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NÚMERO 151

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THE SIMPLE CASE OF SUBSIDIARY-COIN SHORTAGES DRIVEN BY DEFLATION IN POW CAMPS

ABSTRACT

I modify a prototypical random-matching model of money to get a simple model which match observations of POW camps' simple economies –say, the use of cigarettes as currency through macroeconomic fluctuations. One new result is that deflation may bring about a shortage of subsidiary coin in the sense that some exchanges do not occur because an indivisible "cigarette" becomes "too valuable". This result is particularly interesting because it has been argued that currency shortages would be relieved by the resulting increase in the value of money; that is, deflation. However, this argument does not take into account currency indivisibility.

RESUMEN

Un modelo prototipo de dinero basado en apareos aleatorios es modificado con el fin de obtener un simple modelo que reproduzca observaciones de simples economías en campos de prisioneros de guerra –digamos, el uso de cigarrillos como moneda a través de fluctuaciones macroeconómicas. Un nuevo resultado es que deflación puede producir una escasez de moneda subsidiaria en el sentido de que algunos intercambios no ocurren porque un "cigarrillo" indivisible llega ser "muy valioso." Este resultado es particularmente interesante porque ha sido argüido que cualquier escasez de moneda sería mitigada por el resultante incremento en el valor del dinero; o sea, deflación. Sin embargo, este argumento no toma en cuenta la indivisibilidad de la moneda.

Introduction

I modify a prototypical random-matching model of money to match observations of POW camps' simple economies --say, the use of cigarettes as currency. One result is that deflation may bring about a shortage of subsidiary coin in the sense that some exchanges do not occur because an indivisible "cigarette" becomes too valuable to be exchanged for some goods and there is not available low-denomination currency. This result is particularly interesting because it can be argued that currency shortages may be relieved by the resulting increase in the "value" of money or deflation. However, this argument does not take into account currency indivisibility as pointed out by Wallace-Zhou [1997].

The examples exhibited in this paper are so simple that simple arithmetic is enough to work them out. These examples are simple economies of Wallace [1997] but with finite horizon, no exogenous upper-bound on asset holdings, and small costs of handling "money". In those simple economies, the role of media of exchange in equilibrium is played by objects (say, cigarettes) that are useful only to a fraction of agents (say, a fraction of agents is smoker). As far as I know, no researcher has even conjectured this modification which can be arbitrarily small. In this way we do not found any problems because a certain, or sure, last date as it is argued in Wallace [1997] and elsewhere. Moreover, in contrast to Kultti [1995], my examples are robust to the introduction of "handling" costs of giving (or even receiving) cigarettes because they are assumed explicitly to reduce the set of equilibria. Therefore, we do **not** have to work with practically infinite-horizon economies and, consequently, to deal with mathematical difficulties. In turn, finite-horizon economies allow us **not** to assume an exogenous upper-bound for monetary holdings as it has been common practice although it has been also regarded as unappealing.

Perhaps reading the Radford [1945]'s description of the simple economy of a POW camp, one may feel less strong the assumption of random matching of agents in an economy. In particular, the following description of a "transit camp" may give some interest to the model presented in this paper:

> A transit camp was always chaotic and uncomfortable: people were overcrowded, no one knew where anyone else was living, and few took the trouble to find out. Organisation was too slender to include an Exchange and Mart board, and private advertisements were the most that appeared. Consequently, a transit camp was not one market but many. (p. 191-192)

In this context I would like to argue that the assumption of finite horizon may have some justification since prisoners in a transit camp perhaps knew that they would be translated to another camp by some date. In any case, may be helpful to discuss the assumptions and results in the context of POW camps.

I regard my examples altogether as a "simple experiment". We will see two economies with different exogenously-given quantity of indivisible currency, say cigarettes. Otherwise, they are identical. Observing the equilibria, we will see that in the currency-starved economy, some possible transactions do not occur as in the other economy. The intuition is that there exists a shortage of low-denomination currency because of deflation. Moreover, Radford [1945] describes that when cigarette deliveries were interrupted "prices fell, trading declined in volume and became increasingly a matter of barter." And he immediately adds that this "deflationary tendency was periodically offset by the sudden injection of new currency." (p. 195) Perhaps my examples may illustrate these fluctuations although not across time in only one economy but across economies which seems to be a lot more convenient.

