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EVALUATING CONVERGENCE WITH MEDIAN-UNBIASED ESTIMATORS IN PANEL DATA

Abstract.

This paper extends Andrews'(1993) median-unbiased estimation for auto- regressive/unit root time series to panel data dynamic fixed effects models. It is shown that medianunbiased estimation applies straightforwardly to models that include linear time trends as well as to those including more general time specific effects. Using Monte Carlo simulations, median-unbiased LSDV estimators are computed and found to be robust to groupwise heteroskedastic and cross-sectionally correlated disturbances. These estimators are then used to evaluate conditional convergence, in the sense of economics having parallel balanced growth paths, in the cases of 48 USA states, 13 OECD's and two wider samples from Summers and Heston's Penn World Tables, with 57 and 100 countries respectively. Unadjusted LSDV estimates, would support conditional convergence in all samples. Median-Unbiased estimates, however, support conditional convergence only among USA states and OECD countries.

Key words: Dynamic panel data models, median-unbiased estimators, unit roots, Monte Carlo simulations, conditional convergence.

JEL classification: C23, C15, O40

Resumen

Este trabajo extiende el método de estimación mediana-insesgada de Andrews (1993) a modelos dinámicos de panel en efectos fijos. Se muestra que la estimación mediadainsesgada se aplica directamente a modelos de panel que incluyen tendencias temporales lineales o efectos específicos al tiempo. Utilizando simulaciones de Monte Carlo, se encuentra que los estimadores mediana-insesgados son bastante robustos a problemas de heterocedasticidad grupal y correlación de corte transversal. Finalmente estos estimadores son utilizados para evaluar convergencia condicional del ingreso per capita en los casos de 48 estados de USA, 13 países de la OCDE y dos muestras de países de las *Penn World Tables* de Summers y Heston con 57 países y 100 países respectivamente. Estimadores no corregidos son consistentes con convergencia condicional en todos los casos. Sin embargo estimadores mediana insesgados apoyan convergencia condicional solamente en los casos de los estados de USA y los países de la OCDE.

1. Introduction

The bias problem in dynamic panel data models in finite samples has been well documented. Nickell (1981), Sevestre and Trognon (1985), Hsiao (1986) have shown that the magnitude of the asymptotic bias of the LSDV for a small time dimension (T) is appreciable. Beggs and Nerlove (1988) show that the bias becomes larger if the cross-sectional dimension of the panel (N) is also small. The use of LSDV estimators in typical panels (small T and large N) is, therefore, not recommended. Instead, estimators with consistency properties relying on the cross-sectional dimension of the panel being large have been proposed. This is the case of IV [Anderson and Hsiao (1981, 1982), Hsiao (1986)] or GMM [Arellano and Bond (1981)] methods, among others.

The increasing interest of researchers for applying panel data techniques to problems involving cross-country information is creating new problems since those samples generally have larger time dimensions but much shorter cross-sectional dimensions than typical panels and, more important, they may be highly trended as well. In several cases reliable methods can not be implemented because of T being large relative to N. That is the case of GMM methods, i.e. Arellano and Bond (1981), Arellano and Bover (1995), Ahn and Schmidt (1995). This is also the case of 2SLS methods as in Keanc and Runkle (1992). More seriously, in contexts where the AR parameter is high, say 0.95, most estimators may become either biased and/or imprecise. Under these circumstances, most bias formulae given in the papers previously referred, may not be accurate either.

This paper extends Andrews' (1993) median-unbiased estimation for autoregressive/unit root time series to dynamic fixed effects models. Mcdian-unbiased estimation seems to be a reliable method to deal with the bias and efficiency problems in the aforementioned context. This approach is sample specific and is based on the distribution of the LSDV estimator, which is well behaved and has a relatively small variance even in the unit root case. The justification for using median-unbiased estimation in dynamic panel data models is similar to the one for Kivict's (1995) LSDVc estimator. The method exploits the fact that even though the LSDV is inconsistent and biased in finite samples it is however relatively efficient. Kiviet (1995) derives a formula to estimate the bias of the LSDV estimator for finite N and T, and shows that it performs well in a number of experimental designs. Cormeño (1997) finds that in contexts with no exogenous regressors, Kiviet's approximation formula to the bias works quite well for an AR parameter value such as 0.5. However, for higher values, say 0.85, 0.95 or 0.99, the formula produces quite biased and imprecise results, the reason being that implementation of Kiviet's method requires preliminary estimates of the AR parameter, and most if not all estimators are either biased or highly imprecise in such a context.

