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A NON-SUBSTITUTION THEOREM WITH HETEROGENEOUS LABOR

The non-substitution theorem for a static Leontief economy was proven independently for the first time by Samuelson (1951) and Georgescu-Roegen (1951), by means of calculus techniques. Using more general techniques, the theorem was proven by Koopmans (1951) for the case of three industries, but the most perspicuous formulation and proof of this result is due to Arrow (1951). Levahri (1965) provides another proof of the theorem showing that, even though a producer may shift from one activity to another, and back to the first, as the interest rate changes in the same direction, that is not possible for the productive system as a whole. Mirrles (1969) proves the theorem for a dynamic economy which excludes joint production. Stiglitz (1970) admits durable goods within a dynamic economy in balanced growth. One of the aims of the present paper is to prove a (static) non-substitution theorem with heterogeneous labor.[1]

The non-substitution theorem is usually presented as an assertion about profit rates and price systems: under certain conditions, a given profit rate determines uniquely an equilibrium price system for the economy. Yet, this assertion is a logical consequence of a property that economics have under such conditions, to wit, that it is possible to represent the set of all its efficient feasible production processes as a convex cone spanned by a *finite* set of basic activities.

Hence, there is a number of assertions logically related to the non-substitution theorem. The theorem itself can be formulated as follows:[2]

Non-substitution theorem: *There is a finite number of basic activities such that every efficient activity is a nonnegative linear combination of these basic activities, every nonnegative linear combination of these basic activities is efficient, and every nonnegative demand vector can be exactly produced by some efficient process.*

The assumptions under which this theorem is known to hold are the following:[3]

(1) Constant returns to scale: *There is a collection of basic activities, such that every possible state of production is represented by a linear combination of a finite number of the basic activities with nonnegative coefficients. The collection of basic activities from which such combinations are formed need not itself be finite.*

(2) Nonjoint production: *No basic activity has more than one output.*

(3) Labor as the only primary input: *In every basic activity, labor is a nonzero input.*

(4) No product is a primary input: *There is a given supply of labor from outside the system, but none of any product.*

An implicit assumption in the proof of the theorem is

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[2] See Arrow (1951), pp. 158, 164, and Nikaido (1968), pp. 190-1.

[3] See Arrow (1951), p. 155.

(5) Homogeneity of labor: *There is only one kind of labor.*

It is easy to see that the theorem is actually a conjunction of three different propositions:

PROPOSITION 1: *There is a finite number of basic activities such that every nonnegative linear combination of these basic activities is efficient.*

PROPOSITION 2: *There is a finite number of basic activities such that every efficient activity is a nonnegative linear combination of these basic activities.*

PROPOSITION 3: *Every nonnegative demand vector can be exactly produced by some efficient process.*

The import of Proposition 2 is that there is a finite number of basic activities such that the set E of all efficient global processes or activities with nonnegative net outputs is a subset of the convex polyhedral cone K, with vertex at the origin, spanned by those basic activities. This assertion must be compared with the one made by Proposition 1, which is that K is a subset of E. The properly non-substitutional statement is Proposition 2, which is usually coupled with Proposition 3, because Proposition 3 is also a consequence of the assumptions supporting Proposition 2.

My first goal in this paper is to prove Proposition 1 under assumptions which are far more general than the Leontief-type assumptions usually associated to the proof of the non-substitution theorem. A second goal is to prove also Proposition 2 under less stringent assumptions than the usual ones; in particular, my proof will make room for heterogeneous labor. The paper is organized as follows. In section 1, I introduce a fairly general structure representing the productive part of an economy. In section 2, I prove the theorems. Finally, in section 3, I discuss the import and limitations of the present version of the non-substitution theorem here introduced, and the open problems left for further research.

