Milgrom-Roberts game this feature is not of major importance, in so far as initial beliefs only determine whether there will be entry in the first period or not. In any case, after the first period no entry will take place. Note that the results of Fudenberg and Levine[4] do not apply to the present model, for precisely the above type of consideration. See Fudenberg and Levine[4], p.772.

Corollary 6 *Prices may only rise in the initial period. Afterwards, they will fall throughout.*

Proof. Since the critical values x_n rise over time, it must be that the following relations hold:

$$c_H(x_n) = \beta(\tilde{p}_{n-1} - c_L) + c_L$$

$$c_H(x_{n-1}) = \beta(\tilde{p}_{n-2} - c_L) + c_L$$

This from the definition of critical values (subtype x_n is indifferent between providing high or low quality at n), but the critical type at period n must strictly prefer to supply low quality tomorrow given that critical values are rising (hence the continuation payoff $\beta(\tilde{p}_{n-1} - c_L)$).

But then this implies that, from the second period onwards, prices must fall as $c_H(.)$ is strictly decreasing and the critical values increase over time. ■

While in the initial period prices might rise or fall, it must be that prices fall in subsequent periods. The result is, in my view, not intuitive: It means that, after the first period, the negative pull on beliefs exercised by the steadily falling measure of rationals who are willing to supply high quality prevails over the positive effect generated by continued high quality purchases. In other words, there is from the 2nd. period on, a systematic relation between the price today and the price tomorrow over and beyond that implied by individual rationality.

While I cannot provide a satisfactory intuition of why this is so, here are some considerations that seem relevant:

To start with, it is not difficult to make out in what sense the first period is qualitatively different from subsequent periods. In any sale equilibrium, after high quality was provided, the seller will have acquired a reputation, not so, of course, in the initial period. This means that the updating from the first to the second periods will be quite different from the updating in latter periods. In latter periods, the only way beliefs can be updated after a high quality sale is if the critical value at the period, say, x_n , exceeds the reputation the seller is carrying at that time. In the first period, this restriction simply does not apply. Note that in the formal argument above it is the fact that $x_n < x_{n-1} < x_{n-2}$ (together with other considerations) which leads to $\tilde{p}_{n-2} < \tilde{p}_{n-1}$ (not just $x_{n-1} < x_{n-2}$).

Beyond this note that the learning effect is stronger the higher the critical value in the period *relative* to the reputation the agent is carrying. This is certainly a key aspect: It implies that the strength of the learning effects will not be determined by the mass of rationals (which obviously falls as time goes by), but, rather, by the decomposition of that mass in rationals intending to supply high quality and those not intending to do so. This leaves open the possibility that the informativeness of high quality sales may fall as the end of the game is approached, though as will be shown immediately, that is not required for prices to fall. In fact, what is required is that the informativeness rises as high quality sales continue.

The more informative a high quality sale, the lower will tend to be the price at which it takes place, and viceversa. To illustrate, say all rationals intend to provide high quality, then a high quality sale conveys no information at all, but the current price is very high (equal to v_H,its upper bound). If no rationals intend to provide high quality, a high quality sale reveals that the seller is honest, but the price is as low as it can currently be.

It can be shown that prices will fall from n to $n-1$ iff

$$\frac{1-\frac{x_n}{\varepsilon+1}}{1-\frac{x_{n+1}}{\varepsilon+1}} \geq \frac{1-\frac{x_{n-1}}{\varepsilon+1}}{1-\frac{x_n}{\varepsilon+1}}$$

(with ε representing the initial mass of honests; the initial mass of rationals taken to be one). The formula reveals how the dynamics of falling prices operate: At n, the price is relatively high precisely because the quality signal is relatively uninformative. At $n-1$, though a high quality sale will be relatively more informative, the price will be lower than that at n on two counts: One, precisely the fact that the signal at that date is more informative; two, the low informativeness of the preceding high quality sale. Note what would happen if the inequality above is reversed: The latter price would be higher on two counts, one, because of the high informativeness of the preceding high quality sale; and, two, because of the low informativeness of the current high quality sale (note that, in this case, unlike in the previous one, both effects reinforce each other). This shows that increasing informativeness of a high quality sale is not only compatible with falling prices but necessary for such price dynamics to result.