The literature about modeling money using random matching is enormous and difficult. The remotely-direct reference which I am familiar with is Kiyotaki-Wright [1989]. The immediate direct reference is Wallace [1997] as already suggested. Perhaps, this is the simplest model in the published literature and it seems to me an enjoyable reading. As far as I know, the unique reference on modeling shortages of indivisible currency using random matching is Wallace-Zhou [1997]. It presents a concise description of the discussion on currency shortages and exhibit an equilibrium, say under free trade, which is argued to be the "model's representation of currency-shortage observations". Moreover Wallace-Zhou exhibit other equilibrium in which some trade is exogenously restricted, say international autarky is imposed. Both equilibria are not Pareto comparable, but with a representative-agent criterion autarky is preferred. In this sense, their "model rationalizes a prohibition on the export of currency" (p. 557). Finally, I should mention Burdett-Trejos-Wright [July, 1998] who try to follow the story of POW camps in a different random-matching model with the exogenous upper-bound for cigarette holdings of one unit.

This paper is organized as follows. In the next section is described the physical environment. In the third section the examples are given and explained independently from each other for reader's convenience. In the fourth section, some concluding remarks are made. The Figure and the Tables mentioned in text are at the end of the paper.

The Physical Environment

Because my model is a modification of Wallace [1997], I describe first the new and key assumptions. Time is discrete with a **finite** horizon. There exist individual endowments of cigarettes which are durable and indivisible. I assume that a cigarette is completely smocked or not at all, prisoners cannot produce cigarettes, and there exists a positive fraction of prisoners who get utility from smoking. There exists small costs of giving cigarettes. For simplicity, I assume that there does not exist costs in accepting cigarettes.

I describe now the standard assumptions with the appropriate modifications derived of the new assumptions. At each period, there exist $N \geq 3$ goods which are perishable (say, services) and divisible. There exist N types of prisoners. Type i gets utility *only* from consuming service i and possibly from smoking. Type i produces only service $i+1$ (modulo N) at the expense of some disutility. As regards services, notice that this assumption implies that in a pairwise meeting, there is not double coincidence of wants; that is, there cannot be barter with services alone.

The utility at any period is the sum of the utility from consumption and the disutility from producing. I assume also that the utility from smoking adds to the utility at any period. Through the life of the economy, prisoners maximize expected discounted utility.

There is a [0,1] continuum of agents for each type. I assume also equal fractions of smokers for each type. Each prisoner is randomly matched with one other prisoner, where the probability of a prisoner meeting another prisoner with given characteristics is the proportion of those people in the entire population. A service is produced and possibly consumed at the meetings of each period.

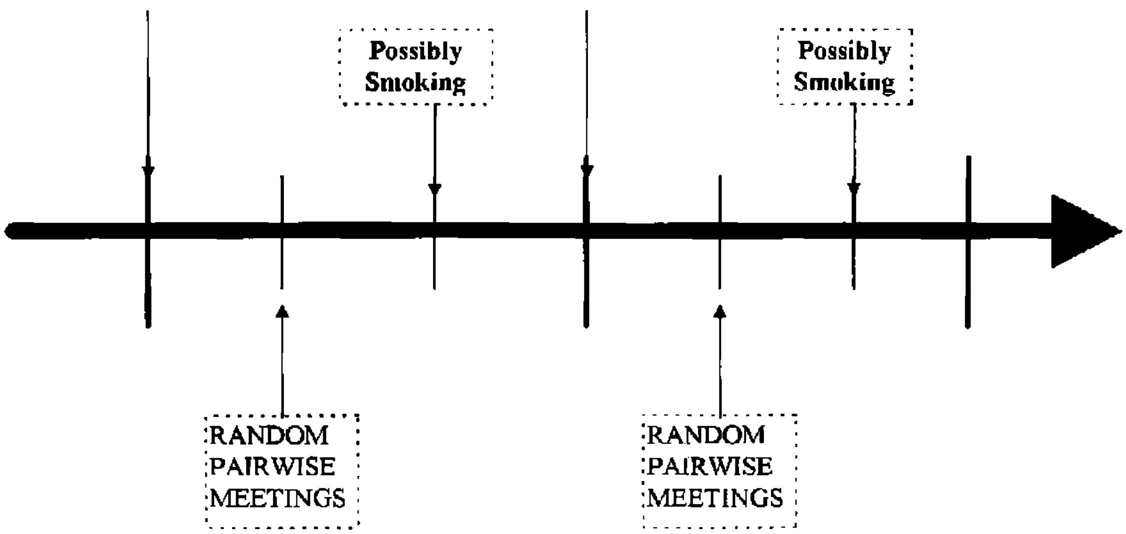
In a meeting prisoners bargain to trade. The bargaining rule assumed is that he potential consumer of a service makes a "take it or leave it" offer. We also assume that if a potential producer is indifferent between accepting or rejecting an offer, he/she accepts the offer.

