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MANAGERIAL PERFORMANCE AND PRODUCT MARKET COMPETITION: THE HICKS CONJECTURE RECONSIDERED

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Managerial Performance and Product Market Competition: The Hicks Conjecture Reconsidered*

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Abstract

This article analyzes the effect that product market competition has on firms' incentives to improve their managerial performance. Product market competition is measured with the number of firms, and managerial performance with the manager's cost-reduction effort of each firm. First, a Cournot oligopoly, in which an arbitrary number of firms compete simultaneously, is analyzed. In this framework, it is found that firms in markets with more competitors have regularly lower managerial performance. Then, the increase in product market competition is analyzed in detail by considering a Stackelberg framework in which an arbitrary number of incumbents foresee the entry of a new firm. The main result of the study is that the managers of incumbent firms increase their cost-reduction efforts in the face of the entry of a firm to the product market, whatever prior level of competition. Therefore, the conclusion points in the direction of the Hicks conjecture: firms increase their managerial performance when product market competition rises.

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Contents

1 Introduction

> "It seems not at all unlikely that people in monopolistic positions will very often be people with sharply rising subjective costs; if this is so, they are likely to exploit their advantage much more by not bothering to get very near the position of maximum profit, than by straining themselves to get very close to it. The best of all monopoly profits is a quiet life."
>
> John R. Hicks (1935, p.8)

The Hick's conjecture holds that a monopoly may have no interest in conducting a superb management since it is costly and it has no competing firm to threaten its dominant position. Following a similar reasoning, Adam Smith (Book 1, Ch. 11) had already stated in 1776 that "Monopoly... is a great enemy to good management." In 1966, Leibenstein coined the term *X-inefficiency* as the loss of efficiency in an industry caused by firms' sub-optimal performance. He recognized that firms may operate above their optimal marginal cost because of internal reasons.[1] During the following years, there appeared to be a consensus in the literature supporting the Hick's conjecture: product market competition increases firms' managerial performance. Willig (1987, p. 481) sustained that "It is conventional wisdom that greater product market competition disciplines firms to have an efficient operation, while market power fosters managerial sloth and X-inefficiency." Nonetheless, during the past years it has not been an easy task to support this conjecture theoretically.

In 1983, Hart was the first one to analyze the problem using a principal-agent model. He found that higher competition increases firms managerial incentives. He emphasized the role of information. His model assumed that higher product market competition alleviates the

[1] It was originally coined *X-inefficiency* because Leibenstein (1966) did not have a clear explanation for this firms' sub-optimal performance, even though he showed that it existed with data from the U.S.

agency problem that exists between the owner and the manager of a firm. However, Scharfstein (1988) showed that Hart's results depended on assuming an infinite risk-averse manager. He showed that Hart's positive effect could be reversed by relaxing this assumption. Hermalin (1992) disentangled the relation into four different effects: information, risk-adjustment, income and value of cost-reduction. He found that all of them were potentially of ambiguous sign, making it hard to support any claim theoretically *a priori*. Since then, there has been no apparent consensus on the answer for this problem in the literature.

In this article we shall develop a new approach that offers a simple explanation. We will model an increase in product market competition by considering a sequential framework in which incumbent firms foresee the entry of a new firm to the market. The focus will be on the effect that the entry of a new firm has on the managerial performance of incumbent firms. In contrast, previous literature has analyzed the effect of product market competition on firms' managerial performance using comparative statics. Furthermore, we will abstract from information, risk and income effects by assuming independence among firms' cost realizations, risk-neutrality and quasilinear utility functions. Given our independence assumption, the strategic interaction among firms will be solely through the product market. We shall find that allowing the incumbents to be *strategic* when product market competition rises is the main cause for our main result: regardless of the number of incumbents, they strengthen their managerial incentives when they foresee the entry of a new firm to the market. Hence, our main finding shall be that firms increase their managerial performance when product market competition rises, whatever the prior level of product market competition.

In order to derive our main result, first, we shall develop a model in which an arbitrary number of firms compete simultaneously in a Cournot fashion. Using a comparative statics analysis, we will find that firms in more concentrated markets exhibit higher managerial performance regularly.[2] However, it shall be noted that using comparative statics implicitly assumes that firms do not react or change their behavior as competition rises. To address this issue we will develop an entry model in which an arbitrary number of incumbent firms precede an entrant in a Stackelberg fashion. Therefore, we will be able to compare two equilibriums: one in which an arbitrary number of firms are established Cournot competitiors with another one in which another firm is to enter the market. Comparing both equilibriums will allow us to examine how a firm changes its managerial performance when product market competition rises. In line with the Hicks conjecture, we shall conclude that every established firm increases its managerial performance when a new firm enters the market, whatever the previous level of product market competition.

The article is organized as follows. Section 2 contains a literature review. In section 3 the base model is presented. In section 4 we will analyze the monopoly case, and in section 5 we shall extend the model to a Cournot duopoly and oligopoly with simultaneous competition. Then, we will develop the entry model in a Stackelberg duopoly and oligopoly in section 6. We will perform a robustness check of the Stackelberg model in section 7. Section 8 contains a discussion of our main assumptions and results, and section 9 presents two testable implications of our results. Finally, section 10 summarizes the most important findings and concludes. The non-trivial proofs of the lemmas and propositions are relegated to the Appendix.

[2]This result is consistent with those of Martin (1993), Hermalin (1994), Schmidt (1997) and Raith (2003). We say *regularly* because this may not be the case in a monopoly for some parameter values. See Propositions 1 and 2 below.

2 Literature Review

The literature that analyzes the effect that product market competition has on managerial incentives contains four articles that measure competition (indirectly) with the number of firms. Martin (1993) develops an oligopoly model in which an exogenous number of firms compete in quantities. Firms' managers are assumed to have private information of their effort and of an exogenous productivity shock. His main result is that the expected marginal cost of firms is higher the more firms operate in the market. Thus, he concludes that competition affects firms' managerial performance negatively. Hermalin (1994) develops a Cournot model in order to prove the existence of asymmetric equilibriums when firms are otherwise identical. He concludes that the relation between the number of firms and managerial performance is ambiguous; it depends on the cost function's convexity and on the demand's elasticity to the number of firms. Under our assumptions, fixed demand's elasticity and convex cost function, he states that more firms imply lower managerial performance for a relatively wide range of parameter values. Both articles support our results using comparative statics in which firms compete in a Cournot fashion.

We will develop a moral hazard model similar to the one used by Schmidt (1997); however, he measures competition as an exogenous parameter. He finds that the effect of competition on managerial effort is in general ambiguous and depends on two effects: value of cost-reduction and threat of liquidation. He develops an example with Bertrand competition in which a higher number of firms is associated with lower managerial effort, except for the monopoly case. Raith (2003) develops a model with free entry and price competition in a Salop circle. In his framework, a more competitive market is characterized by a larger demand or by lower entry costs, since both induce a higher number of firms in equilibrium. He finds that competition increases managerial incentives if and only if more firms in the market imply a larger market-share for each firm in equilibrium. This is the case for a larger demand but not for lower entry costs. In our setting, the demand shall be assumed fixed, causing that a higher number of firms always implies a lower market-share for each firm. Therefore, Schmidt's and Raith's results are also consistent with our results under a Cournot framework .

Several authors analyze the effect of competition on managerial incentives in a differentiated goods duopoly. In this setting, product market competition is often measured by the type of competition and by the substitutability degree of the products produced by both firms. For example, Horn et al. (1994) find a negative relation comparing price competition with quantity competition. Aggarwal and Samwick (1999) analyze the relation between the use of relative performance evaluation contracts and the type of competition. Picollo et al. (2008) find a U-shape relationship and Beiner et al. (2011) an inverted U-shape relationship between product substitutability and managerial incentives. Plehn-Dujowich and Serfes (2010) derive the strategic properties of optimal contracts depending on market demand properties and type of competition. Fershtman and Judd (1985) and Sklivas (1987) find that separating property and management within a firm in a duopoly may give rise to contracts whose objective differs from profit maximization.[3]

Using an eleven country database, Porter (1990) concludes that the relation between the overall level of market competition in a country and its firms' managerial performance is positive. However, the empirical literature on this topic is not conclusive. There is no consensus on the right measure for product market competition or managerial incentives and performance.[4] Nickell (1996) finds a positive relation between firms' productivity growth rate

[3] There is a parallel IO literature that analyzes the link between market competition and innovation. In a strict sense, innovation refers to an increase of the production-possibility frontier, whereas managerial performance refers to how far from the frontier does a firm operate. Nonetheless, this literature is closely related to our case. Usually, there are two contrary effects in the relation between innovation and competition: the *schumpeterian* and *darwinian* effects, proposed by Schumpeter (1942) and Arrow (1962), respectively. Both are closely related to the *scale* and *opportunity cost* effects that we shall find later on, respectively. See Ross and Scherer (1990, Chap. 18), Vickers (1995) and Motta (2004, Chap. 2.3) for a literature review and its relation with managerial incentives.

[4] To measure product market competition the market concentration index, Lerner index, substitutability degree among products, market size, number of firms and entry costs are often used. Managerial incentives are

and competition, measured by the number of firms and firms' profits. Cuñat and Guadalupe (2005) find a positive relation between the pay-performance sensitivity of CEO contracts and industry competition. They use a quasi-natural experiment in which they observe changes in the competition level of an industry due to public policy in Great Britain. Karuna (2007) finds a positive relation of product substitutability degree, market size and concentration with the stock-option payment of CEOs.[5]

3 Model

Consider an industry in which firms produce a homogeneous product and compete in quantities. We measure the level of product market competition with the number of firms, which is given exogenously. The consumers' preferences are such that the inverse demand curve is $P = 1 - Q$, where P is the price of the product and Q its total output. If there are N firms in the industry, then $Q = \sum_{i=1}^{N} q_i$, where q_i is the quantity produced by firm $i = 1, ..., N$. Initially, all firms have marginal cost of production $c_H = c$, where $c \in (0, \bar{c})$. Each firm can hire a manager in order to increase its managerial performance and, hence, lower its marginal cost to $c_L = 0$. Thus, c is interpreted as the difference in efficiency between an efficient firm and an inefficient one. The upper bound $\bar{c}$ assures that all firms will produce at the equilibrium whatever their costs.[6]

The managerial relation within a firm implies a moral hazard problem, since a) a higher managerial effort does not assure a cost reduction, it only increases its probability, and b)

often measured with CEO contracts of large firms, for example, with pay-performance sensitivity, relative performance evaluation or stock-options clauses.

[5]See Van Reenan (2011) for a larger empirical literature review on the relation between product market competition and managerial incentives.

[6]The upper bound $\bar{c}$ will depend on the number of firms in the industry: the more firms, the lower $\bar{c}$ is. From now on, when we make a statement that holds $\forall\, c \in (0, \bar{c})$, which includes equilibrium values that come from industries with different number of firms, then we refer to the lowest $\bar{c}$ possible. Note as well that $\bar{c} \in (0, 1]$ because of the demand function given. Hence, if a statement holds $\forall\, c \in [0, 1]$, then it holds $\forall\, c \in (0, \bar{c})$.

the manager's effort is not observed by the firm's principal, he only observes the final cost realization.[7] The order of the game is as follows. First, the principal of each firm offers his manager a *take it or leave it* contract. Second, the manager of each firm performs cost-reduction effort. Third, cost realizations are publicly known. Finally, each principal decides his firm's output, profits are collected, and contracts fulfilled. We assume that cost realizations are independent among firms, and that all firms are initially symmetric. The probability of reducing the marginal cost of any firm i is given by $P(c_i = c_L) = e_i$, where $e_i \in [0,1]$ shall be interpreted as the manager's effort of firm i.

Every manager incurs in an effort cost $\psi(e_i) = e_i^2/2$. The manager's market is competitive, and all of them are equal. Therefore, a) managers have an outside option of $\bar{u} = 0$, since they have no bargaining power with firms' principals, and b) the matching process between managers and firms is irrelevant. To solve the moral hazard problem, each principal designs a debt-contract in which he pays the manager $b_i \geq 0$ if she lowers the marginal cost and zero otherwise. We assume that all principals and managers are risk-neutral. Hence, we impose a limited liability constraint in the manager's bonus accordingly.

4 Monopoly

Consider an industry with one firm in the product market, say firm i. Let us proceed by backward induction. The monopoly's profits will depend on its realized cost. Let $\Pi_i(c_L)$ and $\Pi_i(c_H)$ be the monopoly profits for low and high cost, respectively. Solving the monopoly problem leads us to the following profits in the last stage

$$\Pi_i(c_i) = \left(\frac{1-c_i}{2}\right)^2. \tag{4.1}$$

[7] From now on, we refer to the principal as *he* and to the manager as *she*.

Prior to the cost realization, the monopoly's expected profits may be written as

$$\mathbb{E}(\Pi_i) = e_i[\Delta\Pi_i - b_i] + \Pi_i(c_H), \tag{4.2}$$

where $\Delta\Pi_i = \Pi_i(c_L) - \Pi_i(c_H)$ is the value of the cost-reduction. For the manager to accept the contract, her individual rationality (IR) constraint must be satisfied, thus

$$e_i b_i - \frac{e_i^2}{2} \geq 0. \tag{IR}$$

As the manager's effort is not observed by the principal, she will perform the effort that maximizes her expected utility. Thus, the following incentive compatibility (IC) constraint must be satisfied

$$e_i = \arg\max_{\hat{e}} \left\{ \hat{e} b_i - \frac{\hat{e}^2}{2} \right\}. \tag{IC}$$

The contract must also satisfy the following limited liability (LL) constraint, which assures that the contract-bonus is non-negative,

$$b_i \geq 0. \tag{LL}$$

Solving (IC), we find that the manager will perform an effort $e_i = b_i$.[8] Since $e_i \in [0,1]$, then (IC) implies (LL). Replace $b_i = e_i$ in (IR) and (4.2) to obtain the following maximization problem of the principal

$$\max_{e_i} \quad e_i[\Delta\Pi_i - e_i] + \Pi_i(c_H) \tag{Max}$$

$$\text{s.a.} \quad \frac{e_i^2}{2} \geq 0. \tag{IR'}$$

[8]As $e_i = b_i$ according to (IC), from now on we will refer to managerial effort and incentives to increase managerial performance indistinctly. Rigorously, we should replace $e_i = b_i$ and solve for b_i to find the subgame perfect equilibrium. However, as will be noted further on, it is easier to solve for e_i, and then replace $b_i = e_i$. Given the equality implied by (IC), both procedures are equivalent.

