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NÚMERO 138

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CHOOSING ONE'S IDENTITY

Abstract

This paper has two parts: In the first part, it presents a way of formalizing identity choice issues. In the main part, it looks at the incentives firms have to differentiate themselves by means of choosing meaningless, worthless 'tags' in the absence of exogenous rules preventing them from imitating each other. It shows that even when one can find a ranking of firms' products (from worse to better sell), 'endogenous' differentiation of this sort might result. A crucial factor in obtaining this result is that not all buyers should be able to observe all tags in the same way, i.e., that observability conditions should vary across buyers.

1 Introduction

Most 'real-world' markets rely on reputation to support trade, and reputations hinge on 'identities', i.e., roughly speaking, on a set of circumstances that allows agents to distinguish those they have interacted with in the past, thus putting them in a position to string together the histories of specific agents and from those histories infer something about who those agents 'really' are.

Conventionally, most economic theory just treat agents' 'identities' as exogenously given, and moreover, it tends to display a strong bias towards assuming that agents are perfectly identified. Occasionally, the opposite extreme assumption is made, namely, that agents are completely anonymous (in fact, it is sometimes claimed that markets are characterized by anonymous transactions). In any case, what is very seldom allowed for is for agents to be able to choose their 'identities'. In practice, it is clear that agents have a degree of control over their identities. Agents can choose their names, and often can change their names. They can choose whether or not to advertise their names, and they can choose where to advertise. They can choose to hide their identities, or simply not to carry any form of identification. Moreover, all agents cannot observe all elements that make up an agent's identity in the same way. Some are effectively in a position to survey the identities of the whole population, while others are only able to observe the identities of those agents in their immediate surroundings. Some can observe better those that are 'near' to them, than those that are farther away. Others, in contrast, can see the wood but not make out its trees.

Given these possibilities (and many others) plus the rather complex payoffs structures underlying most situations, the presumption that either of the extreme cases (full identification or complete anonymity) provides a 'good' description of the general 'identifiability' conditions prevailing in economic interactions appears highly dubious. As usual, in the absence of theory and theory-based evidence, it is hard to make a really informed judgement.

The first and general purpose of this paper is to make a start in filling the gap that would seem to exist in this area by developing a formal, specialized framework to study 'identity' choices.

The first step is to develop a more precise set of concepts to deal with a myriad of rather subtle issues that is hard to address otherwise . For example, what does it mean to be 'identified'? Does it suffice that a person be distinguishable from any other? But then, what about agents that operate under two different names? In fact, what exactly should we call a 'name'? Is a 'name' in essence just a 'tag' serving the aim of differentiating agents or is there more to it? Might 'names' not also serve the purpose of 'associating' groups of agents? (Think of family names). I think it evident that the 'fuzzy' day to day meanings of words like 'name' and 'image' cannot provide a solid basis for theorizing on this subject.

The next step is to note that the choice of identity is strategic. The extent to which an individual can and might want to 'identify' herself will depend on the extent to which her fellows differentiate themselves from her and among themselves. For example, the only 'generic' product in a market is clearly distinguishable from any other product, while the incentives of a criminal to disclose his identity will clearly depend on whether or not he can tell if the person confronting him is an undercover agent or a fellow criminal. Given this strong strategic dimension, it is natural to use game theory to analyze this type of behavior.

In applying the game theoretic formalism a basic task is to identify the type of game which might serve as a benchmark environment to study identity choices. Of course, the 'appropriate' game to study will vary with the more specific questions one pursues within this broader program.

Here I come to the second and main part of this paper: It focuses on the question of whether and under which conditions and to what extent agents can differentiate themselves by means of what I will be calling 'tags', i.e. observable characteristics which carry no semantic meaning and are by themselves not in any way payoff relevant. In a sense, 'tags' are a form of cheap talk but the message they aim to convey is of a very restricted nature, namely, 'I am not someone else'. By means of their choice of tags agents in a population will group themselves in various ways, and these groupings will then determine the type of inferences other agents will draw about unobservable, payoff-relevant characteristics of rivals.

To study these questions, it seems to me that at least in a first approximation it suffices to work with very simple (quasi) static games, though clearly the question of choice of identities, closely linked as it is with the process of reputation formation, can only be fully understood in a truly dynamic context. More specifically, the game studied here is one where one-product sellers start out by simultaneously choosing tag configurations. After this choice they are paired randomly (the matching is random to better concentrate on the question of 'identifiability' -see Section ? below) to buyers with unit demands and sellers proceed to demand a price from buyers. If buyers accept the price is paid and trade takes place. The crucial features are, first, that different buyers value sellers products differently; and secondly, that buyers cannot tell sellers apart except by relying on the tags sellers 'wear' (buyers themselves are perfectly identifiable). The choice of tags will afect buyers beliefs about who is who, and, consequently, sellers ability to extract a payment from buyers. Note an important peculiarity of this model: Unlike what happens in 'marriage' models, the value of a transaction to one of the parties involved, namely sellers, depends only on the price agreed, not directly on the actual identity of buyers. This means, in particular, that there is no obvious 'single crossing structure' underlying the results[1].

[1] As in for example a situation where agent A prefers agent B, and agent C prefers agent D, and vice-versa. Endogenous identification will follow in such an environment quite naturally as the ranking of interaction partners is exactly inverted across individuals.

The statements coming out of this analysis concern the features of the tag space and payoff space (i.e., how many tags there are, who can observe whose tags, who values what how much, etc.) that translate into informative tag patterns, that is tag patterns that actually allow agents to distinguish each other though perhaps not perfectly (readers familiar with the cheap talk literature will note that this is strongly reminiscent of the results of that literature where the focus is precisely on whether or not cheap talk can trascend babbling, or to put it differently, whether something can actually be communicated through cheap talk).

Here is a short overview of the results: First of all, the paper shows that the fact that some buyers prefer one seller's products to another, while others have opposite preferences, is not enough to induce differentiation. What I call partial observability (some buyers not being able to observe some tags) and differential perspectives (differing buyers' ability to observe tags across matches) are required. More specifically, it is shown that in order to obtain any form of strict differentiation (both differentiated sellers strictly prefering to be distinguished) tags have to be public. The simplest form of such differentiation is what is called in the paper 'nested' differentiation and which can arise with a single public tag. Here sellers are grouped in pools including 'very good' and 'very bad' sellers whose members, given the way buyers form their beliefs, strictly prefer not to leave while outsiders strictly prefer not to join. To go beyond this, multiple partially observable public tags are required. In such context, strict differentiation requires pooling of sellers. Finally, it is shown that in order to get complete strict differentiation (each seller on his or her own) it turns out to be necessary for buyers to have complex perspectives (all tags neither public nor private).

In all of this, as already hinted at, the way out-of-equilibrium beliefs are formed is central (perhaps not surprisingly given that this is some form of signalling game). Accordingly, the consistency requirement of sequential equilibrium, the natural solution concept for the game studied, will come to play in many case a very important role.