In contrast, median-unbiased estimation can be implemented for a range of AR parameter values on the interval [-1,1]. Thus, highly persistent AR processes do not poses any problem for the method. In addition, the method can be applied to samples of any finite dimension. Obviously, the limitation of the method is that it only applies to dynamic panel data models with no exogenous regressors.

The rest of the paper is organized as follows. Section 2 extends Andrews' (1993) median unbiased estimation to a panel data context. Section 3 explores the robustness of median-unbiased estimators to heteroskedastic and cross-sectionally correlated disturbances. Section 4 presents an empirical application of the method to the controversial issue of conditional convergence. Finally, Section 5 concludes.

2. Median-unbiased estimation in dynamic panel data

This section extends Andrews' (1993) median-unbiased estimation of first order autoregressive/unit root (AR/UR) time series models to panel data. Specifically, the paper considers a two way dynamic error-components model or dynamic panel data model with no exogenous regressors as described in the panel data literature, e.g. Hsiao (1986), or Baltagi (1995).

Extension of Andrews' approach from time series to a dynamic panel of N cross-sections over T+1 periods is straightforward. It will suffice to show that the LSDV estimator (which is the OLS analogue in panel data) is invariant to the individual specific effects, time specific effects (or time trend coefficients) and the variance of the innovations. Following Andrews' definition of models, consider the latent variable model

$$y_{it}^* = \beta y_{i-1}^* + v_{it}, \qquad i = 1, \dots, N, \ t = 1, \dots, T,$$
 (1)

where $v_{\mu} \sim iid(0, \sigma^2)$, $y_{i0}^* \sim (0, \sigma^2/(1-\beta^2))$ if $\beta \in (-1,1)$, and y_{i0}^* is some constant or random variable if $\beta = 1$.¹ Thus, for each cross-section, the process given by (1) is strictly stationary with mean zero in the former case, and a random walk with arbitrary initial condition in the later case. Define now the following model for y_{μ} , the observable variable:

$$y_{\mu} = \mu_i + \lambda_i + y_{\mu}^*$$
 $i = 1, ..., N, t = 1, ..., T$ (2)

where μ_i and λ_i are individual and time specific effects respectively, which are

¹ The normality assumption is not made since Imhof's (1961) algorithm is not used in this paper. Instead, Monte Carlo simulations will be used.

assumed fixed. From (1) and (2) it can be obtained:

$$y_{il} = \tilde{\mu}_i + \tilde{\lambda}_i + \beta y_{il-1} + v_{il}$$
(3)

where $\mu_i = \mu_i(1 - \beta)$ and $\tilde{\lambda}_i = \lambda_i - \beta \lambda_{i-1}$. In the case $\beta \in (-1,1)$, for each crosssection $(y_{ii} - \lambda_i)$ will be a strictly stationary process with mean μ_i . In this case, $(y_{i0} - \lambda_0) \sim (\mu_i, \sigma^2/(1 - \beta^2))$. For the case $\beta = 1$, $(y_{ii} - \lambda_i)$ is a random walk process with arbitrary initial condition. Performing the Within transformation [Wallace and Hussain (1969)] on model (3) gives:

$$\tilde{\mathcal{Y}}_{ii} = \beta \tilde{\mathcal{Y}}_{ii-1} + \tilde{\mathcal{V}}_{ii}, \tag{4}$$

