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THE OPTIMAL INTERTEMPORAL ALLOCATION OF HEALTH, SAVING, ASSET DISTIBUTION AND GROWTH

Resumen

Tomamos en cuenta la salud en la utilidad esperada, incluyendo la probabilidad de morir y un índice de calidad de vida. Esto implica que las tasas de preferencia intertemporal sean endógenas, y el funcional de utilidad intertemporalmente consistente no cóncavo. Cuando son bajos los salarios y las tasas de interés, la solución óptima es diferente para los pobres, quienes desahorrarán hasta un límite dado por una restricción de crédito, que para los ricos, que sí ahorrarán. La distribución de la riqueza se bifurca, frenando el crecimiento. Son posibles trampas de pobreza en que el ingreso crece a una tasa mayor para los ricos que para los pobres, si nuevas las tecnologías requieren inversión por parte de los participantes, como en el caso del capital humano. Estas trampas no serían detectadas por las pruebas de convergencia usuales.

Abstract

We account for health in expected utility, including the probability of death and a quality of life index. This makes time preference rates endogenous, and time-consistent utility non-concave. When wages and interest rates are low, the optimal solution is different for the poor, who will dissave to a credit constraint, than for the rich, who will save. The income distribution bifurcates, slowing growth. Poverty traps are possible in which income grows faster for the rich than for the poor, if new technologies substituting older technologies require investment from the participants. These would not be detected by convergence tests.

Introduction

How can one explain, in a way useful to the theory of economic growth and development, that the poor do not save, or do not save much? Such behavior can be observed through prolonged periods in the process of economic development. It led the classics to maintain that savings originate from profits and not from the general population. In middle-income Latin American countries, it implies that the majority of the population does not keep a bank account. When the question is posed to the layman, the answer seems straightforward: food and clothing, as well as health and children's needs have a greater priority than saving, quite independently of variations in personal preferences. In other words, there are needs, such as nutrition, whose urgency makes postponing their satisfaction an undesirable choice. It turns out that if we take health and well-being into account when calculating expected values of utility, intertemporal discount rates are endogenized and this viewpoint can be formalized.

The study of the impact of health on economic growth gained impulse in a Nobel prize winning series of works by Fogel (1991, 1992, 1994[a], 1994[b]), who finds that nutrition plays a very important role in productivity increases in the long-term history of economic development. This work led to more recent study on the ways in which health interacts with the economy. Amongst these is the micro-economic study of health as investment in human capital, integrated with the study of education, community, gender, and public health programs (see Schultz, 1997 for one paper in an increasingly wide leterature). Working on economic growth, Knowles and Owen (1995, 1997) include health capital in Mankiw, Romer and Weil's (1992) augmented Solow model, finding health indicators to be more significant then education. Life expectancy is found to promote growth by Barro (1996) and Bloom (1998) in cross-country growth regressions, while Mayer (1998) finds working-age male mortality and fertility-age female mortality promotes growth with 15 to 20 year lags in Mexico over the period 1950-1995. Health may extend the horizon of human capital investment, as in Barro's (1996) tentative growth model incorporating health, which considers a health-dependent depreciation rate for human capital.

Here we are concerned with a theoretical study of the impact of health on saving. An important difference between health and education as forms of human capital is that it is unrealistic to consider that health's role as productive human capital is its main economic function, when in fact its "consumption" role generating human well-being may be more important. Empirical studies (Floud, et al., 1984, 1990, 1996; Pritchett et al., 1996; Steckel, 1995, 1997) have confirmed that health indicators are excellent for representing the kind of well-being that income and consumption indices measure. Indeed, if with reference to an economic growth model a single representative economic good had to be chosen, it is likely that 'health' could be a better choice than some composite basket of consumption goods. It is not hard to think that goods are consumed to produce the best possible states of health and well-being, and that when these states are worse rather than better there is higher malnutrition, discapacity, mortality and so on. If we take these effects into account in expected utility, the agent's basic problem can now be posed as, what is the optimal intertemporal allocation of health and well-being? The difference that the introduction of health makes is that the optimization problem is no longer concave. The behavior of agents bifurcates. At any given interest rate r, there will be a level of health h at which the sum of the intertemporal discount rate ρ with the sum of the endogenous rates of instantaneous mortality $p(h)$ and quality of life loss $q(h)$ (including discapacity adjustment, explained below) equal r. This level of health h corresponds to a level of wealth under which agents will dissave and above which agents will accumulate.

For simplicity, we shall not distinguish between physical and human capital accumulation. Thus productive assets will represent both. Also, we shall abstract from the role of health as productive capital. Nevertheless, since health is best considered as a state of being which is maintained by nutrition, housing, medical care, etc., we shall consider health h as a state variable measuring instantaneous well-being. This implies that health functions as a stock variable producing a flow of utility and which is maintained by consumption, in the absence of which it tends to decay. Thus in our model health functions as a capital with a natural rate of depreciation, while consumption takes the role of investment in health capital.

When expected utility takes health into account in a time-consistent manner, the utility preference functional takes on a well-known form with variable time preference rates (see below). Using these utility functionals we solve a Ramsey-type economic growth problem. The main result is that in the presence of heterogeneous agents with differing wealth, there are two types of solutions. Above a certain threshold of health or wealth families will have an incentive to save and accumulate. Below this threshold it will be optimal to borrow to advance consumption and health. We shall introduce a credit restriction for completeness and to exclude indefinite indebtedness. The results imply the following income distribution dynamics. Under certain circumstances a closed economy may divide into two classes: those who can save and accumulate, and those who do not save but consume their wages directly. If eventually wages increase enough then the lower class will begin to save.

Our results imply that poor families do not save, while rich families do. Such a situation has been studied from other points of view. Galor and Zeira (1993) and Bourguignon (1997) suppose that the poor effectively face lower interest rates than the rich. Of course, such an effect would have to be added to the one we study. Changes in intertemporal rates of substitution have also been considered. For example, the elasticity of marginal utility is decreasing in the Stone (1954) and Geary (1951) model. However these do not affect the sign of the saving rate. Together, these approaches study the possible endogeneity of the growth rate $\gamma = \frac{r-\rho}{\sigma}$. Increasing returns to scale can also be considered, as in Freeman (1996). Here we focus on endogenous discount rates *resulting from the consideration of health*, in which families at lower states of well-being are effectively more impatient than families who are better off.

We give some closed economy results with heterogeneous agents without endogenous discount rates. However, it is too complicated to extend them to endogenous discount rates. Instead we show that when for whatever reason the poor do not save below some level of assets, if there are two classes of technologies available for production, the first with capital and labor, and the second with physical and human capital as factors, wages may not rise, even if the economy (and savers' income) is growing. In this case a poverty trap, or income distribution trap arises in which, within a closed economy, the income of the poor remains constant while the income of the rich grows exponentially. We show that this divergence in incomes would not be detected by the usual convergence regressions.

The results relate to the literature on inequality and growth. In recent studies, Birdsall and Sabot (1995) and Birdsall and Londoño (1997) show that weak human capital accumulation and its unequal distribution are amongst the causes of these problems. They discuss World Bank policies, finding that they do not sufficiently take into account the effect of inequity in the distribution of human capital on growth, and that they have not had a significant impact on poverty. Alesina and Rodrik (1994) and Persson and Tabellini (1994) find a negative effect of inequality on growth in cross-country studies. Perotti (1992, 1994, 1996) tests the main theories. Benabou (1997) cites 23 empirical studies of the relation between inequality and growth, which consistently find that initial inequality is detrimental to long-term growth. Our model gives a new explanation of how inequality slows growth, and its distribution dynamics give an explanation for the dark side of Kuznets' inverted U-curve.

Some additional results emerge from our model. Modelling consumption as investment in health assets implies (independently of time preferences) that where there are more poor the aggregate "consumption function" for transitory increments in income is more Keynesian while where there are less it conforms more to the permanent income life-cycle hypothesis. This predicts a change of the nature of the consumption function, and therefore of stability to economic fluctuations, with economic development. Savers will also present a marginal propensity to immediately consume permanent changes in wealth, since they will invest in well-being when wealth increases. Non-savers will consume any extra wealth immediately.

The plan of the paper is as follows. We first derive the appropriate utility functional in which health intervenes by determining the probability of death, and we expand this conception to include loss of quality of life. Next we pose the corresponding growth model, defining the state equations for health. Before discussing the full solution we consider for purposes of comparison an intertemporal optimization problem with preferences taken over health but with fixed intertemporal discount rates. Behavior is similar to the usual Ramsey model, except that families will not save until they reach a minimum level of well-being, and during this time their relative asset endowment worsens. However, the duration of this phenomenon is measured in years rather than decades. We show that the closed economy problem also behaves similarly, and construct the aggregate propensity to consume. Next we solve the intertemporal optimization problem with endogenous discount rates depending on health. We apply it in a technological context showing the possibility of income traps, and discuss its implications on income distribution dynamics and on the effect of inequality on growth.

The intertemporal utility functional

In our model dynasties form their preferences over *paths of health* $h(t)$ rather than paths of consumption $c(t)$. Thus they maximize

$$U[h] = \int_0^\infty E\left[u(h(t))\right] e^{(n-\rho)t} dt = \int_0^\infty P(t) u(h) e^{(n-\rho)t} dt \tag{1}$$

where $P(t)$ is the probability of being alive at time t, ρ is the intertemporal discount rate and n is the population growth rate, assumed to be constant throughout the paper. In their finite horizon models, Yaari (1965) and Blanchard (1985) work with this functional, with $P(t) = e^{-pt}$, where p is a constant instantaneous probability of death. In our case, we let p depends on health and therefore on welfare.