Of course, the question remains why the improvement in beliefs (the shift of mass to honest types) never compensates the increased willingness of rational sellers to cheat.

The diagram below presents the results of a simulation for eight periods (with parameters $v_H = 10, c_H = 8, c_L = 7, v_L = 4, \beta = 0.2$ and initial beliefs equal to 0.4)

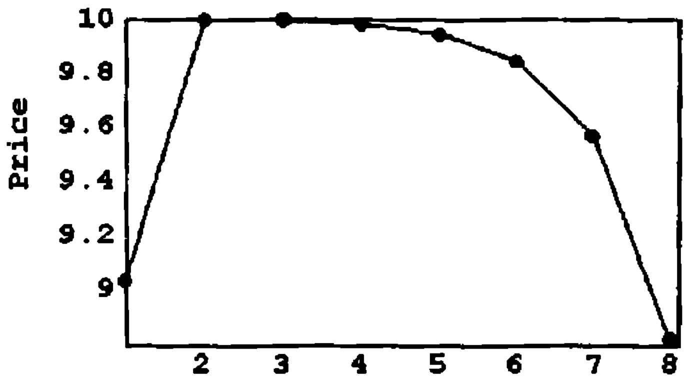

Fig. 5: Time Path of Prices

6 Existence and Uniqueness

As in previous sections of this paper, most of the arguments only retrace arguments presented in Milgrom and Roberts[11] in the context of this modified version of their model.

So, one can show by arguments exactly analogous to the ones presented in Milgrom and Roberts that the above equilibrium is unique. When it comes to existence, one substantial departure arises because, in this model, for any given finite horizon, only in a range of initial beliefs $[0, \bar{\delta}_T]$ (for any given $c_H(.)$) will such a sale equilibrium exist (while in Milgrom and Roberts reputational equilibria existed for any initial beliefs).

In establishing uniqueness, the first thing to note is that no sales can take place after low quality is supplied. Now suppose high quality has been invariably provided in the past, and in the current period the buyer has accepted the price offer of the seller. If the seller provides low quality, the continuation payoff will be zero. If instead the seller decides to supply high quality, given the strategies of the other buyers, a certain pattern of trade will result in the future, that is, a pattern of no sales, sales, and quality and price decisions. The expected present value of payoffs of supplying high quality today is then given by $R(Q_H) - S_{Q_H}(c_H(t_0)) + R(Q_L) - S_{Q_L}(c_L)$, where

$R(Q_H)$ stands for expected present value of high quality sales (including the current one), $S_{Q_H}(c_H(t_0))$ stands for the expected present value of the cost of producing the high quality items sold; $R(Q_L)$ stands for the expected present value of a low quality sale, while $S_{Q_L}(c_L)$ stands for the expected present value of producing the substandard good. Clearly $S_{Q_H}(c_H(t_0))$ is decreasing in t_0, so that if the (rational) seller with subtype t finds it profitable to provide high quality today, so will all sellers with subtypes above t. This implies that the maximized value of future payoffs over all possible strategies will also increase in t_0 as it is the value of the maximum of increasing functions of the above form (including the 0 function). This suffices to establish two key features of any equilibrium: 1) Only pure strategies will be played (since it will be impossible to make a positive measure of rationals indifferent simultaneously -$c_H(.)$ is strictly decreasing), and 2) the strategy of a rational seller will call for high quality to be supplied at a given stage of the game after a given history iff the subtype of that seller exceeds some critical value.