where $\tilde{y}_{ii} = (y_{ii} - y_{i.} - y_{i.} + y_{i.}), \quad \tilde{y}_{ii-1} = (y_{ii-1} - y_{i.-1} - y_{i.-1} + y_{i.-1}),$ and $\tilde{y}_{ii} = (v_{ii} - v_{i.} - v_{i.} - v_{i.} + v_{i.}).$ For each transformed variable, the second, third and fourth terms are the individual (over time for each cross-section), cross-sectional (over cross-sections at a given time), and overall (over both cross-sections and time) means of the corresponding original variables. The OLS estimator applied to the transformation (4) is known as the Within or LSDV estimator (LSDV1 in this paper). This is given by

$$\beta_{ISDV1} = \left(\sum_{i=1}^{N} \sum_{t=1}^{T} \widetilde{y}_{it} \widetilde{y}_{it-1}\right) / \left(\sum_{i=1}^{N} \sum_{t=1}^{T} (\widetilde{y}_{it-1})^{2}\right).$$
(5)

Since the Within transformation sweeps out both individual and time specific effects, β_{ISDV1} is independent of these effects, and so is its distribution. By continuous substitution, as in Hsiao (1986), p.73, it can be obtained from (3):

$$(y_{ii} - \lambda_i) = \frac{1 - \beta'}{1 - \beta} \tilde{\mu}_i + \beta' y_{i0} + \sum_{j=1}^{i-1} \beta'^{-j} v_{ij} \,. \tag{6}$$

Using $\mu_i = \mu_i(1-\beta)$, $y_{i0} = \mu_i + \lambda_0 + bv_{i0}$, with $b = (1-\beta^2)^{-1/2}$, and $\lambda_0 = 0$, expression (6) can be rewritten as

$$y_{ii} = \mu_i + \lambda_i + \beta' b v_{i0} + \sum_{j=1}^{i-1} \beta^{i-j} v_{ij} .$$
⁽⁷⁾

Thus, given the Within transformation, formula (5) only involves weighted sums of *iid* errors. Hence, scaling up the variance of these errors will affect proportionally the numerator and denominator in (5), leaving β_{ISDV1} invariant with respect to σ^2 .

In the unit-root case, given an arbitrary initial condition, y_{i0} , equation (3) becomes

$$y_{ii} = y_{i0} + \lambda_i + \sum_{j=1}^{i} v_{ij} .$$
(8)

In this case, β_{LSDF1} will be independent of y_{i0} and λ_i since these are simply swept out by the Within transformation. Also in this case, β_{LSDF1} will be unaffected by the level of σ^2 by a similar argument to the one given for the stationary case.

Consider now the particular case in which the time specific effects take the form of a simple linear time trend, that is $\lambda_i = \theta t$. In this case, model (3) becomes

$$y_{ii} = \mu_i + \partial t + \beta y_{ii-1} + v_{ii}, \qquad (9)$$

where $\mu_i = \mu_i(1-\beta) + \theta\beta$, and $\tilde{\theta} = \theta(1-\beta)$. In the case $\beta \in (-1,1)$, for each crosssection, y_{ii} is a strictly stationary process around a linear time trend with intercept μ_i and slope θ . The initial condition for each cross-section will be $y_{i0} \sim (\mu_i, \sigma^2/(1-\beta^2))$. For $\beta = 1$, y_{ii} is a random walk process with drift θ for each cross-section and with arbitrary initial condition. Subtracting individual means from (9) gives the Within transformation

$$(y_{ii} - \bar{y}_{i.}) = \theta(t - \bar{t}) + \beta(y_{ii} - y_{i.}) + (v_{ii} - \bar{v}_{i.}), \qquad (10).$$

where t = T(T+1)/2. The OLS estimator of β in (10) is called LSDV2 here. It can be shown that all previous independence or invariance results also apply in this case.

A few remarks are in order here. First, since the Within transformation wipes out individual and time specific effects the previous results hold if these effects were random, as long as they are independent from each other and from y_{i0} and v_{ir} . Second, the invariance results also hold in the cases in which the time specific effects take the form of higher order time trends (i.e. quadratic), or if individual time effects or time trends are allowed for. Finally, the previous results apply in the case of oneway error-components models that exclude time specific effects. In this case $y_{ir} = \mu_i + y_{ir}^*$, and all previous invariance results are obvious. Only models given by (3) and (9) will be considered.