The objective function is globally concave and the restriction does not bind. Therefore, the first order condition is necessary and sufficient. Thus, the optimal managerial effort and contract-bonus in a monopoly are given by

$$e_m = b_m = \frac{\Delta\Pi_i}{2} = \frac{c(2-c)}{8}. \tag{4.3}$$

Note that e_m is an increasing function of c. For a large c, the gain in profits due to a cost-reduction is high, which gives incentives to the manager to perform a higher effort. Furthermore, given the monopoly's cost, the monopoly price is

$$P_m(c_i) = \frac{1+c_i}{2}. \tag{4.4}$$

Therefore, using (4.3) and (4.4) we may compute the expected market price as

$$\mathbb{E}(P_m) = \frac{1+c(1-e_m)}{2} = \frac{8+8c-2c^2+c^3}{16}. \tag{4.5}$$

Note that $\mathbb{E}(P_m)$ is an increasing function of c as well. This is the result of two opposite effects. On the one hand, the monopoly's managerial effort is higher, the larger c is. As the monopoly is more efficient, its expected market price is lower *ceteris paribus*. Nonetheless, on the other hand, a larger c implies, *ceteris paribus*, a higher expected marginal cost, since c_H is higher, and thus a higher expected market price. Alternatively, note that $\mathbb{E}(P_m) = 1/2 + \mathbb{E}(c_i)/2$, and verify that the expected marginal cost $\mathbb{E}(c_i) = c(1-e_m)$ is increasing in c. So, even though a larger c implies a higher managerial effort, it is not sufficient to compensate the efficiency loss via marginal cost, and thus the expected price is higher.

5 Simultaneous Competition: Cournot

5.1 Cournot Duopoly

Now consider a setting in which two firms compete simultaneously in quantities, a Cournot duopoly. Let us proceed by backward induction. Once the firms' costs are realized, both firms play a usual Cournot static game with asymmetric costs in the last period. Let $\Pi_i(c_i, c_j)$ be the profits of firm $i = 1, 2$.[9] Solving for the Cournot equilibrium of two firms with different marginal costs, we obtain

$$\Pi_i(c_i, c_j) = \left(\frac{1 - 2c_i + c_j}{3}\right)^2. \tag{5.1}$$

In order to write the profits of firm i prior to the cost realizations, define the value of the cost-reduction of firm i, given c_j, as

$$\Delta_i \Pi_i(c_j) = \Pi_i(c_L, c_j) - \Pi_i(c_H, c_j) \quad \forall c_j \in \{c_L, c_H\}. \tag{5.2}$$

Then, in the first period, the expected profits of firm i may be written as

$$\mathbb{E}(\Pi_i) = e_i[\mathbb{E}(\Delta_i \Pi_i) - b_i] + \mathbb{E}(\Pi_i | c_i = c_H), \tag{5.3}$$

where

$$\mathbb{E}(\Delta_i \Pi_i) = e_j \Delta_i \Pi_i(c_L) + (1 - e_j) \Delta_i \Pi_i(c_H), \tag{5.4}$$

$$\mathbb{E}(\Pi_i | c_i = c_H) = e_j \Pi_i(c_H, c_L) + (1 - e_j) \Pi_i(c_H, c_H). \tag{5.5}$$

[9] In this and in the following section, we shall only model explicitly the first periods of the game. We will only state the equilibrium profit functions of the last period accordingly. The last stage in which product market competition takes place may be solved as a usual static oligopoly.

Therefore, taking into account the (IR), (IC) and the (LL) constraints, firm i solves

$$\max_{e_i} \quad e_i[\mathbb{E}(\Delta_i\Pi_i) - e_i] + \mathbb{E}(\Pi_i|c_i = c_H) \tag{Max}$$

$$\text{s.a.} \quad \frac{e_i^2}{2} \geq 0. \tag{IR'}$$

The first order condition of this problem serves as the best-response function of firm i, given by

$$e_i(e_j) = \frac{\mathbb{E}(\Delta_i\Pi_i)}{2} = \frac{\Delta_i\Pi_i(c_H) - [\Delta_i\Pi_i(c_H) - \Delta_i\Pi_i(c_L)]e_j}{2} \quad \forall i = 1,2. \tag{5.6}$$

Note that the managerial effort choices of firms are strategic substitutes if $\Delta_i\Pi_i(c_H) > \Delta_i\Pi_i(c_L)$, which is the case in our present framework. The intuition of this condition is straightforward: if it is more profitable for firm i to lower its marginal cost when firm j is high-cost than when it is low-cost, then, if firm j increases its cost-reduction effort, firm i will lower its managerial effort in return, since the cost reduction would be less profitable. Solving the system of equations implied in (5.6), we find that the unique equilibrium is symmetric and the managerial effort of each firm is given by

$$e_c = \frac{\Delta_i\Pi_i(c_H)}{2 + [\Delta_i\Pi_i(c_H) - \Delta_i\Pi_i(c_L)]} = \frac{2c}{9 + 2c^2}\,. \tag{5.7}$$

Compare the latter expression with (4.3), the one for managerial effort in a monopoly. In contrast to the monopoly case, this expression includes the term $\Delta_i\Pi_i(c_H) - \Delta_i\Pi_i(c_L)$ in the denominator. This term is the result of the strategic interaction in contracting decisions due to product market competition.

Proposition 1. *The equilibrium effort done by a firm in a Cournot duopoly is higher than the equilibrium effort done by a monopoly if and only if the cost reduction is sufficiently high. Formally,*

$$e_c \geq e_m \quad \textit{if and only if} \quad c \geq c^* \approx 0.25. \tag{5.8}$$

The proof of Proposition 1 is omitted. The result may be easily obtained from expressions (4.3) and (5.7). Note that in a duopoly $\bar{c} = 1/2$. In our Cournot framework, the relation between the number of firms and their managerial effort is determined by two contrary effects. On the one hand, the *scale effect* establishes that a higher number of firms causes the market-share of each firm to be smaller, and thus their profits as well. Therefore, more firms imply a lower value of cost-reduction, and thus decrease the incentives to lower the marginal cost. On the other hand, the *opportunity cost effect* establishes that the opportunity cost of lowering the marginal cost, high-cost profits, decreases as the number of firms increases. As it is worse to stay with a high cost when the number of firms is more than one, then the cost-reduction effort increases with the number of firms.

Note that in the duopoly case the *scale effect* is fixed; there is only one more firm than in a monopoly. However, the opportunity cost of lowering the marginal cost is negatively related to c since high-cost profits diminish for larger values of c. Therefore, a larger c implies a stronger opportunity cost effect due to the increase of one firm in the market. When c is sufficiently large, $c \geq c^*$, then the opportunity cost effect is stronger than the scale effect, so the effort of a firm in a duopoly will be higher than the monopoly effort. Furthermore, given the cost of each firm, in the last period the market price is given by

$$P_c(c_i, c_j) = \frac{1 + c_i + c_j}{3}. \tag{5.9}$$

Therefore, using (5.7) and (5.9) we can compute the expected market price as

$$\mathbb{E}(P_c) = \frac{2c^2 + 18c + 9}{27 + 6c^2}. \tag{5.10}$$

Corollary 1. *The expected market price of a monopoly is higher than that of a Cournot duopoly.*

The proof of Corollary 1 is omitted. The result may be easily obtained from expressions (4.5) and (5.10). Proposition 1 states that the effort done by each firm in a duopoly may be higher or lower than the effort done in a monopoly. However, we find that the expected market price will always be lower in a duopoly. A monopoly enjoys full market power, whereas in a duopoly, quantity competition drives the market price downward for the usual reasons. Therefore, even though a monopoly may have better management than a firm in a duopoly, the effect that an increase in product market competition has on the market price directly is stronger than the one it has via managerial effort.

5.2 Cournot Oligopoly

In this section we shall generalize the model to an oligopolistic industry in which N firms operate in the product market. Let $\mathscr{N}$ be the set of all the firms in the industry. Initially, all the firms are high-cost. However, after the cost realizations there will be potentially low and high-cost firms. Let $\mathscr{L}$ and $\mathscr{H}$ be the sets of low and high-cost firms after cost realizations, respectively. Define $N = |\mathscr{N}|$, $L = |\mathscr{L}|$ and $H = |\mathscr{H}|$. In other words, of the N total firms, L reduce their marginal cost and H stay with high marginal cost. Then, by definition we have $\mathscr{N} = \mathscr{L} \cup \mathscr{H}$ and $N = L + H$.

The variables L and H are perfectly correlated random variables. Therefore, without loss of generality we shall focus the analysis on the variable L. Let us proceed by backward induction. In the last period, the firms play a usual Cournot static game with asymmetric costs. Let $\Pi_i(c_i, L)$ be the profit function of firm $i \in \mathscr{N}$, given that there are L low-cost firms in the market. Solving for the equilibrium of a Cournot oligopoly consisting of N firms with asymmetric costs c_L and c_H, we find that the profit function of firm i is given by

$$\Pi_i(c_i, L) = \left(\frac{1 - c_i(N+1) + c(N-L)}{N+1} \right)^2. \tag{5.11}$$

Define the costs profile $\mathscr{C}_{-i} = (c_1, ..., c_{i-1}, c_{i+1}, ..., c_N)$ as the distribution of all the firms' costs except firm i. Hence, prior to the cost realizations, the random variable L has the following probabilities conditional on the cost of firm i

$$P(L=l|c_i=c_H) = \sum_{\mathscr{C}_{-i}|L=l} \left[\prod_{k\in\mathscr{L}} e_k \prod_{j\in\mathscr{H}} (1-e_j) \right] \quad \forall l = 0, ..., N-1, \tag{5.12}$$

$$P(L=l|c_i=c_L) = \sum_{\mathscr{C}_{-i}|L=l-1} \left[\prod_{k\in\mathscr{L}} e_k \prod_{j\in\mathscr{H}} (1-e_j) \right] \quad \forall l = 1, ..., N. \tag{5.13}$$

Therefore, we can establish the following

$$P(L=l|c_i=c_H) = P(L=l+1|c_i=c_L). \tag{5.14}$$

In other words, the probability that there are L low-cost firms given that firm i has high cost is the same probability that there are $L+1$ low-cost firms given that firm i has low cost. Note that this is a straight implication of our independence assumption. Define the value of cost-reduction of firm i given that there are L low-cost firms as

$$\Delta_i \Pi_i(L) = \Pi_i(c_L, L+1) - \Pi_i(c_H, L). \tag{5.15}$$

In the first period, the expected profits of firm i can be written as

$$\mathbb{E}(\Pi_i) = e_i \mathbb{E}(\Pi_i|c_i=c_L) + (1-e_i)\mathbb{E}(\Pi_i|c_i=c_H) - e_i b_i \tag{5.16}$$

$$= e_i \Big[\mathbb{E}(\Pi_i|c_i=c_L) - \mathbb{E}(\Pi_i|c_i=c_H) \Big] + \mathbb{E}(\Pi_i|c_i=c_H) - e_i b_i, \tag{5.17}$$

where the difference in brackets is

$$\mathbb{E}(\Pi_i|c_i=c_L) - \mathbb{E}(\Pi_i|c_i=c_H) = \sum_{l=1}^{N} P(L=l|c_i=c_L)\Pi_i(c_L,l) - \sum_{l=0}^{N-1} P(L=l|c_i=c_H)\Pi_i(c_H,l). \tag{5.18}$$

Change the counter of the sum in the first term of the right-hand side of (5.18) from l to $l+1$, and use (5.14) and (5.15) to rewrite (5.18) as

$$\mathbb{E}(\Pi_i|c_i = c_L) - \mathbb{E}(\Pi_i|c_i = c_H) = \mathbb{E}(\Delta_i\Pi_i). \tag{5.19}$$

Replacing (5.19) in (5.17) leads us to the following expected profits of firm i

$$\mathbb{E}(\Pi_i) = e_i[\mathbb{E}(\Delta_i\Pi_i) - b_i] + \mathbb{E}(\Pi_i|c_i = c_H). \tag{5.20}$$

Note that expression (5.3) of the duopoly case is a particular case of (5.20). Therefore, taking into account the (IR), (IC) and (LL) constraints and solving the maximization problem of firm i, we find its best-response function, given by

$$e_i(e_{-i}) = \frac{\mathbb{E}(\Delta_i\Pi_i)}{2} = \frac{1}{2}\sum_{l=0}^{N-1} P(L = l|c_i = c_H)\Delta_i\Pi_i(l) \quad \forall i \in \mathcal{N}. \tag{5.21}$$

Lemma 1. *In the unique symmetric equilibrium of a Cournot oligopoly with N firms, the managerial effort performed by every firm is*

$$e_c(N) = \frac{\Delta_i\Pi_i(0)}{2 + (N-1)[\Delta_i\Pi_i(0) - \Delta_i\Pi_i(1)]} = \frac{Nc}{2}\left(\frac{2 + c(N-2)}{(N+1)^2 + Nc^2(N-1)}\right). \tag{5.22}$$

The proof of the previous lemma may be found in the Appendix. Lemma 1 states the effort of each firm in the unique symmetric equilibrium. However, it does not exclude the existence of other asymmetric equilibriums. Given the initial symmetry of the firms, there is no clear mechanism by which the firms could reach an asymmetric equilibrium. Therefore, we delimit the analysis to the symmetric equilibrium.[10] Replacing $N = 1$ and $N = 2$ in (5.22) results in the equilibrium efforts that we previously obtained for a monopoly and a duopoly, given

[10] Hermalin (1994) proves the existence of asymmetric equilibriums in a more general context. However, the analysis of asymmetric equilibriums goes beyond the reach of this study.

in expressios (4.3) and (5.7), respectively. As all the firms are symmetric in the oligopoly, the managerial effort in equilibrium depends only on the value of cost-reduction when no other firm lowers its cost, $\Delta_i\Pi_i(0)$, and on its comparison with the value of cost-reduction when another firm lowers its cost pondered by the number of rival firms, $(N-1)[\Delta_i\Pi_i(0) - \Delta_i\Pi_i(1)]$. This last term is the result of the strategic interaction in contracting decisions due to product market competition. Now, let us use comparative statics to analyze the effect that the number of firms has on the managerial equilibrium effort.