Prisoners can not commit themselves to future actions. Any prisoner can exit any meeting with the cigarettes with which arrived to the meeting, and cannot be forced to render any service at any meeting. Types and inventories are known in each meeting. Trading histories of agents are private information

Finally, the assumed timing of actions for an economy of two periods is indicated in figure 1.

Figure 1

Timing of actions assuming an economy of two periods



Two Examples

Introduction

As is standard, the equilibrium concept is a Nash equilibrium (in fact, a perfect-subgame equilibrium) with rational expectations.

I describe next the particular assumptions which constitute my examples. There are two periods, say period 1 and period 2. The utility functions of a prisoner at any period are

$$u[x] \equiv \begin{cases} (x)^{1/2} & \textit{if the prisoner consumes x of his preferred service} \\ 0 & \textit{otherwise} \end{cases}$$

$$s[\sigma] \equiv \begin{cases} 9\,(\sigma)^{1/2} & \textit{if a smoker smokes } \sigma \textit{ cigarettes} \\ 0 & \textit{if a nonsmoker smokes } \sigma \textit{ cigarettes} \end{cases}$$

$$l[y] \equiv -y \quad \textit{if he produces y of his specialized service}$$

The discount factor is $\beta = 0.9$. One half of agents is smoker. There are N = 3 types of agents and services. The cost of giving n cigarettes is $\varepsilon[n] = 0.1$. In both economics, each prisoner is endowed with one cigarette at period 1. Moreover, ***in Economy A*** each prisoner is endowed with zero "cigarettes" and ***in Economy B*** each prisoner is endowed with one "cigarette" at period 2.

Next, I describe the main differences of trade across economies. At the first period, in a single-coincidence meeting between two nonsmokers: Trade does **not** occur in Economy A: *The potential consumer's best strategy is not to sale his cigarette for his/her preferred service at the "reservation price" of the potential producer and both prisoners do not have "change"! Cigarettes are too scarce and, consequently, too valuable to be media of exchange.* Notice that in the model, cigarettes are the only objects which may play the role of media of exchange. In contrast, trade occurs in Economy B: *The potential producer accepts to purchase one cigarette at the "price" offered by the potential consumer. Notice that cigarettes are doubtless media of exchange.*

All the trading strategies in meetings with single coincidence of wants are described in Table 2 for period 1 and in Table 3 for period 2.

Table 2

Best strategies of a potential consumer in a single-coincidence meeting at the initial period.

Economies A and B t = 1		Potential Consumer (with one cigarette): Smoker	Potential Consumer (with one cigarette): Nonsmoker
Potential Producer (with one cigarette)	Smoker	**NO Trade**	**TRADE**
	Nonsmoker	**NO Trade**	**NO in A** **YES in B**

Table 3

Best strategies of a potential consumer in a single-coincidence meeting at the last period.

Economies A and B T = 2		**Potential Consumer**	
		Smoker (with one cigarette)	Nonsmoker (with 1, 2, 3 cigarettes)
Potential Producer	Smoker (with one cigarette)	**NO Trade**	**TRADE**
	Nonsmoker (with 1, 2, 3 cigarettes)	**NO Trade**	**Whatever!**

Notice that in meetings without single coincidence of wants trade does not occur. Also notice that gifts of cigarettes by nonsmokers at the last period are not optimal actions because of the cost of giving cigarettes.

Economy A

To look for all the equilibria we proceed backwardly; that is, we start at the end of the last period T = 2 to determine optimal strategies and then we go backwards to determine the other optimal strategies. However, notice that it is simpler to verify that the trading strategies are individual optimal.

The easiest thing is to realize that at the end of the last period smokers smoke their cigarettes stocks. As regards nonsmokers we can assume that they also smoke or that they freely dispose their cigarettes.