Median-unbiased estimation is implemented as follows. (i) For a given sample size, a mapping between different AR parameter values and the corresponding median of the distribution of the LSDV estimator needs to be obtained. (ii) The previous mapping is then used to correct actual LSDV estimates. The median-unbiased estimate is the value of the AR parameter for which the median of the distribution of the LSDV is the actual LSDV estimate. This implies subtracting a median-bias from the actual LSDV estimate. Fractiles of the LSDV1 and LSDV2 for a few relevant AR parameter values and sample sizes are shown in Table A.1. They have been tabulated using Monte Carlo simulations with 20001 replications. The sample sizes chosen correspond to those of the actual panels of per capita income that will be used in the empirical application later in Section 6. Monte Carlo simulations have been chosen instead of Imohf's (1961) algorithm for practical reasons. The computational work has been made using GAUSS programs. Figure 1 in the Appendix presents the median-bias of the LSDV estimator.

The previous results show, numerically, the existence of a monotonically increasing relationship between true AR parameter values and the median (and other fractiles as well) of the distribution of the LSDV estimators. Also, they show that the downward biases are sizable. In particular, for a given sample size, the bias becomes larger as the true AR coefficient approaches one. In the same way, the median (and mean) biases become larger and the 90% confidence intervals become wider the shorter is the time dimension of the samples. For the sample dimensions considered, using LSDV estimators is likely to produce downward biased point estimates of the AR parameter. Moreover, those estimates may be consistent with stationary processes when in fact they are non-stationary.

3. Robustness of median-unbiased estimators

This section explores the robustness of median-unbiased estimators to groupwise heteroskedasticity and cross-sectional correlation problems. This exercise seems to be necessary because these problems are likely to be present in practice. Table 1 describes the relevant design. The variance levels of the error term for each cross section were obtained from the covariance matrix (diagonal elements) of actual samples of output per capita in logarithms, after removing time and individual effects. Notice that the highest ratio of maximum to minimum variances is considerably large (124 times approximately) and corresponds to the more heterogeneous 100-country sample. The values for the minimum and maximum off-diagonal elements of the correlation matrix of disturbances were obtained in the same way, but they were truncated to values between -0.1 to +0.1 approximately. The average absolute value of these elements is between 0.05 to 0.06.

SAMPLE	RATIO MAX/MIN	MAX/MIN		
DIMENSIONS	VARIANCE	CORRELATION		
(N = 48, T+1 = 63)	37.83	- 0.11 / + 0.11		
(N - 13, T + 1 = 120)	9.02	- 0.10 / ± 0.10		
$(N-57, T+\overline{1}=41)$	106.31	- 0.11 / + 0.11		
(N - 100, T + 1 = 31)	123.77	- 0.11 / + 0.11		

Design of Heteroskedasticity and Cross Sectional Correlation

In general, the results show that the median-unbiased estimators are quite robust to the simultaneous presence of groupwise heteroskedaticity and cross-sectional correlation problems. Table A.2 in the Appendix, shows the relevant fractiles. In all cases, the median fractiles are practically unaffected. The 90% intervals, though, are widened moderately. In particular, they become wider the shorter the time dimension of the samples. It should be noticed that Andrews (1993) median-unbiased estimators in single time series are also found to be quite robust to non-*iid* and non-normal error structures.

4. An empirical application to conditional convergence

The empirical work on convergence is controversial. The cross-section regression approach (as in Baumol (1986), Barro(1991), Barro and Sala-i-Martin (1992), Mankiw, Romer and Weil (1992), among others) has been criticized on several aspects such as the wasting of useful information across time by averaging growth rates and its underlying assumption of homogeneity across countries. More seriously, the uniform finding that economies converge at the rate of about 2 percent per year almost everywhere can be shown as evidence that this statistical apparatus is flawed [Quah (1993a, 1993b)]. Also, Evans (1996, 1997) shows formally that the cross-section regression approach in fact produces biased results.