Proposition 2. *Let $e_c(n)$ be the effort level in the unique symmetric equilibrium of a Cournot oligopoly, given in* (5.22)*, for the case of $N = n$ firms. Then,*

$$e_c(n) \geq e_c(n+1) \quad \forall n \geq 2. \tag{5.23}$$

The proof of the previous proposition may be found in the Appendix. Proposition 1 established that the effort done by a firm in a duopoly could be higher or lower than the effort done by a monopoly, depending on c. Proposition 2 states that this result no longer holds for industries with more than one firm. If we compare an industry with N firms and another one with $N+1$ firms, then the effort performed by a manager in the first industry will always be higher than by one in the second, except for the monopoly case. Furthermore, note that the expected marginal cost of each firm in equilibrium is $\mathbb{E}(c_i) = c(1-e_c(n))$ with $c \in (0, \bar{c})$ fixed. Hence, the expected marginal cost is increasing in the number of firms when there are at least two firms in the market. This result is due to the scale effect, which states that an increase in the number of firms lowers the cost-reduction effort of managers. Therefore, even though the opportunity cost effect increases the effort with the number of firms, the scale effect is stronger.

This result may seem somewhat troubling, since it states that two firms are sufficient for the opportunity cost effect to lose importance. In other words, it seems that product market competition may increase the managerial performance of a firm just in the case of a monopoly. According to Proposition 2, an increase in product market competition will always diminish the managerial performance of a firm, as long as there are at least two firms in the industry. Thus far we have compared an industry with N firms to one with $N+1$ firms. Intuitively, the N firms of the first industry *are not the same* N firms of the second industry (plus one), since all firms are symmetric in the $N+1$-firm industry. In other words, we are comparing two industries with different levels of product market competition. However, if we want to address the question of how an increase in product market competition affects the managerial performance of a firm, then we must analyze how a firm *reacts* to an increase in product market competition, i.e., to the entry of a new firm.

6 Increase in Competition: Stackelberg

In this section let us consider an industry with the same characteristics as before, except one. In this case, consider an industry with N incumbent competitors and one entrant firm. This setting will allow us to analyze the effect that firm entry has on the incumbents' managerial effort. We will compare this new equilibrium with the previous one to analyze how the manager of an established firm changes its effort when a firm is to enter the market. In other words, we address the effect that an increase in product market competition has on the managerial effort of previously established firms. The order of the game is as follows: in period 1) the incumbent firms decide their respective bonus-contracts and their managers their respective effort levels simultaneously; 2) incumbents' costs are publicly realized; 3) incumbent principals make output decisions simultaneously; 4) the entrant firm decides its bonus-contract and its manager her effort after observing the incumbents' aggregate output;

5) the entrant principal decides his firm's output. We consider the order of the game as exogenous.[11] We shall refer to this sequential setting as *Stackelberg*.

6.1 Stackelberg duopoly

First, consider the case in which we have an initial monopoly and another firm enters the market. Thus, we have a Stackelberg duopoly with one incumbent firm and one entrant. Let firm 1 be the incumbent and firm 2 the entrant. Let us proceed by backward induction. In the fifth period, the entrant firm knows c_1, q_1 and c_2, and produces q_2 according to

$$q_2(c_2,q_1) = \arg\max_{q_2} \{\Pi_2 = (1-q_1-q_2-c_2)q_2\} = \frac{1-q_1-c_2}{2}. \tag{6.1}$$

Therefore, in the fifth period the profits of the entrant firm are given by

$$\Pi_2(c_2,q_1) = \left(\frac{1-q_1-c_2}{2}\right)^2. \tag{6.2}$$

In the fourth period, the entrant firm knows c_1 and q_1; c_2 is yet to be realized. Its expected profits may be written as

$$\mathbb{E}(\Pi_2) = e_2[\Delta_2\Pi_2(q_1) - b_2] + \Pi_2(c_H,q_1), \tag{6.3}$$

where $\Delta_2\Pi_2(q_1) = \Pi_2(c_L,q_1) - \Pi_2(c_H,q_1)$ is the entrant's value of cost-reduction given q_1. Maximizing (6.3) with respect to e_2 subject to the (*IC*), (*IR*) and (*LL*) constraints results in the entrant's best response for effort given by

$$e_2(q_1) = \frac{\Delta_2\Pi_2(q_1)}{2} = \frac{c(2-2q_1-c)}{8}. \tag{6.4}$$

[11] In the following section, it will be observed that our results are robust to alternative game orders.

In the third period, when the entrant firm has not entered the market yet, the incumbent knows c_1.[12] The incumbent's expected profits conditional on c_1 and c_2 are

$$\mathbb{E}(\Pi_1|c_1,c_2) = [1-q_2(c_2,q_1)-q_1-c_1]q_1. \tag{6.5}$$

Recall that $P(c_2 = c_L) = e_2(q_1)$. Hence, by the Law of Iterated Expectations (LIE), we may write the incumbent's expected profit given c_1 as

$$\mathbb{E}(\Pi_1|c_1) = e_2(q_1)\mathbb{E}(\Pi_1|c_1,c_2 = c_L)+[1-e_2(q_1)]\mathbb{E}(\Pi_1|c_1,c_2 = c_H). \tag{6.6}$$

Replacing (6.1) and (6.4) in (6.6) we obtain

$$\mathbb{E}(\Pi_1|c_1) = q_1\left[\frac{1-q_1-2c_1+c}{2}-\frac{c^2(2-2q_1-c)}{16}\right]. \tag{6.7}$$

Hence, the incumbent maximizes (6.7) with respect to q_1 to find its optimal production level, given by

$$q_1(c_1) = \frac{c^3-2c^2+8c+8-16c_1}{4(4-c^2)}. \tag{6.8}$$

Note that the expected value of the incumbent's profits in (6.7) is taken over the entrant's marginal cost. We take the liberty of writing $\Pi_1(c_1) = \mathbb{E}(\Pi_1|c_1)$. Hence, replace (6.8) in (6.7) to find the following incumbent's profits, given c_1,

$$\Pi_1(c_1) = \frac{(c^3-2c^2+8c+8-16c_1)^2}{32(16-4c^2)}. \tag{6.9}$$

In the first period, the incumbent does not know its own marginal cost. Its expected profits may be written as

$$\mathbb{E}(\Pi_1) = e_1[\Delta_1\Pi_1-b_1]+\Pi_1(c_H), \tag{6.10}$$

[12] Recall that we assumed $c \in (0,\bar{c})$ in such a way that the upper bound $\bar{c}$ assures that every firm produces in equilibrium. In other words, $\bar{c}$ is such that it is always profitable for the entrant firm to enter the market, regardless of what the final cost realizations are.

where $\Delta_1\Pi_1 = \Pi_1(c_L) - \Pi_1(c_H)$ is the incumbent's value of cost-reduction. Maximizing (6.10) with respect to e_1 and subject to the (*IC*), (*IR*) and (*LL*) constraints results in the incumbent's optimal managerial effort given by

$$e_s = \frac{\Delta_1\Pi_1}{2} = \frac{c(c^3 - 2c^2 + 8)}{8(4 - c^2)}. \tag{6.11}$$

Therefore, in the sub-game perfect equilibrium, the incumbent's and entrant's respective effort levels are given by e_s and $e_2(q_1(c_1))$, and their respective production levels by $q_1(c_1)$ and $q_2(c_2, q_1(c_1))$.

Proposition 3. *A monopoly increases its effort to reduce its marginal cost if another firm enters the market. Formally,*

$$e_s > e_m \quad \forall c \in (0, \bar{c}). \tag{6.12}$$

The proof of Proposition 3 is omitted. The result may be easily obtained from expressions (4.3) and (6.11). In Proposition 1, we found that a firm in a duopoly can perform a higher or lower effort than a monopoly, depending on c. In that case, we cannot say that the monopoly is one of the firms in the duopoly. In order to do so, we would need to assume that the monopoly is *blind* to the entry of the second firm. However, in the present Stackelberg duopoly, we find the managerial effort that an incumbent does in a duopoly. Therefore, Proposition 3 establishes how a monopoly would react if another firm enters the market. We find that a monopoly's manager always increases her cost-reduction effort when a second firm is to enter the market.

The latter result is due to a *strategic effect*. In a usual Stackelberg duopoly, the leading firm takes advantage of its position by being aggressive. It produces more output before the other firm enters the market, so the second firm is not able to produce a large output, since it would lower the price too much. In this setting, the strategic effect exploits the position of the Stackelberg leader. From (6.4) note that, if the incumbent produces a higher output, the best response of the entrant's manager is to lower her effort. A lower entrant's managerial effort will translate into a higher expected marginal cost, and thus a lower output. Hence, the incumbent has incentives to increase its managerial effort in order to decrease its marginal cost and thus increase its output. Allowing the incumbent firm to anticipate the entrant adds a strategic effect, which, together with the opportunity cost effect, is larger than the scale effect.

Now let us we compute the expected market price. Once the cost realizations are made, the final market price is given by $P_s = 1 - q_1(c_1) - q_2(c_2, q_1(c_1))$. First, compute the expected production of the incumbent firm as

$$\mathbb{E}(q_1) = e_s q_1(c_L) + [1 - e_s] q_1(c_H). \tag{6.13}$$

Replace (6.8) and (6.11) in (6.13) to obtain

$$\mathbb{E}(q_1) = \frac{c^5 - 2c^4 + 12c^3 - 32c + 32}{4(4 - c^2)^2}. \tag{6.14}$$

To find the expected entrant's effort, replace $q_1 = \mathbb{E}(q_1)$ in expression (6.4) and obtain

$$\mathbb{E}(e_2) = \frac{c(4 - 3c^2)(c^3 - 2c^2 + 8)}{16(4 - c^2)^2}. \tag{6.15}$$

Note that the entrant's expected cost is given by $\mathbb{E}(c_2) = c[1 - \mathbb{E}(e_2)]$. Use this fact and (6.14) to compute the expected entrant's production, given by

$$\mathbb{E}(q_2) = \frac{1 - \mathbb{E}(q_1) - \mathbb{E}(c_2)}{2} = \frac{-3c^7 + 6c^6 - 16c^5 - 8c^4 + 80c^3 - 96c^2 - 128c + 128}{32(4 - c^2)^2}. \tag{6.16}$$

Finally, use (6.13) and (6.16) to compute the expected market price as $\mathbb{E}(P_s) = 1 - \mathbb{E}(q_1) - \mathbb{E}(q_2)$, given by

$$\mathbb{E}(P_s) = \frac{3c^5 - 6c^4 + 20c^3 + 32c^2 - 96c - 32}{32(4 - c^2)}. \tag{6.17}$$

Proposition 4. *The expected market price in a monopoly is higher than in a duopoly, Cournot or Stackelberg. Furthermore, the expected price in a Cournot duopoly is higher than in a Stackelberg one. Formally,*

$$\mathbb{E}(P_m) > \mathbb{E}(P_c) > \mathbb{E}(P_s) \quad \forall c \in (0, \bar{c}). \tag{6.18}$$

The proof of Proposition 4 is omitted. The first inequality follows from Corollary 1, and the second one may be easily obtained from expressions (5.10) and (6.17). In Corollary 1 we found that the market price in a Cournot duopoly is lower than in a monopoly. In Proposition 4 we find that in a Stackelberg duopoly the market price is even lower. This result is due to the higher effort done by the incumbent firm. In other words, the Stackelberg duopoly market price is lower than the monopoly one for the usual reasons of increasing the number of firms. However, it is even lower than the Cournot duopoly market price because the strategic effect causes the incumbent firm to perform a higher effort, which lowers the expected marginal cost, and thus the expected market price as well.

6.2 Stackelberg oligopoly

In this section let us consider the generalization of the Stackelberg duopoly of the previous section. Consider an industry with N established firms and assume that another firm enters the market. Thus, we have N leading firms and one follower, denoted by f. Let $\mathscr{N}$ be the set of all the incumbent firms, and $\mathscr{L}$ and $\mathscr{H}$ be the sets of low and high-cost incumbents after cost realizations, respectively. Define $N = |\mathscr{N}|$, $L = |\mathscr{L}|$ and $H = |\mathscr{H}|$. Let $Q_N = \sum_{i \in \mathscr{N}} q_i$ be the total output of the incumbent firms at the end of the third period. Let us proceed by backward induction. In the sixth period, the entrant firm knows L, Q_N and c_f, and produces

q_f according to the following program

$$q_f(c_f, Q_N) = \arg\max_{q_f} \left\{\Pi_f = (1 - Q_N - q_f - c_f)q_f\right\} = \frac{1 - Q_N - c_f}{2}. \tag{6.19}$$

Therefore, in the fifth period the profits of the entrant firm are given by

$$\Pi_f(c_f, Q_N) = \left(\frac{1 - Q_N - c_f}{2}\right)^2. \tag{6.20}$$

In the fourth period, the entrant firm knows L and Q_N; c_f is yet to be realized. Its expected profits may be written as

$$\mathbb{E}(\Pi_f) = e_f[\Delta_f \Pi_f(Q_N) - b_f] + \Pi_f(c_H, Q_N), \tag{6.21}$$

where $\Delta_f \Pi_f(Q_N) = \Pi_f(c_L, Q_N) - \Pi_f(c_H, Q_N)$ is the entrant's value of cost-reduction given Q_N. Maximizing (6.21) with respect to e_f subject to the (*IC*), (*IR*) and (*LL*) constraints results in the entrant's best response for effort given by

$$e_f(Q_N) = \frac{\Delta_f \Pi_f(Q_N)}{2} = \frac{c(2 - 2Q_N - c)}{8}. \tag{6.22}$$

In the third period, any incumbent firm $i \in \mathcal{N}$ knows c_i and L. Let $\Pi_i(c_i, q_{-i}, c_f)$ be the profits of an incumbent firm, where $q_{-i} = Q_N - q_i$ is the outoput of all other incumbent firms. Then, by the LIE, the expected profits of firm i may be written as

$$\mathbb{E}(\Pi_i | c_i) = e_f(Q_N)\Pi_i(c_i, q_{-i}, c_L) + [1 - e_f(Q_N)]\Pi_i(c_i, q_{-i}, c_H), \tag{6.23}$$

where

$$\Pi_i(c_i, q_{-i}, c_f) = [1 - q_{-i} - q_f(c_f, Q_N) - q_i - c_i]q_i. \tag{6.24}$$