1. THE PRODUCTIVE STRUCTURE

I shall define a type of structures representing productive structures of arbitrary economies by means of the definition of a set-theoretic predicate.[4] What I intend to model by means of this mathematical framework is the group of "producers", understood as the different kinds of trades of the economy, which should rather be thought of as the different kinds of basic processes available, disregarding the problem of who takes the production decisions in the economy. Usually, the "producers" are denoted by means of a finite number of numerals $1, \ldots, \eta$, while their corresponding production possibility sets (usually interpreted as the activities available to these producers) are denoted by $X_1, \ldots, X_\eta$, respectively. The elements of any X_h ($1 \leqslant h \leqslant \eta$) are nonnegative $2\mu + \nu$-dimensional vectors of the form $[\mathbf{x}, \underline{\mathbf{x}}, \overline{\mathbf{x}}]$, where the ν-dimensional vector

[4] The reader interested in this methodology may take a look at Suppes (1957), Sneed (1971) or García de la Sienra (1994).

$\mathbf{x}$ is intended to represent the expenditures of labor of the process, the μ-dimensional vector $\underline{\mathbf{x}}$ stands for the inputs, and the μ-dimensional vector $\overline{\mathbf{x}}$ denotes the outputs. The vector $\hat{\mathbf{x}} = \overline{\mathbf{x}} - \underline{\mathbf{x}}$ shall represent the net outputs of the process. The set $X = \sum_h X_h$ is called the *global production possibility set* and its members are called *global processes*. The set of all processes in X having nonnegative net outputs shall be denoted by X^+. I shall write the netput form of process $[\mathbf{x}, \underline{\mathbf{x}}, \overline{\mathbf{x}}]$ as $\tilde{\mathbf{x}} = [-\mathbf{x}, \hat{\mathbf{x}}]$.

An analysis of the production processes in any economy reveals that goods and services are the result of chains of production processes. For instance, putting a bottle of milk in your table requires processes of cattle breeding, milking cows, pasteurizing the milk, bottling it, transporting it, shelfing it, selling it, and so on, just to mention but a few of the processes involved. The whole process, that starts, say, with cattle breeding and ends with your buying the bottle, can be seen as a single production process composed of several basic processes. These basic processes are characterized by their not being decomposable into processes of different types; that is, we may think, as an idealization,[5] that these processes can be operated at an arbitrarily low intensity (non-increasing returns to scale), but are not the sum of (nonzero) processes of a different kind. Processes are of the *same kind* if and only if they employ both labor power and other inputs of the same type; that is, if they have positive entries in the same positions.

I shall make the very natural assumption that there are basic processes. But notice that my concept of a basic process is more general than the one used above in connection with the standard formulation of the non-substitution theorem, where a basic process is one in which only one kind of good is produced as output. An analogous of a basic process in this sense, with heterogeneous labor, would be one in which there is only one kind of labor input and only one kind of product. Processes with these characteristics are indeed basic in my sense and will be called *simple*. But a basic process can also be complex if, for instance, has more than one kind of labor input or more than one kind of product (joint production). This is empirically the case when you have an activity that requires the simultaneous operation of two different trades (for instance, the usual TV news broadcasting technology requires simultaneously the operation of cameras and news-reading), or that yields several joint products at the same time (for instance, beef and leather, or both). Non-basic processes will also be called *composite*. Notice that within a basic complex process we may have a kind of operation which is independent of the other. For instance, roof painting may require the support of someone to hold the stairs, but stairs-holding does not require the support of the painter. It is empirically observable that most operations are independent because in industry, for example, they can be performed in a series, one after the other, in a production line.

DEFINITION 1. By a *productive structure* I mean a structure of the form $\langle X_1, \ldots, X_\eta \rangle$ such that, for every $h \in \{1, \ldots, \eta\}$:

(A1) X_h is a closed convex cone in the linear space $\mathbb{R}^{2\mu+\nu}$.

(A2) X_h is a set of basic activities.

[5] For a discussion of the relevant concept of idealization in this context, see García de la Sienra (1994).