Several alternative approaches have been proposed since then. Evans (1994, 1996, 1997, 1998) and Evans and Karras (1996a, 1996b) have proposed several methods that exploit the cross-sectional and/or time series dimensions of the data to test endogenous against exogenous growth and to estimate growth regressions consistently. In general, they find evidence supporting exogenous growth theories in

samples such as the USA states, 13 OECD countries and a 48-country sample from the Penn World Tables. They find conditional convergence, in the sense of economies having parallel balanced growth paths. Absolute convergence does not hold even in the case of the USA states.

On the other hand, studies of wider and more heterogeneous samples of economies have produced mixed results. Lee, Pesaran and Smith (1995) using a unit root approach cannot reject the hypothesis of non-stationary output processes in 3 samples of countries. This result is in spite of their high time series estimates of convergence rates (about 20% per year), which they attribute to highly upward biases in their point estimates. In contrast, Islam (1995) using the same 98-country sample as in Mankiw, Romer and Weil (1992) finds much faster conditional converge rates if a dynamic panel data model is used instead of a cross-section or pooled regression model. Particularly, he uses the Minimum Distance (MD) and Least Squares Dummy Variable (LSDV) estimators obtaining similar results. Yet Islam's results using the LSDV estimator, might still be significantly biased in favor of high convergence rates since he uses data spaced five years apart, which results in a very short time dimension of the sample (only 5 points in time). Lee, Pesaran and Smith (1998) and Maddala and Wu (1997) have pointed out that imposing the restriction that economies have identical auto regressive and time trend parameters, as in the case of Islam, can produce very misleading results on convergence.

This section uses median-unbiased estimators in panel data to evaluate conditional convergence in the sense of economies having parallel balanced growth paths. It is found here, that even when the assumption of equal auto regressive coefficients and common trends is imposed a priori, convergence in the sense given previously is likely to happen only in samples of relatively homogeneous countries once the biases are corrected. Four panels of yearly per capita income are studied. They include 48 USA states for the period 1929-1991, 13 OECD countries during 1870-1989, 57 countries over the period 1950-1990 and 100 countries, including the previous 57 countries, during 1960-1990. The last two samples are taken from Summers and Heston's Penn World Tables (PWT), version 5.6. Additional information on these samples is provided in the Appendix.

Models (3) and (9) given before are used. These models are useful to characterize whether deviations of per capita output of economies around a common trend are stationary or not. In the first case, the dynamics of output per capita of economies will be consistent with conditional convergence. Even though conditional convergence can be taken as evidence in favor of exogenous growth models, it should be pointed out a similar output dynamics would be consistent with a technological imitation mechanism as in endogenous growth models. Interestingly, an endogenous growth model that explicitly takes into account the interdependence among cconomies will also predict conditional convergence, i.e. see Howitt and Aghion (1998). Thus, the distinction among exogenous and endogenous growth models could not be made on the basis of convergence results only.

The results are shown in Table 2. The estimates of the AR coefficient in these models are labeled LSDV1 and LSDV2 respectively. Uncorrected estimates are obtained using the actual data samples. The corresponding median-unbiased estimates have been obtained by correcting the actual LSDV for the median-bias, as explained in section 3. They are computed using the fractiles under groupwise heteroskedasticity and cross-sectional correlation since these problems are presumed to be present in the actual data. Also, median-unbiased estimates of the AR coefficient from fractiles under *iid* errors are reported for comparison. The implied rates of convergence are reported in both cases. They are approximately equal to one minus the AR parameter value. The rates reported in parenthesis in the last column arc obtained using the 0.05th and 0.95th fractiles and can be interpreted as the lower and upper values of the AR coefficient whose 90% probability intervals would be consistent with the actual LSDV estimates. In most cases they have been computed by linear interpolation. It can be seen that the uncorrected LSDV1 and LSDV2 estimates are consistent with conditional convergence in all samples. However, after the median-bias correction, conditional convergence holds only in the cases of the USA states and OECD countries, and takes place at much slower rates than the ones implied by the uncorrected estimates. In the case of the PWT samples of countries, the median-unbiased estimates do not support conditional convergence.