Replace (6.19), (6.22) and (6.24) in (6.23) to obtain

$$\mathbb{E}(\Pi_i|c_i) = q_i \left[\frac{1 - q_{-i} - q_i - 2c_i + c}{2} - \frac{c^2(2 - 2q_{-i} - 2q_i - c)}{16} \right]. \tag{6.25}$$

Maximize (6.25) with respect to q_i to obtain the following best response of an incumbent firm in the production stage

$$q_i(q_{-i}) = \frac{c^3 - 2c^2 + 8c + 8 - 16c_i}{4(4 - c^2)} - \frac{q_{-i}}{2}. \tag{6.26}$$

Summing for all i in both sides of the previous expression and solving for Q_N in equilibrium we obtain

$$Q_N(L) = \frac{N(c^3 - 2c^2 + 8c + 8) - 16c(N - L)}{2(N + 1)(4 - c^2)}. \tag{6.27}$$

Using the latter expression and the fact that $q_{-i} = Q_N - q_i$ for any $i \in \mathcal{N}$, from (6.26) we may obtain the production of firm i in equilibrium given by

$$q_i(c_i, L) = \frac{c^3 - 2c^2 + 8c + 8 - 16c_i(N + 1) + 16c(N - L)}{2(4 - c^2)(N + 1)}. \tag{6.28}$$

Then, according to (6.25), we can write the expected profits of firm i conditional on its own cost as

$$\mathbb{E}(\Pi_i|c_i) = q_i \left[\frac{1 - Q_N - 2c_i + c}{2} - \frac{c^2(2 - 2Q_N - c)}{16} \right]. \tag{6.29}$$

Note that the expected value of the latter expression is taken over the entrant's marginal cost. We take the liberty of writing $\Pi_i(c_i, L) = \mathbb{E}(\Pi_i|c_i)$. Replacing (6.27) and (6.28) in (6.29) accordingly results in the following profits of firm i

$$\Pi_i(c_i, L) = \frac{[c^3 - 2c^2 + 8c + 8 - 16c_i(N + 1) + 16c(N - L)]^2}{32(4 - c^2)(N + 1)^2}. \tag{6.30}$$

Let us follow the same procedure as in section 5 to obtain the expected profits of an incumbent firm in the first period, prior to the cost realizations. Hence, from (5.20) we obtain

$$\mathbb{E}(\Pi_i) = e_i[\mathbb{E}(\Delta_i\Pi_i) - b_i] + \mathbb{E}(\Pi_i|c_i = c_H), \tag{6.31}$$

where $\Delta_i\Pi_i(L) = \Pi_i(c_L, L+1) - \Pi_i(c_H, L)$ is the value of cost-reduction of an incumbent firm. Therefore, maximizing (6.31) with respect to e_i subject to the (IR), (IC) and (LL) constraints, we obtain the best-response function of firm i, given by

$$e_i(e_{-i}) = \frac{\mathbb{E}(\Delta_i\Pi_i)}{2}. \tag{6.32}$$

Lemma 2. *In the unique symmetric equilibrium among incumbents of a Stackelberg oligopoly with N incumbent firms, the managerial effort performed by every incumbent firm is*

$$e_s(N) = \frac{Nc}{2}\left(\frac{c^3 - 2c^2 - 8c + 8 + 8cN}{(N+1)^2(4-c^2) + 8c^2N(N-1)}\right). \tag{6.33}$$

The proof of the previous lemma may be found in the Appendix. Note that replacing $N = 1$ in (6.33) results in the expression we previously obtained for the duopoly case, given in expression (6.11). As for the managerial equilibrium effort of a Cournot oligopoly, in this setting we cannot discard the existence of asymmetric equilibriums. However, following the same reasoning as before, we focus only in the symmetric equilibrium regarding incumbent firms. Now, let us analyze the effect that the entry of a firm to the product market has on the managerial equilibrium effort of incumbents.

Proposition 5. *Every firm in a Cournot oligopoly of N firms increases its managerial effort if another firm is to enter the market, whatever the number of firms. Formally, let* $e_s(n)$ *be the symmetric equilibrium effort given in* (6.33) *for the case of* $N = n$ *incumbent firms, then*

$$e_s(n) > e_c(n) \quad \forall n \geq 1. \tag{6.34}$$

The proof of the previous proposition may be found in the Appendix. Proposition 5 states our most important result: the manager of a firm in a Cournot market will increase her cost-reduction effort if a firm is to enter the market, whatever the number of firms already in the market. In other words, for any prior level of product market competition, a firm will improve its management if product market competition rises. This result is a generalization of Proposition 3. Whatever the initial number of firms, the strategic effect plus the opportunity cost effect are always stronger than the scale effect when a firm enters the market; thus, every existing firm increases its effort to reduce marginal cost.

Our result emphasizes the importance of modeling the increase in product market competition as a transition between a market with less competition to one with more competition. The main reason is the Stackelberg leaders' aggressive strategy. In this framework, the question of how product market competition affects a firm's managerial performance can be appropriately addressed. In the previous Cournot framework, the question we addressed was how is the managerial performance of firms in highly competitive industries compared to that of firms in less competitive industries. Although it may appear to be a subtle difference, it has a resounding effect on the given answer. Even though the Cournot framework gave us the first approach to the scale and opportunity cost effects, we ignored the strategic effect. The strategic effect resulted in being the key to understanding the mechanism behind the positive effect that product market competition has on managerial performance.

7 Robustness Check: Alternative Orders

In this section let us replicate our Stackelberg oligopoly model using two alternative orders. First, we shall consider an intercalated order in which the entrant firm signs its contract and its manager performs effort prior to the production stage of the incumbents. Nonetheless, we will assume that the entrant produces after the incumbent firms do so. This order is intuitive if we consider that a contracting stage may take less time than changing production plans. Hence, under this framework, we allow for the entrant firm to sign its contract before the incumbents can change production. Secondly, we consider a scenario in which the entrant is an already existing firm that is entering the market of a new product. Hence, the entrant may sign its managerial contract at the same time as its competitors, but it is not able to produce simultaneously, since presumably it takes it more time to set up for production. We will find that our main result in Proposition 5 holds using any of these two alternative orders: every firm in an oligopoly increases its managerial effort if another firm is to enter the product market, whatever the number of already existing firms. This fact emphasizes that the main drive of our result is our sequential framework in the product market.

7.1 Intercalated Order

Let us consider the following order: in period 1) the incumbents decide their respective bonus-contracts and their managers their respective effort levels simultaneously; 2) the entrant decides its bonus-contract and its manager her effort; 3) all firms' costs are publicly realized; 4) incumbent principals make output decisions simultaneously; 5) the entrant principal decides his firm's output after observing the aggregate output of the incumbents. Let us proceed by backward induction. Once all the firms' costs are realized, the firms play a Stackelberg game with N leaders and one follower with asymmetric costs c_L and c_H during the last two periods.[13] Let $\Pi_i(c_i, L, c_f)$ be the profits of firm $i \in \mathscr{N}$ and $\Pi_f(c_f, L)$ be the profits of firm f. Solving for the equilibrium of this sub-game we obtain the following profit function for any incumbent, say firm i

$$\Pi_i(c_i, L, c_f) = \frac{1}{2}\left(\frac{1 + c_f - 2c_i(N+1) + 2c(N-L)}{N+1}\right)^2. \tag{7.1}$$

Likewise, the profit function of firm f is

$$\Pi_f(c_f, L) = \left(\frac{1 - c_f(2N+1) + 2c(N-L)}{2(N+1)}\right)^2. \tag{7.2}$$

Define the costs profile $\mathscr{C} = (c_1, ..., c_N)$ as the distribution of all the incumbents' costs. Prior to the cost realizations, the random variable L has the following unconditional probability

$$P(L = l) = \sum_{\mathscr{C}|L=l}\left[\prod_{k\in\mathscr{L}} e_k \prod_{j\in\mathscr{H}} (1 - e_j)\right] \quad \forall l = 0, ..., N. \tag{7.3}$$

Define the value of the cost-reduction of firm f, given L, as

$$\Delta_f \Pi_f(L) = \Pi_f(c_L, L) - \Pi_f(c_H, L). \tag{7.4}$$

The expected profits of firm f in the second period may be written as

$$\mathbb{E}(\Pi_f) = e_f \mathbb{E}(\Pi_f | c_f = c_L) + (1 - e_f)\mathbb{E}(\Pi_f | c_f = c_H) - e_f b_f, \tag{7.5}$$

where

$$\mathbb{E}(\Pi_f | c_f) = \sum_{l=0}^{N} P(L = l)\Pi_f(c_f, l). \tag{7.6}$$

[13] We do not solve these periods explicitly. The following equilibrium profit functions may be computed in the same fashion as we did for the equilibrium profit functions in section 6.2.

Therefore, we may rewrite expression (7.5) as

$$\mathbb{E}(\Pi_f) = e_f \mathbb{E}(\Delta_f \Pi_f) + \mathbb{E}(\Pi_f | c_f = c_H) - e_f b_f. \tag{7.7}$$

Firm f maximizes (7.7) with respect to e_f subject to the (IR), (IC) and (LL) constraints. The first order condition of this problem serves as the best-response function of firm f given by

$$e_f(e_{-f}) = \frac{\mathbb{E}(\Delta_f \Pi_f)}{2}, \tag{7.8}$$

where $\mathbb{E}(\Delta_f \Pi_f)$ may be written as

$$\mathbb{E}(\Delta_f \Pi_f) = e_i \mathbb{E}(\Delta_f \Pi_f | c_i = c_L) + (1 - e_i) \mathbb{E}(\Delta_f \Pi_f | c_i = c_H) \quad \text{for any } i \in \mathcal{N}. \tag{7.9}$$

Define the following function

$$\Delta_i \Delta_f \Pi_f(L) = \Delta_f \Pi_f(L) - \Delta_f \Pi_f(L+1). \tag{7.10}$$

Use the fact that $P(L = l | c_i = c_H) = P(L = l + 1 | c_i = c_L)$ from (5.14) to rewrite the best-response function of firm f in (7.8) as

$$e_f(e_{-f}) = \frac{\mathbb{E}(\Delta_f \Pi_f | c_i = c_H) - \mathbb{E}(\Delta_i \Delta_f \Pi_f) e_i}{2} \quad \text{for any } i \in \mathcal{N}. \tag{7.11}$$

Now let us turn to the first period. Define the value of cost-reduction of firm i, given c_f, as

$$\Delta_i \Pi_i(L, c_f) = \Pi_i(c_L, L+1, c_f) - \Pi_i(c_H, L, c_f). \tag{7.12}$$

Following the same reasoning that we used in section 5 to obtain expression (5.20) we can write the expected profits of firm i conditional on the cost of firm f as

$$\mathbb{E}(\Pi_i|c_f) = e_i[\mathbb{E}(\Delta_i\Pi_i|c_f) - b_i] + \mathbb{E}(\Pi_i|c_i = c_H, c_f). \tag{7.13}$$

By the LIE we can write the expected profits of firm i as

$$\mathbb{E}(\Pi_i) = e_f\mathbb{E}(\Pi_i|c_f = c_L) + (1-e_f)\mathbb{E}(\Pi_i|c_f = c_H) - e_i b_i. \tag{7.14}$$

Define the following functions

$$\Delta_f\Delta_i\Pi_i(L) = \Delta_i\Pi_i(L, c_H) - \Delta_i\Pi_i(L, c_L), \tag{7.15}$$

$$\Delta_f\Pi_i(c_i, L) = \Pi_i(c_i, L, c_H) - \Pi_i(c_i, L, c_L). \tag{7.16}$$

Replacing (7.13) in (7.14) and using the last two definitions we obtain

$$\mathbb{E}(\Pi_i) = -e_f\Big[e_i\mathbb{E}(\Delta_f\Delta_i\Pi_i) + \mathbb{E}(\Delta_f\Pi_i|c_i = c_H)\Big] + e_i\mathbb{E}(\Delta_i\Pi_i|c_f = c_H) + \mathbb{E}(\Pi_i|c_i = c_H, c_f = c_H) - e_i b_i. \tag{7.17}$$

However, firm i can foresee the behavior of the entrant firm and take into account its best-response. Therefore, replace $e_f = e_f(e_{-f})$, given in (7.11), in (7.17) to obtain

$$\begin{aligned}\mathbb{E}(\Pi_i) = {} & \frac{\mathbb{E}(\Delta_i\Delta_f\Pi_f)\mathbb{E}(\Delta_f\Delta_i\Pi_i)}{2}e_i^2 + \mathbb{E}(\Pi_i|c_i = c_H, c_f = c_H) - \frac{\mathbb{E}(\Delta_f\Pi_f|c_i = c_H)\mathbb{E}(\Delta_f\Pi_i|c_i = c_H)}{2} - e_i b_i \\ & - \frac{1}{2}\Big[\mathbb{E}(\Delta_f\Pi_f|c_i = c_H)\mathbb{E}(\Delta_f\Delta_i\Pi_i) - \mathbb{E}(\Delta_f\Pi_i|c_i = c_H)\mathbb{E}(\Delta_i\Delta_f\Pi_f)\Big]e_i + \mathbb{E}(\Delta_i\Pi_i|c_f = c_H)e_i. \end{aligned} \tag{7.18}$$

Maximizing (7.18) with respect to e_i subject to the (IR), (IC) and (LL) constraints leads us to the best-response function of firm $i \in \mathscr{N}$, given by

$$e_i(e_{-i}) = \frac{2\mathbb{E}(\Delta_i\Pi_i|c_f = c_H) + \mathbb{E}(\Delta_f\Pi_i|c_i = c_H)\mathbb{E}(\Delta_i\Delta_f\Pi_f) - \mathbb{E}(\Delta_f\Pi_f|c_i = c_H)\mathbb{E}(\Delta_f\Delta_i\Pi_i)}{2\Big(2 - \mathbb{E}(\Delta_i\Delta_f\Pi_f)\mathbb{E}(\Delta_f\Delta_i\Pi_i)\Big)} \tag{7.19}$$

Lemma 3. *In the unique symmetric equilibrium among incumbents with N incumbent firms, the managerial effort performed by every incumbent firm is*

$$e_s'(N) = \frac{c}{4}\left(\frac{8N(N+1)^2+8N(N+1)^2(N-1)c-2(2N+1)(N-1)c^2-(2N+1)(2N^2-N+3)c^3}{2(N+1)^4+4N(N^2-1)(N+1)c^2-(2N+1)(N^2+1)c^4}\right). \quad (7.20)$$

The proof of the previous lemma may be found in the Appendix. Following the same reasoning as in the previous sections, we focus only in the symmetric equilibrium regarding incumbent firms.