Let the instantaneous probability of death be $p(h(t))$, a decreasing function of the present level of health. It can be shown that $P(t)$, the probability of being alive at time t, satisfies the differential equation

$$\frac{dP}{dt}(t) = -p(h(t))P(t). \tag{2}$$

Now, instead of $P(t)$ we can introduce $P(t)Q(t)$ where, besides a probability of death, we include a quality of life index Q satisfying $0 \leq Q \leq 1$, which includes discapacity adjustment.[1] For example, having lost a leg or suffered from malnutrition, the same level of instantaneous health h (afforded by a steady consumption c) will yield now a diminished utility $Qu(h)$. Introducing a the differential equation analogous to (2),

$$\frac{dQ}{dt}(t) = -q(h(t))Q(t). \tag{3}$$

$q(h(t))$ represents an endogenous instantaneous rate of loss of quality of life. Putting mortality and quality of life loss rates together by defining

$$\phi(h) = p(h) + q(h), \tag{4}$$

and substituting in (1), the utility functional maximized by the agents is

$$U[h] = \int_0^\infty u(h(t)) e^{-(\rho-n)t} \exp[-\textstyle\int_0^t \phi(h(s))ds] dt. \tag{5}$$

The assumptions on ϕ are

$$\phi \geq 0, \lim_{h\to\infty} \phi = 0, \phi' \leq 0, \phi'' \geq 0, \lim_{h\to\infty} \phi' = p_0 \geq 0. \tag{6}$$

where p_0 is now a minimum instantaneous probability of death. We assume throughout that

$$\rho + p_0 > n, \tag{7}$$

[1] Ideas such as these are implemented in QALYs (Quality Adjusted Life Years; see Spilker, 1996 for a bibliographical reference) and DALYs (Disability Adjusted Life Years; see Murray, 1996, for a discussion), which have been introduced in the search for health indices appropriate for resource allocation, and which attempt to put being alive, disable or dead on one scale. In our case this scale is implicit in the use of expected utility, which sets the utility of being dead at zero.

for the utility functional (1) to be finite.

To get an idea of the scale of these effects, let us compare three individuals in the prime of their working life at 30. Suppose their life expectancies are 50, 60 and 80. Concomitantly let us suppose that individuals expect that if they are alive at their expected age of death, their quality of life will be 0.4. Using a model with constant p and q for simplicity, the contribution of mortality and life quality to the discount rates are shown in Table I. To these rates a time preference rate ρ can be added. It is clear that these discount rates are most sensitive at the poor end of the wealth spectrum. However, once inequality is present, a small difference in discount rates can maintain it.

Table I. Simple estimates of contributions of mortality and life quality loss rates to endogenous discount rate (percentage points)

Life Expectancy at 30	50	60	80
Mortality rate contribution	5.0	3.3	2.0
Life quality loss rate contribution	4.6	3.1	1.8
Total contributions	9.6	6.4	3.8

These heuristic calculations are only meant to show that the effects are plausible. Nevertheless, that the probability of being alive in the future may diminish with present deprivation is well documented. For example, Bidani and Ravallion (1997) find in a cross-country study that people with an income below US$2 per day have a life expectancy nine years shorter than those above this income level (which still includes a lot of poor). Realistic calculations would need to take into account epidemiology, mortality by ages, quality of life or discapacity indices, treatment costs and income.

We now comment on the nature of the utility functional with endogenous time preference (5). The concept of time preference originates with Boehm-Bawerk and Fisher (1907, 1930), who originates in his theory of the interest rate the idea that discount rates may depend on wealth. Formalized as the theory of recursive utility, Koopmans (1960) and others (Beals and Koopmans, 1969; Uzawa, 1968) show by assuming limited non-complementarity over time that welfare functions exist with variable time preference rates. The theory is presented extensively in Uzawa (1988). In the continuous case, a typical recursive utility functional is given by (5) with h replaced by c. The function $\phi(h)$ represents an instantaneous discount rate due to health. The discount rate between two moments of time t_1, t_2 is the average of the instantaneous discount rates $\phi(h(s))$ for $t_1 \leq s \leq t_2$.

There are two strands to the theory, corresponding to the sign of ϕ'. In the first, the rich are more impatient than the poor, while in the second the reverse holds. The justification for the first position (followed by the authors mentioned above except for Fisher) is that an increase in future consumption will give more weight to present consumption (Epstein, 1987). This strand of the theory is mathematically somewhat simpler since it gives rise to a concave functional and therefore unique equilibria, and has been developed extensively; see for example Becker et al (1989) and Becker and Boyd (1992), who study optimal accumulation paths with multiple capital goods, and Joshi (1995), who introduces uncertainty. The second line, in which the poor are more impatient than the rich, has been studied theoretically in the context of capital accumulation and growth by Mantel (1995). This line finds empirical backing in Lawrance (1991), who attempts to estimate the effect of income on time preferences in the US by using panel data. Though measurement error is a problem due to the delicate nature of these calculations, she concludes that there are considerable differences in the intertemporal preferences of rich and poor households: "...three to five percentage points (...). Controlling for race and education widens this difference (...) from 12 percent for white, college-educated families in the top 5 percent of the labor income distribution to 19 percent for non-white families without an education whose labor incomes are in the bottom fifth percentile." These differences are even bigger than those in Table I. Fukao and Hamada (1989) combine the two strands of endogenous time preferences, studying the evolution of capital ownership under the supposition that the poor and the very rich are more impatient than those in between.

We have fundamented an endogenous time preference in which health and well-being considerations make the poor more impatient than the rich. Although we do not model them, other phenomena can also be thought to reduce the planning horizon of the poor, such as increased uncertainty (relative to wealth), indivisibility of goods, transaction costs, etc. Some of these may be thought to be included in ϕ.

Intertemporal optimization of health and well-being

We begin by assuming that each dynasty's per-capita assets, which may include physical and human capital, obeys the usual equation

$$\dot{a} = (r - n)a + w - c, \tag{8}$$

where a are assets per capita, w are wages and c is consumption. The role of consumption is to raise the level of health and well-being level, off-setting its natural tendency to diminish and decay. Thus, we set

$$\dot{h} = -(\varkappa + n)h - \varpi + c. \tag{9}$$

Here $\varkappa$ is the rate of decay of health (for individuals), and ϖ an additional absolute rate of decrease. It will represent a minimum level of consumption. Although we could consider that some forms of consumption could have an additional, direct effect on utility, for example by writing $u = u(h, c_2)$ (but not $\phi = \phi(h, c_2)$ since that would mean that the instantaneous discount rate would depend on consumption decisions), for simplicity our agents form their preferences over h only.

Some health assets h are non-tradeable, such as health per se and nutrition. Others are tradeable in that they are the direct result of maintaining a stock of semi-durable and durable goods which can be sold. At the extreme of durability, houses have traditionally been treated as productive capital yielding utility. From our point of view, they can be treated in either way, as productive capital (assets which can yield a stream of consumption) or as non-productive capital (generating well-being directly). Any assets considered as productive capital enter the production function. For simplicity we have excluded, however, effects that well-being may have on production (e.g. health in human capital models).

We have thus arrived at the specification of the family's optimization problem,

Problem 1

$$\begin{array}{ll} \max\limits_{a(0),h(0),c(t)} & U(a(0),h(0),d(0)) = \int_0^\infty u(h)e^{-d}dt \\ s.t. & \dot{a} = (r-n)a + w - c \\ & \dot{h} = -(\varkappa + n)h - \varpi + c \\ & \dot{d} = \phi(h) + \rho - n \\ & a \geq a_{\min},\ h \geq 0 \\ & a(0) + h(0) = R_0,\ d(0) = d_0 \\ & \lim_{t\to\infty}(a+h)e^{-\bar{r}t} = 0 \end{array} \tag{10}$$

Here $\phi(h) + \rho$ is the instantaneous discount rate at any time. We have set

$$d(t) = (\rho - n)t + \int_0^t \phi(h(s))ds \tag{11}$$

so that the formulation of the optimization problem fits the usual general form. Thus d is a state variable of our problem, and its differential equation is the derivative of (11). The problem is invariant to the initial condition d_0, which multiplies the utility functional by a constant. Nevertheless it will be useful to retain this parameter. Since an increase in d causes a decrease in utility, its multiplier will be negative. We prefer to keep this unusual sign and retain the usual notation e^{-d} rather than writing e^d with d tending to $-\infty$ for discounts.

Problem 1 is dynamically consistent since the integral of d essentially only depends on future considerations, with the past only entering as a constant factor to which the maximization problem is invariant. Thus there is no incentive later to change any decision taken now about the future, since the problem faced later is not different to the one considered from the present time.

Let us go through some of the details of Problem 1. The constraint $h \geq 0$ sets a lower bound on levels of health. We shall find that dissavers find it optimal to reach $h = 0$ in finite time, if they are not credit constrained. Thus to be able to exclude exponential indebtedness to infinity or health being run down to zero, we have introduced a credit restriction $a \geq a_{\min}$, where we assume $a_{\min} \leq 0$.

For simplicity we have introduced no investment function in h (its state equation is linear in c). Therefore wealth can jump between a and h, so in effect we are abstracting from some types of dynamics. The assumption is equivalent to supposing that the adjustment between a and h is fast relative to the accumulation of capital. Such jumps will be shown to occur at $t = 0$, so that balance between a and h is attained instantly (which is why $a(0)$ and $h(0)$ appear as optimization variables) and what matters is the initial asset constraint. Let the total assets of the dynasty be

$$R = a + h. \tag{12}$$

The initial assets constraint is represented by $a(0) + h(0) = R_0$. The transversality condition is also stated in terms of the total assets R.

In contrast to the usual Ramsey problem the addition of a constant modifies the utility function, so that the absolute level of u utility matters, as does the level of ϕ.

Before solving this relatively complicated problem, we solve two simpler ones for comparison purposes and to clarify our understanding. These are an finite-horizon intertemporal optimization problem with preferences over health but without endogenous time preferences (Problem 2), and a typical Ramsey-type problem with preferences over consumption and a minimum consumption requirement (Problem 3).

Problem 2

$$\begin{array}{ll} \max\limits_{a(0),h(0),c(t)} & U[h] = \int_0^\infty u(h) e^{(n-\rho-p_0)t} dt \\ s.t. & \dot{a} = (r-n)a + w - c \\ & \dot{h} = -(\varkappa + n)h - \varpi + c \\ & a(0) + h(0) = R_0, \; a \geq a_{\min} \end{array} \tag{13}$$

Problem 3

$$\begin{array}{ll} \max\limits_{c(t)} & U[c] = \int_0^\infty u(c - \varpi) e^{(n-\rho-p_0)t} dt \\ s.t. & \dot{a} = (r-n)a + w - c \\ & a(0) = a_0, \; a \geq a_{\min}, \; c \geq \varpi \end{array} \tag{14}$$

p_0 represents a constant instantaneous probability of death. All of the nonstandard features in Problem 2 have been explained for Problem 1. We have introduced a term ϖ (and a redundant consistency restriction $c \geq \varpi$) in Problem 3 following Stone (1954) and Geary (1951), to force a minimum consumption level in the case when u satisfies the Inada conditions at 0. This brings Problem 3 closer to Problem 2, in which the term ϖ induces this minimum consumption naturally. The transversality condition for Problem 3 is $\lim_{t\to\infty} a e^{-(\bar{r}-n)t} = 0$, where $\bar{r} = \frac{1}{t}\int_0^t r(s)ds$. For Problem 2 we ask instead that $\lim_{t\to\infty} R e^{-(\bar{r}-n)t} = 0$ (nothing would change if we retained the original condition, but this one is more natural). We shall assume the viability condition

$$(r-n)a_{\min} + w - \varpi > 0, \tag{15}$$

which means that a family at the credit constraint can sustain positive consumption on the basis of its wages.