TABLE 2:

Sample (N, T+1)/ Estimator	UNCO	RRECTED	MEDIAN-UNBIASED		
	Estimates	Implied Rate of Convergence	Estímates	Implied Rate of Convergence	
<u>US States</u> (48,63)				<u> </u>	
LSDV 1	0.9109	9.3	0.9511	5.0 (2.6-7.5)	
			0.9505	5.1 (3.3-6.8)	
LSDV 2	0.8811	12.7	0.9188	8.5 (5.9-11.3)	
			0.9177	8.9 (7.3-9.4)	
<u>OECD</u> (13,120)			<u>-</u> <u>-</u> <u>-</u> <u>-</u> <u>-</u>		
LSDV I	0.9576	4.3	0.979 2	2.1 (0.0 - 4.2)	
			0.9788	2.1 (1.0 - 3.0)	
LSDV 2	0.9686	3.2	0.9938	0.6 (0.0 - 2.4)	
			0.9938	0.6 (0.1 - 1.0)	
<u>PWT-1</u> (57,41)					
LSDV 1	0.9569	4.4	1.0000	0.0	
			1.0000	0.0	
LSDV 2	0.9555	4.6	1.0000	0.0	
			0000.1	0.0	
<u>PWT – 2</u> (100,31)		<u> </u>			
LSDV 1	0.9537	4.7	0000.1	0.0	
			1.0000	0.0	
LSDV 2	0.9533	4.8	1.0000	0.0	
			1.0000	0.0	

Uncorrected and Median-Unbiased Estimates of AR Coefficients

*In the last two columns and for each estimator, the values in the first row are computed using fractiles under groupwise heteroskedasticity and cross-sectional correlation. The values in the second row correspond to fractiles under *iid* errors.

5. Conclusion

This paper has implemented median-unbiased estimation in dynamic panel data models. First, it has been shown that Andrews' (1993) approach applies straihtforwardly to LSDV estimators in dynamic fixed effects models with no exogenous regressors. Second, median-unbiased LSDV estimators have been found to be quite robust to groupwise heteroskedastic and cross-sectional correlated innovations. Relative to alternative estimation or bias-correction techniques, the method has the advantage that it can be applied to panels of any finite cross-sectional and time dimensions and with highly persistent dynamics as would be the case of most cross-country panels.

The empirical application has found that unadjusted LSDV estimators would be consistent with conditional convergence in the four samples studied. Medianunbiased estimates, however, support conditional convergence, only for the USA states and OECD samples, and at much slower rates than unadjusted LSDV estimates would imply. APPENDIX

Data Sources

The USA states sample includes real per capita personal income of 48 contiguous states over the period 1929-1991. The data source is The U.S. Department of Commerce. See Evans and Karras (1996a) for details.

The OECD sample includes per capita real gross domestic product during the period 1870-1989 for Australia, Austria, Belgium, Canada, Denmark, Finland, France, Italy, Norway, Sweden, United Kingdom, United States and West Germany. The Data source is Angus Maddison (1991). See Evans (1996, 1998) for details.

The other two samples include per capita gross domestic product in constant international prices (RGDPCH) from the Penn World Tables 5.6 by Summers and Heston (1991, 1993). Countries with complete information over the periods 1950-1990 and 1960-1990, except oil and centrally planned countries were selected. The PWT-1 sample includes 57 countries. They are Egypt, Kenya, Mauritius, Morocco, Nigeria, South Africa, Uganda, Canada, Costa Rica, Dominican Republic, El Salvador, Guatemala, Honduras, Mexico, Nicaragua, Panama, Trinidad and Tobago, The United States, Argentina, Bolivia, Brazil, Chile, Colombia, Ecuador, Guyana, Paraguay, Peru, Uruguay, Venezuela, India, Japan, Pakistan, Philippines, Sri-Lank, Thailand, Austria, Belgium, Cyprus, Denmark, Finland, France, West Germany, Greece, Iceland, Ireland, Italy, Luxembourg, Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, Turkey, United Kingdom, Australia, New Zealand.