Proposition 6. *Every firm in a Cournot oligopoly of N firms increases its managerial effort if another firm is to enter the market. Formally, let $e_s'(n)$ be the symmetric equilibrium effort given in* (7.20) *for the case of $N = n$ incumbent firms, then*

$$e_s'(n) > e_c(n) \quad \forall n \geq 1. \quad (7.21)$$

The proof of the previous proposition may be found in the Appendix. Proposition 6 states the same result that we found in section 6.2. Hence, if we allow for the entrant to sign a contract with its manager before the incumbents can change their production, our main finding is not altered: when a new firm enters the market, the optimal response of an incumbent is to increase its managerial effort.

7.2 Simultaneous Contracts and Sequential Production

Now let us consider the following order: in period 1) the incumbents and the entrant decide their respective bonus-contracts and their managers their respective effort levels simultaneously; 2) all firms' costs are publicly realized; 3) incumbent principals make output decisions simultaneously; 4) the entrant principal decides his firm's output after oberving the incumbents' aggregate output. Note that the last two periods of the game are the same as in our intercalated order of the previous section. Let us proceed by backward induction and use results that we obtained in the previous section. In the last period, the entrant firm obtains profits $\Pi_f(c_f, L)$ given by (7.2). In the third period, an incumbent obtains profits $\Pi_i(c_i, L, c_f)$ given by (7.1). According to (7.11), the entrant firm has the following best-response for effort in the first period

$$e_f(e_{-f}) = \frac{\mathbb{E}(\Delta_f \Pi_f | c_i = c_H) - \mathbb{E}(\Delta_i \Delta_f \Pi_f) e_i}{2} \quad \text{for any } i \in \mathcal{N}. \tag{7.22}$$

We may write the expected profits of an incumbent firm $i \in \mathcal{N}$ in the first period as in (7.17)

$$\mathbb{E}(\Pi_i) = -e_f \Big[e_i \mathbb{E}(\Delta_f \Delta_i \Pi_i) + \mathbb{E}(\Delta_f \Pi_i | c_i = c_H) \Big] + e_i \mathbb{E}(\Delta_i \Pi_i | c_f = c_H) + \mathbb{E}(\Pi_i | c_i = c_H, c_f = c_H) - e_i b_i. \tag{7.23}$$

Maximizing the latter expression with respect to e_i subject to the (IR), (IC) and (LL) constraints leads us to the best-response function of firm $i \in \mathcal{N}$, given by

$$e_i(e_{-i}, e_f) = \frac{\mathbb{E}(\Delta_i \Pi_i | c_f = c_H) - \mathbb{E}(\Delta_f \Delta_i \Pi_i) e_f}{2}, \tag{7.24}$$

where e_{-i} denotes the effort of all incumbent firms different to i. Expressions (7.22) and (7.24) characterize the equilibrium effort levels of the incumbents and the entrant.[14]

Lemma 4. *In the unique symmetric equilibrium among incumbents with N incumbent firms, the managerial effort performed by every incumbent firm is*

$$e_s''(N) = \frac{c}{4} \left(\frac{8N(1-c+cN)(N+1)^2 - c^2 N(2N+1)(2-c+2cN)}{2(N+1)^4 + 4N(N-1)(N+1)^2 c^2 - c^4 N^2 (2N+1)} \right). \tag{7.25}$$

[14] In the previous section, we replaced (7.22) in (7.23) and solved for the equilibrium effort of an incumbent, since incumbents could foresee the best-response of the entrant. However, under our present framework with simultaneous contracts this is not possible.

The proof of the previous lemma may be found in the Appendix. Following the same reasoning as in the previous sections, we focus only in the symmetric equilibrium, regarding incumbent firms.

Proposition 7. *Every firm in a Cournot oligopoly of N firms increases its managerial effort if another firm is to enter the market. Formally, let $e_s''(n)$ be the symmetric equilibrium effort given in* (7.25) *for the case of $N = n$ incumbent firms, then*

$$e_s''(n) > e_c(n) \quad \forall n \geq 1. \tag{7.26}$$

The proof of the previous proposition may be found in the Appendix. Proposition 7 states the same result that we found in section 6.2. Hence, this second alternative order does not change our main finding either. Under this order we assumed that the entrant is an already existing firm, and thus it can contract its manager at the same time as the incumbents. Nonetheless, our main result holds: when a new firm enters the market, incumbent firms increase their managerial performance.

8 Discussion

We have developed a model using a short-run framework in which the number of firms N is fixed. If instead we would have assumed free entry with an entry cost $F > 0$, the results of the model would have been the same. Assume an entry cost $F > 0$ such that the number of firms in equilibrium is N. In this case, N is endogenously determined and depends on F. Hence, instead of assuming an exogenous increase in the number of firms in the industry, assume that the entry cost drops to a point in which only one more firm enters the product market. In such a case, the measure for product market competition would be F, and it would be reflected in N. Note that the results of the model would be qualitatively the same. An alternative would be to consider a larger drop in the entry cost in such a way that more than one firm could enter the market. However, we are interested in analyzing a marginal increase in product market competition, i.e., the entry of *one firm*.

As shown in the literature review, all of the previous articles that addressed the same question as ours use comparative statics. This is common practice in the literature. In section 5 we have argued that doing comparative statics of the number of firms presumes that firms do not react to the increase in product market competition. In contrast, we would expect that when incumbent firms realize that the market is about to become more competitive, they would change their managerial contracts and production plans accordingly. Therefore, we have developed the model in a sequential Stackelberg framework. This has allowed us to compare a prior equilibrium in which N firms are established Cournot competitors with a new one in which the established firms become incumbents. Comparing these two equilibriums allowed us to analyze how managerial performance is affected by an increase in competition, i.e., how do incumbent firms strategically change their managerial contracts in the face of the entry of a new firm to the product market.

We considered the order of our sequential framework as exogenous. First, we considered a completely sequential order in which the incumbents' contracting and production stages take place before the entrant firm makes contracting and production decisions. This order is natural in the sense that the incumbents can foresee the entrant firm, and thus change their managerial contracts and production plans accordingly. Nonetheless, this is not the only possible order. Hence, we have also analyzed two alternative orders. First, it may be the case that the order is intercalated: the entrant firm may contract its manager and perform cost-reduction effort prior to the incumbents' production stage. We have argued that this order may be intuitive if a contracting stage takes less time than changing production. Secondly, it may be the case that the incumbents and the entrant make simultaneous contracting and managerial decisions, but that the entrant decides its output after the incumbents do so. This second order may be intuitive if we consider that the entrant firm is an already existing firm entering a new product's market. We have showed that our main result holds using both alternative orders. Note that production is sequential between the incumbents and the entrant in all the orders that we have considered, whereas the contracting stage differs between them. This fact shows that our results are driven by the strategic effect that originates from modelling the entry of a firm as a production follower in a Stackelberg fashion.[15]

9 Testable Implications

The most important testable implication of our model derives from the comparison of our simultaneous with our sequential framework. Under simultaneous competition, we have found that firms in markets with more competitors have lower managerial performance than firms in more concentrated markets. Therefore, our model predicts a positive correlation between market concentration and managerial performance. However, this would be the case for a cross-section analysis in which we compare multiple industries. As we have shown, this conclusion does not hold when we analyze an increase in product market competition in one particular industry.

In our sequential framework we have found that incumbents increase their managerial performance when a firm is to enter the market. Therefore, our model predicts that if a firm enters a market, then the firms that were already in the market will increase their managerial incentives and thus their cost-reduction efforts. This is the case for a panel analysis, in which we compare multiple industries in different periods of time, and thus observe different levels of product market competition for the same industry. This subtle difference may be one of

[15] It may be shown that, if we assume a sequential contracting stage with simultaneous production, our results no longer hold. However, this setting is not intuitive given our research question. The product market competition stage must be assumed sequential, since we wish to analyze the effect that an increase in product market competition has on managerial incentives.

the causes that explains the difficulty of proving empirically that product market competition rises managerial performance.

Another testable implication of our model concerns the comparison between a monopoly and a duopoly. If we compare monopolistic industries with duopolist ones in a cross-section analysis, our model predicts that firms in duopolies will have higher managerial performance than monopolies only in industries in which an improved management causes a large marginal cost reduction. In cotrast, in a panel analysis, our model predicts that when a firm enters the market of a monopoly, then the former monopolist will strengthen his managerial incentives in order to increase its manager's cost-reduction effort. Nevertheless, we have also argued that the product market price in a duopoly should always be lower than in a monopoly, whatever the managerial effort of firms, incumbent or entrant.

10 Conclusion

Our aim has been to offer a novel explanation to the Hicks conjecture: product market competition increases firms' managerial performance. The conjecture lies in the fact that, even though the intuition behind this idea may seem clear at first sight, it has not been an easy task to formalize it in a model. Furthermore, even though the available empirical evidence may seem to point in the conjecture's direction, it is not conclusive overall. In order to solve this issue, we have done the following.

First, we developed a simultaneous framework in which we observed that firms operating in more competitive markets have lower managerial performance. This result is consistent with previous findings; however, it points in the opposite direction of the conjecture. We recognized that this fact is due to the *increase* in product market competition that was considered. An exogenous increase in the number firms via a comparative statics analysis assumes implicitly that firms do not react to the rise of competition. We have argued that this is not a very realistic scenario if we want to analyze one particular industry. Therefore, we modeled the increase in product market competition explicitly via firm entry. By analyzing a sequential framework, we have studied the strategic reaction of incumbent firms to an increase in competition. Our main finding has been that incumbent firms increase their managerial performance when a firm enters the market, whatever the prior level of product market competition. This conclusion goes in line with the conjecture: product market competition incentives managerial effort. Furthemore, we have argued and shown that our main result is caused by the strategic effect that arises from modelling the production stage sequentially. Hence, our model shows the importance of using a sequential framework in which the incumbents can react strategically to an increase in competition.

Previous literature has emphasized the role of different effects in the agency relation within the firm, e.g. information, risk-adjustment, income and threat of liquidation effects. However, few articles model explicitly product market competition. The ones that do so analyze competition increases using comparative statics. Our analysis is novel in the way that it explicitly models the increase in product market competition. Our moral hazard model may seem oversimplified, since we avoided many of the latter effects by assuming risk-neutrality, independent cost realizations and quasilinear utility. Nevertheless, our aim has been to introduce a new channel by which product market competition affects managerial effort; this is via the *strategic effect* that arises due to the sequential framework in the product market. A further step would be to develop a sequential model using a more robust principal-agent framework in order to analyze the interaction between more effects; however, this goes beyond the reach of our present study.

Appendix

Claim 1 below is used to prove Lemmas 1, 2, 3 and 4.

Claim 1. *Let $e_i = e \; \forall i \in \mathscr{N}$ and $F : \mathscr{L} \to \mathbb{R}$ be a function. If there exist two functions $f^1, f^2 : \mathscr{N} \times \mathbb{N} \to \mathbb{R}$ that satisfy*

(a) $F\left(\frac{N-1-k}{2}\right) = f^1(N,k)$ *if $N-k$ is odd;*

(b) $F(l) + F(N-1-k-l) = 2 \cdot f^1(N,k)$ *if $N-k$ is odd;*

(c) $(-1)^{N-1-l-k}F(l) + (-1)^l F(N-k-1-l) = (-1)^{l+1}(N-k-1-2l) \cdot f^2(N,k)$ *if $N-k$ is even,*

then

$$\mathbb{E}(F|c_i = c_H) = F(0) - (N-1)[F(0) - F(1)]e. \tag{10.1}$$

Proof of Claim 1. Let $F : \mathscr{L} \to \mathbb{R}$ be a function. Let f^1 and $f^2 : \mathscr{N} \times \mathbb{N} \to \mathbb{R}$ be functions that satisfy properties (a), (b) and (c) listed in Claim 1. The expected value of F, conditional on $c_i = c_H$, is given by

$$\mathbb{E}(F|c_i = c_H) = \sum_{l=0}^{N-1} P(L = l|c_i = c_H)F(l). \tag{10.2}$$

Note that the variable L has a Binomial distribution if $e_i = e \; \forall \; i \in \mathscr{N}$. Hence,

$$\mathbb{E}(F|c_i = c_H) = \sum_{l=0}^{N-1} \binom{N-1}{l} e^l (1-e)^{N-1-l} F(l). \tag{10.3}$$

Using the Binomial Theorem, we may write

$$\begin{aligned} e^l(1-e)^{N-1-l} &= \sum_{k=0}^{N-1-l} \binom{N-l-1}{k} (-1)^{N-1-l-k} (e)^{N-1-k} \qquad (10.4)\\ &= \binom{N-l-1}{0} (-1)^{N-l-1} (e)^{N-1} + \ldots + \binom{N-l-1}{k} (-1)^{N-l-1-k} e^{N-1-k} + \ldots \\ &\quad + \binom{N-l-1}{N-l-1} (-1)^0 (e)^l. \qquad (10.5) \end{aligned}$$

Therefore, $e^l(1-e)^{N-1-l}$ is a sum of $N-l$ terms; it is a polynomial that contains the terms from e^l to e^{N-1}. The same is true for $\binom{N-1}{l} e^l (1-e)^{N-1-l} F(l)$, then (10.3) is a a sum of N polynomials. The l-th polynomial contains the terms from e^l to e^{N-1}. Note that e^{N-1} appears in each one of the N polynomials, e^{N-2} appears in $N-2$ polynomials (all except when $l = N-1$), whereas e^0 only appears in one polynomial (when $l = 0$). In other words, e^l appears in the first $l+1$ polynomials in the sum of (10.3); equivalently, e^{N-1-k} appears in $N-k$ polynomials. For example, the term in (10.3) that contains e^{N-1} is given by

$$\left[\sum_{l=0}^{N-1}\binom{N-1}{l}\binom{N-l-1}{0}(-1)^{N-l-1}F(l)\right]e^{N-1}. \tag{10.6}$$

In general, the term in (10.3) that contains e^{N-1-k} for any $k=0,1,...,N$ is given by

$$\left[\sum_{l=0}^{N-1-k}\binom{N-1}{l}\binom{N-l-1}{k}(-1)^{N-l-1-k}F(l)\right]e^{N-1-k}. \tag{10.7}$$

Therefore, we may rewrite (10.3) as

$$\mathbb{E}(F|c_i=c_H)=\sum_{k=0}^{N-1}\left[\sum_{l=0}^{N-1-k}\binom{N-1}{l}\binom{N-1-l}{k}(-1)^{N-1-l-k}F(l)\right]e^{N-1-k}. \tag{10.8}$$

Define the function $C(N,k)$ as

$$C(N,k)=\sum_{l=0}^{N-1-k}\binom{N-1}{l}\binom{N-1-l}{k}(-1)^{N-1-l-k}F(l). \tag{10.9}$$

Note that $C(N,k)$ is the coefficient of e^{N-1-k} in the sum in (10.8). To prove Claim 1, we shall prove that the coefficients $C(N,k)$ equal zero for any term that includes e to a greater power than one. Note that $C(N,k)$ is a sum of $N-k$ terms. The proof is divided in two parts.