The solutions for fixed time preferences

We begin with Problem 2. We use the usual abbreviation $\gamma = \frac{r-\rho-p_0}{\sigma}$, and suppose throughout, when assuming an exogenous rate of interest, that $r < \frac{\rho+p_0-n\sigma}{1-\sigma}$ so that $\gamma < r - n$. This is the usual assumption for the utility functional (1) to be finite.

Theorem 1 *Consider Problem 2 and suppose that r and w are exogenous and constant. There are two types of solutions, according to whether the family is credit con-*

strained or not. Unconstrained solutions (Type 1) are given by

$$a = \frac{\varkappa + n + \gamma}{r - n - \gamma} h - \frac{w - \varpi}{r - n}, \; h = h_0 e^{\gamma t}, \; c = (\varkappa + n + \gamma) h + \varpi, \tag{16}$$

where $h_0 = \frac{r-n-\gamma}{r+\varkappa}\left[R_0 + \frac{w-\varpi}{r-n}\right]$. *The level of assets R below which agents are initially constrained is*

$$R_{\min} = \frac{r + \varkappa}{\varkappa + n + \gamma} a_{\min} + \frac{(r - n - \gamma)(w - \varpi)}{(\varkappa + n + \gamma)(r - n)}. \tag{17}$$

Constrained (Type 2) solutions are given by

$$a = a_{\min}, \; h = h_{\text{eq}} + (h_0 - h_{\text{eq}}) e^{-(\varkappa + n)t}, \; c = (r - n) a_{\min} + w, \tag{18}$$

where $h_{\text{eq}} = \frac{(r-n)a_{\min} + w - \varpi}{\varkappa + n}$ *is the equilibrium level of h and* $h_0 = R_0 - a_{\min}$. *The corresponding constrained asset equilibrium level is* $R_{\text{eq}} = h_{\text{eq}} + a_{\min}$. *We have*

$$R_{\text{eq}} > R_{\min} \Leftrightarrow \frac{\gamma((r - n) a_{\min} + w - \varpi)}{(\varkappa + n)(\varkappa + n + \gamma)} > 0. \tag{19}$$

Families evolve between the two types of solutions as follows.

Case 1: $\gamma > 0$. *If* $R_0 \geq R_{\min}$, *the family will follow an unconstrained solution, while if* $R_0 < R_{\min}$, *the family will initially be constrained, but after a finite time will begin saving.*

Case 2: $\gamma = 0$. *If* $R_0 \geq R_{\min}$, *the family is unconstrained and has constant R, while if* $R_0 < R_{\min}$, *it is constrained and R tends to* $R_{\text{eq}} = R_{\min}$ *in infinite time.*

Case 3: $\gamma < 0$. $R_0 > R_{\min}$, *initially the family will dissave, following an unconstrained solution, while if* $R_0 \leq R_{\min}$ *initially it will be constrained. Savers will eventually be constrained if* $R_{\text{eq}} < R_{\min}$, *which holds if* $\gamma < -(\varkappa + n)$, *and non-savers will eventually save if* $R_{\text{eq}} > R_{\min}$, *which holds if* $\gamma > -(\varkappa + n)$.■

For purposes of comparison, we give the solution to Problem 3. All proofs (and some additional results) are in the appendix.

Theorem 2 *Consider Problem 3 and suppose that r and w are exogenous and constant.*

Case 1: $\gamma > 0$. *The family will follow an unconstrained solution given by*

$$a = -\frac{w - \varpi}{r - n} + \left(a_0 + \frac{w - \varpi}{r - n}\right) e^{\gamma t}, \; c = \varpi + (c_0 - \varpi) e^{\gamma t}, \tag{20}$$

with $c_0 = (r - n - \gamma)(a_0 + \frac{w-\varpi}{r-n}) + \varpi$.

Case 2: $\gamma = 0$. *The two types of solution are identical, and will be followed by any family.*

Case 3: $\gamma < 0$. *If* $a > a_{\min}$ *the family will dissave, following the unconstrained solution of equation (20) until* $a = a_{\min}$, *and then will follow the constrained solution*

$$a = a_{\min}, c = (r - n) a_{\min} + w. \tag{21}$$

If $a = a_{\min}$ *initially then the family will follow the constrained solution from the beginning.*■

The main contrasts between the solutions to Problems 2 and 3 are the following. Firstly, the introduction of the well-being state variable h has resulted in some non-saving results. Since agents optimize an investment portfolio containing health and assets, when the asset stock h is too low, it must be increased before investment will occur in productive capital. In the case of economic growth, when $\gamma > 0$, modelling families as solving Problem 3 implies they all save, while modelling families as solving Problem 2 implies they will begin to save only after they have reached a certain minimum level of total wealth $R_{\min}$. Secondly, in the case $\gamma = 0$, constrained families are not identical but differ in their levels of well-being, although this converges. Third, parameters such as $\varkappa$ and ϖ have been added naturally into consideration, affecting the equilibrium levels and rates of convergence of the system.

The non-saving results are relatively weak because the exponent governing the exponential convergence to saving is $\varkappa$, which is the exponent governing the decay of individual well-being in state equation (9) for h. Therefore, the decay represented by $\varkappa$ cannot have a half-time longer than a fraction of an individual's lifetime. This does not explain non-saving behavior observed through prolonged periods in the process of economic development.

The propensity of the poor to consume will be high once they reach the credit constraint. Non-savers, who consume their full income, will spend any additional income immediately, while savers will increase h immediately but also postpone some consumption. Suppose there is a small unexpected permanent increase in wealth ΔR_0 at $t = 0$ and let $C = (n_1c_1 + n_2c_2)N$ be the aggregate consumption. Differentiating equations (16) and (18) with respect to R_0, there will be a jump in h (attained by an instantaneous burst in consumption), and a corresponding increase in permanent consumption, so

$$\begin{aligned} \Delta C|_{t=0} &= \Delta h_0 = [\frac{r-n-\gamma}{r+\varkappa}n_1 + n_2]\Delta R_0, \\ \Delta C(t) &= (\varkappa + n + \gamma)\frac{r-n-\gamma}{r+\varkappa}e^{\gamma t}n_1\Delta R_0. \end{aligned} \tag{22}$$

The aggregate marginal propensity for immediate consumption is $\frac{r-n-\gamma}{r+\varkappa}n_1 + n_2$, which is between zero and one. Thus for permanent changes in wealth the effect on consumption is a mixture of what the permanent income hypothesis and the Keynesian aggregate consumption function predict, even for the non-constrained families, because they increase their capital h immediately. Suppose instead there is a small transitory increase in wealth, that is, a fluctuation in income with zero net effect in wealth, which we may represent as Δw with $\int_0^\infty \Delta w e^{-(\bar{r}-n)t}dt = 0$. This will not affect the rich, while the poor, who are constrained, will transmit through consumption the change of wealth to their assets h, so

$$\Delta C(t) = n_2 \Delta w(t). \tag{23}$$

Now the aggregate marginal propensity to consume is n_2 and the aggregate consumption function is Keynesian. Thus our model predicts that economies with non-savers have a Keynesian aggregate consumption function with propensity to consume proportional to their number, while the propensity to consume is larger for permanent than for temporary changes in income, because of the increased investment in h.[2]

Let us now give some closed economy results relating to Problem 2. Thus, we suppose that there is a production function $F(K, L)$ and that the aggregate of the family assets a equals K. Let the rate of depreciation of capital be δ. To observe non-saving behavior we must model inequality of distribution, for if all families are equal, then they must own assets and the interest rate must rise to a level at which there is an incentive to save. Thus we consider the dynamics of Problem 2 for a closed economy consisting of two sets of identical families, in one of which families are more wealthy than in the other. Suppose that each set of families grows at the same rate n and that the proportion of families in each set is n_1 and n_2 respectively ($n_1 + n_2 = 1$), while the total population is N. Let a_i, h_i, R_i $i = 1, 2$, represent the variables corresponding to families in the first and second sets respectively, and suppose that $R_{10} > R_{20}$ (the first set of families has a higher initial wealth). Let $g_i = (r + \varkappa)^{1/\sigma} h_i$, and define $a = n_1 a_1 + n_2 a_2$, and similarly h, R and g. The aggregate capital per capita in the economy is $k = a$, $\gamma = \frac{r-\rho-p_0}{\sigma}$ as before.

Theorem 3 *Consider Problem 2 for a closed economy (r and w are endogenous).*

(1) Suppose first that all families are saving. Introducing the change of variables $g = (r + \varkappa)^{1/\sigma} h$, we obtain the system of simultaneous equations

$$\frac{\dot{g}}{g} = \gamma, \tag{24}$$

$$\left\{1 - \frac{1}{\sigma} f''(k)(r + \varkappa)^{-\frac{1+\sigma}{\sigma}} g\right\} \dot{k} = f(k) - (\delta + n)k$$
$$-(\varkappa + n + \gamma)(r + \varkappa)^{-1/\sigma} g - \varpi. \tag{25}$$

where $r = f'(k) - \delta$. The loci of $\dot{g} = 0$ and $\dot{k} = 0$ in the (k, g) plane are given by:

$$\dot{g} = 0 \Leftrightarrow k = k^*, \text{ where } f'(k^*) = \rho + p_0, \tag{26}$$

$$\dot{k} = 0 \Leftrightarrow g = \frac{\sigma (r + \varkappa)^{1/\sigma} (f(k) - (\delta + n)k - \varpi)}{r + \sigma(\varkappa + n) - \rho + p_0} \tag{27}$$

The phase-diagram is of the type of the usual Ramsey diagram (see Figures 1a and 1b) except for a possible asymptote for k in the case when $\sigma(\varkappa + n) < \rho + p_0$, in which case k does not go beyond $k_{\max}$ (where $f'(k_{\max}) = \rho + p_0 - \sigma(\varkappa + n)$) even when h becomes large. However, the qualitative behavior of the solutions is unaffected by this asymptote.