The PWT-2 sample has 100 countries including (in addition to the previous 57 countries): Algeria, Benin, Burkina Faso, Burundi, Cameroon, Cape Verde, Central, Chad, Comoros, Congo, Gabon, Gambia, Ghana, Guinea, Guinea Bissau, Ivory Coast, Lesotho, Madagascar, Malawi, Mali, Mauritania, Mozambique, Namibia, Rwanda, Senegal, Seychelles, Togo, Tunisia, Zambia, Zimbabwe, Jamaica, Bangladesh, Hong Kong, Indonesia, Israel, Jordan, Korea, Malaysia, Singapore, Syria, Taiwan, Fiji, Papua New.

Smpl. size	N = 48; T+1 = 63			N = 13; $T+1 = 120$		
AR / Fract	0.05	0.5	0.95	0.05	0.5	0.95
LSDV1						
0.89	0.8371	0.8553	0.8718	0.8481	0.8723	0.8932
0.91	0.8566	0.8739	0.8895	0.8688	0.8918	0.9113
0.93	0.8759	0.8925	0.9071	0.8897	0.9109	0.9287
0.95	0.8952	0.9104	0.9240	0.9107	0.9302	0.9463
0.97	0.9131	0.9277	0.9404	0.9311	0.9490	0.9629
0.99	0.9301	0.9437	0.9556	0.9501	0.9660	0.9782
0.999	0.9376	0.9505	0.9617	0.9583	0.9732	0.9844
LSDV2						
0.89	0.8365	0.8546	0.8711	0.8481	0.8710	0.8915
0.91	0. 856 2	0.8732	0.8888	0.8681	0.8905	0.9094
0.93	0.8753	0.8919	0.9063	0.8892	0.9099	0.9273
0.95	0.8945	0.9097	0.9233	0.9098	0.9291	0.9448
0.97	0.9124	0.9271	0.9397	0.9297	0.9474	0.9614
0.99	0.9296	0.9431	0.9550	0.9490	0.9647	0.9771
1.00	0. 93 74	0.9505	0.9617	0.9579	0.9727	0.9837

Table A.1:

Fractiles of LSDV Estimators of the AR Coefficient in Panel Data

Smpl. size	N = 57; T+1 = 41			N = 100; $T+1 = 31$		
AR / Fract	0.05	0.5	0.95	0.05	0.5	0.95
LSDV1		· · ·				
0.89	0.8110	0.8333	0.8535	0.7916	0.8121	0.8314
0.91	0.8296	0.8515	0.8708	0.8089	0.8293	0.8478
0.93	0.8476	0.8687	0.88 77	0.8263	0.8460	0.8643
0.95	0.8657	0.8857	0.9036	0.8435	0.8621	0.8794
0.97	0.8827	0.9019	0.9188	0.8592	0.8777	0.8944
0.99	0.8988	0.9173	0.9334	0.8751	0.8926	0.9086
0.999	0.9058	0.9237	0.9395	0.8817	0.8990	0.9146
LSDV2						
0.89	0.8101	0.8326	0.8529	0.7912	0.8117	0.8307
0.91	0.8285	0.8504	0.8700	0.8083	0.8285	0.8472
0.93	0.8469	0.8679	0.8864	0.8259	0.8454	0.8633
0.95	0.8649	0.8848	0.9028	0.8426	0.8617	0.8790
0.97	0.8820	0.9011	0.9183	0.8588	0.8771	0.8938
0.99	0.8981	0.9165	0.9326	0.8 744	0.8920	0.9080
1.00	0.9059	0.923 8	0.9395	0.8818	0.8991	0.9147

Table A.1 (Continued)