1.0.1 Marketing and Trade Mark Legislation

Note the close connection between the subject matter of this paper and a lot of the issues 'marketing' deals with. After all, we are talking about a form of product differentiation, though here the primary concern is with the 'observability' and 'identifiability' of products rather than just their attributes (as in more conventional analysis); with how products are perceived rather than just what they are. In fact, I do not think it farfetched to claim that the focus of 'marketing' is precisely on the 'image' of products and producers rather than their actual properties, and this is also the focus of the analysis in this paper.

Another important area of application is trade mark protection. Surprisingly (at least to me) many of the rather esoteric sounding questions dealt with in this paper seem to have counterparts in the legal discussions in the courts concerning trade mark infringement. For example, it is apparently a primary concern of the law to define whether a mark is really being observed by buyers who might be misled. Two equal marks observed by different sets of buyers do not constitute a case of infringement. This type of consideration relates directly to the central claim in this paper: That the fact that buyers can observe different things to different extents is crucial in determining firms incentives to imitate each other.

In a sense, this paper can be seen as an attempt to get an analytical hold on the manifold ambiguities and problems facing the law in defining exactly what a trademark is or should be.

2 A Framework for Studying Identities

2.1 Basic Formalization

The purpose of this section is to develop an abstract scheme of categories on which to base the analysis of identity choices.

The starting point is a set of 'subjects', $S = \{1, 2, 3, ...N\}$, and a set of agents, $A = \{1, 2, 3, ...M\}$.

A 'subject' will be designated by small case letters, an agent by capital letters. A subject i is basically a list of 'attributes' $A_i = (x_i^1, x_i^2 x_i^3, ..., x_i^m)$. An 'attribute' is here any feature of the subject that is relevant in some way or another to some agent's decision in the economy. Denoting the space of attributes $\mathcal{A}$, the map of subjects' attributes is then a correspondence $\Lambda : S \to \mathcal{A}$. Say, there are m attributes, and each takes values on the real line, then the space of attributes is R^m,with $x^k = R \cup \{NA\}$,where NA is a 'null value' denoting non-applicability (more on nulls below). A subject's list of attributes is simply a point in that space.

An agent K is an observing entity that, as usual, takes decisions. Every agent will also be a subject (i.e., for every entry in the agents' list there is a corresponding entry in the subjects' list, so that $M \geq N$), but not the other way around: There might be subjects that do not take decisions (say, a product). Often the set of subjects and agents will coincide, but, in principle, it seems important to allow for the distinction to study phenomena like 'branding', or even to allow for agents not being able to observe their own attributes.

Now, a crucial distinction is between 'observable' and 'non-observable' attributes. As the attributes of a given subject observable to one agent might not be observable to another, for each agent K one divides each subject's attributes into two subsets, O_{Ki}, the set of attributes of subject i observable to agent K, and N_{Ki}, the set of attributes of subject i not observable by agent K.

'Observable' means here pretty much what it means in ordinary language, but there is a point that deserves some elaboration: In modelling certain situations one might want to allow for attributes to be hidden while in other cases one might prefer to assume that certain attributes just do not apply to certain subjects or that agents are able to drop attributes altogether from their list. The question is how to deal with such situations. The hiding of the attribute might itself be observable, in which case it would seem appropriate to model hidden attributes by means of 'null values', i.e., a token appearance (a kind of parch) that does not allow the other agent to assess the level of the attribute . On the other hand, hiding the value of an attribute in such a way that the other agent cannot tell whether the attribute is hidden or just not present, seems equivalent to making that attribute not observable. Finally, the case of dropping attributes or instances where attributes do not apply (e.g., tall sugar) is somewhat more ambiguous. If agents are aware that a particular attribute does not apply to some subject, another 'null value' (distinct from the 'null value' denoting observable hiding and with the meaning 'attribute does not apply') would seem to be called for. If agents are not aware, again unobservability would seem appropriate.

Say there are three attributes, x^1, x^2, x^3, then the space of attributes is $R^{3'}$, with $x^k = R \cup \{N\} \cup \{NA\}$, $k = 1, 2, 3$, where N is the 'null value' representing non-observability while NA stands for the 'null value' representing non-applicability. Say only the first attribute value of subject i is observable by agent K (but all three attributes apply), then $O_{Ki} = \{x_i^1, N, N\}$

I define the 'field of vision' of agent K to be the mapping from subjects to the list of attributes observable by that agent, $\Psi_K : S \rightarrow \mathcal{A}'$, where $\mathcal{A}'$

stands for the attribute space extended to allow for non- observability, so that $\Psi_K(i) = O_{Ki}$.

Note that this formalization can capture something like 'coarse vision', i.e., an agent not being able to make out the exact value of an attribute. Say an agent can only distinguish whether the value of attribute a_i is either in the interval $[0,1]$ or in $(1,+\infty)$. Then one can create an additional observable attribute $\widetilde{a}_i$ such that it takes one value when the original attribute falls in the first interval and another when the original attribute falls in the other interval, while letting the original attribute be now unobservable.

2.2 Substantial Attributes versus Tags

Though this is the core of the framework proposed here, there are a series of important elaborations. First of all, one should distinguish between attributes that are payoff relevant in some way, either because acquiring them is costly or because some agents directly care about them, and those that are not payoff relevant in any way. I will call the latter 'tags' while referring to the former as 'substantive'. As already noted in the introduction, the analytical part of this paper will deal with choice of 'tags' rather than 'substantive characteristics'.

2.3 Symbolic versus Nominal Attributes

In fact, it will deal with a very special class of tags, namely those that do not have any 'semantic meaning'. In principle, any attribute, whether payoff relevant or not, might carry what I call semantic meaning. That is, might carry an implicit message. For example, a tag (now the literal thing) might carry a written legend. Or, the color red might be associated with left-wing politics. This latter example illustrates quite well what 'semantic meaning' entails and what it does not: If the color red is associated with left-wing politics because agents believe that a lot of left-wingers have chosen to wear red, then this is not a case of 'semantic meaning': If left-wingers had chosen instead to wear brown, brown would just as readily have been associated with them. 'Semantic meaning' comes into play only if the attribute by itself has the effect of conveying a particular message independent of who else is carrying it: If a left-winger can make his political convictions known by carrying red even if no other left-winger is wearing the color, then red is meaningful: It means 'I am a leftwinger'. The difference is somewhat subtle (specially in dynamic treatments as it might be argued that red has come to mean left-wing because in the past the color was used by left-wingers) but I think of the utmost importance in many situations. Take the two designations COCA COLA and ARCI COLA. Certainly, the producers of the latter product in adding the word COLA were not aiming to mislead buyers into believing that their product was the real thing (COCA COLA) but rather were telling them something about the type of drink on offer (namely a COLA drink). They were doing this by sharing an observable attribute (the designation COLA) with a well known drink. Since the distinction is important I introduce an additional term, symbols, to designate attributes with semantic meaning. Note that symbols are not interchangeable, quite unlike tags: A rose by the name of something stinky would not smell as sweet.