(2) Suppose now that not all families save. Then only the families in the second set do not save (since there must be positive assets in the economy). The per-capita amount of capital in the economy is $k = n_1 a_1 + n_2 a_{\min}$. Two equations describe the

[2] Using the Yaari (1965) and Blanchard (1985) finite horizon model in a monetary economy with Keynesian unemployment, Rankin and Scalera (1995) find that a positive probability of death increases the short- and long-run multipliers of government spending. This will also hold in our model, if more families become non-savers.

variables of the first set of families, equation (24) with g_1 instead of g, and

$$\left(\frac{1}{n_1} - \frac{1}{\sigma} f''(k)(r + \varkappa)^{-(1+\sigma)/\sigma} g_1\right) \dot{k} = f(k) + \left(\frac{n_2}{n_1} r - \frac{1}{n_1} n - \delta\right) k$$
$$-(\varkappa + n + \gamma)(r + \varkappa)^{-1/\sigma} g_1 - \varpi. \qquad (28)$$

The behavior of families in set 2, who are not saving, is described by

$$a_2 = a_{\min}, \dot{h}_2 = -(\varkappa + n)h_2 + (r - n)a_{\min} + w - \varpi. \blacksquare \qquad (29)$$

One important feature of the solution when distribution is not equal is that while the second set of families does not save, *the relative distribution of real assets worsens*, since a_1 grows while a_2 remains at $a_{\min}$. Another feature is that distribution affects wealth in the following sense. Suppose the number of non-saving families increases while the number of saving families remains unchanged. Then there will be a higher demand for capital, so in the closed economy interest rates will be higher and the saving families will become wealthier. Thus *families with the same initial wealth will become richer in poorer societies*.

Theorem 3 allows us to understand the nature of the solution of the system in the case of unequal distribution by using first one and then another Ramsey-type phase diagram. Near the equilibrium behavior is governed by the phase diagrams in Figures 1a or 1b. If some non-saving behavior occurs for the case of a growing economy, the growth path will first be governed by a similar phase diagram, derived using (28) instead of (25). If the economy begins with a suboptimal level of capital, a trajectory staring near the bottom left-hand corner will be chosen which reaches the corresponding trajectory in Figures 1a or 1b at a point at which $R_2 = R_{\min}$. The g_1 axis is proportional to the g axis, with the constant of proportionality also depending on the solution to the problem. This procedure does not represent a full graphical solution since the identities which make the two graphs fit together are not obtained graphically, but it does give a good qualitative idea of the solutions. The system has four equations, and detailed comparisons of distribution and rate of convergence along the trajectories would require further analysis or a numerical study. Since the credit restriction implies less investment in non-productive assets h, lifting it should slow the transition to equilibrium but yields a Pareto increase in well-being.

Notice that linearization at the steady state would not capture non-saving behavior, since by the time steady state is reached all families are saving.

The solutions with endogenous time preferences

Before solving Problem 1 we must state some additional assumptions regarding the relationship between functions u and ϕ. The utility function $u(h)$ is defined on some interval $[0, \infty)$ on which it satisfies the usual conditions $u(0) = 0$, $u \geq 0$, $u' > 0$, $u'' < 0$, and $\lim_{h \to \infty} u' = 0$ (we shall not require the remaining Inada condition $u'(0) = \infty$). We have already stated the basic properties of ϕ in (6). Besides, we shall need the assumption that as agents become very rich ϕ changes slower than the rate of change of u, as we expect of the behavior of time preference. This takes the following form.

$$\text{Let } \Xi = -\frac{\phi' u}{u'} > 0. \text{ Then } \lim_{h\to\infty} \Xi = 0. \tag{30}$$

Write

$$\sigma(h) = -\frac{hu''}{u'}, \theta(h) = -\frac{h\phi''}{\phi'} \tag{31}$$

for the respective elasticities. Examples of functions satisfying (30) are

$$u = \frac{h^{1-\sigma_0} - h_{\min}^{1-\sigma_0}}{1-\sigma_0}, \phi = p_0 + \rho_1 \frac{h^{1-\theta_0}}{\theta_0 - 1} \tag{32}$$

for any $h \in [h_{\min}, \infty)$, where $\theta_0 > \max\{1, \sigma_0\}$, $h_{\min}$ is chosen below empirically relevant values and u and ϕ are extended appropriately on $[0, h_{\min}]$. For these,

$$\lim_{h\to\infty} \Xi = \lim_{h\to\infty} \rho_1 h^{\sigma_0 - \theta_0} \frac{h^{1-\sigma_0} - h_0^{1-\sigma_0}}{1-\sigma_0} = 0. \tag{33}$$

As we shall find below, along an unconstrained solution of Problem 1 h is chosen so that the marginal benefits of an extra unit of investment in a or h are equal. We shall find that the corresponding equation is

$$(r + \varkappa)\mu = u' e^{-d} + \phi' \nu, \tag{34}$$

where μ and ν (which is negative) are the multipliers of h and d. On the left and right hand sides are the marginal benefits of investment in a (lost interest plus saved health depreciation) and h (marginal utility of health plus marginal utility due to decreased mortality and disability) respectively. We shall find it useful to work with a change of variables in which we introduce

$$\chi = \frac{u' e^{-d}}{(r + \varkappa)\mu}, \tag{35}$$

the proportion of the benefits which is marginal utility of health. Recall definition (30) for Ξ. We shall also use the shorthand

$$\Psi = \phi + \rho + \Xi, \tag{36}$$

for a quantity which will play the role of a discount rate. Observe that

$$\Psi' = \phi' - \frac{u'(u'\phi' + u\phi'') - u\phi' u''}{u'^2} = \frac{\sigma - \theta}{h}\Xi \tag{37}$$

is negative in the example (32) given above.

Write $\tilde{h}(t; h_0)$ for the constrained agent's solution (18) for h. We shall need the auxiliary function $V(h_0, t)$ representing the utility of this health trajectory,

$$V(h_0, t) = \int_0^t u(\tilde{h}(t; h_0)) \exp[-\textstyle\int_0^t \phi(\tilde{h}(s; h_0)) ds] dt. \tag{38}$$

We write $V(h_0)$ for $V(h_0, \infty)$.

Theorem 4 *Consider Problem 1. (1) Unconstrained (Type 1) solutions satisfy the sys-*

tem of differential equations

$$\frac{\dot{h}}{h} = \frac{r - n - \chi(\Psi - n) - \dfrac{\dot{r}}{r + \varkappa}}{\chi\sigma + (1 - \chi)\theta}, \tag{39}$$

$$\dot{\chi} = \left[r - \sigma\frac{\dot{h}}{h} - \phi - \rho - \frac{\dot{r}}{r + \varkappa} \right] \chi, \tag{40}$$

$$\dot{R} = (r - n)R - (r + \varkappa)h + w - \varpi. \tag{41}$$

and the constraints $0 \leq \chi \leq 1$. *If* $h \to \infty$, *then* $\lim_{t\to\infty} Re^{-\bar{r}t} = 0,\ \lim_{t\to\infty} \chi = 1$.

(2) Suppose that r *and* w *are exogenous and constant, with* $r < \frac{\rho + p_0 - n\sigma}{1-\sigma}$ *as before. A phase diagram (see Figures 2a and 2b) can be constructed for subsystem (39), (40). The loci of* $\dot{h} = 0$ *and* $\dot{\chi} = 0$ *are given by*

$$\dot{h} = 0 \Leftrightarrow \chi(h, r) = \frac{r - n}{\Psi - n}, \tag{42}$$

$$\dot{\chi} = 0 \Leftrightarrow \chi(h, r) = \frac{W}{W + \Xi}. \tag{43}$$

where $W = \frac{\theta}{\sigma}(\phi + \rho) + (1 - \frac{\theta}{\sigma})r - n$. *Define* (h^*, χ^*, R^*) *by*

$$h^* = \phi^{-1}(r - \rho), \tag{44}$$

$$\chi^* = \frac{r - n}{r - n + \Xi(h^*)}, \tag{45}$$

$$R^* = \frac{(r + \varkappa)h^* - w + \varpi}{r - n}. \tag{46}$$

Above h^* *the discount rate* ϕ, *which includes mortality and life quality loss, is less than* r, *so families have an incentive to save, while below they do not. The equilibrium quantities* h_{eq}, R_{eq}, h^*, R^* *satisfy*

$$R_{eq} \lesseqgtr R^* \Leftrightarrow h_{eq} \lesseqgtr h^* \Leftrightarrow a_{min} \lesseqgtr R^* - h^*. \tag{47}$$

On the plane (h, R) *the solution diagram can be of one of three types.*

Case I (see Figure 2a). Suppose $R_{eq} < R^*$. *Solutions constrained for all time run along the line* $R = h + a_{min}$ *for* $R \in [a_{min}, R_{eq}]$. *There are two branches representing optimal solutions containing unconstrained portions. The first, which is decreasing for values* $R \in [R_{eq}, R^*)$, *eventually joins the line* $R = h + a_{min}$. *The second is the stationary point* (h^*, R^*). *The third is increasing and runs unconstrained along the values* $R \in (R^*, \infty)$. *Some of its properties have been stated above.*

Case II. $R_{eq} = R^*$. *This is as in Case I, without the decreasing branch.*

Case III (see Figure 2b). $R_{eq} > R^*$. *In this case the branch tending to infinity sets off from the fully constrained solution along* $R = h + a_{min}$, *and this branch covers the values* $R \in [a_{min}, \infty)$.