It is now shown inductively that any sequential equilibrium must agree with the one described in the text at all stages $m \leq n$ (note that in establishing this, we will also proving that the critical values x_n at any stage are independent of the length of the game). Induction proceeds on n, and the case $n = 0$ (initial period) is immediate. Fix an equilibrium and assume the result holds for $n = k$. The value of entering stage k with subtype t and reputation x is given by $V_k(t,x)$. Say at stage $k+1$ the history is H^+ and reputation is $\overline{x}$. If $x \geq \overline{x}$ is the reputation that would result if the seller supplied high quality at stage $k+1$, then a seller of subtype t would choose to supply high quality or not according as

$$c_H(t) - c_L - \beta V_k(t,x) \gtreqless 0$$

Since reputation must be consistent with the seller's equilibrium strategy, it must be that for $t \geq \overline{x}$,

$$c_H(t) - c_L - \beta V_k(t,x) \gtreqless 0 \;\; as \;\; t \gtrless x$$

It is easy to check using the monotonicity of V_k, that the unique $x \geq \overline{x}$ for which this holds is $x = \overline{x} \vee x_{k+1}$. Hence the conditional probability that high quality will be supplied at stage $k+1$ given history H^+ is $p_{k+1}(\overline{x})$. Obviously, so long as sellers charge expected value, buyers will buy. It has thus been established that if history affects play only through reputations for stages $m \leq k$, it will do so as well at $k+1$.

Now, as far as existence is concerned: In the two period case the range of initial beliefs for which there existed a sale equilibria was given by $\left[0, \overline{\delta}_2\right]$ (given $c_H(.)$). Basically, this restriction reflected the conditions that must be satisfied in order for there to be pooling and hence sales in equilibrium. These conditions were threefold: First, updated beliefs had to be in the range where there is a sale equilibrium in the initial period (namely, $\left[0, \overline{\delta}_1\right]$); secondly, the individual rationality constraint of the rational type had to be satisfied; and thirdly, the individual rationality constraint of low quality suppliers had to be satisfied. These conditions generalize for longer horizons as follows: Initial beliefs for horizon T have to be in the range $\delta_T^T \in \left[0, \overline{\delta}_T\right]$ with $\overline{\delta}_T = \min\left[1, \widetilde{\delta}_T, \widehat{\delta}_T, \overline{\overline{\delta}}_T\right]$ with these arguments given to the solutions to

$$\begin{aligned}
\delta_{T-1}\left(x_T, \widehat{\delta}_T\right) &= \overline{\delta}_{T-1} \\
p_T^T\left(x_T, \widetilde{\delta}_T\right) &= c_H - \beta\widetilde{V}\left(\widetilde{\delta}_T, x_T\right) \\
p_T^T\left(x_T, \overline{\overline{\delta}}_T\right) &= c_L
\end{aligned}$$

These formulas refer to the first period of a game of horizon T, hence the superscript T. The following proposition provides some guidance as to which initial beliefs are acceptable:

Proposition 7 *If a sale equilibrium exists for given initial beliefs for a given horizon T', it will also exists for those same initial beliefs at any $T > T'$.*

Proof. The proof proceeds by showing that each of the above magnitudes $\widetilde{\delta}_T, \widehat{\delta}_T, \overline{\overline{\delta}}_T$ must lie above $\overline{\delta}_{T-1}$.

It is first shown that $\widehat{\delta}_T > \overline{\delta}_{T-1}$.

$$\delta_{T-1}\left(x_T, \widehat{\delta}_T\right) = \frac{(1-x_T)}{(1-x_T)\widehat{\delta}_T + \left(1-\widehat{\delta}_T\right)} = \overline{\delta}_{T-1}$$

This can be rewritten

$$\widehat{\delta}_T = \frac{\overline{\delta}_{T-1}}{1 - x_T + \overline{\delta}_{T-1}x_T}$$

The denominator of this expression is clearly smaller than one, so that $\widehat{\delta}_T > \overline{\delta}_{T-1}$.

Next it is shown that $\overline{\overline{\delta}}_T > \overline{\overline{\delta}}_{T-1}$. Since it must be that

$$p_T^T\left(x_T, \overline{\overline{\delta}}_T\right) = c_L$$

$$p_{T-1}^{T-1}\left(x_{T-1}, \overline{\overline{\delta}}_{T-1}\right) = c_L$$

Now if $\overline{\overline{\delta}}_T = \overline{\overline{\delta}}_{T-1}$ then since $x_T < x_{T-1}$(note that these values do not depend on the length of the game), and the initial period price is falling in x, it would be that

$$p_{T-1}^{T-1}\left(x_{T-1}, \overline{\overline{\delta}}_{T-1}\right) < p_T^T\left(x_T, \overline{\overline{\delta}}_T\right)$$

So, since initial period price is falling in initial beliefs, in order for equality to be restored we must have $\overline{\overline{\delta}}_T > \overline{\overline{\delta}}_{T-1}$.