(A3) Inaction is possible; i.e. $\widetilde{\mathbf{0}} \in X_h$.

(A4) Labor is productive; i.e. $\forall[\mathbf{x}, \underline{\mathbf{x}}, \overline{\mathbf{x}}] \in X_h$: if $\mathbf{x} \geqslant \mathbf{0}$ then $\overline{\mathbf{x}} \geqslant \overline{\mathbf{0}}$.

(A5) Labor is indispensable; i.e. $\forall[\mathbf{x}, \underline{\mathbf{x}}, \overline{\mathbf{x}}] \in X_h$: if $\overline{\mathbf{x}} \geqslant \overline{\mathbf{0}}$ then $\mathbf{x} \geqslant \mathbf{0}$.

(A6) There is a global process that has a positive net output.

(A7) There is no land of Cockaigne; i.e. $\mathbf{x} = \mathbf{0}$ implies $\underline{\mathbf{x}} = \underline{\mathbf{0}}$ and $\overline{\mathbf{x}} = \overline{\mathbf{0}}$.

LEMMA: *For every process $\widetilde{\mathbf{x}} \in X^+$, there is an efficient process $\widetilde{\mathbf{y}}^* \in X^+$, such that $\widehat{\mathbf{y}}^* \geqq \widehat{\mathbf{x}}$ and $\mathbf{y}^* \leqq \mathbf{x}$.*

Proof: The strategy of the proof is simple. I will show, first, that the set

$$E(\widetilde{\mathbf{x}}) \equiv \{\widetilde{\mathbf{y}} \in X^+ \mid \widehat{\mathbf{y}} \geqq \widehat{\mathbf{x}} \text{ and } \mathbf{y} \leqq \mathbf{x}\}$$

is compact and, second, I will use the fact that the function $\varphi : E(\widetilde{\mathbf{x}}) \to \mathbb{R}$, that assigns to every vector $\widetilde{\mathbf{y}} \in E(\widetilde{\mathbf{x}})$ the inner product of this vector with the $2\mu + \nu$ vector of wages and prices whose entries are all 1s, is continuous. Thus, it will follow (by Weierstrass theorem) that φ has a maximum $\widetilde{\mathbf{y}}^*$. It will be easy to see that $\widetilde{\mathbf{y}}^*$ is an efficent point in X^+.

Since $\widetilde{\mathbf{0}} \in X^+$, in order to prove that $E(\widetilde{\mathbf{x}})$ is bounded, notice that the set

$$F \equiv \{\mathbf{y} \mid [\mathbf{y}, \underline{\mathbf{y}}, \overline{\mathbf{y}}] \in E(\widetilde{\mathbf{x}})\}$$

is bounded (because $\mathbf{0} \leqq \mathbf{y} \leqslant \mathbf{x}$ for every $[\mathbf{y}, \underline{\mathbf{y}}, \overline{\mathbf{y}}] \in E(\widetilde{\mathbf{x}})$). Now, if $E(\widetilde{\mathbf{x}})$ were unbounded, there would be an unbounded sequence $(\widetilde{\mathbf{y}}_k)$ in $E(\widetilde{\mathbf{x}})$. At any rate, $(\widetilde{\mathbf{y}}_k)$ has an unbounded subsequence –call it also $(\widetilde{\mathbf{y}}_k)$– such that $(\|\widetilde{\mathbf{y}}_k\|)$ is increasing and unbounded. Nevertheless, the corresponding sequence of labor input vectors, $(\mathbf{y}_k)$, can be seen to converge to a limit $\mathbf{y}$ (not necessarily in F) because F is bounded. Let

$$\widetilde{\mathbf{z}}_k = (\|\widetilde{\mathbf{y}}_k\| + 1)^{-1}\widetilde{\mathbf{y}}_k.$$