The next easiest thing is to notice that nonsmokers do not produce in exchange for cigarettes at the last period. They exchange their entire cigarette stock to smokers only for his/her favorite service. Otherwise, they would not been optimizing. For convenience, we postpone the computation of the expected utilities for nonsmokers until we determine the cigarettes stocks in equilibrium of smokers at period 2.

As regards smokers they do not sell any cigarette to nonsmokers at the last period because nonsmokers do not render any service in the last period. We determine next that smokers do not sell any cigarette to smokers for his/her favorite service. The first thing is to notice that every prisoner arrive at most with two cigarettes at the meetings of period 2. They were endowed with one cigarette, possibly they bought one, and possibly the smoke part or all of their cigarettes hold at period 1. Hence, the possible inventories for any prisoner are zero, one, or two cigarettes. Now for this possible cigarettes inventories, we determine next that a smoker do not sell cigarettes to another smoker for his/her favorite service as indicated in Table 4.

TABLE 4

Optimal number of cigarettes sold (z*) by a smoker potential consumer to a smoker at the last period.

Economy B t = 2 = T		Possible number of cigarettes hold before the meetings by a smoker potential consumer		
		1	2	3
Possible number of cigarettes hold before the meetings by a smoker potential producer	1	$z^* = 0$	$z^* = 0$	$z^* = 0$
	2	$z^* = 0$	$z^* = 0$	$z^* = 0$
	3	$z^* = 0$	$z^* = 0$	$z^* = 0$

For reader's convenience, I indicate in Table 4 the cigarettes inventories and strategies that occur in equilibrium. Next, I explain why the indicated strategies are optimal. As regards selling cigarettes, notice that if a potential consumer has zero cigarettes, the best thing that he/she can do is to sell zero cigarettes. In Table 5 we have the payoffs for the possible trading strategies of a smoker potential consumer with two cigarettes in a meeting with a smoker potential producer without cigarettes.

Table 5

Possible payoffs in a meeting between a smoker potential consumer with 2 cigarettes and a smoker potential producer with zero cigarettes at the last period.

Economy A $t = 2 = T$	**Payoffs**	
Number of cigarettes given by the potential consumer	**Potential consumer**	**Potential Producer**
0	$9\,(2)^{1/2} \approx 12.73$	0
1	$9\,(2)^{½} + 9\,(1)^{½} - \varepsilon[1] = 11.9$	0
2	$\left[9\,(2)^{½}\right]^{½} - \varepsilon[2] \approx 3.56$	0

Hence, the optimal trading strategy of a smoker in that meeting is not to sell any cigarette. Now, it seems that decreasing marginal utility from smoking implies that the optimal trading strategy is the same in the other meetings with different cigarettes stocks. In any case, it can be verified directly.

Therefore, we conclude that a smoker do not sell their cigarette stock in the last period. Going backwards, we proceed now to compute smokers' expected optimal utilities.

Let $E_2^{smoker}[U*|m$ be the expected optimal utility of a smoker with m cigarettes after the endowments of period 2 are made but before the meetings of period 2. Since any producer is left indifferent between producing to buy cigarettes or not,

$$E_2^{smoker}[U*|m] = \begin{cases} s^{smoker}[0] = 0 & if\ m = 0 \\ s^{smoker}[1] = 9 & if\ m = 1 \\ s^{smoker}[2] = 9\,(2)^{1/2} & if\ m = 2 \\ \ldots \end{cases}$$

Let $v_1^{smoker}[n]$ be the expected optimal utility of a nonsmoker with n cigarettes after the meetings of period 1 but before smoking at period 1. Then, because discounting and smoking smoothing,

$$v_1^{smoker}[n] = \begin{cases} s^{smoker}[0] + \beta \cdot 0 = 0 & if\ n = 0 \\ s^{smoker}[1] + \beta \cdot 0 = 9 & if\ n = 1 \\ s^{smoker}[1] + \beta \cdot s_2^{smoker}[1] = 9 + \beta \cdot 9 & if\ n = 2 \\ \ldots \end{cases}$$

Now let $y_1^{smoker} \equiv v_1^{smoker}[2] - v_1^{smoker}[1] = \beta \cdot 9$, which is the quantity of service rendered in equilibrium by a smoker in exchange for one cigarette at period 1.

Notice that in a single coincidence meeting at t = 1, a smoker does **not** sell his/her unique cigarette to a smoker. If he/she would sell it, the payoff would be $u[y_1^{smoker}] = (\beta \cdot 9)^{1/2} - \varepsilon[1] < 3$ instead of $s^{smoker}[1] = 9$.