2.4 Discretionary Attributes versus Fixed Attributes or Characteristics

And last but certainly not least, some attributes will be chosen, some not; in other words, some will be 'fixed', some 'discretionary'. This should best be understood as a 'stylized' version of a more realistic scenario in which it might be costly to choose attributes, and more costly to choose some than others.

These considerations (with the exception of 'coarse vision') partition the list of attributes of a 'subject' in various ways, some of which will vary with the particular agent observing the subject (tags/substantive attributes;observable/not observable attributes) while others won't (characteristic/discretionary attributes). Of course, there are other distinctions that might be relevant (for example., concerning how easily observable attributes can be remembered; how easily 'communicable' they are; etc.), but the above seem to make up the minimal framework for thinking about identity choices at least in a (quasi) static environment.

2.5 What is an Identity?

As an illustration of the above formalism at work, and for general reference in the remaining of the paper, I go into a brief discussion of the meaning of an 'identity'.

At the most general level, the 'identity' of an subject in the eyes of an agent K is a set of observable values of attributes of a subject i that allows K to distinguish this particular subject from all others. More formally,

Definition 1 *The 'identity' of subject i in the eyes of agent K, I_{iK}, is the smallest set of attributes values $(x_i^{z_1}, x_i^{z_2}, ..., x_i^{z_o}) \subset A_i$ such that*

$$i)\; x_i^{z_n} \in O_{Ki},\; n = 1, ..., o$$

$$ii)\; for\; each\; j \neq i, \exists\; n\; s.t.\; x_j^{z_n} \neq x_i^{z_n}$$

A subject i is then identified in the eyes of an agent K if and only if $O_{Ki} \neq O_{Kj}$, $\forall\, j \neq i$, or, equivalently, if and only if $\Psi_K^{-1}(O_{Ki}) = \{i\}$.

Note that the use of 'null values' allows for identification by means of non-observables, i.e., allows an agent to distinguish between subjects i and j by means of an attribute that is observable to the agent for one but not for the other subject. Note further that the concept of identification proposed here does not imply that an agent knows everything about the subject in question. For identification in the above sense to imply that, agents must know the map of subjects' attributes with certainty. Nor does the notion of identity proposed here imply that in the eyes of the agent there is a 1:1 mapping from observables to agents. For example, in a situation where the agent is not certain about the map of attributes, an agent might not be able to tell whether two distinct sets of observables correspond to one or two subjects.

It is perhaps useful to define a identification mapping for agent K as the mapping from attributes observable by K to subjects, i.e., $\Upsilon_K : \mathcal{A}' \rightarrow S \cup \{\emptyset\}$,and call the set $\Upsilon_K(O_{Ki})$ the 'identification set' of subject i in the eyes of agent K. Note that $\Psi_K^{-1}(O_{Ki}) = \Upsilon_K(O_{Ki})$, so that a subject i is identified iff his identification set only contains him.

One can define further the 'degree of (direct) identification of a subject i in the eyes of agent K' by taking the inverse of the cardinality of that agent's identification set in the eyes of K, $\frac{1}{|\Upsilon_K(O_{Ki})|}$. A subject is 'under-identified' if its degree of identification is below one.

2.6 Direct versus Indirect Identification

One must draw an important distinction at this point: One can keep track of a player's identity by making sure one sticks to him or her over time -

say, if I know that to start with I was alone with a given person in a room, I can be certain I am still with the same person just by making sure no one goes in or out of the room. Direct observation of the subject's features is not involved in this procedure. In other words, knowing the 'matching procedure' can sometimes suffice to identify a subject even if none of the subject's attributes are directly observable.

Defining a 'matching function' $M : A \rightarrow \Delta S$, i.e. a mapping from the space of agents to the space of probability distributions over subjects. Clearly, deterministic matching (i.e., $M : A \rightarrow S$) will lead automatically to identification without observation, in the sense that the agent will know who she is facing regardless of what she can observe. More generally, the nature of the matching will define a probability distribution over subjects giving the likelihood that an agent is matched with a given subject conditional on that agent's direct observations. Decisions will be based on this probability distribution reflecting 'total identification', i.e., the 'sum' of direct and indirect identification.

3 Association

There is another role observable attributes might play besides identifying a subject, and that is to associate it with a wider class of subjects. The association can take place in various ways: For example, it may be a result of under-identification. In such situations, a subject is necessarily associated with all other subject from which it cannot be distinguished. 'Pooling' is the usual term to designate this form of association. But there is association by over-identification as well. Subjects who are identified might be seen as being connected by virtue of sharing certain attributes (say a producer assigns his proprietary name to its products; sharing a family name or caste designation; franchising; academic affiliations, etc.).

The simple scheme proposed here is, seems to me, adequate to deal with issues of identification in the sense this term was just defined. A more comprehensive theory of 'image'- this latter term understood as an agent's beliefs regarding the payoff relevant attributes of a subject, would have to take into consideration issues of association by over-identification and would probably require a more elaborate foundation that the one just presented.

4 Identification by Means of Tags

As an application of the system of categories just outlined, I look at the question of whether and under which conditions 'tags', i.e., observable attributes that are not payoff-relevant and carry no semantic meaning, can actually be informative, or, in other words, can serve to differentiate subjects. I look at a simple game which is (quasi) static in the sense that the really interesting choices take place simultaneously at the beginning of the game, and in which all subjects are agents (can observe and be observed). Most relevantly, given that the focus of the discussion here is on the potential informativeness of tags, agents will be assumed to have complete freedom to choose tags, in particular, it will not be assumed that the rules of the game prevent agents from choosing the same tags as their fellows.

As mentioned in the introduction, tags are a form of cheap talk, though one which does not rely on 'semantic meaning'. Own names, including logos and other distinctive markings, can reasonably be described as tags, as their meanings are mostly of only secondary importance and people seldom have strong preferences over this kind of items (in fact, in this age of 'globalization', businesses in choosing own names and logos are very concerned to try and come up with as 'neutral' a designation as possible, lest they get tangled up in all kind of cultural sensibilities). So, the following games can be understood as depicting situations in which agents are choosing own names[2].

The rest of the paper will deal with a seller-buyer game in which the value of the item on sale to each buyer varies by seller. An important feature of this game is that the seller cares only for the price, not for who is paying it, while the buyer also cares for the likely identity of the seller[3]. Also, it will be

[2] It might be objected that trademark law prevents agents from adopting the names of their fellows. The answer to that must be that this paper is precisely about the manifold ambiguities and problems facing the law in defining exactly what a trademark is. After all, even if the law could succeed in precisely identifying a set of attributes' values as a 'trademark', there is still the problem that was is observable varies from agent to agent. Moreover, these problems and others are aggravated by the fact that nowadays many businesses operate internationally, a circumstance that not only generates the usual problems of trading across different legal environments, but in this case also implies dealing with very different cultural and informational standards making the problem of defining a 'trademark' even more intractable.

[3] It bears repeating that games in which, unlike what happens in buyer-seller interactions, the value of a transaction depends for both sides on the identities of both the parties involved ('mating games') operate very differently.

assumed throughout that buyers are perfectly identified, leaving for further work the analysis of two-sided identification problems.