Since X^+ is a cone, $\widetilde{\mathbf{z}}_k \in X^+$. Moreover, $(\widetilde{\mathbf{z}}_k)$ is bounded because $\|\widetilde{\mathbf{z}}_k\| \leqslant 1$. Hence, without loss of generality, we may assert that (a subsequence of) $(\widetilde{\mathbf{z}}_k)$ converges to a point $\widetilde{\mathbf{z}}$ which must belong to X^+ because X^+ is closed. Since $(\|\widetilde{\mathbf{z}}_k\|)$ is increasing, $\widetilde{\mathbf{z}} \neq \widetilde{\mathbf{0}}$ and so, due to the productivity and/or indispensability of labor, $\overline{\mathbf{z}} \geqslant \overline{\mathbf{0}}$. On the other hand, since $(\mathbf{y}_k) \to \mathbf{y}$ as $k \to \infty$,

$$\begin{aligned}
\mathbf{z} &= \lim_{k \to \infty} \mathbf{z}_k \\
&= \lim_{k \to \infty} (\|\widetilde{\mathbf{y}}_k\| + 1)^{-1} \cdot \lim_{k \to \infty} \mathbf{y}_k \\
&= 0 \cdot \mathbf{y} \\
&= \mathbf{0}.
\end{aligned}$$

Hence, given that labor is indispensable, $\widetilde{\mathbf{z}} = \widetilde{\mathbf{0}}$. This contradiction shows that $E(\widetilde{\mathbf{x}})$ is bounded.

Now, it is easy to see that $E(\widetilde{\mathbf{x}})$ is closed, because the limit of any convergent sequence of points of $E(\widetilde{\mathbf{x}})$ is in X^+, which is closed, and also satisfies the conditions for belonging to $E(\widetilde{\mathbf{x}})$.

Clearly, since φ is continuous, it has a maximum at a point $\tilde{\mathbf{y}}^*$ in $E(\tilde{\mathbf{x}})$. I claim that this point is actually efficient in X^+. Suppose, on the contrary, that there is a $\tilde{\mathbf{y}}^{**}$ which is more efficient than $\tilde{\mathbf{y}}^*$. Then $\varphi(\tilde{\mathbf{y}}^{**}) > \varphi(\tilde{\mathbf{y}}^*)$, which is impossible because φ had reached a maximum at $\tilde{\mathbf{y}}^*$. □

2. EFFICIENCY AND NON-SUBSTITUTION

Probably due to its outstanding implications regarding the uniqueness of a price vector for the economy, the non-substitution theorem has received a great deal of attention. Yet, its reciprocal, labeled here as Theorem 1, also has important implications for the existence of price vectors. As it is shown in García de la Sienra (1996), any productive structure in which the conditions of Theorem 1 hold has at least one system of price and wages $(\mathbf{p}, \mathbf{w})$ such that the value of any netput $\hat{\mathbf{x}}$ at these prices, $\mathbf{p}\hat{\mathbf{x}}$, is proportional to the wages paid to the labor power that produced this netput, $\mathbf{wx}$. What the non-substitution theorem adds is that this pair $(\mathbf{p}, \mathbf{w})$ is unique up to similarity transformations (multiplication by a scalar). Hence, the economic implications of the following result should also be appreciated.

THEOREM 1: *There is a finite number of basic activities such that every nonnegative linear combination of these basic activities is efficient; i.e.* $\exists(\tilde{\mathbf{x}}_1, \ldots, \tilde{\mathbf{x}}_\eta) \in X_1 \times \cdots \times X_\eta$: $K(\tilde{\mathbf{x}}_1, \ldots, \tilde{\mathbf{x}}_\eta) \subseteq E$.

Proof: Let $\tilde{\mathbf{x}}$ be an efficient global process and let $\{\tilde{\mathbf{x}}_h\}$ be a family of basic processes such that $\tilde{\mathbf{x}} = \sum_h \tilde{\mathbf{x}}_h$. I will show that the convex cone $K(\tilde{\mathbf{x}}_1, \ldots, \tilde{\mathbf{x}}_\eta)$ spanned by these processes is a set of efficient activities.