First, assume that $N-k$ is odd. Then, there is a central term in the sum in (10.9); the $\frac{N-k-1}{2}+1$-th term splits the sum in two halves with the same number of terms each. This means that there are $\frac{N-k-1}{2}$ terms before and after this central term. Note that in the central term $l=\frac{N-k-1}{2}$. Take the term in (10.9) in which $l=\frac{N-k-1}{2}+\varepsilon$ with $\varepsilon\in\mathbb{N}$ and $\varepsilon\leq\frac{N-k-1}{2}$. This term may be written as

$$\frac{(N-1)!}{\left(\frac{N-1-k}{2}+\varepsilon\right)!\left(\frac{N-1-k}{2}-\varepsilon\right)!(k)!}(-1)^{\frac{N-1-k}{2}+\varepsilon}\cdot F\left(\frac{N-1-k}{2}+\varepsilon\right). \tag{10.10}$$

Note that if we take $-\varepsilon$ instead of ε, then the symmetrical term is the one in which $l=\frac{N-k-1}{2}-\varepsilon$, given by

$$\frac{(N-1)!}{\left(\frac{N-1-k}{2}-\varepsilon\right)!\left(\frac{N-1-k}{2}+\varepsilon\right)!(k)!}(-1)^{\frac{N-1-k}{2}-\varepsilon}\cdot F\left(\frac{N-1-k}{2}-\varepsilon\right). \tag{10.11}$$

Note that $sgn((-1)^{\frac{N-1-k}{2}-\varepsilon}) = sgn((-1)^{\frac{N-1-k}{2}+\varepsilon})$.[16] Therefore, both terms have the same coefficient on the function $F(l)$. This means that the term that includes $F(0)$ has the same coefficient as the term that includes $F(N-1-k)$; the same holds for $F(1)$ and $F(N-2-k)$, and so on. This allows us to rewrite $C(N,k)$ as

$$C(N,k) = \sum_{l=0}^{\frac{N-1-k}{2}-1} \binom{N-1}{l}\binom{N-1-l}{k}(-1)^{N-1-l-k}\Big[F(l)+F(N-1-k-l)\Big] + \binom{N-1}{\frac{N-1-k}{2}}\binom{\frac{N-1+k}{2}}{k}(-1)^{\frac{N-1-k}{2}} \cdot F\left(\frac{N-1-k}{2}\right), \quad (10.12)$$

where

$$F(l)+F(N-1-k-l) = 2\cdot f^1(N,k), \quad (10.13)$$

$$F\left(\frac{N-1-k}{2}\right) = f^1(N,k). \quad (10.14)$$

Therefore, replacing the last two expressions in (10.12) results in

$$C(N,k) = \frac{f^1(N,k)\cdot(N-1)!}{k!}\cdot\Phi(N,k), \quad (10.15)$$

where

$$\Phi(N,k) = \frac{(-1)^{\frac{N-1-k}{2}}}{\left(\frac{N-1-k}{2}\right)!^2} + \sum_{l=0}^{\frac{N-k-1}{2}-1} \frac{2(-1)^{N-1-k-l}}{(N-l-1-k)!l!}. \quad (10.16)$$

To complete the first part of the proof, in which we assumed $N-k$ odd, we shall prove that $\Phi(N,k)=0$ for all $k \le N-3$. In other words, we shall prove that the coefficient of the term in which e^{N-1-k} appears is zero $\forall N-1-k \ge 2$. Let $N-k = 2n+1$ where $n \in \mathbb{N}$. Rewrite (10.16) to obtain

$$\Phi(n) = \frac{(-1)^n}{n!^2} + \sum_{l=0}^{n-1} \frac{2(-1)^{2n-l}}{(2n-l)!l!}. \quad (10.17)$$

Disaggregating the sum in the latter expression, we can rewrite $\Phi(n)$ as

$$\Phi(n) = \frac{1}{(2n)!0!} - \frac{1}{(2n-1)!1!} + \ldots + \frac{(-1)^n}{(n)!^2} + \ldots - \frac{1}{(2n-1)!1!} + \frac{1}{(2n)!0!} = \sum_{h=0}^{2n}(-1)^h\frac{1}{(2n-h)!h!}. \quad (10.18)$$

[16] By symmetrical we mean the following. For example, suppose $N-k=15$, then $l=0,1,\ldots,14$. The central term is the 8-th term in which $l=7$. The term in which $l=6$ is symmetrical to the one in which $l=8$ (with $\varepsilon=1$), $l=5$ is symmetrical to $l=9$ ($\varepsilon=2$), and so on.

Multiply both sides of (10.18) by $(2n)!$ to obtain

$$(2n)!\Phi(n) = \sum_{h=0}^{2n} (-1)^h \binom{2n}{h}. \tag{10.19}$$

Separating the binomial coefficient, we can rewrite (10.19) as

$$(2n)!\Phi(n) = \sum_{h=0}^{2n} (-1)^h \left[\binom{2n-1}{h-1} + \binom{2n-1}{h} \right]. \tag{10.20}$$

Disaggregating the sum in the right-hand side of (10.20) it can be seen that $(2n)!\Phi(n) = 0 \ \forall n \in \mathbb{N}$. Note that $(2n)! > 0 \ \forall n \in \mathbb{N}$, then we have that $\Phi(n) = 0 \forall n \in \mathbb{N}$. By definition, this implies $\Phi(N,k) = 0 \forall k \leq N-3$, hence we can conclude that the coefficient $C(N,k) = 0 \ \forall N-1-k \geq 2$.

Now we turn to the second part of the proof; assume $N-k$ is even. Hence, the sum in $C(N,k)$, given by (10.9), has $N-k$ terms and can be split in two halves with the same number of terms each. The $\frac{N-k}{2}$-th term, in which $l = \frac{N-k}{2} - 1$, is the last term of the first half. Following the same logic as in the first part of the proof, take the term of (10.9) in which $l = \frac{N-k}{2} - \varepsilon$ with $\varepsilon \in \mathbb{N}$ and $\varepsilon \leq \frac{N-k}{2}$. This term may be written as

$$\frac{(N-1)!}{\left(\frac{N-k}{2} - 1 + \varepsilon\right)! \left(\frac{N-k}{2} - \varepsilon\right)! (k)!} (-1)^{\frac{N-k}{2} - 1 + \varepsilon} \cdot F\left(\frac{N-k}{2} - \varepsilon\right). \tag{10.21}$$

Note that if we take $\varepsilon - 1$ instead of $-\varepsilon$, then the symmetrical term is the one in which $l = \frac{N-k}{2} + \varepsilon - 1$, given by

$$\frac{(N-1)!}{\left(\frac{N-k}{2} - \varepsilon\right)! \left(\frac{N-k}{2} - 1 + \varepsilon\right)! (k)!} (-1)^{\frac{N-k}{2} - \varepsilon} \cdot F\left(\frac{N-k}{2} - \varepsilon\right). \tag{10.22}$$

Note that $sgn((-1)^{\frac{N-k}{2} - 1 + \varepsilon}) \neq sgn((-1)^{\frac{N-k}{2} - \varepsilon})$.[17] Hence, both terms have the same coefficient on the function $F(l)$ but with different sign. This means that the term that includes $F(0)$ has the same coefficient as the term that includes $F(N-1-k)$ but with opposite sign; the same holds for $F(1)$ and $F(N-2-k)$, and so on. This allows us to rewrite $C(N,k)$ as

$$C(N,k) = \sum_{l=0}^{\frac{N-k}{2} - 1} \binom{N-1}{l} \binom{N-1-l}{k} \Big[(-1)^{N-1-l-k} F(l) + (-1)^l F(N-k-1-l) \Big], \tag{10.23}$$

where

$$(-1)^{N-1-l-k} F(l) + (-1)^l F(N-k-1-l) = (-1)^{l+1} (N-k-1-2l) \cdot f^2(N,k). \tag{10.24}$$

[17] By symmetrical we mean the following. For example, suppose $N-k = 12$, then $l = 0, 1, ..., 11$. The last term of the first half is the 6-th term in which $l = 5$. The term in which $l = 5$ is symmetrical to the one in which $l = 6$ (with $\varepsilon = 1$), $l = 4$ is symmetrical to $l = 7$ ($\varepsilon = 2$), and so on.

Therefore, replacing the latter expression in (10.23) results in

$$C(N,k) = \frac{f^2(N,k)\cdot(N-1)!}{k!}\cdot\Lambda(N,k) \tag{10.25}$$

where

$$\Lambda(N,k) = \sum_{l=0}^{\frac{N-k}{2}-1}(-1)^{l+1}\frac{N-k-1-2l}{(N-k-1-l)!l!}. \tag{10.26}$$

To complete the second part of the proof, in which we assumed $N-k$ even, we shall prove $\Lambda(N,k)=0$ for every $k\le N-3$. Let $N-k=2n$, where $n\in\mathbb{N}$. Rewrite $\Lambda(t,k)$ as

$$\Lambda(n) = \sum_{l=0}^{n-1}(-1)^{l+1}\frac{2n-1-2l}{(2n-1-l)!l!}. \tag{10.27}$$

Multiply both sides of (10.27) by $(2n-1)!$ to obtain

$$(2n-1)!\Lambda(n) = \sum_{l=0}^{n-1}(-1)^{l+1}(2n-1-2l)\binom{2n-1}{l} = \sum_{l=0}^{n-1}(-1)^{l+1}(2n-1-2l)\left[\binom{2n-2}{l-1}+\binom{2n-2}{l}\right]. \tag{10.28}$$

Disaggregate the sum in (10.28) and note that the contiguous terms share a binomial coefficient. Thus, we obtain

$$(2n-1)!\Lambda(n) = \sum_{l=0}^{n-2}(-1)^{l+1}2\binom{2n-2}{l}+(-1)^n\binom{2n-2}{n-1}. \tag{10.29}$$

Disaggregating the sum in (10.29) we obtain the following series

$$(2n-1)!\Lambda(n) = -2\frac{(2n-2)!}{(2n-2)!0!}+2\frac{(2n-2)!}{(2n-3)!1!}+...+2(-1)^{n-1}\frac{(2n-2)!}{n!(n-2)!}+(-1)^n\frac{(2n-2)!}{(n-1)!^2}. \tag{10.30}$$

Separate the first $n-1$ terms in (10.30) and regroup to obtain

$$(2n-1)!\Lambda(n) = -\frac{(2n-2)!}{(2n-2)!0!}+\frac{(2n-2)!}{(2n-3)!1!}+...+(-1)^n\frac{(2n-2)!}{(n-1)!^2}+...+\frac{(2n-2)!}{1!(2n-3)!}-\frac{(2n-2)!}{0!(2n-2)!}. \tag{10.31}$$

Hence, the series in (10.31) can be rewritten as

$$(2n-1)!\Lambda(n) = \sum_{i=1}^{2n-2}(-1)^{i+1}\binom{2n-2}{i} = \sum_{i=1}^{2n-2}(-1)^{i+1}\left[\binom{2n-3}{i-1}+\binom{2n-3}{i}\right]. \tag{10.32}$$

Disaggregating the sum in (10.32) it can be seen that $(2n-1)!\Lambda(n)=0\ \forall n\in\mathbb{N}$. Note that $(2n-1)!>0\ \forall n\in\mathbb{N}$, then we have $\Lambda(n)=0\ \forall n\in\mathbb{N}$. By definition, this implies $\Lambda(N,k)=0\ \forall k\le N-3$, hence we can conclude that the coefficient $C(N,k)=0\ \forall N-1-k\ge 2$. This concludes the second part of the proof. Thus far, we have proved that the coefficients $C(N,k)$ of the polynomial given in (10.10) are zero for all the terms that include e^{N-1-k} with $N-1-k \geq 2$. Therefore, we can rewrite expression (10.8) as

$$\mathbb{E}(F|c_i = c_H) = F(0) - (N-1)[F(0) - F(1)]e. \tag{10.33}$$

Q.E.D

Proof of Lemma 1. In the symmetric equilibrium $e_i = e_c(N)\ \forall i \in \mathscr{N}$. From expression (5.21) we have

$$e_c(N) = \frac{\mathbb{E}(\Delta_i \Pi_i)}{2} = \frac{1}{2}\sum_{l=0}^{N-1} P(L = l|c_i = c_H)\Delta_i \Pi_i(l) \quad \forall i \in \mathscr{N}. \tag{10.34}$$

Let

$$f^1(N,k) = \frac{Nc(2-c+ck)}{(N+1)^2}, \tag{10.35}$$

$$f^2(N,k) = \frac{2Nc^2}{(N+1)^2}. \tag{10.36}$$

Using the last two functions, verify that the function $\Delta_i \Pi_i(l)$ satisfies conditions (a) - (c) of Claim 1. Therefore, by Claim 1 we can write

$$\sum_{l=0}^{N-1} P(L = l|c_i = c_H)\Delta_i \Pi_i(l) = \Delta_i \Pi_i(0) - (N-1)[\Delta_i \Pi_i(0) - \Delta_i \Pi_i(1)]e_c(N). \tag{10.37}$$

Hence, replacing the last expression in (10.34) we obtain

$$e_c(N) = \frac{\Delta_i \Pi_i(0)}{2 + (N-1)[\Delta_i \Pi_i(0) - \Delta_i \Pi_i(1)]}. \tag{10.38}$$

Replacing expression (5.11) in (10.49) leads us to the effort level in the symmetric equilibrium, given by

$$e_c(N) = \frac{Nc}{2}\left(\frac{2 + c(N-2)}{(N+1)^2 + Nc^2(N-1)}\right). \tag{10.39}$$