Next, we determine the expected utilities of nonsmokers. Notice that at the start of period two, there are not smokers with 2 or 3 cigarettes. That is, if $p_2^{smokers}[k]$ denotes the fraction for each type of prisoners who are smokers,

$$p_2^{smokers}[2] = p_2^{smokers}[3] = \ldots = p_2^{smokers}[k+2] = 0 \ .$$

Let $E_2^{nons}[U*|m]$ be the expected optimal utility of a nonsmoker with m cigarettes after the endowments of period 2 are made but before the meetings of period 2. To compute these expected utilities, we use the appropriate values of Table 6.

Table 6

. Payoffs according to cigarette stocks in meetings between a nonsmoker potential consumer and a smoker potential producer (last period).

Cigarettes stocks		**Number Of Cigarettes sold**	**Payoffs**	
***Potential* Consumer**	**Potential Producer**	**Z***	**Potential Consumer**	**Potential Producer**
1	0	1	$[9 - 0]^{1/2} - \varepsilon[1] = 3 - \varepsilon[1] \approx 2.9$	0
1	1	1	$[9\,(2)^{1/2}\ \ 9]^{1/2} - \varepsilon[1] = \approx 1.92$	9
2	0	2	$[9\,(2)^{1/2} - 0]^{1/2} - \varepsilon[2] = \approx 3.56$	0
2	1	2	$[9\,(3)^{1/2} - 9]^{1/2} - \varepsilon[2] = \approx 2.56$	9
3	0	3	$[9\,(3)^{1/2} - 0]^{1/2} - \varepsilon[3] = \approx 3.94$	0
3	1	3	$[9\,(4)^{1/2} - 9]^{1/2} - \varepsilon[3] \approx 2.9$	9

This table gives the payoffs (according to cigarette stocks) in meetings at the last period between a nonsmoker consumer and a smoker producer. Then,

$$E_2^{nons}\left[U^*|0\right] = 0\ ,$$

$$E_2^{nons}\left[U^*|1\right] \approx \frac{p_2^{smoker}[0]}{N}2.9 + \frac{p_2^{smoker}[1]}{N}1.92\ \text{, and}$$

$$E_2^{nons}\left[U^*|2\right] \approx \frac{p_2^{smoker}[0]}{N}3.56 + \frac{p_2^{smoker}[1]}{N}2.56\ .$$

Let $v_1^{nons}[n]$ be the expected optimal utility of a nonsmoker with n cigarettes after the meetings of period 1 but before the end of period 1. Then,

$$v_1^{nons}[0] = \beta\ E_2^{nons}[U^*|0] = 0\ ,$$
$$v_1^{nons}[1] = \beta\ E_2^{nons}[U^*|1] \approx \frac{\beta}{N}\left(p_2^{smoker}[0]\cdot 2.9 + p_2^{smoker}[1]\cdot 1.92\right)\text{, and}$$
$$v_1^{nons}[2] = \beta\ E_2^{nons}[U^*|2] \approx \frac{\beta}{N}\left(p_2^{smoker}[0]\cdot 3.56 + p_2^{smoker}[1]\cdot 2.56\right).$$

Notice that in a single coincidence meeting at t = 1, a nonsmoker **does** sell his/her unique cigarette to a smoker since

$$v_1^{nons}[1] < \frac{\beta}{3}\cdot 3 < u\left[y_1^{smoker}\right] - \varepsilon[1] = \sqrt{\beta}\cdot 3 - \varepsilon[1]\ .$$

(Actually, $v_1^{nons}[1] - u\left[y_1^{smoker}\right] \approx -2.42$.)

Let

$$y_1^{nons} \equiv v_1^{nons}[2] - v_1^{nons}[1]$$

which is the quantity of service rendered by a nonsmoker in exchange for one cigarette at period 1. Since $y_1^{nons} < 4$, a smoker does **not** sell his/her unique cigarette to a nonsmoker potential producer at period 1

Notice that $p_2^{smoker}[0] = \frac{1}{2} - p_2^{smoker}[1]$ and $p_2^{smoker}[1] = (1 - \frac{1}{2})(\frac{1}{2})/N = \frac{1}{12}$.