Let

$$\mathbf{L} \equiv \begin{bmatrix} x_{11} & \ldots & x_{1\eta} \\ \vdots & & \vdots \\ x_{\nu 1} & \ldots & x_{\nu\eta} \end{bmatrix}$$

and

$$\mathbf{N} \equiv \begin{bmatrix} \hat{x}_{11} & \ldots & \hat{x}_{1\eta} \\ \vdots & & \vdots \\ \hat{x}_{\mu 1} & \ldots & \hat{x}_{\mu\eta} \end{bmatrix},$$

where the hth column of $\mathbf{L}$ is the vector of labor inputs of activity $\tilde{\mathbf{x}}_h$, and the hth column of $\mathbf{N}$ is that of net outputs. Hence, $\tilde{\mathbf{x}} = [-\mathbf{Lq}, \mathbf{Nq}]$ for some positive vector $\mathbf{q}$.

Let $\tilde{\mathbf{y}}$ be an arbitrary element of $K(\tilde{\mathbf{x}}_1, \ldots, \tilde{\mathbf{x}}_\eta)$, so that $\tilde{\mathbf{y}} = [-\mathbf{Ls}, \mathbf{Ns}]$ for some nonnegative $\mathbf{s}$, and suppose that $\tilde{\mathbf{y}}$ is not efficient. By the Lemma, there is an efficient process $\tilde{\mathbf{z}}$ such that $\hat{\mathbf{z}} \geqq \hat{\mathbf{y}}$ and $\mathbf{z} \leqq \mathbf{y}$, with one of these inequalities being actually strict. Let

$$\mathbf{u} = (1-\alpha)\mathbf{Lq} + \alpha\mathbf{Ls}$$

and

$$\hat{\mathbf{u}} = (1-\alpha)\mathbf{Nq} + \alpha\mathbf{Ns}$$

for $\alpha < 0$. If α is very small, $(1-\alpha)\mathbf{q} + \alpha\mathbf{s} \geqslant \mathbf{0}$ and so $\tilde{\mathbf{u}} = [-\mathbf{u}, \hat{\mathbf{u}}] = [-\mathbf{L}((1-\alpha)\mathbf{q} + \alpha\mathbf{s}(\mathbf{N}((1-\alpha)\mathbf{q}+\alpha\mathbf{s})]$ is a global activity. Let $\beta \equiv -(\alpha/(1-\alpha))$. Then $0 < \beta < 1$ and

$$(1-\beta)[-\mathbf{u}, \hat{\mathbf{u}}] + \beta[-\mathbf{z}, \hat{\mathbf{z}}] = [-((1-\beta)\mathbf{u} + \beta\mathbf{z}), (1-\beta)\hat{\mathbf{u}} + \beta\hat{\mathbf{z}}]$$

is also a global process. But

$$(1-\beta)\mathbf{u} + \beta\mathbf{z} = \mathbf{Lq} - \beta(\mathbf{y}-\mathbf{z}) \leqq \mathbf{Lq}$$

and

$$\begin{aligned}(1-\beta)\hat{\mathbf{u}} + \beta\hat{\mathbf{z}} &= \frac{1}{1-\alpha}((1-\alpha)\mathbf{Nq} + \alpha\mathbf{Ns}) + \beta\hat{\mathbf{z}} \\ &= \mathbf{Nq} + \frac{\alpha}{1-\alpha}\mathbf{Ns} + \beta\hat{\mathbf{z}} \\ &= \mathbf{Nq} - \beta\mathbf{Ns} + \beta\hat{\mathbf{z}} \\ &= \mathbf{Nq} + \beta(\hat{\mathbf{z}} - \mathbf{Ns}) \\ &= \mathbf{Nq} + \beta(\hat{\mathbf{z}} - \hat{\mathbf{y}}) \\ &\geqq \mathbf{Nq}\end{aligned}$$