4.1 A Benchmark Buyer-Seller Game

There are N buyers and sellers. Subjects and agents coincide here, i.e., $S = A$. To start with buyers and/or sellers are allowed to choose simultaneously 'tags', i.e., observable attributes that carry neither meaning nor are in any way payoff relevant. These 'tags' will be the only observable attributes in the game. Sellers and buyers are then matched 1:1 randomly. That is, the matching function of ,say, seller i, $M(i)$, will assign probability $\frac{1}{N}$ to being matched with any given buyer, probability 0 to being matched with another seller. Similarly for buyers. After being matched, agents will observe each others' tags, and then proceed to trade an indivisible object for a price p. The trading procedure will involve the seller making a price offer to the buyer, and the buyer accepting or rejecting it. If the offer is rejected, both get payoffs of 0. If it is accepted, the seller gets a payoff p, while the buyer gets a payoff $E(v|O_{BS}, M(B)) - p$, where the first expression denotes the expected value of the item in the eyes of the buyer given what he can observe and his matching distribution. Of course, I will assume that the value to a buyer of the item being sold varies according to who the seller is (Note that the symmetric assumption does not apply: The value of the transaction to a seller depends only on the price. This will turn out to be very important for the ability of tags to separate agents). I will denote the value of the item of seller M for buyer N by $v(M, N)$. Note that such a value will be both an attribute of M as well as of N. As mentioned, these attribute will be unobservable. The exact field of vision of each agent will vary from case to case.

Note that sellers when confronted with non-identified buyers will solve a standard price discrimination problem. If buyers are identified, on the other hand, a seller will always set $p = E(v|O_{BS}, M(B))$.

The objective is now to find what properties of the subject-attributes and observability mappings lead to some degree of identification among agents.

The solution concept that will be used is the notion of sequential equilibrium (though the dynamics involved are in some sense secondary, the interesting stage being the initial simultaneous choice of tags).

5 Single Tag

Let there be M sellers and M buyers. Assume that players can only observe what happens within their own match. Let the value of the product of seller m to buyer n be given by $v(n,m)$. Assume that there is only one observable attribute which is in itself not payoff relevant and meaningless and can take values $1,...,M$. $T(m)$ stands for the value of this tag adopted by seller m. Assume further that all buyers can observe each seller's tag, i.e., $O_{B_nS_m} = \{T(m)\} \forall n,m$. Finally, assume that all buyers are identifiable by all sellers.

The question is: Does there exist a sequential equilibrium such that there is some degree of identification among sellers? The answer is no except in the extreme case where all sellers would earn the same payoff when identified.

To see that there are no separating equilibria: The payoff of seller m will be given in a separating equilibrium by

$$\sum_n p(B_n)\, v(n,m)$$

where $p(B_n)$ stands for the probability that seller m be matched with buyer n.

Assume that there is a seller o whose payoff is highest. Then any seller $m \neq o$ has an incentive to deviate and adopt the same tag value as seller o. This regardless of out-of-equilibrium beliefs as the deviation will not be registered by buyers who can only observe what happens in his or her own match.

So, the only possible equilibria will entail identity pooling. The payoff to a seller under such a candidate equilibrium is then

$$\sum_n p(B_n) \sum_m p(S_m)\, v(n,m)$$

where $p(S_m)$ stands for the probability that a buyer assigns to being matched with seller m. Note that this last expression can be rewritten in the following way,

$$\sum_m p(S_m) \sum_n p(B_n)\, v(n,m)$$

In words, a seller's payoff under pooling is just a convex combination of sellers' payoffs when identifiable. Hence, since $p(S_m) = \frac{1}{M}\ \forall m$, if buyers observe a tag value different from the one specified in the candidate equilibrium and their beliefs remain unchanged, or shift in such a way as to increase the weight assigned to sellers with payoffs under identifiability below this magnitude, deviating will not pay. Otherwise no equilibrium exists.

Note that this result is independent of the particular matching probabilities involved. Moreover, it is independent of the exact configuration of reservation values.

5.0.1 Public Tags

The question is here: What degree of identification could be achieved if tags were public in the sense of buyers being able to observe not only the tag values of the seller they have been matched with but also those of all the other sellers? Again, full differentiation is not possible (except for the limiting case in which all sellers' identification payoffs are the same). The argument is even more straightforward here than in the case with 'private' transactions: Assume all sellers are identified. Since they can be ranked according to their identification payoffs and since pooling payoffs are just convex combinations of identification payoffs, deviating and pooling with a higher ranked seller is always profitable. This is always so because the consistency requirement pins down out of equilibrium beliefs completely in this scenario (unlike what happened in the previous case): If a buyer observes an individual deviation, he can ascertain the exact identity of the deviant by observing the tags of the non-deviating sellers.

A full pooling equilibrium can be supported by beliefs' structures analogous to the ones specified for the case of 'private' tags.

5.0.2 Differentiation with Public Tags

It would be wrong, though, to draw from the previous section the conclusion that whether tags are public or private is irrelevant, as the following example makes plain.

Say there are three buyers and three sellers, with seller 1 being the top one, seller 2 the middle one and seller 3 the bottom seller (ranked according to identification payoffs). As before there is only one tag that sellers choose. Assume now that the identification payoff of seller 2, denote it by $v(2)$[4],

[4] $v(n) = \frac{1}{3}v(n,1) + \frac{1}{3}v(n,2) + \frac{1}{3}v(n,3)$

satisfies the following inequality,

$$v(2) > \frac{1}{3}v(1) + \frac{1}{3}v(3) + \frac{1}{3}v(2)$$

The following pattern of identification is an equilibrium with public tags (but not with private tags):

The top and the bottom sellers share one tag value while the middle seller differentiates herself. The argument is straightforward: If the middle seller imitates the top and bottom sellers, she will obtain the payoff given by the RHS of the above inequality. Whether imitating the middle seller is profitable for the top seller will depend on out of equilibrium beliefs. If we assume that buyers when they register a deviation from the top and bottom sellers' pool, believe the bottom player to be the one deviating, then the top seller will not want to follow this course of action. Finally, note that it does not pay for the bottom seller to deviate and imitate the middle seller either (under the previously specified out-of-equilibrium beliefs)[5][6].

Note that none of this would work if tags were private (even if the top seller's tag value is taken to be non-discretionary, the bottom seller would still want to imitate the middle player).

More generally, for any number of sellers and buyers (≥ 3), it can be shown that any equilibrium with differentiation must follow the 'nested' pattern suggested by the example. More precisely,

Proposition 2 *For any $N > 2$, in an equilibrium with differentiation, it cannot be that all the identification values of sellers in a given pool be above all the identification values of sellers in another pool.*

Proof. Assume the contrary. Take the seller with the lowest identification value of both pools. That seller will have an incentive to imitate the higher pool, as tags are public and, hence, deviating in that manner cannot possibly induce beliefs that make the lowest seller worse off. ■

It would seem that (endogenous) differentiation in this game is generally impossible with a single private tag (evidently observability plays no role

[5] Of course, there is a pooling equilibrium here as well. And these two are the only possible configurations, as can be easily verified.