Q.E.D

Proof of Proposition 2. Let $n \in \mathbb{N}$ be the number of firms in the product market. Let $e_c(n)$ be the effort level in the unique symmetric equilibrium of a Cournot oligopoly, given in (5.22). Note that the firm with the lowest possible profits is a high-cost firm that competes with $n-1$ low-cost firms. Under Cournot competition with constant marginal costs, the equilibrium profits are the square of the produced quantity of each firm. Replace $L = N-1$ and $c_i = c_H$ in expression (5.11) and verify that all firms will produce positive quantities if and only if $c < 1/N$. Hence, in a Cournot oligopoly $\bar{c} = 1/N$. Since Proposition 2 compares a market with n firms to one with $n+1$, let $c < 1/(n+1)$. From expression (5.22) we obtain that $e_c(n) \geq e_c(n+1)$ if and only if $g^1(n) \geq 0$, where

$$g^1(n) = n^2(2-c^3+2c^2-4c)+n(2-c^3+2c^2-6c)-(2-c). \tag{10.40}$$

We shall prove by induction that $g^1(n) \geq 0 \; \forall \, n \geq 2$ with $c < 1/(n+1)$. Verify that $g^1(2) \geq 0 \; \forall \; c < 1/3$. Assume $g^1(k) \geq 0$ for some $k > 2$ with $c < 1/(k+1)$ fixed, then we shall prove that $g^1(k+1) \geq 0 \; \forall k > 2$. Note that

$$g^1(k+1) = g^1(k) + g^2(k), \tag{10.41}$$

where

$$g^2(k) = 2(k+1)(2-c^3+2c^2-4c)-2c. \tag{10.42}$$

Note that $g^1(k) \geq 0$ by assumption. Hence, in order to prove $g^1(k+1) \geq 0$, we shall prove by induction $g^2(k) \geq 0$ $\forall k > 2$ with $c < 1/k+1$. Verify that $g^2(3) \geq 0 \; \forall \; c < 1/4$. Assume $g^2(h) \geq 0$ for some $h > 3$ with $c < 1/(h+1)$ fixed, then we shall prove that $g^2(h+1) \geq 0 \; \forall h > 3$. Note that

$$g^2(h+1) = g^2(h) + g^3, \tag{10.43}$$

where

$$g^3 = 2(2-c^3+2c^2-4c). \tag{10.44}$$

Note that $g^2(h) \geq 0$ by assumption and that $g^3 \geq 0 \; \forall \; c < 1/(h+1) < 1/4$. Hence, $g^2(h+1) \geq 0$. Therefore, we can conclude that $g^2(h) \geq 0 \; \forall h > 3$ with $c < 1/(h+1)$ and that $g^1(k) \geq 0 \; \forall k > 2$ with $c < 1/(k+1)$. Hence, $g^1(n) \geq 0 \; \forall n > 2$. We had already proved $g^1(2) \geq 0$, thus we obtain that $e_c(n) \geq e_c(n+1) \; \forall n \geq 2$. Q.E.D

Proof of Lemma 2. In the symmetric equilibrium $e_i = e_s(N) \; \forall i \in \mathscr{N}$. From expression (6.32) we have

$$e_s(N) = \frac{1}{2} \sum_{l=0}^{N-1} P(L = l | c_i = c_H) \Delta_i \Pi_i(l) \quad \forall i \in \mathscr{N}. \tag{10.45}$$

Let

$$f^1(N,k) = \frac{Nc(c^3 - 2c^2 + 8 + 8ck)}{(N+1)^2(4-c^2)}, \tag{10.46}$$

$$f^2(N,k) = \frac{16Nc^2}{(N+1)^2(4-c^2)}. \tag{10.47}$$

Using the last two functions, verify that the function $\Delta_i\Pi_i(l)$ satisfies conditions (a) - (c) of Claim 1. Therefore, by Claim 1 we can rewrite (10.45) as

$$2e_s(N) = \Delta_i\Pi_i(0) - (N-1)[\Delta_i\Pi_i(0) - \Delta_i\Pi_i(1)]e_s(N). \tag{10.48}$$

Solving for $e_s(N)$ and using the definition of $\Delta_i\Pi_i(l)$ and expression (6.30), we obtain

$$e_s(N) = \frac{Nc}{2}\left(\frac{c^3 - 2c^2 - 8c + 8 + 8cN}{(N+1)^2(4-c^2) + 8c^2N(N-1)}\right). \tag{10.49}$$

Q.E.D

Proof of Proposition 5. Let $n \in \mathbb{N}$ be the number of incumbent firms in the product market. Let $e_c(n)$ and $e_s(n)$ be the effort levels in the unique symmetric equilibrium of a Cournot and a Stackelberg oligopoly, given in (5.22) and (6.33), respectively. Recall that $\bar{c} \in (0,1]$; hence, every statement that holds $\forall\ c \in [0,1]$ holds $\forall\ c \in (0,\bar{c})$ as well. From expressions (5.22) and (6.33), we obtain that $e_s(n) > e_c(n)$ if and only if $\xi^1(n) > 0$, where

$$\xi^1(n) = (c^2+4)n^3 + (c^4 - 2c^3 + 9c^2 - 8c + 8)n^2 + (-c^4 + 2c^3 - 9c^2 + 8c + 4)n - c^2. \tag{10.50}$$

We shall prove by induction that $\xi^1(n) > 0\ \forall\ n \geq 1$ with $c \in [0,1]$. Verify that $\xi^1(1) > 0\ \forall\ c \in [0,1]$. Assume $\xi^1(k_1) > 0$ for some $k_1 > 1$ with $c \in [0,1]$ fixed, then we shall prove that $\xi^1(k_1+1) > 0\ \forall k_1 > 1$. Note that

$$\xi^1(k_1+1) = \xi^1(k_1) + \xi^2(k_1), \tag{10.51}$$

where

$$\xi^2(k_1) = 3(c^2+4)k_1^2 + 2(c^4 - 2c^3 + 10c^2 - 8c + 14)k_1 + c^2 + 16. \tag{10.52}$$

Note that $\xi^1(k_1) > 0$ by assumption. Hence, in order to prove $\xi^1(k_1+1) > 0$, we shall prove by induction $\xi^2(k_1) > 0\ \forall k_1 > 1$. Verify that $\xi^2(2) > 0\ \forall\ c \in [0,1]$. Assume $\xi^2(k_2) > 0$ for some $k_2 > 2$ with $c \in [0,1]$ fixed, then we shall prove that $\xi^2(k_2+1) > 0\ \forall k_2 > 2$. Note that

$$\xi^2(k_2+1) = \xi^2(k_2) + \xi^3(k_2), \tag{10.53}$$

where

$$\xi^3(k_2) = 6(c^2+4)k_2 + 3c^2 + 12. \tag{10.54}$$

Note that $\xi^2(k_2) > 0$ by assumption and $\xi^3(k_2) > 0\ \forall k_2 > 2$ with $c \in [0,1]$. Therefore, we can conclude that $\xi^i(k_i) > 0\ \forall k_i > i$ with $i = 1,2$. Hence, $\xi^1(n) > 0\ \forall n > 1$. We had already proved $\xi^1(1) > 0$, thus we obtain that $e_s(n) > e_c(n)\ \forall n \geq 1$.

Q.E.D

Proof of Lemma 3. First, using definitions (7.10) and (7.15), respectively, verify the following

$$\Delta_i \Delta_f \Pi_f(L) = \frac{c^2(2N+1)}{(N+1)^2}, \tag{10.55}$$

$$\Delta_f \Delta_i \Pi_i(L) = \frac{2c^2 N}{(N+1)^2}. \tag{10.56}$$

Note that both functions do not depend on L. Hence, $\Delta_{if} := \mathbb{E}(\Delta_i \Delta_f \Pi_f)$ and $\Delta_{fi} := \mathbb{E}(\Delta_f \Delta_i \Pi_i)$ are constants, given by (10.55) and (10.56), respectively. In the symmetric equilibrium, $e_i = e_s'(N)\ \forall i \in \mathscr{N}$. Rewrite the best-response function for effort given in (7.19) as

$$\Delta_{fi}\Delta_{if} e_s'(N) - 2e_s'(N) = \frac{1}{2}\Big[\Delta_{fi}\mathbb{E}(\Delta_f \Pi_f | c_i = c_H) - \Delta_{if}\mathbb{E}(\Delta_f \Pi_i | c_i = c_H) - 2\mathbb{E}(\Delta_i \Pi_i | c_f = c_H)\Big], \tag{10.57}$$

where

$$\mathbb{E}(\Delta_f \Pi_f | c_i = c_H) = \sum_{l=0}^{N-1} P(L = l | c_i = c_H)\Delta_f \Pi_f(l), \tag{10.58}$$

$$\mathbb{E}(\Delta_f \Pi_i | c_i = c_H) = \sum_{l=0}^{N-1} P(L = l | c_i = c_H)\Delta_f \Pi_i(c_H, l), \tag{10.59}$$

$$\mathbb{E}(\Delta_i \Pi_i | c_f = c_H) = \sum_{l=0}^{N-1} P(L = l | c_i = c_H)\Delta_i \Pi_i(l, c_H). \tag{10.60}$$

Define the following function

$$\delta(L, c_i) = \Delta_{fi} \cdot \Delta_f \Pi_f(L) - \Delta_{if} \cdot \Delta_f \Pi_i(c_i, L). \tag{10.61}$$

Using the latter definition, we can rewrite (10.57) as

$$(2-\Delta_{fi}\Delta_{if})e_s(N) = -\frac{1}{2}\sum_{l=0}^{N-1} P(L=l|c_i=c_H)\delta(l,c_H) + \sum_{l=0}^{N-1} P(L=l|c_i=c_H)\Delta_i\Pi_i(l,c_H). \tag{10.62}$$

Let

$$f_\delta^1(N,k) = \frac{c^3(2N+1)}{(N+1)^4}\Big[2(1+ck)(N-1)+c(3N+1)\Big], \tag{10.63}$$

$$f_\delta^2(N,k) = \frac{2c^4(2N+1)(N-1)}{(N+1)^4}. \tag{10.64}$$

Using the last two functions, verify that function $\delta(L,c_H)$ satisfies properties (a) - (c) of Claim 1. Therefore, by Claim 1 we can write

$$\sum_{l=0}^{N-1} P(L=l|c_i=c_H)\delta(l,c_H) = \delta(0,c_H)-(N-1)[\delta(0,c_H)-\delta(1,c_H)]e_s'(N). \tag{10.65}$$

Furthermore, let

$$f_\Delta^1(N,k) = \frac{2Nc(1+ck)}{(N+1)^2}, \tag{10.66}$$

$$f_\Delta^2(N,k) = \frac{4Nc^2}{(N+1)^2}. \tag{10.67}$$

Using the last two functions, verify that function $\Delta_i\Pi_i(L,c_H)$ satisfies properties (a) - (c) of Claim 1. Therefore, by Claim 1 we can write

$$\sum_{l=0}^{N-1} P(L=l|c_i=c_H) = \Delta_i\Pi_i(0,c_H)-(N-1)[\Delta_i\Pi_i(0,c_H)-\Delta_i\Pi_i(1,c_H)]e_s'(N). \tag{10.68}$$

Replacing expressions (10.65) and (10.68) in (10.62), and solving for $e_s'(N)$ results in

$$e_s'(N) = \frac{2\Delta_i\Pi_i(0,c_H)-\delta(0,c_H)}{2\Big(2-\Delta_{fi}\Delta_{if}+(N-1)\big[\Delta_i\Pi_i(0,c_H)-\Delta_i\Pi_i(1,c_H)\big]\Big)-(N-1)\big[\delta(0,c_H)-\delta(1,c_H)\big]}. \tag{10.69}$$

Replacing expressions (7.1) and (7.2) in (10.69) leads us to the expression for the managerial effort of an incumbent firm in the sub-game perfect symmetric equilibrium, given by

$$e'_s(N) = \frac{c}{4}\left(\frac{8N(N+1)^2+8N(N+1)^2(N-1)c-2(2N+1)(N-1)c^2-(2N+1)(2N^2-N+3)c^3}{2(N+1)^4+4N(N^2-1)(N+1)c^2-(2N+1)(N^2+1)c^4}\right). \tag{10.70}$$

Q.E.D

Proof of Proposition 6. Let $n \in \mathbb{N}$ be the number of incumbent firms in the product market. Let $e_c(n)$ and $e'_s(n)$ be the effort levels in the unique symmetric equilibrium of a Cournot and a Stackelberg oligopoly with intercalated order, given in (5.22) and (7.20), respectively. Note that the firm with lower possible profits is a high-cost firm that enters a market with n low-cost firms. Under Stackelberg competition with constant marginal costs, the equilibrium profits of the entrant firm are the square of its produced quantity. Replace $c_f = c$ and $L = N$ in expression (7.2) and verify that all firms will produce positive quantities if and only if $c < 1/(2N+1)$. Hence, in a Stackelberg oligopoly $\bar{c} = 1/(2N+1)$. Since Proposition 5 compares Cournot and Stackelberg oligopolies, let $c < 1/(2n+1)$. From expressions (5.22) and (7.20), we obtain that $e'_s(n) > e_c(n)$ if and only if $\xi^1(n) > 0$, where

$$\begin{aligned}\xi^1(n) = {} & 4n^6+4(4-2c+c^2)n^5+2(12-6c+2c^3-c^4)n^4+(16+2c-17c^2+10c^3-5c^4)n^3\\ & +(4+10c-21c^2+8c^3-4c^4)n^2+c(6-11c+2c^2-c^3)n+2c-3c^2.\end{aligned} \tag{10.71}$$

We shall prove by induction that $\xi^1(n) > 0 \; \forall \, n \geq 1$ with $c < 1/(2n+1)$. Proposition 3 states $e_s(1) > e_c(1)$, hence $\xi^1(1) > 0$. Assume $\xi^1(k_1) > 0$ for some $k_1 > 1$ and $c < 1/(2k_1+1)$, then we shall prove that $\xi^1(k_1+1) > 0$ $\forall k_1 > 1$ with $c < 1/(2k_1+1)$. Note that

$$\xi^1(k_1+1) = \xi^1(k_1) + \xi^2(k_1), \tag{10.72}$$

where

$$\begin{aligned}\xi^2(k_1) = {} & 24k_1^5+(140-40c+20c^2)k_1^4+(336-128c+40c^2+16c^3-8c^4)k_1^3+(412-74c\\ & -11c^2+54c^3-27c^4)k_1^2+(256-62c-73c^2+62c^3-31c^4)k_1+64+2c-45c^2+24c^3-12c^4.\end{aligned} \tag{10.73}$$