In each case the branches run through the values of R *on* $[a_{min}, \infty)$. *Given an initial value* R_0, *the solution path is given by the trajectory on the corresponding branch initiating at* R_0. *Branches along which* R *is increasing (respectively decreasing) describe families who eventually save (respectively become credit constrained).*

Further properties of the solutions are the following. Along increasing unconstrained solutions, h, $R \to \infty$, $\chi \to 1$, the growth rates of h and R tend to γ_∞ and $\frac{h}{R} \to \frac{r-n-\gamma_\infty}{r+\varkappa} > 0$. The linear approximation to (39), (40) abiut the stationary unstable point (h^*, χ^*, R^*) can have complex roots (as in Figures 2a and 2b). Decreasing unconstrained solutions will reach the credit constraint in finite time, and in its absence, the constraint $h = 0$. If a solution is first unconstrained and then constrained, R is decreasing and the solutions switch at the curve

$$\chi = \chi_1(h) = -\frac{u'(h)}{(r+\varkappa)V'(h)}, \tag{48}$$

at a value $h \geq h_{eq}$. If instead a solution is first constrained and then unconstrained, R is increasing and the solutions join on the curve

$$\chi = \chi_2(h) = \frac{u'(h)\left[-(\varkappa+n)h + (r-n)a_{\min} + w - \varpi\right]}{(r+\varkappa)u(h)}. \tag{49}$$

Finally, d is obtained by integration.■

Observe that when the complex roots mentioned above exist, as in the examples provided in Figure 2, it can happen that families who eventually save may initially sacrifice their health and well-being, while families who eventually do not may initially choose higher levels of health and well-being, since the future will be discounted at a higher rate. This kind of behavior is only explained by endogenous preferences, and not, for example, by a dependence of interest rates on assets or by a changing elasticity of substitution σ. The model also explains that the poor often do have a reserve in the sense that they may divert their income stream to emergency uses if necessary, in effect borrowing from their level of health and well-being (such an effect is not obtained in Problem 3). Another phenomenon explained by the model is the existence of bonded peons, as in prerevolutionary Mexican Haciendas, who would on each payday borrow at high interest rates to buy basic foodstuffs, accumulating a debt that could never be repaid and was inherited by the next generation.

Since whether families eventually save or not depends on their initial wealth, the theorem implies a bifurcation in income dynamics. If the poor are not wealthy enough, families will divide into two sets, one in which they eventually save, and another in which they are eventually constrained. After some time has passed, the non-saving families will tend to approximately similar incomes, and will not have any earnings from capital. Mechanisms such as those described in Banerjee and Newman (1991) or Loury (1981), in which there exist stochastic phenomena which make families richer or poorer, could convert these distributions into continuous distributions. However, endogenous intertemporal preferences resulting from health further skew the distributions or introduce more than one peak. Only a raise in the wage level will change the non-savers into savers. This could happen, for example, when enough capital has been invested by the saving families. If this is the case, non-saving behavior will not be observed in the steady states, although it will be a feature of the transition.

It may be mentioned that qualitatively similar income distribution results can be obtained in a model in which agents choose over consumption paths using the utility functional (5) with c instead of h. Here consumption would proxy for states of well-being, and the stability of preferences would be an ex-post result of consumption smoothing rather than a clear assumption (since the instantaneous discount rate depends on the consumption decision!).

Figures 2a and 2b describe two possible states of asset dynamics which may exist in an open economy, depending on its parameters. These characterize an important difference between underdevelopment, in which people who are poor enough will not have the incentives to save, and development, in which wages are high enough for the general population to embark on life-time savings.

The closed economy problem including heterogenous agents with endogenous time preferences is difficult. It cannot be reduced to two-dimensional phase diagrams by means of changes of variables. To atempt to classify the possible equilibria or to perform a numerical study is beyond the scope of this work. However, it would not be hard to construct a stationary state in which one set of families does not save while another owns all the capital. This is conceivable even in a model with growth, so long as the poorer set of families is sufficiently behind in health to find unattractive an interest rate near the equilibrium value given by the discount rate of the wealthier families. However, we consider it more natural to model $\phi(h) = p_0$ for h greater than some $\overline{h}$, in which case once the poorer families reach $\overline{h}$ all families have the same discount rate and we are back to the usual models. To show the possibilities which can arise with endogenous discount rates, we show in the next section that under some special assumptions about technological change, which are nevertheless quite characteristic of underdevelopment, it is possible for the income of the poor and the rich to grow at different rates.

A poverty trap

In a growing economy families whose income is mainly return on capital (physical or human) will see their income grow at rate γ, while non-savers will see their income deteriorate until it reaches the floor provided by wages, and then grow at the rate that wages grow. Thus the crucial question becomes whether the saving and investment of the better-off segments of the population will result in a general increase of wages, raising the income of the poor enough for them to begin saving. Once this happens, their relative income will improve. How long wages take to increase will thus be a factor determining qualitative changes in income distribution.

The purpose of this section is to show that if two sets of technologies coexist, one which is capital intensive and requires investment on the part of the participants (e.g. in human capital), and the other based on capital and labor, it is possible that wages will not rise, so that the income of the segment of the population who saves will grow exponentially, while non-savers' income will remain constant. Let

$$F(K_1, K_2, L) = A_1 K_1 + A_2 K_2^{\alpha} L^{1-\alpha}. \tag{50}$$

F is a production function consisting of two technologies which are perfect substitutes for each other. The first technology has capital (physical and human) as sole factor, while the second has labor and capital as factors. If K_1 (and also K_2) are thought of as including physical and human capital, each would have diminishing returns. For example, we may think of $A_1 K_1$ as representing a late twentieth century modern sector, and $A_2 K_2^{\alpha} L^{1-\alpha}$ as representing more traditional production and manufacturing. The coexistence of substitute technologies of different degrees of advancement is typical of underdevelopment.

To simplify the application of the model with endogenous preferences, we chose some $\phi(h)$ satisfying

$$\phi(h) = p_0 \text{ for } h \geq \overline{h},\ \phi(h_{\text{eq}}) > A_1 - \rho - p_0, \tag{51}$$

with $h_{\text{eq}} < \overline{h}$ and extend ϕ so that it satisfies conditions (6) and (30).

We suppose as for Theorem (3) that the population consists of two sets of families described by the parameters $n,\ n_1,\ n_2,\ N$. On intervals on which ϕ is constant the solutions to Problem 1 coincide with the solutions to Problem 2 given by theorem (2).

Theorem 5 *Consider the closed economy populated with dynasties solving Problem 1 with production function F given by (50). Let* $R_{20} = R_{\text{eq}}$ *and suppose that* $R_{10} - \frac{r+\varkappa}{r-n-\gamma} h_{10} - \frac{w-\varpi}{r-n}$ *where* $h_{10} \geq \overline{h}$, *and that* R_{10} *is sufficiently large for investment to occur in both forms of production. The trajectories*

$$\begin{aligned}
a_1 &= \tfrac{\varkappa+n+\gamma}{r-n-\gamma} h_1 - \tfrac{w-\varpi}{r-n} & a_2 &= a_{\min} \\
h_1 &= h_{10} e^{\gamma t} & h_2 &= h_{20} \\
c_1 &= (\varkappa + n + \gamma) h_1 + \varpi & c_2 &= (r-n) a_{\min} + w \\
d_1 &= (\rho + p_0) t & d_2 &= (\rho + \phi(h_{\text{eq}})) t
\end{aligned} \tag{52}$$

$$\begin{gathered}
r = A_1,\ w = (1-\alpha) A_2^{1/(1-\alpha)} \left(\frac{\alpha}{A_1}\right)^{\alpha/(1-\alpha)},\ L = N, \\
K_1 = \left(n_1 a_1 + n_2 a_{\min} - \left[\frac{\alpha A_2}{A_1}\right]^{1/(1-\alpha)}\right) N,\ K_2 = \left[\frac{\alpha A_2}{A_1}\right]^{1/(1-\alpha)} N.
\end{gathered} \tag{53}$$

describe the behavior of the economy.■

Such a solution would be impossible if dynasties solved Problems 2 or 3. Its main feature is that the well-being h_1 of the first set of families, who are better off, grows exponentially (and so also approximately their consumption and assets), while the well-being h_2 of the second set of families, who are worse off and do not save, remains constant.

We now show that conventional convergence studies would miss the unbalanced growth present in this model. To see this suppose that the term $A_1 K_1$ written here is a simplified representation of a technology using physical and human capital, in which each has diminishing returns. Suppose that the schooled portion of the population in underdeveloped economies is negligible, so that in these economies $K_1 = 0$. while the unschooled portion in developed economies is negligible, so that here $K_2 = 0$. We can suppose that the developed world is converging to some balanced growth path given by an expanded version of $A_1 K_1$. At the same time, the underdeveloped countries will be converging to a zero growth path given by the production function $A_2 K_2^{\alpha} L^{1-\alpha}$, in which the capital to labor ratio K_2/L tends to some constant. When a convergence study is carried out on this world economy, convergence is detected, because each subset of countries is converging. But wages never rise and incomes diverge exponentially.

The example points out that when technological transition dynamics (in human capital investment or technological absorption) have a longer term than capital to labor ratio adjustment, convergence studies will detect the second process, while conclusions about the first can be false.

Inequality and growth

There are several theories explaining the relationship between inequality and growth. One strand centers on the effects that distribution has through the political balance of power (Bertola, 1993; Alesina and Rodrik, 1994, Persson and Tabellini, 1994, Benabou, 1995). In these redistributive pressures slow growth. In another strand, originated by Loury (1981), a credit restriction for investment in human capital makes investment by the poor suboptimal. Our model with endogenous time preferences gives an alternative theory. In an economy with solution diagram given by Figure 2a, the more inequality there is, the more families will find themselves in the class of non-savers. These families will not contribute to economic growth with their saving and investment, will provide cheap labor which will slow capital accumulation, and will have a low demand for goods. Thus, at best, the transition to optimal levels of capital will be slower the more unequal asset distribution is, and at worst, it may be possible for the poorest sectors of the population to become marginalized from the growing sectors of the economy, as in Theorem 5. If wages rise enough for some saving to begin, the presence of a credit restriction and diminishing returns to human capital investment would imply families would invest as much as possible in human capital. Now for some time it would be through the credit restriction that inequality would slow human capital accumulation and growth (see Mayer, 1999, for a numerical study of these effects).

Final remarks

We have explained several important phenomena without introducing any hypotheses extraneous to neoclassical endogenous growth theory. The only addition has been a full account of the role of health in expected utility. Here we include not only the probability of death but a quality of life index. This implies that time preferences are endogenous, with the poor being more impatient than the rich. The credit restriction only softens the effects of these assumptions.

The optimal intertemporal allocation of health is different for the poor than for the rich. For the poor, life quality loss and mortality may be too high to postpone consumption. For the rich, saving is worthwhile, because wealth is enough to offset unhealth. This formalizes the layman's intuition on the problem of non-saving, as stated in the introduction. As a consequence, families will save only when their wealth is above a critical level, and below this level they will dissave and borrow, if the credit markets allow. A functional definition of poverty thus arises: savers versus non–savers. This distinction disappears if wages rise enough. Its presence characterizes an important difference between development and underdevelopment. Further, it gives an explanation of marginalization, which is so prevalent in underdevelopment, since families who do not own capital (especially human capital) have a diminished access to many institutions, especially working opportunities.