Finally, it is shown that $\widetilde{\delta}_T > \widetilde{\delta}_{T-1}$. It must be that

$$p_T^T\left(x_T, \widetilde{\delta}_T\right) - c_H + \beta \widetilde{V}_{T-1}^T\left(\widetilde{\delta}_T, x_T\right) = 0$$

$$p_{T-1}^{T-1}\left(x_{T-1}, \widetilde{\delta}_{T-1}\right) - c_H + \beta \widetilde{V}_{T-2}^{T-1}\left(\widetilde{\delta}_{T-1}, x_{T-1}\right) = 0$$

Note that the last condition after multiplication by β becomes

$$\beta p_{T-1}^{T-1}\left(x_{T-1}, \widetilde{\delta}_{T-1}\right) - \beta c_H + \beta^2 \widetilde{V}_{T-2}^{T-1}\left(\widetilde{\delta}_{T-1}, x_{T-1}\right) = 0$$

In what follows two results will be used: One, that the x_n 's do not depend on the game horizon; and two, that $x_n \geq \overline{x}_n$ when $\overline{\delta}_T^T \leq \delta_T^T$ (the proof is exactly analogous to the one in Milgrom and Roberts[11], p.311). Again, if $\widetilde{\delta}_T = \widetilde{\delta}_{T-1}$, we would have $p_{T-1}^{T-1}\left(x_{T-1}, \widetilde{\delta}_{T-1}\right) < p_T^T\left(x_T, \widetilde{\delta}_T\right)$. Moreover we have

$$\beta \widetilde{V}_{T-1}^T\left(\widetilde{\delta}_T, x_T\right) = \beta p_{T-1}^T - \beta c_H + \beta^2 \widetilde{V}_{T-2}^T\left(\widetilde{\delta}_T, x_{T-1}\right)$$

But by the properties of sale equilibrium established previously, it must be that

$$p_{T-1}^T \geq c_H$$

$$\widetilde{V}_{T-2}^T\left(\widetilde{\delta}_T, x_{T-1}\right) = \widetilde{V}_{T-2}^{T-1}\left(\widetilde{\delta}_T, x_{T-1}\right)$$

So that one can conclude that

$$\beta \widetilde{V}_{T-1}^{T}\left(\widetilde{\delta}_T, x_T\right) \geq \beta^2 \widetilde{V}_{T-2}^{T-1}\left(\widetilde{\delta}_T, x_{T-1}\right)$$

So in order to establish equality it must be that $\widetilde{\delta}_T > \widetilde{\delta}_{T-1}$, since initial price will be falling in initial beliefs and $\widetilde{V}_{T-1}^{T}\left(\widetilde{\delta}_T, x_T\right)$ can be shown to be falling in this magnitude as well. To see this, recall that $x_T \geq \overline{x}_T$ when $\overline{\delta}_T^T \leq \delta_T^T$. But these two inequalities imply that

$$p_T(x) < \overline{p}_T(x)$$

which in turn implies that prices are lower throughout, and the result follows. ■

An rather immediate related result is the following:

Corollary 8 *As $T \to \infty$, $\overline{\delta}_T \to 1$ if $\mathbf{x} = 0$ (where bold x is as defined in the main text).*

Proof. If $\mathbf{x} = 0$, then initial price will be given by $p_\infty = v_H$ for any initial beliefs $\in [0, 1]$. But then the individual rationality for honests will be satisfied at any initial belief since

$$v_H - c_H + \frac{\beta (v_H - c_H)}{1 - \beta} > 0$$

So will be the constraint requiring initial price to be above c_L. ■

The previous results show that in effect lower $\delta's$ are 'better' for existence than larger ones, in the sense that the smaller δ, the shorter the horizon required for a sale equilibrium to exist.