Note that $\xi^1(k_1) > 0$ by assumption. Hence, in order to prove $\xi^1(k_1+1) > 0$, we shall prove by induction $\xi^2(k_1) > 0$ $\forall k_1 > 1$ with $c < 1/(2k_1+1)$. Verify that $\xi^2(2) > 0$ for $c < 1/5$. Assume $\xi^2(k_2) > 0$ for some $k_2 > 2$ and $c < 1/(2k_2+1)$, then we shall prove that $\xi^2(k_2+1) > 0$ $\forall k_2 > 2$ with $c < 1/(2k_2+1)$. Note that

$$\xi^2(k_2+1) = \xi^2(k_2) + \xi^3(k_2), \tag{10.74}$$

where

$$\xi^3(k_2) = 120k_2^4 + (800 - 160c + 8c^2)k_2^3 + (2088 - 624c + 240c^2 + 48c^3 - 24c^4)k_2^2 \\ + (2512 - 692c + 178c^2 + 58c^3 - 78c^4)k_2 + 820 - 320c - 69c^2 + 107c^3 - 78c^4. \tag{10.75}$$

Note that $\xi^2(k_2) > 0$ by assumption. Hence, in order to prove $\xi^2(k_2+1) > 0$, we shall prove by induction $\xi^3(k_2) > 0$ $\forall k_2 > 2$ with $c < 1/(2k_2+1)$. Verify that $\xi^3(3) > 0$ for $c < 1/7$. Assume $\xi^3(k_3) > 0$ for some $k_3 > 3$ and $c < 1/(2k_3+1)$, then we shall prove that $\xi^3(k_3+1) > 0$ $\forall k_3 > 3$ with $c < 1/(2k_3+1)$. Note that

$$\xi^3(k_3+1) = \xi^3(k_3) + \xi^4(k_3), \tag{10.76}$$

where

$$\xi^4(k_3) = 480k_3^3 + (3120 - 480 + 24c^2)k_3^2 + (7056 - 1728c + 504c^2 + 96c^3 - 48c^4)k_3 \\ + 5520 - 1476c - 426c^2 + 106c^3 + 54c^4. \tag{10.77}$$

Note that $\xi^3(k_3) > 0$ by assumption. Hence, in order to prove $\xi^3(k_3+1) > 0$, we shall prove by induction $\xi^4(k_3) > 0$ $\forall k_3 > 3$ with $c < 1/(2k_3+1)$. Verify that $\xi^4(4) > 0$ for $c < 1/9$. Assume $\xi^4(k_4) > 0$ for some $k_4 > 4$ and $c < 1/(2k_4+1)$, then we shall prove that $\xi^4(k_4+1) > 0$ $\forall k_4 > 4$ with $c < 1/(2k_4+1)$. Note that

$$\xi^4(k_4+1) = \xi^4(k_4) + \xi^5(k_4), \tag{10.78}$$

where

$$\xi^5(k_4) = 1440k_4^2 + (7680 - 960c + 48c^2)k_4 + 10656 - 2208c + 528c^2 - 96c^3 - 48c^4. \tag{10.79}$$

Note that $\xi^4(k_4) > 0$ by assumption. Hence, in order to prove $\xi^4(k_4+1) > 0$, we shall prove by induction $\xi^5(k_4) > 0\ \forall k_4 > 4$ with $c < 1/(2k_4+1)$. Verify that $\xi^5(5) > 0$ for $c < 1/11$. Assume $\xi^5(k_5) > 0$ for some $k_5 > 5$ and $c < 1/(2k_5+1)$, then we shall prove that $\xi^5(k_5+1) > 0\ \forall k_5 > 5$ with $c < 1/(2k_5+1)$. Note that

$$\xi^5(k_5+1) = \xi^5(k_5) + \xi^6(k_5), \tag{10.80}$$

where

$$\xi^6(k_5) = 2880k_4 + 9120 - 960c + 48c^2. \tag{10.81}$$

Note that $\xi^5(k_5) > 0$ by assumption. Hence, in order to prove $\xi^5(k_5+1) > 0$, we shall prove by induction $\xi^6(k_5) > 0\ \forall k_5 > 5$ with $c < 1/(2k_5+1)$. Verify that $\xi^6(6) > 0$ for $c < 1/13$. Assume $\xi^6(k_6) > 0$ for some $k_6 > 6$ and $c < 1/(2k_6+1)$, then we shall prove that $\xi^6(k_6+1) > 0\ \forall k_6 > 6$ with $c < 1/(2k_6+1)$. Note that

$$\xi^6(k_6+1) = \xi^6(k_6) + 2880 > 0. \tag{10.82}$$

Therefore, we can conclude that $\xi^i(k_i) > 0\ \forall k_i > i$ with $i = 1, ..., 6$. Hence, $\xi^1(n) > 0\ \forall n > 1$. We had already proved $\xi^1(1) > 0$, thus we obtain that $e_s(n) > e_c(n)\ \forall n \geq 1$.

Q.E.D

Proof of Lemma 4. In the symmetric equilibrium, $e_i = e_s''(N)\ \forall i \in \mathscr{N}$. Then, let the equilibrium efforts of the incumbents and the entrant be $e_s''(N)$ and $e_f(N)$, respectively. According to (7.22) and (7.24), the equilibrium is characterized by the following equations

$$e_s''(N) = \frac{\mathbb{E}(\Delta_i \Pi_i | c_f = c_H) - \mathbb{E}(\Delta_f \Delta_i \Pi_i) e_f(N)}{2}, \tag{10.83}$$

$$e_f(N) = \frac{\mathbb{E}(\Delta_f \Pi_f | c_i = c_H) - \mathbb{E}(\Delta_i \Delta_f \Pi_f) e_s''(N)}{2}. \tag{10.84}$$

Let

$$f^1(N,k) = \frac{c(2N+1)(2+c+2ck)}{4(N+1)^2}, \tag{10.85}$$

$$f^2(N,k) = \frac{c^2(2N+1)}{(N+1)^2}. \tag{10.86}$$

Using the last two functions, verify that the function $\Delta_f \Pi_f(L)$ satisfies conditions (a) - (c) of Claim 1. Therefore, by Claim 1 we can write

$$\mathbb{E}(\Delta_f \Pi_f | c_i = c_H) = \Delta_f \Pi_f(0) - (N-1)[\Delta_f \Pi_f(0) - \Delta_f \Pi_f(1)] e_s''(N). \tag{10.87}$$

In the proof of Lemma 3 we showed that the function $\Delta_i \Pi_i(L, c_H)$ satisfies properties (a) - (c) of Claim 1. Hence, we may write

$$\mathbb{E}(\Delta_i \Pi_i | c_f = c_H) = \Delta_i \Pi_i(0, c_H) - (N-1)[\Delta_i \Pi_i(0, c_H) - \Delta_i \Pi_i(1, c_H)] e_s''(N). \tag{10.88}$$

Furthermore, in the proof of Lemma 3 we show that $\mathbb{E}(\Delta_i \Delta_f \Pi_f) = \Delta_{if}$ and $\mathbb{E}(\Delta_f \Delta_i \Pi_i) = \Delta_{fi}$, where Δ_{if} and Δ_{fi} are given by (10.55) and (10.56), respectively. Using this fact and equations (10.87) and (10.88), we may write the equation system implied by equations (10.83) and (10.84) as

$$2e_s''(N) = \Delta_i \Pi_i(0, c_H) - (N-1)[\Delta_i \Pi_i(0, c_H) - \Delta_i \Pi_i(1, c_H)] e_s''(N) - \Delta_{fi} e_f(N), \tag{10.89}$$

$$2e_f(N) = \Delta_f \Pi_f(0) - (N-1)[\Delta_f \Pi_f(0) - \Delta_f \Pi_f(1)] e_s''(N) - \Delta_{if} e_s''(N). \tag{10.90}$$

Replace (10.90) into (10.89) and solve for $e_s''(N)$ to obtain

$$e_s''(N) = \frac{2\Delta_i \Pi_i(0, c_H) - \Delta_{fi} \Delta_f \Pi_f(0)}{4 + 2(N-1)[\Delta_i \Pi_i(0, c_H) - \Delta_i \Pi_i(1, c_H)] - \Delta_{fi}(N-1)[\Delta_f \Pi_f(0) - \Delta_f \Pi_f(1)] - \Delta_{fi} \Delta_{if}}. \tag{10.91}$$

Replace expressions (10.55) and (10.56) into (10.91) and use (7.4) and (7.10) to obtain

$$e_s''(N) = \frac{c}{4} \left(\frac{8N(1-c+cN)(N+1)^2 - c^2 N(2N+1)(2-c+2cN)}{2(N+1)^4 + 4N(N-1)(N+1)^2 c^2 - c^4 N^2 (2N+1)} \right). \tag{10.92}$$

Q.E.D

Proof of Proposition 7. Let $n \in \mathbb{N}$ be the number of incumbent firms in the product market. Let $e_c(n)$ and $e_s''(n)$ be the effort levels in the unique symmetric equilibrium of a Cournot and a Stackelberg oligopoly with simultaneous contracts and sequential production, given in (5.22) and (7.25), respectively. Recall that $\bar{c} \in (0,1]$; hence, every statement that holds $\forall\ c \in [0,1]$ holds $\forall\ c \in (0, \bar{c})$ as well. From expressions (5.22) and (7.25), we obtain that $e''_s(n) > e_c(n)$ if and only if $\xi^1(n) > 0$, where

$$\xi^1(n) = 8cn^5 + 4(c^3 - 2c^2 + 6c + 2)n^4 + 2(-c^5 + 2c^4 - 6c^2 + 6c + 16)n^3 \\ + (-3c^5 + 6c^4 - 11c^3 - 2c^2 - 16c + 40)n^2 + (-c^5 + 2c^4 - 6c^3 - 12c + 16)n + c^3 - 2c^2. \quad (10.93)$$

We shall prove by induction that $\xi^1(n) > 0 \ \forall \ n \geq 1$ with $c \in [0,1]$. Verify that $\xi^1(1) > 0 \ \forall \ c \in [0,1]$. Assume $\xi^1(k_1) > 0$ for some $k_1 > 1$ with $c \in [0,1]$ fixed, then we shall prove that $\xi^1(k_1+1) > 0 \ \forall k_1 > 1$. Note that

$$\xi^1(k_1 + 1) = \xi^1(k_1) + \xi^2(k_1), \quad (10.94)$$

where

$$\xi^2(k_1) = 40ck_1^4 + 16(c^3 - 2c^2 + 11c + 2)k_1^3 + 2(-3c^5 + 6c^4 + 12c^3 - 42c^2 + 130c + 72)k_1^2 \\ + 2(-6c^5 + 12c^4 - 3c^3 - 36c^2 + 70c + 104)k_1 - 6c^5 + 12c^4 - 13c^3 - 22c^2 + 16c + 88. \quad (10.95)$$

Note that $\xi^1(k_1) > 0$ by assumption. Hence, in order to prove $\xi^1(k_1+1) > 0$, we shall prove by induction $\xi^2(k_1) > 0 \ \forall k_1 > 1$. Verify that $\xi^2(2) > 0 \ \forall \ c \in [0,1]$. Assume $\xi^2(k_2) > 0$ for some $k_2 > 2$ with $c \in [0,1]$ fixed, then we shall prove that $\xi^2(k_2+1) > 0 \ \forall k_2 > 2$. Note that

$$\xi^2(k_2 + 1) = \xi^2(k_2) + \xi^3(k_2), \quad (10.96)$$

where

$$\xi^3(k_2) = 160ck_2^3 + 48(c^3 - 2c^2 + 16c + 2)k_2^2 + 4(-3c^5 + 6c^4 + 24c^3 - 66c^2 + 302c + 96)k_2 \\ - 18c^5 + 36c^4 + 34c^3 - 188c^2 + 616c + 384. \quad (10.97)$$

Note that $\xi^2(k_2) > 0$ by assumption. Hence, in order to prove $\xi^2(k_2+1) > 0$, we shall prove by induction $\xi^3(k_2) > 0 \ \forall k_2 > 2$. Verify that $\xi^3(3) > 0 \ \forall \ c \in [0,1]$. Assume $\xi^3(k_3) > 0$ for some $k_3 > 3$ with $c \in [0,1]$ fixed, then we shall prove that $\xi^3(k_3+1) > 0 \ \forall k_3 > 3$. Note that

$$\xi^3(k_3 + 1) = \xi^3(k_3) + \xi^4(k_3), \quad (10.98)$$

where

$$\xi^4(k_3) = 480ck_3^2 + 96(c^3 - 2c^2 + 21c + 2)k_3 + -12c^5 + 24c^4 + 96c^3 - 264c^2 + 1208c + 384. \quad (10.99)$$

Note that $\xi^3(k_3) > 0$ by assumption. Hence, in order to prove $\xi^3(k_3+1) > 0$, we shall prove by induction $\xi^4(k_3) > 0 \; \forall k_3 > 3$. Verify that $\xi^4(4) > 0 \; \forall \, c \in [0,1]$. Assume $\xi^4(k_4) > 0$ for some $k_4 > 4$ with $c \in [0,1]$ fixed, then we shall prove that $\xi^4(k_4+1) > 0 \; \forall k_4 > 4$. Note that

$$\xi^4(k_4+1) = \xi^4(k_4) + 960ck_4 + \xi^5, \quad (10.100)$$

where

$$\xi^5 = 96c^3 - 192c^2 + 2496c + 192. \quad (10.101)$$

Note that $\xi^4(k_4) > 0$ by assumption. Verify that $\xi^5 > 0 \; \forall \, c \in [0,1]$. As the second term of (10.101) is positive, it can be seen that $\xi^4(k_4+1) > 0 \; \forall k_4 > 4$. Therefore, we can conclude that $\xi^i(k_i) > 0 \; \forall k_i > i$ with $i = 1, \ldots, 4$. Hence, $\xi^1(n) > 0 \; \forall n > 1$. We had already proved $\xi^1(1) > 0$, thus we obtain that $e_s''(n) > e_c(n) \; \forall n \geq 1$.

Q.E.D

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