What happens in a meeting with single coincidence of wants" and both prisoners are nonsmokers? The potential consumer's best strategy is **not** to sell his/her unique cigarette since

$$v_1^{nons}[1] > u\left[y_1^{nons}\right] - \varepsilon[1]$$

The difference $v_1^{nons}[1] - u\left[y_1^{nons}\right] \approx 0.129$

Economy B

As in Economy A, we proceed backwardly from the end of period 2 to look for all the equilibria. For the argument's sake and to make this subsection independent of section 2.3, we will go step by step at the expense of many repetitions of statements of section 3.2. First, notice that at the end of the last period smokers smoke their cigarettes stocks. As regards nonsmokers we can assume that they also smoke or that they freely dispose their cigarettes.

Second, notice that nonsmokers do not produce in exchange for cigarettes at the last period. They exchange their entire cigarette stock to smokers only for his/her favorite service. For convenience, we postpone the computation of the expected utilities for nonsmokers until we determine the cigarettes stocks in equilibrium of smokers at period 2.

As regards smokers they do not sell any cigarette to nonsmokers at the last period because as already mentioned nonsmokers do not render any service for cigarettes in the last period. We determine next that smokers do not sell any cigarette to smokers for his/her favorite service. The first thing is to notice that every prisoner arrive at the meetings of period 2 with one, two, or three cigarettes. They were endowed with one cigarette, possibly they bought one, and possibly the smoke part or all of their cigarettes hold at period 1. Moreover, they were endowed with one cigarette at the start of period 2. Hence, the possible cigarette inventories for any prisoner are one, two, or three cigarettes. Now for this possible cigarettes inventories, we determine next that a smoker do not sell cigarettes to another smoker for his/her favorite service as indicated in Table 7.

Table 7

Optimal number of cigarettes sold (z^*) by a smoker potential consumer to a smoker at the last period.

Economy B t = 2 – T		Possible number of cigarettes hold before the meetings by a smoker potential consumer		
		1	2	3
Possible number of cigarettes hold before the meetings by a smoker potential producer	1	$z^* = 0$	$z^* = 0$	$z^* = 0$
	2	$z^* = 0$	$z^* = 0$	$z^* = 0$
	3	$z^* = 0$	$z^* = 0$	$z^* = 0$

Actual number of cigarettes hold by smokers in equilibrium	***Strategies actually played in equilibrium***

For reader's convenience, I indicate in Table 7 the cigarettes inventories and strategies that occur in equilibrium. Next, I explain why the indicated strategies are optimal. In Table 8 we have the payoffs for the possible trading strategies of a smoker potential consumer with three cigarettes in a meeting with a smoker potential producer with one cigarettes.

Table 8

Possible payoffs in a meeting between a smoker potential consumer with 3 cigarettes and a smoker potential producer with 1 cigarettes at the last period.

Economy B t = 2 = T	Payoffs	
Number of cigarettes given by the potential consumer	Potential consumer	Potential Producer
0	$9\cdot(3)^{1/2} \approx 15.59$	9
1	$\left[9(2)^{1/2}-9\right]^{1/2}+9\cdot(2)^{1/2}-\varepsilon[1]$ ≈ 14.65	9
2	$\left[9(3)^{1/2}-9\right]^{1/2}+9\cdot(1)^{1/2}-\varepsilon[2]$ ≈ 11.56	9
3	$\left[9(4)^{1/2}-9\right]^{1/2}-\varepsilon[3]=2.9$	9

Hence, the optimal trading strategy of a smoker in that meeting is not to sell any cigarette. Now, it seems that decreasing marginal utility from smoking implies that the optimal trading strategy is the same in the other meetings with different cigarettes stocks. In any case, it can be verified directly and in fact has been accomplish partially in the last section.

Therefore, we conclude that a smoker do not sell their cigarette stock in the last period. Going backwards, we proceed now to compute smokers' expected optimal utilities.