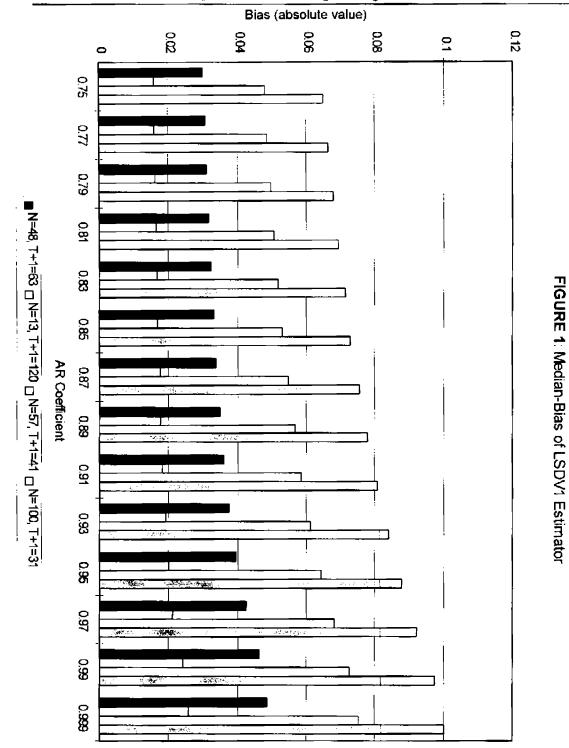
Table A.2:

Fractiles of the LSDV Estimators Under Groupwise Heteroskedasticity & Cross Sectional Correlation

Smpl. Size	N = 48; T+1 = 63			N = 13; $T+1 = 120$		
AR / Fract	0.05	0.5	0.95	0.05	0.5	0.95
LSDV1						
0.89	0.8305	0.8549	0.8788	0.8447	0.8718	0.8959
0.91	0.8488	0.8733	0.8953	0.8674	0.8922	0.9145
0.93	0.8691	0.8915	0.9128	0.8866	0.9107	0.9293
0.95	0.8885	0.9098	0.9292	0.9086	0.9300	0.9484
0.97	0.9071	0.9267	0.9451	0.9301	0.9486	0.9640
0.99	0.9252	0.9426	0.9594	0.9484	0.9659	0.9796
0.999	0.9327	0.9502	0.9663	0.9557	0.9729	0.9853
LSDV2						
0.89	0. 830 0	0.8543	0.8777	0.8439	0. 8 707	0 .89 47
0.91	0. 8 479	0.8727	0.8945	0.8665	0.8908	0.9126
0.93	0 .868 1	0.8909	0.9123	0.8863	0.9097	0.9 28 1
0.95	0.8883	0.9092	0.9286	0.9081	0.9288	0.9468
0.97	0.9068	0.9260	0.9446	0.9291	0. 94 74	0.9628
0.99	0.9247	0.9421	0.9591	0.9476	0.9649	0.9785
1.00	0.9327	0.9499	0.9657	0.9563	0.9726	0.9852

Smpl. Size	N = 57; $T+1 = 41$			N = 100; $T+1 = 31$		
AR / Fract	0.05	0.5	0.95	0.05	0.5	0.95
LSDV1						
0.89	0.7979	0.8322	0.8643	0.7762	0.8099	0.8464
0.91	0.8160	0.8491	0.8818	0.7932	0.8286	0.862 7
0.93	0.8356	0.8678	0.8987	0.8127	0.8442	0.8787
0.95	0.8538	0.8836	0.9144	0.8282	0.8596	0.8940
0.97	0. 8 704	0.8997	0.9281	0.8454	0.8759	0.9063
0.99	0.8879	0.9151	0.9418	0.8619	0.8896	0.9196
0.999	0 .8 963	0.9225	0.9476	0.8689	0.8976	0.9263
LSDV2						
0.89	0.7973	0.8313	0.8635	0.77 59	0.8095	0.8458
0.91	0.8154	0.8486	0.8811	0.7930	0.8280	0.8622
0.93	0.8353	0.8671	0.8978	0.8121	0.8437	0.8780
0.95	0.8531	0.8827	0.9138	0.8280	0.8592	0.8935
0.97	0.8698	0.8993	0.9276	0.8451	0. 8 755	0.9059
0.99	0.8869	0.9143	0.9411	0.8614	0.8891	0.9191
1.00	0 .89 49	0.9222	0.947 6	0.8677	0.8968	0.9269

Table A.2: (Continued)



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