[6] In fact, fixing the tag value of the top seller would immediately result in an equilibrium of the above variety since then out-of-equilibrium beliefs will be pinned down exactly (by the consistency requirement of sequential equilibrium) in the way just mentioned.

in this conclusion), and can only be achieved via a single public tag in the fashion just described. Admittedly, this is an odd form of differentiation which does not seem at first glance (or even at second glance) to have a counterpart in practice. This is hardly surprising, as identities are seldom such a one-dimensional affaire. The next natural step is then to look at multiple tags.

6 Multiple Tags and Partial Observability

With multiple tags, observability comes into play. Clearly, if all tags are observable by all buyers and all can be freely chosen, then introducing additional tags will not make a difference. It turns out though that even with multiple private tags (a seller's tags are observable by a buyer only when that buyer is matched with the particular seller) and only partial fields of vision in each match (i.e., not all tag values of the seller are observable by the buyer matched with him) differentiation can only result in situations where the differentiated players are indifferent between pooling and separation. With public tags (buyers can observe a seller's tags even if they are not currently matched with him) basically the same result obtains in the two-seller case. In the general case, though, essential differentiation appears possible.

6.1 Private Tags Case

The results are obtained under the following simplifying assumptions:

Assumption 1 Assume that if $T_z \in O_{S_n B_l}$ then $T_z \in O_{S_m B_l}$, for all seller pairs (S_m, S_n), and all buyers B_l.

This assumption is made in order to exclude situations where buyers can distinguish between sellers regardless of sellers' tag choices. Nothing essential hinges on it. It is introduced just to simplify the exposition.

Also, and more substantially, it is necessary to restrict out of equilibrium beliefs under a particular set of deviations. This is done because it would seem very hard to say anything general in the private tags case. That is the content of the assumption below.

Assumption 2 Let T' be obtained from T by setting some tags values of (say) seller n that differed under the original profile equal to those of seller m (in T). If $T(B,n,m,T') \neq T(B,n,m,T)$ but $T(B,n,m,T') \neq \emptyset$ (where $T(B,m,n,T)$ stands for the set of tags in the profile T whose values differ as between sellers m and n, and are observable by buyer B), buyers simply ignore the discordances, and form their beliefs following the tag values that remain unchanged from T,as they would have if there had not been a deviation from T to T'.

This restriction means that when a buyer is confronted with a deviation that equalizes some but not all the tag values of a given seller to the values of another, the buyer continues to use the remaining differences to distinguish between the two sellers involved.

I will start with the two seller case, and then I shall show how to generalize the result to the case of three or more sellers.

Proposition 3 *Under the above assumptions, with two sellers only, for any subset T_S of the set of tag values in the equilibrium profile T such that $T_z(n) \neq T_z(m)$, define the set of buyers $B(T_S,m,n,T)$ as consisting of those buyers who can only observe divergent tags in this subset. Then the profile T forms part of an equilibrium iff*

$$\sum_{l\in B(T_S,m,n,T)} v(m,l) = \sum_{l\in B(T_S,m,n,T)} v(n,l) \quad \forall\, T_S$$

Proof. Note that it is enough to concentrate on deviations of the kind described in the assumption above concerning out-of-equilibrium beliefs, i.e., deviations that equate a group of divergent tag values among two sellers all. in either one or the other direction. Other deviations can always be made unprofitable by choosing appropriate out-of-equilibrium beliefs.

1) Let $B(z,m,n,T)$ be the set of buyers for whom $T_z \in O_{S_nB}$, and $T_z(n) \neq T_z(m)$. Now, let

$$B(m,n,T) = \bigcup_{z\in T(m,n,T)} B(z,m,n,T)$$

with $T(m,n,T) = \{z|T_z(n) \neq T_z(m)\}$, then it must be that

$$\sum_{l\in B(m,n,T)} v(m,l) = \sum_{l\in B(m,n,T)} v(n,l)$$

This follows since, clearly, either seller n or m can deviate and set all his or her tag values equal to those of the other. No buyer will be able to detect such a deviation, and, hence, any buyer when matched with the deviant seller will assume that he or she is being matched with the seller being imitated as tags are assumed private (i.e., buyers can only observe tag values of the seller they are being matched with).

If now the RHS expression in the equation above would exceed the LHS expression, player n would have an incentive to deviate and imitate m. In the reverse situation, player m would have the incentive to imitate n. More precisely, note that the general expression for the expected payoff (ex ante, i.e., before the matching) of a seller n is

$$\sum_B p(B) \sum_S p\left(S|T^n, T^{-n}, \Psi^B\right) v(S, B)$$

where $p(B)$ is the probability that a seller is matched with buyer B; $p\left(S|T^n, T, \Psi^B\right)$ is the probability that seller B assigns to having been matched with seller S conditional on actually being matched with seller n, given the tag profile and the observability conditions for buyer B. A seller n deviates from T_E trying to imitate another seller m by equalizing (say) all divergent tag values to those of the latter seller. The payoff to n under T_E is given by

$$\sum_{B\in N(m,n,T)} p(B) \left(\frac{1}{2}v(n, B) + \frac{1}{2}v(m, B)\right) + \sum_{B\in B(m,n,T)} p(B)\, v(m, B)$$

and his payoff from this deviation can be written

$$\sum_{B\in N(m,n,T)} p(B) \left(\frac{1}{2}v(n, B) + \frac{1}{2}v(m, B)\right) + \sum_{B\in B(m,n,T)} p(B)\, v(n, B)$$

where $N(m, n, T)$ stands for the set of buyers who could not distinguish between n and m under T_E, and who consequently cannot distinguish between sellers under the deviation either; $B(m, n, T)$, on the other hand, stands for the set of buyers who could originally distinguish between sellers but no longer can after the deviation. The payoffs from m trying to imitate in the same way n are exactly reversed. Hence, since the first sum in each expression cancels out in the comparison, and since the probability of being matched with a seller is a constant, the condition for there not to be incentives for either player reduces to

$$\sum_{B\in B(m,n,T)} v(m, B) = \sum_{B\in B(m,n,T)} v(n, B)$$

The same logic will apply in what follows, but I will not go over the details.

2) Let $T(B,m,n,T)$ be the set of tags in the profile T whose values differ as between sellers m and n, and are observable by buyer B. If one can partition buyers into K sets $\{G_k\}_{k-1}^{K}$ such that for any two sets G_k, G_l in this partition,

$$\bigcup_{B \in G_k} T(B,n,m,T) \cap \bigcup_{B \in G_l} T(B,n,m,T) = \emptyset \qquad ((*))$$

then within each such group it must be that

$$\sum_{l \in G_l} v(m,l) = \sum_{l \in G_l} v(n,l)$$

This follows since seller n or m can set all tag values in, say, $\bigcup_{B \in G_k} T(B,n,m,T)$, equal to the values of the other seller. Since buyers in any other partition cell cannot observe this tags, their beliefs will not change in anyway as a consequence of this deviation. Buyers within the partition cell when matched with the deviant will again assume that they are dealing with the seller being imitated.