7 Conclusions

While prices cannot separate types here given the absence of something like the 'single crossing' condition of conventional signalling models, it is still interesting, seems to me, that in this type of environment nothing resembling the 'bonding' story of the infinite horizon models emerges. The 'recursive' nature of the equilibrium would seem crucial in obtaining this negative result.

The price dynamics, though at first not surprising, on closer examination are remarkably regular in the end-phase of the game. Also, the 'continuity' of sales, i.e., the fact that sporadic no-sale episodes are not possible within strings of sales, represents a departure from the dynamics described in Milgrom and Roberts, and one that can clearly be attributed to the 'pre-trade' communication taking place through prices.

On the other hand, it is clear that the model studied here can only be considered a first approximation to the subject. In particular, the fact that the cost structure of rational types is assumed to systematically differ from that of honest types might strike some as ad-hoc. And, quite honestly, it is: Assuming that honest types share the cost structure postulated here for rational types generates a series of technical problems that I am still trying to solve. This feature of the model is introduced not because I feel that it provides a good description of actual cost structures, but purely as a device to obtain unique price paths.

Nevertheless, it seems to me that the results of this model, preliminary as they are, are suggestive enough to justify further research into reputational models of this kind.

A Proof of Proposition 1

Consistency of Beliefs If low quality is supplied, buyers know that the seller must be rational, for honest sellers never provide low quality. Consistency of beliefs is evident as a rational seller of type t will only supply high quality in the region $[0, \min\left[1, \widetilde{\delta}, \widehat{\delta}, \overline{\overline{\delta}}\right]]$, if

$$c_H(t) - c_L \leq \beta(\widetilde{p}_1 - c_L)$$

Then the formula

$$\delta_1(H) = \frac{\rho_2(H)\,\delta_2}{\rho_2(H)\,\delta_2 + (1-\delta_2)}$$

is just Bayes' Rules given initial beliefs δ_2.

Sequential Rationality

Buyers:

At any time, buyers should never pay more than the expected value of the good. Clearly, it always pays to buy at a price below v_L.

Sellers:

By the assumption that price deviations leave beliefs unchanged, sellers must charge the expected value of the good to the buyers in an equilibrium at each period. If low quality was supplied, then buyers will not buy at any price above v_L. Since $v_L < c_L$, it just does not pay to sell. The condition $c_H(s) - c_L \leq \beta(\widetilde{p}_1 - c_L)$ just says that the gain from supplying low quality today is smaller than the loss associated with doing so.

If $\delta > \min\left[1, \widetilde{\delta}, \widehat{\delta}, \overline{\overline{\delta}}\right]$, then either the constraint $\delta \leq \overline{\overline{\delta}}$ ($\widehat{\delta} = \min\left[1, \widetilde{\delta}, \widehat{\delta}, \overline{\overline{\delta}}\right]$) is violated, or the individual rationality constraint for the honest type is violated ($\widetilde{\delta} = \min\left[1, \widetilde{\delta}, \widehat{\delta}, \overline{\overline{\delta}}\right]$), or $\widetilde{p}_2(\delta) < c_L$. The price at $t = 2$ cannot be below c_L, for, in that case, sellers for whom it does not pay to supply high quality[12] would rather not sell. This cannot represent equilibrium behavior, as the sellers who are not selling are earning zero profits, while they would earn positive profits if they mimicked the behavior of the high quality suppliers (that is charged their price and supplied high quality). This must be so since the individual rationality constraint of honest types must be satisfied

[12] There must be some, for otherwise, if everyone is supplying high quality, p_2 cannot be below v_H, under the pricing rule being used.

at the selling price (the high quality price), but this implies that the individual rationality constraint of the rational type with the highest costs is also satisfied at that price.