The nature of health and well-being as an asset implies that temporary changes in income give rise to a Keynesian aggregate consumption function if there are non-savers. Both savers and non-savers will show a propensity to consume permanent changes in income, because they will invest in well-being when wealth increases (part and all of the change in wealth respectively). Thus in the process of development the consumption function will tend to change from the Keynesian to a permanent income form. Underdevelopment is therefore linked to stronger economic fluctuations not cushioned by the availability of savings to smooth consumption and demand.

The implications of our model describe some facets of underdevelopment. When the poor cannot afford a long enough planning horizon, families will tend to divide into two classes, savers and non-savers, according to the initial level of wealth. The non-savers eventually have very similar asset, income, health and well-being levels, while the savers will tend to maintain their initial distribution of wealth. For the lower income families, the possibility of saving depends on wages increasing, or on transfers from the remaining population (such as public education and health). Poverty traps may develop if new technologies substituting older technologies require investment from the participants, so that to participate in growth the poor must first invest, for example in human capital (unlike the case of unskilled labor, where there are no preconditions on participants). In these only the income of sectors of population who are able to save can grow. More generally, the time wages take to rise, which may depend on technological absorption and human capital investment lags, may be independent of the transitional dynamics of the aggregate capital to labor ratio usually measured in studies on convergence. While wages rise, growth will be slowed by the low consumption and investment levels of the poor population. This effect will be stronger the greater the level of inequality in asset distribution.

Appendix

Proof of Theorem 1. We apply Pontryagin's Maximum Principle. The Hamiltonian is:

$$H = u(h)e^{-(\rho+p_0-n)t}+\lambda((r-n)a+w-c)+\mu(-(\varkappa+n)h-\varpi+c)+\eta(a-a_{\min}) \quad (54)$$

The first order conditions are:

$$\begin{aligned} 0 &= H_c = -\lambda + \mu, \\ -\dot{\lambda} &= H_a = (r-n)\lambda + \eta, \\ -\dot{\mu} &= H_h = u'e^{-(\rho+p_0-n)t} - (\varkappa + n)\mu. \end{aligned} \quad (55)$$

Observe that the Hamiltonian is linear in c. This has the implication that the variables a and h may jump, keeping R constant. Since the Hamiltonian is concave in the state variables, the jumps may only occur at $t = 0$ (Kamien and Schwartz, 1981). Observe also that maximizing $U(a(0), h(0))$, subject to the restriction $a(0)+h(0) = R_0$ will lead to $\frac{\partial U}{\partial a(0)} = \frac{\partial U}{\partial h(0)}$, i.e. to $\lambda = \mu$ at $t = 0$. Thus the problem is well posed.

There are two types of solutions, corresponding to $\eta = 0$ and $\eta > 0$.

Type 1 solutions: $\eta = 0$. In this case $\lambda = \mu = \mu_0 e^{-(r-n)t}$. Substituting μ in the remaining equation, differentiating logarithmically and dividing by σ,

$$\frac{\dot{h}}{h} = \gamma - \frac{\dot{r}}{\sigma(r+\varkappa)}. \quad (56)$$

In the case when r and w are exogenous and constant, $h = h_0 e^{\gamma t}$. Using equation (41) for R, we can solve to obtain (16) and the expression for h_0 in terms of R_0. c is obtained from the differential equation for h. When $a = a_{\min}$, the level of R is $R_{\min}$.

Type 2 solutions: $\eta > 0$. In this case $a = a_{\min}$. Hence $c = (r-n)a_{\min} + w$, and $\dot{h} = -(\varkappa+n)h + (r-n)a_{\min} + w - \varpi$, so we obtain (18). h tends to an equilibrium level h_{eq}, so also R tends to an equilibrium level, given by R_{eq}.

Combinations of the two types of solutions when r and w are constant. The solutions are continuous in R.

Case 1: $\gamma > 0$. In the Type 1 (unrestricted) solutions, a and h are increasing, so these cannot reach a point where a becomes constrained. However, if $R_0 < R_{\min}$, initially the family will follow a Type 2 solution, but since $R_{\text{eq}} > R_{\min}$, in finite time it will switch to a Type 1 solution.

Case 2: $\gamma = 0$. Type 1 solutions have constant R, so are followed indefinitely if $R_0 \geq R_{\min}$. Otherwise a Type 2 solution in which R tends to $R_{\text{eq}} = R_{\min}$ in infinite time occurs.

Case 3: $\gamma < 0$. The Type 1 (unrestricted) solutions of a and h are decreasing. Hence if $R_0 > R_{\min}$ (respectively $R_0 \leq R_{\min}$) the family will begin with a Type 1 (respectively type 2) solution. The rest is clear.■

Proof of Theorem 2. We use the solutions to the usual problem, replacing c with $c+\varpi$.■

Proof of Theorem 3. The solutions will only be of Type 1, because $k > 0$. By equation (56) g satisfies $\frac{\dot{g}}{g} = \frac{\dot{h}}{h} + \frac{\dot{r}}{\sigma(r+\varkappa)} = \gamma$ and therefore (24). Using $f(k) = rk + w + \delta k$, equation (41) can be stated as $\dot{h} + \dot{k} = f(k) - (\delta + n)k - (\varkappa + n)h - \varpi$. Hence

$$\dot{k} = f(k) - (\delta + n)k - (\varkappa + n)h - \varpi - \left(\gamma - \frac{f''(k)\dot{k}}{\sigma(r+\varkappa)}\right)(r+\varkappa)^{-1/\sigma} g, \quad (57)$$

from which equation (25) is obtained. From these equations the loci of $\dot{g} = 0$ and $\dot{k} = 0$ are obtained.

We now turn to the case with unequal distribution. g_1 satisfies equation (24). As long as families in the second set do not save, $f(k) = rk + w + \delta k = \frac{N_1}{N_1+N_2} r a_1 + w$. Hence

$$\begin{aligned} \frac{N_1+N_2}{N_1}\dot{k} + \dot{h}_1 &= \dot{a}_1 + \dot{h}_1 = (r-n)a_1 - (\varkappa+n)h_1 + w - \varpi \\ &= \frac{N_1+N_2}{N_1}(f(k) - (\delta+n)k) - \frac{N_2}{N_1}w - (\varkappa+n)h_1 - \varpi. \quad (58) \end{aligned}$$

Substituting the equation for $\dot{h}_1$,

$$\begin{aligned} \frac{N_1+N_2}{N_1}\dot{k} &= \frac{N_1+N_2}{N_1}(f(k) - (\delta+n)k) - \frac{N_2}{N_1}(f(k) - rk) \\ &\quad -(\varkappa+n)h_1 - \varpi - \left(\gamma - \frac{f''(k)\dot{k}}{\sigma(r+\varkappa)}\right)(r+\varkappa)^{-1/\sigma} g_1, \quad (59) \end{aligned}$$

from which we obtain (28).

Once both sets of families save, g_i satisfy equation (24), $k = \sum_1^2 \frac{N_i}{N_1+N_2} a_i$, and we obtain in a similar way

$$\dot{k} = f(k) - (\delta + n)k - \varpi - \sum_{i=1}^{2} \frac{N_i}{N_1+N_2}\left(\varkappa + n + \gamma - \frac{\dot{r}}{\sigma(r+\varkappa)}\right) h_i. \quad (60)$$

Writing $g = \frac{1}{N_1+N_2}\sum_{i=1}^{2} N_i g_i$, we obtain the same aggregate system as when distribution is equal.■

Proof of Theorem 4. The maximization problem is bounded by the one obtained replacing ϕ with 0. Thus (see Cesari, 1983) a solution exists and it satisfies the usual first-order conditions. The problem is not convex because of the term e^{-d}, so the solutions to the first-order conditions need not be unique. We define the Hamiltonian

$$H = ue^{-d} + \lambda((r-n)a + w - c) + \mu(-(\varkappa+n)h - \varpi + c) + \nu(\phi + \rho - n) + \eta(a - a_{\min}). \quad (61)$$

The first-order conditions are:

$$\begin{aligned} 0 &= H_c = -\lambda + \mu, \\ -\dot{\lambda} &= H_a = (r-n)\lambda + \eta, \\ -\dot{\mu} &= H_h = u'e^{-d} - (\varkappa+n)\mu + \phi'\nu, \\ -\dot{\nu} &= H_d = -ue^{-d}. \end{aligned} \quad (62)$$

Observe that $U(R_0, d_0) = e^{-d_0}U(R_0, 0)$, so the maximization process is invariant under changes to d_0. Differentiating with respect to d_0,

$$\nu(0) = -e^{-d_0}U(R_0, d_0) = -U(R_0, d_0). \quad (63)$$

From the differential equation for ν, writing $\nu(\infty) = \lim_{t\to\infty} \nu$,

$$\nu(\infty) - \nu(0) = \int_0^\infty ue^{-d} dt = U(R_0, d_0) = -\nu(0). \tag{64}$$

Therefore $\nu(\infty) = 0$ and $\nu(t) < 0$ for all t. As before, the condition $\lambda = \mu$ holds initially since we maximize in $a(0)$, $h(0)$ subject to the restriction $a(0) + h(0) = R_0$. Jumps can only occur at $t = 0$ because H is concave in a and h, while d cannot jump. We now solve the first-order conditions.

We begin by examining unconstrained solutions with $\eta = 0$. In this case $\lambda = \mu = \mu_0 e^{-(\bar{r}-n)t}$. Thus we obtain equation (34). Since each of the terms in (34) is positive, $0 \le \chi \le 1$. Moreover, since $\lim_{t\to\infty} \phi'(h(t))\nu(t) = 0$, if h is bounded away from zero, the solution satisfies $\lim_{t\to\infty} \chi = 1$ (otherwise we may have $\mu \to 0$). Differentiating (34),

$$\frac{d}{dt}((r + \varkappa)\mu) = u''\dot{h}e^{-d} - u'e^{-d}\dot{d} + \phi''\dot{h}\nu + \phi'\dot{\nu}. \tag{65}$$

Dividing (65) by $(r + \varkappa)\mu$ and substituting $\frac{\nu}{(r+\varkappa)\mu} = \frac{1-\chi}{\phi'}$ and $\dot{\nu}$ (see (62)),

$$\frac{\dot{r}}{r + \varkappa} - (r - n) = \left[\frac{hu''}{u'}\chi + \frac{h\phi''}{\phi'}(1 - \chi)\right]\frac{\dot{h}}{h} + \left[\frac{u\phi'}{u'} - \phi(h) - \rho - n\right]\chi. \tag{66}$$

Thus we obtain (39). Equation (40) is obtained by differentiating the definition for χ. Together they are equivalent to the original first order conditions, except that they may admit additional solutions not satisfying the constraints for χ.