Now, since

$$\begin{bmatrix} -(\mathbf{Lq} - \beta(\mathbf{y}-\mathbf{z})) \\ \mathbf{Nq} + \beta(\hat{\mathbf{z}} - \hat{\mathbf{y}}) \end{bmatrix} \geqq \begin{bmatrix} -\mathbf{Lq} \\ \mathbf{Nq} \end{bmatrix},$$

the assumption that $\tilde{\mathbf{y}}$ is not efficient implies that

$$\begin{bmatrix} -(\mathbf{Lq} - \beta(\mathbf{y}-\mathbf{z})) \\ \mathbf{Nq} + \beta(\hat{\mathbf{z}} - \hat{\mathbf{y}}) \end{bmatrix} \geqslant \begin{bmatrix} -\mathbf{Lq} \\ \mathbf{Nq} \end{bmatrix} = \tilde{\mathbf{x}},$$

with at least one equality being strict. But this is impossible because $\tilde{\mathbf{x}}$ was supposed to be efficient. □

The economic meaning of the non-substitution theorem boils down to the assertion that there is only one efficient way of getting things done. We shall see that whenever the basic processes are simple, i.e. when they are constituted by only one kind of labor input, and only one kind of material output (nonjoint production), then non-substitution holds. This is the meaning of the following theorem, which is the reciprocal of Theorem 1.

THEOREM 2: *Suppose that all basic processes are simple. Then there is a finite number of basic activities such that every efficient activity is a nonnegative linear combination of these basic activities; i.e.* $\exists(\tilde{\mathbf{x}}_1, \ldots, \tilde{\mathbf{x}}_\eta) \in X_1 \times \cdots \times X_\eta : E \subseteq K(\tilde{\mathbf{x}}_1, \ldots, \tilde{\mathbf{x}}_\eta)$.

Proof: By Theorem 1, there is a finite number of basic nonzero (in fact, simple) activities $\tilde{\mathbf{x}}_1, \ldots, \tilde{\mathbf{x}}_\eta \in X_1 \times \cdots \times X_\eta$ such that $K(\tilde{\mathbf{x}}_1, \ldots, \tilde{\mathbf{x}}_\eta) \subseteq E$. We will show that, in fact, $E \subseteq K(\tilde{\mathbf{x}}_1, \ldots, \tilde{\mathbf{x}}_\eta)$.

Let $\mathbf{L}$ and $\mathbf{N}$ be matrices as in the proof of Theorem 1, where $x_{hh} = 1$ and $x_{hk} = 0$ for $h \neq k$ (since there is no joint production), and let $\tilde{\mathbf{x}}$ be any element of $K(\tilde{\mathbf{x}}_1, \ldots, \tilde{\mathbf{x}}_\eta)$, so that $\tilde{\mathbf{x}} = [-\mathbf{Lq}, \mathbf{Nq}]$ for some positive vector $\mathbf{q}$. Let $\tilde{\mathbf{z}}$ be an arbitrary element of E and $\{\tilde{\mathbf{z}}_h\}$ a decomposition of $\tilde{\mathbf{z}}$ in simple basic activities; i.e. $\tilde{\mathbf{z}} = \sum_h r_h \tilde{\mathbf{z}}_h$ for $r_h \geqslant 0$.

By Theorem 1, the cone $K(\widetilde{\mathbf{z}}_1, \ldots, \widetilde{\mathbf{z}}_n)$ is a set of efficient activities and, since $\widetilde{\mathbf{z}}_h \in K(\widetilde{\mathbf{z}}_1, \ldots, \widetilde{\mathbf{z}}_n)$ for every h, the basic activities $\widetilde{\mathbf{z}}_h$ themselves are efficient. I will show that there is a nonnegative vector $\mathbf{t}$ such that $\widetilde{\mathbf{z}} = [-\mathbf{Lt}, \mathbf{Nt}]$.