Uniqueness Note that the equation (derived from (1),(2),(3) and (4))

$$(c_H(s) - c_L)\left[(1-s)\delta_2 + (1-\delta_2)\right] - (1-s)\delta_2\beta(v_L - c_L) = \beta(1-\delta_2)(v_H - c_L) \tag{5}$$

is monotone falling in s for $s \in [0,1)$, i.e., the sale equilibria are unique (if they exist). Again, from equations (1)-(4), one can write

$$\delta_1 = \frac{\left[1 - c_H^{-1}\left(\beta\left(\widetilde{p}_1(\delta_1) - c_L\right) + c_L\right)\right]\delta_2}{\left[1 - c_H^{-1}\left(\beta\left(\widetilde{p}_1(\delta_1) - c_L\right) + c_L\right)\right]\delta_2 + (1-\delta_2)} \tag{6}$$

Since δ_2 enters linearly, this equation takes the value 0 at $\delta_2 = 0$, the value 1 at $\delta_2 = 1$, and is continuous, it follows that the solution to the equation $\delta_1(\delta) = \overline{\delta}$ is unique, if it exists. Since the candidate selling equilibrium is unique, it follows that $\delta_1(\delta)$ takes only one value in the range $[0,1)$ for $\delta \in [0,1]$. Further note that $\delta_1(\delta)$ in $[0,1] \times [0,1)$ must be strictly increasing (since this equation takes the value 0 at $\delta_2 = 0$, and the value 1 at $\delta_2 = 1$, and is continuous), and, so, $\widetilde{p}_2(\delta)$ must be strictly decreasing[13], and c_H $\beta(\widetilde{p}_1(\delta) - c_H)$ strictly increasing. It follows that, if a solution to the equation $\widetilde{p}_2(\delta) = c_H - \beta(\widetilde{p}_1(\delta) - c_H)$ exists, it must be unique. Such a solution exists if sale equilibria exist, since $\widetilde{p}_2(0) = v_H$, while $c_H - \beta(\widetilde{p}_1(0) - c_H) < c_H$, if sale equilibria exist.

Since all sale equilibria must be pooling (for the same general reason as in the one-shot game: It will always be advantageous and always be feasible

[13] To see this: Note that

$$\begin{aligned} p_2 &= \delta_2\left[\rho_2(H)v_H + (1-\rho_2(H))v_L\right] + (1-\delta_2)v_H \\ \textit{with } \rho_2(H) &= 1 - c_H^{-1}\left[\beta(p_1 - c_L) + c_L\right] \end{aligned}$$

Note that $\rho_2(H)$ is falling in initial beliefs as p_1 is falling in that variable. Hence the expression in square brackets multiplying initial beliefs is falling. In the equation for p_2 the weight on the smaller expression is increasing while that on the bigger one (v_H) is falling, so, the expression overall must be falling.

for the rational type to mimic the strategy of the honest type in any candidate separating equilibrium), they must take the above form. By definition then, there cannot be sale equilibria outside the region $[\min\left[1,\widetilde{\delta},\widehat{\delta},\overline{\overline{\delta}}\right],1]$. Note finally that there cannot be pooling no-sale equilibria in the region $[0,\min\left[1,\widetilde{\delta},\widehat{\delta},\overline{\overline{\delta}}\right]]$ because of the refinement being used here: It pays to deviate to some price below the expected value of the good but above its cost. ■

B Proof of Proposition 3

The argument is practically identical with that in Milgrom-Roberts, except that there is the additional complication that sales can only take place if the individual rationality constraint of the honest type is satisfied. I tackle this problem by proceeding as if this constraint were satisfied at all moments and then verifying that in the resulting candidate equilibrium this is in effect so. Since it will be shown that the asymptotic price will equal v_H ($> c_H$) so long as sales take place, it is immediate that this constraint will be satisfied from then on. The only real issue is whether trading will take place to start with, i.e., whether this constraint will be satisfied in an initial period. This turns out to depend on initial beliefs.