Consider the case in which r and w are exogenous and constant. We construct the (h, χ) quadrants of the phase diagram of the system given by equations (39) to (41) (see Figures 2a and 2b). The loci (42) and (43) are easily derived. The conditions given for $\phi(0)$ and ρ imply that for $h = 0$, $0 < \frac{r-n}{\Psi-n} < 1$, while as $h \to \infty$, $\frac{r-n}{\Psi-n}$ exceeds 1, justifying the graph of $\dot{h} = 0$ (which is strictly increasing in the case $\theta > \sigma$). Here we have used the assumption $\phi'' > 0$, which implies $\theta > 0$; otherwise there could be sign changes where $\chi\sigma + (1 - \chi)\theta = 0$. The locus $\dot{\chi} = 0$ has the form $\chi = \frac{W}{W+\Xi}$ stated in (43), with the assumption on $\phi(0)$ implying $W(0) > 0$. Since $\Xi > 0$, the graph stays in $0 \le \chi \le 1$ when W, which is monotonically decreasing, is non-negative. If W is bounded away from zero as $h \to \infty$ (this happens if $\frac{\theta-\sigma}{\theta}(r - n) < \rho - n$), $\frac{W}{W+\Xi} \to 1$ because $\Xi \to 0$ as $h \to \infty$ by (30) (see Figure 2b). If instead there is some finite value at which $W = 0$, we obtain a graph as in Figure 2a. A little algebra shows that the two loci intersect only at $h = h^*$, corresponding to $\chi = \chi^*$, defined in (44) and (45). Since the curves intersect only once, the general form of both diagrams is fully determined. For $h < h^*$, $\dot{\chi} < 0$ along the $\dot{h} = 0$ curve, and viceversa. This defines the sign of $\dot{\chi}$ in the corresponding regions, while the signs for $\dot{h}$ above and below the $\dot{h} = 0$ curve are easy to determine.

The statement of the theorem defines the stationary values (h^*, χ^*, R^*). We show that in the subsystem given by equations (39), (40), (h^*, χ^*) is an unstable point. Let $B(h, \chi) = \chi\sigma + (1 - \chi)\theta > 0$ be the denominator of $\frac{\dot{h}}{h}$. The linearization of the system at this point is

$$\begin{pmatrix} \dot{h} \\ \dot{\chi} \end{pmatrix} = \begin{pmatrix} -\dfrac{h\Psi'\chi}{B} & -\dfrac{h(\Psi-n)}{B} \\ \sigma\dfrac{\Psi'\chi}{B} - \phi' & \sigma\dfrac{(\Psi-n)}{B} \end{pmatrix}_{(h^*,\chi^*)} \begin{pmatrix} h-h^* \\ \chi-\chi^* \end{pmatrix}. \tag{67}$$

The conditions for ϕ imply $\Psi(0) > \lim_{h\to\infty}\Psi = \rho + p_0 > n$ by assumption (7). Hence the determinant and trace are positive, so the stationary point is unstable. The system has complex eigenvalues if

$$\begin{aligned} B^2(tr(M)^2 - 4\det(M)) &= (-h\Psi'\chi + \sigma(\Psi-n))^2 \\ &\quad -4\left(-h\Psi'\chi\sigma(\Psi-n) + (\sigma\Psi'\chi - \phi')h(\Psi-n)\right) \\ &= (-(\sigma-\theta)\Xi\chi + \sigma(\Psi-n))^2 + 4\phi' h(\Psi - n) < 0. \end{aligned} \tag{68}$$

Substituting Ψ and χ at the critical point, the condition is

$$[(\theta-\sigma)\Xi\frac{r-n}{r-n+\Xi} + \sigma(r+\Xi-n)]^2 < 4\frac{u'\Xi}{u}h(r+\Xi-n). \tag{69}$$

This holds if

$$\max[\frac{(\sigma-\theta)^2\Xi(r-n)^2}{h(r+\Xi-n)^3}, \frac{\sigma^2(r+\Xi-n)}{h\Xi}] < \frac{u'}{u}. \tag{70}$$

Functions u and ϕ can be chosen to satisfy this by first choosing r, n, h^*, and at this point σ, θ, u and Ξ. Then u' and ϕ' can be scaled up (keeping u and Ξ constant) until the inequality is satisfied, and the functions u and ϕ extended to $[0,\infty)$.

Now we show that the optimal unconstrained trajectories $R(t)$ are monotonic, increasing strictly to infinity if $R_0 > R^*$; remaining constant if $R_0 = R^*$ and decreasing strictly to the credit constraint if $R_0 < R^*$. $R(t)$ cannot first decrease (or stay equal) and then increase. This is because, by eliminating all subintervals on which R first decreases (or stays equal) and then attains the same value, we obtain a new control trajectory, with possible jumps in a, h (permissible in a non-optimal trajectory) for which we have an increasing function $R^1(t) > R(t)$ which dominates and therefore affords a higher well-being state h then $R(t)$. It is also impossible for a path $R(t)$ to first increase and then decrease. If it did, there would be values t_1, t_2, for which $R(t_1) = R(t_2)$, $R(t) > R(t_2)$ for $t_1 < t < t_2$ and R is decreasing after t_2 (using the first part of this paragraph). By replacing the controls after t_2 with an indefinite repetition of the controls used in (t_1, t_2), we obtain a new control trajectory, with possible jumps in a, h which, for which total wealth follows a trajectory $R^1(t) > R(t)$ for $t > t_2$, which must therefore increase the family's utility. Hence $R(t)$ is monotonic. It must be strictly monotonic or constant, because if it is constant in an interval then h is constant by equation (41), so $(h,\chi,R) = (h^*,\chi^*,R^*)$. But this solution is unstable and cannot be reached unless it holds for all time. Further, the increasing solutions $R(t)$ must tend to infinity, and the decreasing solutions to negative infinity, since there are no other stationary equilibria. In the absence of a credit restriction together with the viability restriction (15), the solution would reach and have to shift to the restriction $h = 0$ in finite time. We show the solution never reaches $h = 0$ by contradiction. For if it does then at this point R is decreasing so by its state equation at that time $a \leq -\frac{w-\varpi}{r-n}$. But this contradicts the viability restriction (15), which implies $a_{\min} > -\frac{w-\varpi}{r-n}$. Thus a decreasing solution must reach the credit restriction. Observe now that two solutions $R_1(t)$, $R_2(t)$, cannot cross. This is because then we could choose a control giving $R(t) = \max\{R_1(t), R_2(t)\}$, which would improve one of the two solutions. Therefore if $R_0 > R^* > 0$, the corresponding solution $R(t)$ must tend to infinity (it cannot decrease without crossing the constant solution), and if $R_0 < R^*$ the corresponding solution $R(t)$ decreases to the credit restriction.

Clearly if $R \to \infty$ so does h, otherwise there would be a lot of wasted utility. We have seen that if $h \to \infty$ $\chi \to 1$ (see Figures 2a and 2b), so by equation (40) $\frac{\dot{h}}{h} \to \gamma_\infty = \frac{r-p_0-\rho}{\sigma}$. Since the solution for R satisfying the transversality condition when $h = h_0 e^{\gamma_\infty t}$ is $R = \frac{r+\varkappa}{r-n-\gamma_\infty} h_0 e^{\gamma_\infty t} - \frac{w-\varpi}{r-n}$, R tends to the same growth rate and $\frac{h}{R} \to \frac{r-n-\gamma_\infty}{r+\varkappa}$.

Now we examine the conditions which must hold if the decreasing unrestricted solution meets the credit constraint at time $t = T$. Along the unconstrained solution $\lambda = \mu$ so each of these is the marginal value for assets R. Along the constrained solution this marginal value is given by $V'(h_T)$ in current value, where h_T is the value of h at which the constrained solution begins. Assets cannot jump between a and h at $t = T$, because, if a rises, then to the right we do not have a credit restriction, while if a diminishes to $a_{\min}$, raising h, an unnecessary suboptimal interval of the solution would be introduced. Hence $h(T^-) = h_T$ and a and h are continuous at $t = T$. Now, the marginal utility of raising or lowering h_T must be the same on either side of $t = T$, for otherwise it is worth shifting T to shift the solution up or down a little. Thus we must have $\mu(T) = e^{-d(T)} V'(h(T))$, where we discount to $t = 0$. This implies

$$\chi(T) = \frac{u'(h(T))e^{-d(T)}}{(r+\varkappa)\mu(T)} = \frac{u'(h(T))}{(r+\varkappa)V'(h(T))}. \tag{71}$$

This gives a curve on the (h, χ) plane on which the unconstrained and constrained solutions must meet. Since along the constrained solutions R rises if $h_T < h_{eq}$, we must have $h_T \geq h_{eq}$.

We now consider the opposite situation, when the restricted solution becomes unrestricted, which can hold if $R_{eq} > R^*$. As before, there are no jumps, and, if we initiate the unconstrained solution at $t = T$ with $d = \int_0^t \phi(\tilde{h}(s; h_0))ds$, where $h_0 = R_0 - a_{\min}$,

$$\mu(T) = \frac{\frac{\partial V}{\partial t}(h_0, T)}{\frac{\partial \tilde{h}}{\partial t}(T; h_0)} = \frac{u(\tilde{h}(T; h_0))e^{-\int_0^t \phi(\tilde{h}(s;h_0))ds}}{\frac{\partial \tilde{h}}{\partial t}(T; h_0)}, \tag{72}$$

so at $t = T$

$$\chi = \frac{u'(h)e^{-d}}{(r+\varkappa)\mu} = \frac{u'(h)\frac{\partial \tilde{h}}{\partial t}}{(r+\varkappa)u(h)}, \tag{73}$$

where $\frac{\partial \tilde{h}}{\partial t} = -(\varkappa+n)h + (r-n)a_{\min} + w - \varpi$ is the derivative of h along the constrained solution. Clearly $h_T < h_{eq}$, since the solution only reaches h_{eq} at $t = \infty$.

We are now ready to complete the description of the phase diagram. To see (47) observe that (h_{eq}, R_{eq}) is the intersection of the lines $\dot{R} = 0$ and $R = h + a_{\min}$ on the (h, R) plane, and the first line has a larger slope than the second.