By Axiom A6 and the Lemma we may assume, without any loss of generality, that $\widetilde{\mathbf{x}}$ has a positive net output. Thus, since $\widehat{x}_{hk} \leqslant 0$ for $h \neq k$, due to nonjoint production and the Hawking-Simon condition, $\mathbf{N}$ is invertible and $\mathbf{N}^{-1}$ is nonnegative. Therefore, equation $\mathbf{Ns} = \widehat{\mathbf{z}}_h$ has a nonnegative solution $\mathbf{s}$. Clearly, $\widetilde{\mathbf{y}} \equiv [-\mathbf{Ls}, \mathbf{Ns}] \in K(\widetilde{\mathbf{x}}_1, \ldots, \widetilde{\mathbf{x}}_n)$ and, since $\widehat{\mathbf{z}}_h$ is simple and nonnegative, $\mathbf{Ns} = s_h\widehat{\mathbf{x}}_h$ and so all components of vector $\mathbf{s}$ are zero, except the h-th one, which is precisely s_h. Thus, $\mathbf{Ls} = s_h\mathbf{x}_h$ and $\mathbf{z}_h \leqq \mathbf{Ls}$ because $\widetilde{\mathbf{z}}_h$ is efficient. The next goal is to show that, in fact, $\mathbf{z}_h = \mathbf{Ls}$, but this follows immediately by the argument given in the proof of Theorem 1, assuming that $\widetilde{\mathbf{y}}$ is not efficient, so that $\mathbf{z}_h \leqslant \mathbf{Ls}$.

Hence, we have $\widetilde{\mathbf{z}}_h = [-\mathbf{Ls}, \mathbf{Ns}] = [-s_h\mathbf{x}_h, s_h\widehat{\mathbf{x}}_h]$. It is immediate that $\widetilde{\mathbf{z}} = \sum_h r_h\widetilde{\mathbf{z}}_h = \sum_h[-r_hs_h\mathbf{x}_h, r_hs_h\widehat{\mathbf{x}}_h]$. Setting $t_h = r_hs_h$, we have $\widetilde{\mathbf{z}} = \sum_h t_h\widetilde{\mathbf{x}}_h$ with $t_h \geqslant 0$. This shows that $E \subseteq K(\widetilde{\mathbf{x}}_1, \ldots, \widetilde{\mathbf{x}}_n)$. □

3. CONCLUSIONS

I have thus extended the non-substitution theorem for the static case with heterogeneous labor and established a result with important implications for models more general than the usual Leontief model. In particular, the result holds for productive structures in which activities are basic but not necessarily simple; i.e. in which both joint production and heterogeneous labor are allowed. On the other hand, it seems to me that the non-substitution theorem here established paves the way for a reformulation of the Leontief model of the labor theory of value with more than one primary factor. In his review of my book (García de la Sienra 1992), professor Boris Levin (1994, p. 349) said that the non-substitution theorem

> works with no more than one primary factor. The applicability of this theorem even to a linear dynamic process with technological change is questionable. The author considers such a situation to be fully explained. The serious economist refers to "causal indeterminacy", the "constant price assumption", "perfect foresight", etc. And of course, the possibility of increased returns to scale is not addressed at all.

Certainly, in that book I was considering only the general linear static case of the labor theory of value and therefore I never considered such a situation "to be fully explained". It seems to me that the present result establishes the validity of the non-substitution theorem for the static case of the Leontief model with more than one primary factor. The constant price "assumption" is actually a logical consequence of this theorem: for a proof, see García de la Sienra (1996) and use the fact that both $\mathbf{L}$ and $\mathbf{N}$ have semipositive inverses. I do not claim that the present result solves the problems that plague the dynamic Leontief models. Quite another matter is whether an analogous result holds also for these models, and still another one is whether such a result could help to solve those problems. But this question lies beyond the scope of the present paper and it will be the topic for future research.

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