3) Take now the finest partition G^* satisfying condition $(*)$, then it must be that for any set G_l^* in this finest partition,

$$\sum_{l \in B\left(T\left(\tilde{B},m,n,T\right)\right)} v(m,l) = \sum_{l \in B\left(T\left(\tilde{B},m,n,T\right)\right)} v(n,l)$$

$$\forall \tilde{B} \in G_l^*$$

where

$$B\left(T\left(\tilde{B},m,n,T\right)\right) = \{\, B \mid T(B,m,n,T) \subseteq T\left(\tilde{B},m,n,T\right)\}$$

It is here where the restriction on out-of-equilibrium beliefs postulated above comes into play. Take a buyer $B \in G_l^*$ such that $T(B,m,n,T) \subset T\left(\tilde{B},m,n,T\right)$. If seller n, say, decides to set all tags in $T(B,m,n,T)$ equal to those of the other seller, player B will be led to belief in the usual fashion that he is matched with seller m when actually matched with seller n. Player $\tilde{B}$ on the other hand will note the deviation. The restriction on beliefs introduced here simply says that a player like $\widetilde{B}$ will continue to differentiate sellers m and n using the originally divergent tag values he still registers. The restriction also guarantees that in the case where $T(B,m,n,T) \nsubseteq T\left(\widetilde{B},m,n,T\right)$ and say n sets all tags in $T(B,m,n,T) \cap T\left(\widetilde{B},m,n,T\right)$equal to those of m, both buyers' beliefs will remain unchanged.

This three type of situations exhaust all possible scenarios that might emerge when a seller m decides to equalize some or all divergent tag values as between him and another seller n,for a given tag values profile T. Hence, if all the equations of the above type are satisfied, no seller will have an incentive to deviate (in the particular way considered) from the profile T. Since other deviations (i.e., ones where tags take values that do not figure in T) can be excluded straightforwardly by specifying out-of-equilibrium beliefs appropriately, it is clear that if these conditions are satisfied, T must form part of an equilibrium. With private tags, the reverse implication is obvious. ■

In generalizing to more than two players, the only difference is that now one must allow for pools of players, i.e., groups of players who cannot be distinguished by some buyers. In order to do that, let me introduce some notation: If given a set of sellers P, a tag values' profile T and a buyer B, it is the case that for any two sellers S, S' in the set and for all $T_z \in O_B, T_z(S) = T_z(S')$, then P forms a pool in the eyes of buyer B, and I write P_{SB}, meaning the pool of sellers in the eyes of B that contains seller S.

Proposition 4 *Under the previous two assumptions, a tag values profile T forms part of an equilibrium iff for any subset T_S of the set of tag values in T such that $T_z(n) \neq T_z(m)$, it is the case that*

$$\sum_{l \in B(T_S,m,n,T)} \sum_{S \in P_{ml}} \frac{1}{|P_{ml}|} v(S,l) = \sum_{l \in B(T_S,m,n,T)} \sum_{S \in P_{nl}} \frac{1}{|P_{nl}|} v(S,l)$$

Proof. The argument is exactly analogous to the one in the proof of the two-seller case. ■

Note one difference between the two-sellers case and the general case: In the two-seller case one could interpret the conditions on equilibrium tag profiles as implying that there could be differentiation only if sellers were indifferent between being fully identified and pooled. In the more general case this interpretation is clearly no longer tenable. Now, it is only required that a seller be indifferent between being identified with her group and being identified with any other group distinguishable from hers. This suggests that as the number of sellers increases, the scope for this form of 'indifferent' differentiation increases as well. This insofar as the restrictions on the space of reservation values become less stringent as the number of potential pooling arrangements expand.

6.2 Public Tags

With public tags, as already pointed out in the introduction to this section, the situation is more complex. The extra possibilities arise because many deviations that went unnoticed in the private tags will now be registered.

One important difference is that now assumption 2 follows from the consistency requirement in the definition of sequential equilibrium. Here is an intuitive explanation: Say, there are two sellers and two tags. Under the original tag profile, both tag values differ between sellers. Now, one the sellers deviates and equates one of her tag values to the value of this tag for the other seller under the original profile. A buyer matched with the deviant can tell there was a deviation just as she would in the private tags case. The difference is that now this buyer must conclude that she is matched with the deviant for otherwise she would have to presume that both sellers deviated simultaneously.. The overall pattern of tag values she now observes (which she could not observe with private tags) cannot be explained in any other way. It can be shown though that the consistency requirement excludes such simultaneous deviations (see Kreps and Wilson ?).

In the two-seller case this is the only substantial difference:

Proposition 5 *With public tags, two sellers, and under assumption 1, a tag profile forms part of an equilibrium iff the following conditions hold:*

$$\sum_{l\in B(T_S,m,n,T)} v\left(m,l\right) = \sum_{l\in B(T_S,m,n,T)} v\left(n,l\right) \quad \forall\ T_S$$

Proof. While, as said, the logic underlying this result is basically the same as with private tags, the arithmetic is somewhat different: When a seller 'pools' with the other, since now tags are public, buyers invariably realize that a deviation occurred, and, hence, average out their reservation values across pooled sellers. Take the case where all divergent tags are equalized by seller n. His payoff from this deviation is now

$$\sum_B p(B)\left(\frac{1}{2}v(n,B)+\frac{1}{2}v(m,B)\right)$$

while his payoff from staying the course is

$$\sum_{B\in N(n,m,T)} p(B)\left(\frac{1}{2}v(n,B)+\frac{1}{2}v(m,B)\right)+\sum_{B\subset B(n,m,T)} p(B)\,v(n,B)$$

The payoff from an analogous deviation for m is exactly the same as the deviation payoff for n. The payoff from staying the course for m differs from that for n only in that the second sum is over $v(m,B)$ rather than over $v(n,l)$. It is easy to see that factoring and cancelling terms the condition reduces to the same expression as with private tags.

The rest of the argument is the same as with private tags. ■

With more than two sellers, it no longer seems to be the case that differentiation can only take place if sellers are indifferent. The difference with the two-sellers case is that buyers cannot tell exactly who deviated; they now have to form beliefs regarding the likely identity of the deviator from among the members of the relevant pool. As those beliefs do not appear to be in any way restricted by the requirement of consistency, I will work in what follows with the scenario most favorable to differentiation and assume that a buyer who registers an equalizing deviation will assume that the deviator is the member of the deviant's pool with the lowest reservation value. I will denote that seller S^{*}_{ml}, where m stands for the seller who is actually deviating. Generally, the inequality defining whether a deviation for a seller n who tries to imitate m is profitable should be written

$$\sum_{l\in B(T_S,m,n,T)}\sum_{S\in P_{ml}}\frac{1}{|P_{ml}|+1}v(S,l)+$$

$$\frac{1}{|P_{ml}|+1}\sum_{S\in P_{nl}} p\left(S|P_{nl}\setminus n\right)v(S,l)\leq$$

$$\sum_{l\in B(T_S,m,n,T)}\sum_{S\in Pnl}\frac{1}{|P_{nl}|}v(S,l)$$

In what follows I will take $p\left(S^{*}_{nl}|P_{nl}\setminus n\right)=1$.