Case I. Suppose $R_{eq} < R^*$. Time consistency implies that solutions containing unconstrained portions but including constrained portions trace on each other if the initial condition R_0 is changed in a neighborhood not including R^*. Therefore on the (h, R) plane the unions of these trajectories form two branches (corresponding to solutions with $R_0 < R^*$ and with $R_0 > R^*$) or the point set (h^*, R^*). The two branches must meet at (h^*, R^*). For suppose R_0 is slightly below R^*. Then a constant solution with h slightly less than h^* is feasible, but the optimal solution will bring health forward and so approximate h^* even better. Therefor initial values of h approximate h^* if R_0 approximates R^*. A similar argument holds from above. Besides these solutions, there exists the fully constrained solution along $R = h + a_{\min}$, for initial values $a_{\min} \le R_0 < R_{eq}$.

Cases II and III follow similarly. In each case the branches described in the cover the values R on $[a_{\min}, \infty)$ defining the function $h_0(R_0)$ giving the initial value of h_0 in terms of R_0. Thus, given any initial value R_0, the solution path is given by the trajectory on the corresponding branch initiating at the given level of R.■

Proof of Theorem 5. The functions a_i, h_i, c_i, d_i, $i = 1, 2$, solve optimization Problem 1, using the formulas in Theorem 2, since along these trajectories the instantaneous discount rates $\phi(h_i) + \rho$ are constant. Since returns on capital must be equal, we have $r = F_{K_1} = F_{K_2}$, so $r = A_1 = \alpha A_2 (L/K_2)^{1-\alpha}$. From here we obtain r and $\frac{K_2}{L}$ and therefore K_2. w is obtained from $w = F_L = (1-\alpha) A_2 (K_2/L)^\alpha$ and the expression for K_2/L. Finally, since $K_1 + K_2 = (n_1 a_1 + n_2 a_{\min}) N$, we can now obtain K_1, which is positive if

$$h_{10} > \frac{r - n - \gamma}{\varkappa + n + \gamma} \left(\frac{1}{n_1} \left[\frac{\alpha A_2}{A_1} \right]^{1/(1-\alpha)} + \frac{w - \varpi}{r - n} - \frac{n_2}{n_1} a_{\min} \right) \tag{74}$$

and thus for large enough R_{10}.■

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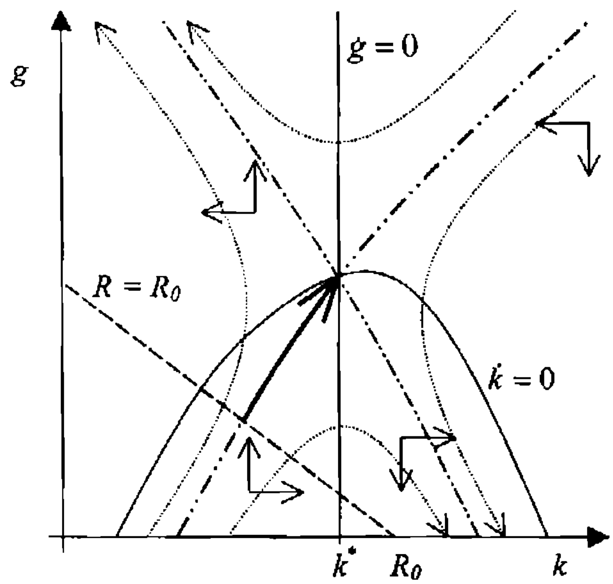

Figure 1a. Phase diagram for k, g, when $\dot{k} = 0$ has no asymptote.

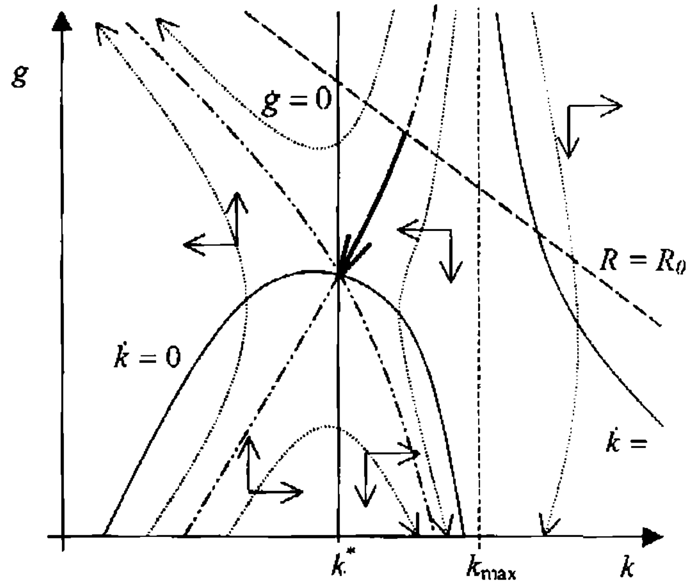

Figure 1b. Phase diagram for k, g, when $\dot{k} = 0$ has an asymptote at k_{max}.

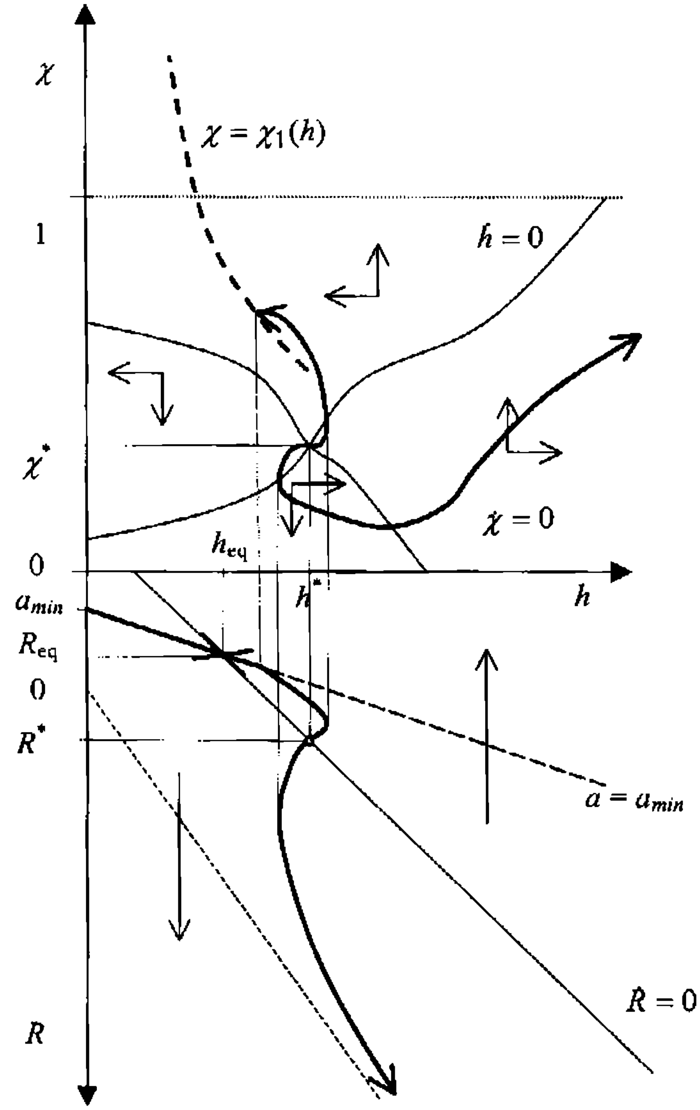

Figure 2a. Dynamics of h, χ, R when subjective discount rates are endogenous. Case $R_{eq} < R^{*}$. The income distribution bifurcates. (The locus $\dot{\chi} = 0$ corresponds to the case when $W(h)$ becomes zero for finite h.)

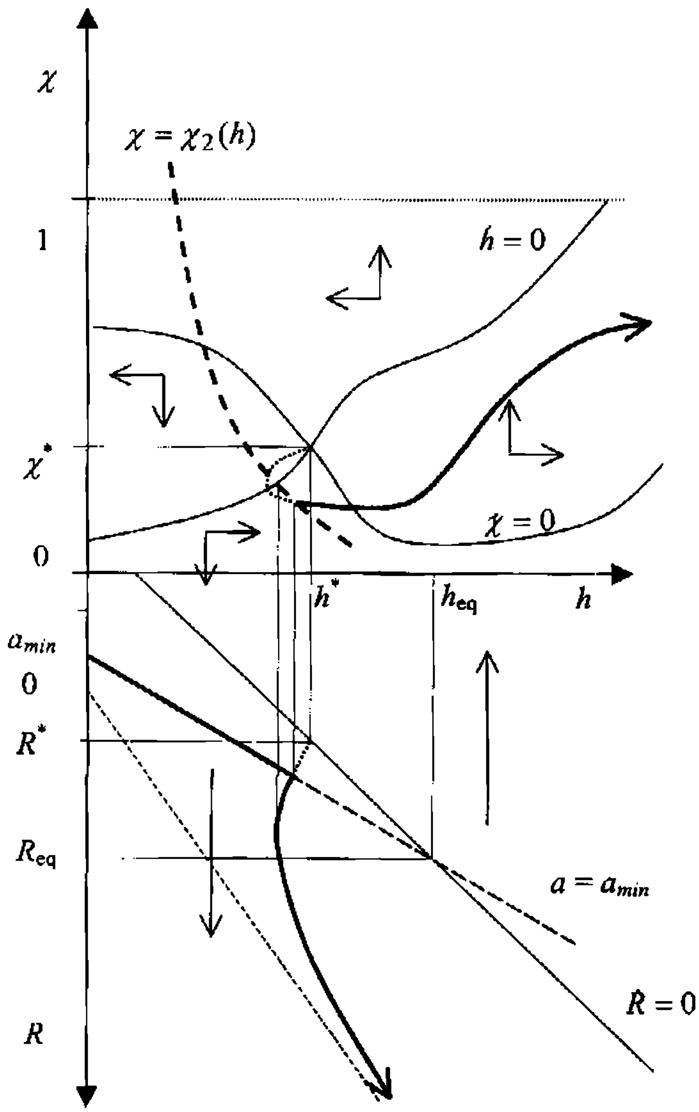

Figure 2b. Dynamics of h, χ, R when subjective discount rates are endogenous. Case $R_{eq} > R^*$. All families eventually save (The locus $\dot{\chi} = 0$ corresponds to the case when $\overline{W}(h)$ is bounded away from zero.)