Define the following function

$$g(t,v) = \max\left[\beta v, \max\left(v_H - c_H(t) + \beta v\right), v_H - c_L\right]$$

Note that $v_H = \widetilde{p}_n(1), V_{m+1}(t,1) = g(t, V_m(t,1))$ and g is increasing in v. Moreover, it is the case that

$$|g(t,v_1) - g(t,v_2)| \leq \beta |v_1 - v_2|$$

To see this: Start by noting that

$$g(t,v) = \max[(v_H - c_H(t) + \beta v), v_H - c_L] \ \forall v$$

since $v_H - c_H(t) > 0$. Let now w.l.of g. $v_1 > v_2 > 0$. There are three subcases:

a) $\max\{(v_H - c_H(t) + \beta v_2), v_H - c_L\} = v_H - c_H(t) + \beta v_2$
$\Rightarrow \max\{(v_H - c_H(t) + \beta v_1), v_H - c_L\} = v_H - c_H(t) + \beta v_1$
and so the inequality we are trying to establish reduces to

$$\beta|v_1 - v_2| \leq \beta|v_1 - v_2|$$

b) $\max\{(v_H - c_H(t) + \beta v_1), v_H - c_L\} = v_H - c_L$
$\Rightarrow \max\{(v_H - c_H(t) + \beta v_2), v_H - c_L\} = v_H - c_L$
and so the inequality we are trying to establish takes the form

$$0 \leq \beta|v_1 - v_2|$$

c) $\max\{(v_H - c_H(t) + \beta v_1), v_H - c_L\} = v_H - c_H(t) + \beta v_1$ and
$\max\{(v_H - c_H(t) + \beta v_2), v_H - c_L\} = v_H - c_L$
We want to show that

$$|c_L - c_H(t) + \beta v_1| \leq \beta|v_1 - v_2|$$

but if this inequality does not hold then

$$c_L - c_H(t) + \beta v_1 \quad > \quad \beta(v_1 - v_2)$$

$$\Rightarrow \quad -c_H(t) + \beta v_2 > -c_L$$

which contradicts the hypothesis.□
So, g is a contraction mapping, and as such has a unique fixed point $\overline{v}(t)$.
Let $\mathbf{x}$ be as in the main text, then

$$If \; t \leq \mathbf{x} \qquad \overline{v}(\mathbf{t}) = \frac{(v_H - c_H(\mathbf{x}))}{1-\beta}$$

$$If \; t > \mathbf{x} \qquad \overline{v}(\mathbf{t}) = \frac{(v_H - c_H(\mathbf{t}))}{1-\beta}$$

To see this: If $t \leq \mathbf{x}$,

$$c_H(t) - c_L \geq \frac{\beta}{1-\beta}(v_H - c_H(t)) \Rightarrow$$

$$v_H - c_H(t) + \beta\frac{v_H - c_H(t)}{1-\beta} < v_H - c_L$$

Using this in the definition of g, one then gets the result. By a similar logic, the result in the case $t > \mathbf{x}$ obtains.□
With this, one can state the following lemma:

Lemma 9 $v < \overline{v}(t) \Rightarrow v < g(t,v) < \overline{v}(t)$

Proof. Let $v < \overline{v}(t)$ then

$$g(t,v) < g(t,\overline{v}(t)) = \overline{v}(t)$$

Also,

$$\begin{aligned} \overline{v}(t) - g(t,v) &= g(t,\overline{v}(t)) - g(t,v) \\ &\leq \beta(\overline{v}(t) - v) \\ &< \overline{v}(t) - v \end{aligned}$$

So

$$g(t,v) > v$$

■

Note:

1)

$$c_H(1) - c_L < \frac{\beta}{1-\beta}(v_H - c_H(1)) \Rightarrow \mathbf{x} < 1$$

2)

$$V_o(t,1) \equiv 0 \leq \overline{v}(t)$$

3)

$$V_m(t,1) \leq \overline{v}(t)$$

After these preliminaries, the proof proper can be presented:

It proceeds by a rolling induction on the following propositions,

(P0) $V_n(t,x)$ is continuous in (t,x) and non-decreasing in t.
(P1) $V_n(t,x)$ is non-decreasing in x.
(P2) $x_{n+1} \leq x_n$'

(P0) has already been established in the main text. Define $x_o \equiv 1$.

(P1)

$$V_{m+1}(t,x) = \max\{\beta V_m(t,x),$$
$$\max\{\tilde{p}_m(x) - c_H(t) + \beta V_m(t, x \vee x_{m+1}), \tilde{p}_m(x) - c_L\}\}$$

Since ρ_{m+1} is non-decreasing in x, so is $\tilde{p}_{m+1}$. Hence, (P1) follows.