Proposition 6 *With public tags, under assumption 1, and with three or more sellers, any tag profile satisfying*

$$\sum_{l\in B(T_S,m,n,T)}\sum_{S\in(P_{ml}\cup S^*_{nl})}\frac{1}{|P_{ml}|+1}v(S,l) \leq \sum_{l\in B(T_S,m,n,T)}\sum_{S\in P_{nl}}\frac{1}{|P_{nl}|}v(S,l)$$

$$\sum_{l\in B(T_S,m,n,T)}\sum_{S\in(P_{nl}\cup S^*_{ml})}\frac{1}{|P_{nl}|+1}v(S,l) \leq \sum_{l\in B(T_S,m,n,T)}\sum_{S\in P_{ml}}\frac{1}{|P_{ml}|}v(S,l) \quad \forall T_S$$

represents an equilibrium and vice-versa[7].

Proof. *These are just the inequalities that have to be satisfied if an equalizing deviation is not to be profitable for either of two differentiated players, n,m.* ■

While the previous 'proposition' is hardly informative, some interesting corollaries follow from the formulas:

Corollary 7 *Given two differentiated players n and m, if $P_n = P_{nl}$, $P_m = P_{ml}$ $\forall\, l \in B(T_s, m, n, T)$ then differentiation has to be nested.*

Proof. Under this condition, one can rewrite the payoff from an equalizing deviation by, say, player n aiming to imitate a player m, as follows

$$\sum_{S\in(P_m\cup S^*_n)}\frac{1}{|P_{ml}|+1}\sum_{l\in B(T_S,m,n,T)}v(S,l)$$

This is just a convex combination of the payoffs under identification in the eyes of buyers in $B(T_S, m, n, T)$. Similarly for m. ■

Note that this result also suggests the possibility that a seller might remain differentiated from another even if the identification payoffs of all the

[7] More generally, the expression should be written

$$\sum_{l\in B(T_S,m,n,T)}\sum_{S\in P_{ml}}\frac{1}{|P_{ml}|+1}v(S,l)+$$
$$\frac{1}{|P_{ml}|+1}\sum_{B\in P_{nl}}p(S|P_{nl}\setminus n)\,v(S,l)\leq$$
$$\sum_{l\in B(T_S,m,n,T)}\sum_{S\in P_{nl}}\frac{1}{|P_{nl}|}v(S,l)$$

sellers in the other seller's pool are all above each of the identification payoffs of the sellers in his pool.

Another corollary characterizes conditions under which whether differentiation via T_S is sustainable will only depend on the cardinality of the pools of the differentiated players, their identification payoffs and the payoffs of the other sellers in the deviant's pool.

Yet another corollary states that if two sellers are differentiated via T_S and are not pooled in the eyes of buyers in $B(T_S, m, n, T)$ then the differentiation must be indifferent.

Corollary 8 *a) Differentiation via T_S between two sellers m, n in the eyes of buyers in $B(T_S, m, n, T)$ will only depend on the size of their respective pools, their identification payoffs and those of the other sellers in the deviant's pool if*

$$\sum_{l\in B(T_S,m,n,T)} \left\{ \frac{v(m,l)+v(S^*_{nl},l)}{|P_{ml}|+1} - \frac{v(n,l)}{|P_{nl}|} \right\} >$$

$$\sum_{l\in B(T_S,m,n,T)} \left\{ \frac{v(m,l)}{|P_{ml}|} - \frac{v(m,l)+v(S^*_{ml},l)}{|P_{nl}|+1} \right\}$$

b) If two sellers m and n are differentiated via T_S but $|P_{nl}| = |P_{ml}| = 1$ $\forall\, l \in B(T_S, m, n, T)$, this differentiation must be indifferent.

Proof. A) Rewriting the conditions above in the following way

$$\sum_{l\in B(T_S,m,n,T)} \frac{1}{|P_{nl}|} \sum_{S\in Pnl\backslash n} v(S,l) \geq$$

$$\sum_{l\in B(T_S,m,n,T)} \left\{ \frac{1}{|P_{ml}|+1} \left(\sum_{S\in P_{ml}\backslash m} v(S,l) \right) + \frac{v(m,l)+v(S^*_{nl},l)}{|P_{ml}|+1} - \frac{v(n,l)}{|P_{nl}|} \right\}$$

$$\sum_{l\in B(T_S,m,n,T)} \frac{1}{|P_{nl}|+1} \sum_{S\in P_{nl}\backslash n} v(S,l) \leq$$

$$\sum_{l\in B(T_S,m,n,T)} \left\{ \frac{1}{|P_{ml}|} \left(\sum_{S\in Pml\backslash m} v(S,l) \right) - \frac{v(n,l)+v(S^*_{ml},l)}{|P_{nl}|+1} + \frac{v(m,l)}{|P_{ml}|} \right\}$$

Clearly, the expression on the LHS in the first inequality is larger than the corresponding expression in the second inequality. Also, the summation on the RHS of the first inequality is smaller than the corresponding summation in the second inequality. Hence if the remaining expression on the RHS of the first inequality is also smaller than the corresponding expression in the second inequality, the two inequalities will be satisfied regardless of the specific values taken by the other expressions.

b)If now these last mentioned expressions are equal and there is no pooling (hence the remaining expressions all take the value zero), clearly the only way the two inequalities can be satisfied simultaneously is if each holds with equality. ■

The main lesson from this is though that there can be 'essential' differentiation (as opposed to indifferent differentiation) so long as there is partial pooling and all tags are public.

7 Differing Perspectives and Differentiation

So far I have not defined formally what public resp. private means. In order to do that, the formal framework has to be extended. The easiest way to do this would seem to be to work with two partitions of a subject's set of attributes: One divides that set into observable and non-observable subsets within a match, while the other divides it into observable and non-observable subsets across matches. That is, one defines the subset of subject i's attributes O_{Ki}^{Pu} that can be observed by agent K whether or not that agent is matched with that subject. The set O_{Ki}^{Pr}, on the other hand, includes the attributes of subject i that are only observable by agent K when the latter is matched with the former.

The public tag case then corresponds to a situation in which $O_{BS}^{Pu} = O_{BS}^{\mathrm{Pr}}\ \forall\ S, \forall\ B$. The situation with $O_{BS}^{Pu} = \{\emptyset\}\ \forall\ S, \forall\ B$ represents the private tags case. It is convenient also to define a public tag for a buyer B as one which belongs to both the above sets, i.e., if $T_z \in O_{BS}^{Pu} \cap O_{BS}^{\mathrm{Pr}}$ then T_z is public for B. Similarly define a private tag for buyer B as one that belongs only to the set O_{BS}^{Pr}. An 'open' tag for buyer B would be one that belongs to O_{BS}^{Pu} but not to O_{BS}^{Pr}.

Another distinction that will turn out to be of relevance in the discussion that follows is that between the three situations $O_{BS}^{Pu} \subset O_{BS}^{\mathrm{Pr}}, O_{BS}^{Pu} \supset O_{BS}^{\mathrm{Pr}}$ and $O_{BS}^{Pu} \cap O_{BS}^{\mathrm{Pr}} \neq \emptyset$ but $O_{BS}^{Pu} \not\subseteq O_{BS}^{\mathrm{Pr}}$ or $O_{BS}^{Pu} \not\supseteq O_{BS}^{\mathrm{Pr}}$. In a sense each of these scenarios describes a perspective of the buyer. Call the first constellation a 'narrow' perspective (or 'myopic', or 'frog' perspective), the second a 'wide' perspective (or 'far-sighted', or 'bird' perspective), and the third a 'mixed' perspective.

7.1 Essential Differentiation in the Two-Sellers Case with Differing Perspectives

It has been shown that essential differentiation can take place with more than two sellers in the multiple-tags, partial-observability scenario. Here it will be shown that such differentiation can also take place in the two-seller case if at least some buyers have narrow, wide or mixed perspectives. In these cases, the result will depend on the choice of out-of-equilibrium beliefs.

First, I show a preliminary result:

Proposition 9 *If there are two buyers who have either all public or all private perspectives then differentiation must be indifferent.*

Proof. Take a subset T_S of the set of all tags $\{z|T_Z(n) \neq T_Z(m)\}$. Now consider an equalizing deviation by, say, n, setting $T_Z^D(n) = T_Z(m)\ \forall\ z \in T_S$. The set of buyers $B(T_S, m, n, T)$ can be partitioned in two subsets, the set of those with public perspectives and the set of those with private perspectives, $B^{\Pr}(T_S, m, n, T)$ and $B^{Pu}(T_S, m, n, T)$, respectively.

The inequality defining whether the deviation is profitable can be written

$$\sum_{B \in B^{\Pr}(T_S,m,n,T)} v(m,B) + \sum_{B \in B^{Pu}(T_S,m,n,T)} \left(\frac{1}{2}v(n,B) + \frac{1}{2}v(m,B)\right) \leq \sum_{B \in B(T_S,m,n,T)} v(n,B)$$

A similar expression obtains for the corresponding deviation by the other seller. It is then straightforward to show that the only way both inequalities can be satisfied is if they both hold with equality. ■

One thing to note is that now even though differentiation must be indifferent, it is no longer the case that this implies that the payoffs under identification in the eyes of buyers in $B(T_S, m, n, T)$ must be the same.

More complex observability conditions might deliver essential differentiation in this two-seller case for all belief formation rules except the symmetric rule just used.

To illustrate this last point, take the case with two tags, one of which is public, the other private (in the eyes of some buyer), but both observable within a match, i.e., a case of narrow perspective. Say, to start with, only tag 2 is supposed to differ. If now seller 1, say, deviates equalizing all his tags to those of the other seller, this buyer when matched with the deviant will realize that a deviation has taken place, and, moreover, will know who the deviant is, but will still not know with whom she is currently matched. The following diagram illustrates what this buyer observes

$$\begin{array}{lcl} (S1) & & (S2) \\ T_1(2) & & ? \\ T_2(2) & \neq & T_2(2) \end{array}$$

The inequality means that the buyer expected these two tag values to differ; in other words, she is aware a deviation has taken place. The brackets around the buyer designations denote the fact that the buyer, of course, cannot directly observe them.

If now the buyer has a wide perspective so that after the deviation she observes

$$\begin{array}{lcl} (S1) & & (S2) \\ ? & & T_1(2) \\ T_2(2) & \neq & T_2(2) \end{array}$$

again an analogous issue arises.

The question is now: Is it reasonable to require that this buyer forms beliefs in exactly the same way when seller m imitates seller n, as when the opposite happens, and to require, moreover, that the buyer thinks it equally likely the seller she is currently matched with is 1 or 2? If so, then the result previously mentioned applies. Otherwise, one could get strict differentiation.

The consistency requirement does not seem to require that beliefs take this form in this kind of situations (assumption ? above takes care of other situations), unlike the situation with all public tags. This because of the extra piece of information $T_1(2)$ in each case. Similar considerations apply to the 'mixed' case.

Proposition 10 *In the two-seller case, if buyers' perspectives are narrow, wide or mixed there can be strict differentiation.*

Proof. For symmetric beliefs rules, the relevant inequalities can be written

$$\sum_{B\in B^{Pr}(T_S,m,n,T)} v(m,B) + \sum_{B\in B^{Pu}(T_S,m,n,T)} \left(\frac{1}{2}v(n,B)+\frac{1}{2}v(m,B)\right) +$$
$$\sum_{B\in B^{W,N,M}(T_S,m,n,T)} (\alpha v(n,B)+\beta v(m,B)) \leq \sum_{B\in B(T_S,m,n,T)} v(n,B)$$
$$\sum_{B\in B^{Pr}(T_S,m,n,T)} v(n,B) + \sum_{B\in B^{Pu}(T_S,m,n,T)} \left(\frac{1}{2}v(n,B)+\frac{1}{2}v(m,B)\right) +$$
$$\sum_{B\in B^{W,N,M}(T_S,m,n,T)} (\alpha v(n,B)+\beta v(m,B)) \leq \sum_{B\in B(T_S,m,n,T)} v(m,B)$$

These inequalities can be satisfied strictly so long as $\alpha \neq \frac{1}{2}$[8]. ■

This shows that even in the two-sellers case, there might be scope for differentiation, so long as the observability conditions are sufficiently complex.

An important corollary of the previous proposition is that there might be full strict identification of some sellers in the eyes of some buyers in the presence of complex perspectives. Note that this could not happen with all public for each buyer in which case pooling was necessary in order for strict differentiation to emerge.

8 Conclusions

The results show that even in the absence of rules prohibiting agents from imitating each other ('trade mark laws') and even in a game without some natural separating structure (in contrast to 'mating' games), so long as the observability conditions are complex enough and there is some heterogeneity in tastes, one can expect some degree of 'natural' or 'endogenous' differentiation. This even in the two-seller case, and even if it is possible to rank sellers using identification payoffs. On the other hand, strict differentiation can only be complete (in the eyes of at least some buyers) if perspectives are sufficiently complex, otherwise a degree of pooling (and publicity) is key to sustain strict differentiation.

[8] I have assumed symmetric belief formation. A fortiori, if the statement holds for symmetric rules, it holds for asymmetric ones.

Though it is dangerous to jump from such abstract analysis into practical issues, it would seem that some variant of this type of analysis could help decide thorny, subtle questions that arise in the implementation of trade mark laws (for example, it could help decide which set of characteristics of a business are really 'distinctive' and hence should be protected). Moreover, as already pointed in the introduction, many questions that arise in deciding the marketing strategies of a firm are closely related to the subject matter of this paper, first and foremost, of course, the choice of trade mark itself.