(P2)

Since (P1) holds for $n = m+1$. for all x (by induction hypothesis),

$$V_{m+1}(x,x) \leq V_{m+1}(x,1)$$

This follows from the lemma. Using the lemma again, and the fact-that

$$\begin{aligned} V_{m+1}(x_{m+1}, x_{m+1}) &= g(x_{m+1}, V_{m+1}(x_{m+1}, x_{m+1})), \\ & \tilde{p}_m(x_{m+1}) - c_H(x_{m+1}) + \beta V_{m+1}(x_{m+1}, x_{m+1}) \\ &= \tilde{p}_m(x_{m+1}) - c_H(x_{m+1}) + \beta g(x_{m+1}, V_{m+1}(x_{m+1}, x_{m+1})) \\ &\geq \tilde{p}_m(x_{m+1}) - c_H(x_{m+1}) + \beta V_m(x_{m+1}, x_{m+1}) \end{aligned}$$

follows from the definition of x_{m+1} and the continuity of V_m. The other inequality follows from the lemma. If $x_{m+1} = 1$, then there is nothing to prove. If $x_{m+1} < 1$,RHS ≥ 0 (follows from the definition of x_{m+1} and the continuity of V_m). Hence, by monotonicity of $\tilde{p}_m$ and c_H, and the continuity and monotonicity of V_{m+1} and the definition of x_{m+2},

$$\begin{aligned} 0 &= \tilde{p}_{m+1}(x_{m+2}) - c_H(x_{m+2}) + \beta V_{m+1}(x_{m+2}, x_{m+2}) \\ &\leq \tilde{p}_m(x_{m+1}) - c_H(x_{m+1}) + \beta V_{m+1}(x_{m+1}, x_{m+1}) \end{aligned}$$

In other words, $x_{m+2} \leq x_{m+1}$, with strict inequality if $x_{m+1} > [0 \vee \mathbf{x}]$. This proves the monotonicity of critical values.□

Now, for the rest of the argument:

Since $\{x_n\}$ is bounded, non-increasing, it has a limit $x^* \geq 0$. If seller supplies high quality (not having failed to do so previously), he acquires a reputation $x \geq x^*$. For any $x \geq x^*$ and any n,

$$\begin{aligned}\rho_n(x) &= \frac{\varepsilon + [1-(x \vee x_n)]}{\varepsilon + (1-x)} \\ &\geq \quad \rho_n(x^*) = \frac{\varepsilon + [1-(x^* \vee x_n)]}{\varepsilon + (1-x^*)}\end{aligned}$$

Since $x_n \to x^*$, RHS$\to 1$ as $n \to \infty$. Hence, $\rho_n(.)$ converges uniformly to 1 for $x \geq x^*$. Since

$$\begin{aligned}V_n(t,x) &= \max\{\beta V_{n-1}(t,x), \\ &\quad \max\{\,\widetilde{p}_n(x) - c_H(t) + \beta V_{n-1}(t, x \vee x_n), \widetilde{p}_n(x) - c_L\}\}\end{aligned}$$

is a contraction mapping (apply Blackwell's sufficiency conditions to check this), it is the case that

$$\lim_{n\to\infty} V_n(t,x) = V_\infty(t,x) \quad for\ all\ x \geq x^*$$

But $V_\infty(t,x) = g(t, V_\infty(t,x))$, so $V_\infty(t,x) = \overline{v}(t)$. It follows that the limiting condition guaranteeing that a high quality sale will take place is

$$-c_H(t) + \beta\overline{v}(t) > -c_L$$

In other words, whenever $t > \mathbf{x}$, high quality will be provided, i.e., $x^* = \lim x_n = [0 \vee \mathbf{x}]$. Since $\mathbf{x} < 1$, Q_H will be sold in the limit, even if not in the short run.

If $x^* = 0$, then clearly it will always pay to sell regardless of initial beliefs. On the other hand, if $x^* > 0$, then an honest type might prefer not to sell if

$$\widetilde{p}_T(\mathbf{x}) < \frac{c_H - \beta v_H}{1-\beta}$$

where T stands for the initial period. ■

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