Let $E_2^{smoker}[U^*|m$ be the expected optimal utility of a smoker with m cigarettes after the endowments of period 2 are made but before the meetings period 2. Since any producer is left indifferent between producing to buy cigarettes or not,

$$E_2^{smoker}[U^*|m] = \begin{cases} \ldots \\ s^{smoker}[1] = 9 & \text{if } m = 1 \\ s^{smoker}[2] = 9\,(2)^{1/2} & \text{if } m = 2 \\ s^{smoker}[3] = 9\,(3)^{1/2} & \text{if } m = 3 \\ \ldots \end{cases}$$

Let $v_1^{smoker}[n]$ be the expected optimal utility of a nonsmoker with n cigarettes after the meetings of period 1 but before smoking at period 1. Then, because discounting and smoking smoothing and taking into account the endowment of one cigarette at period two,

$$v_1^{smoker}[n] = \begin{cases} s^{smoker}[0] + \beta \cdot s^{smoker}[1] = \beta \cdot 9 & \text{if } n = 0 \\ s^{smoker}[1] + \beta \cdot s^{smoker}[1] = 9 + \beta \cdot 9 & \text{if } n = 1 \\ s^{smoker}[2] + \beta \cdot s_2^{smoker}[1] = 9(2)^{1/2} + \beta \cdot 9 & \text{if } n = 2 \\ \ldots \end{cases}$$

Notice that smokers smoke all their cigarette stocks at the end of period 1. Hence, each smoker arrive with one cigarette at the meetings of period 2. If $p_2^{smokers}[k]$ denotes the fraction for each type of prisoners who are smokers, $p_2^{smokers}[1] = 1/2$ and

$$p_2^{smokers}[0] = p_2^{smokers}[2] = p_2^{smokers}[3] = \ldots = p_2^{smokers}[k+2] = 0 \ .$$

Now let $y_1^{smoker} \equiv v_1^{smoker}[2] - v_1^{smoker}[1] = 9 \cdot (\sqrt{2}-1)$, which is the quantity of service rendered in equilibrium by a smoker in exchange for one cigarette at period 1.

Notice that in a single coincidence meeting at t = 1, a smoker does **not** sell his/her unique cigarette to a smoker. If he/she would sell it, the payoff would be $u[y_1^{smoker}] = (9 \cdot [\sqrt{2}-1])^{1/2} - \varepsilon[1] < 5$ instead of $s^{smoker}[1] = 9$.

Next, we determine the expected utilities of nonsmokers. Let $E_2^{nons}[U^* | m]$ be the expected optimal utility of a nonsmoker with m cigarettes after the endowments of period 2 are made but before the meetings of period 2. To compute these expected utilities, we use the appropriate values of Table 6. This table gives the payoffs (according to cigarette stocks) in meetings at the last period between a nonsmoker consumer and a smoker producer. Then,

$$E_2^{nons}[U^* | 1] \approx \frac{p_2^{smoker}[1]}{N} 1.92 ,$$

$$E_2^{nons}[U^* | 2] \approx \frac{p_2^{smoker}[1]}{N} 2.56 , \text{ and}$$

$$E_2^{nons}[U^* | 3] \approx \frac{p_2^{smoker}[1]}{N} 2.9 .$$

Let $v_1^{nons}[n]$ be the expected optimal utility of a nonsmoker with n cigarettes after the meetings of period 1 but before the end of period 1. Then, taking into account the endowment of one cigarette at period two,

$$v_1^{nons}[0] = \beta\, E_2^{nons}[U^* | 1] \approx \frac{\beta}{N} \left(p_2^{smoker}[1] \cdot 1.92\right) ,$$

$$v_1^{nons}[1] = \beta\, E_2^{nons}[U^* | 2] \approx \frac{\beta}{N} \left(p_2^{smoker}[1] \cdot 2.56\right) , \text{ and}$$

$$v_1^{nons}[2] = \beta\, E_2^{nons}[U^* | 3] \approx \frac{\beta}{N} \left(p_2^{smoker}[1] \cdot 2.9\right) .$$

Notice that in a single coincidence meeting at t = 1, a nonsmoker does sell his/her unique cigarette to a smoker since

$$v_1^{nons}[1] < \frac{\beta}{3} \cdot 2 < u[y_1^{smoker}] + v_1^{nons}[0] - \varepsilon[1] = \sqrt{\beta} \cdot 3 - \varepsilon[1] .$$

Actually, $v_1^{nons}[1]-u[y_1^{smoker}]\approx -2.42$.

Let

$$y_1^{nons}\equiv v_1^{nons}[2]-v_1^{nons}[1]$$

which is the quantity of service rendered by a nonsmoker in exchange for one cigarette at period 1. Since $y_1^{nons}<1$, a smoker does **not** sell his/her unique cigarette to a nonsmoker potential producer at period 1

What happens in a meeting with single coincidence of wants" and both prisoners are nonsmokers? The potential consumer's best strategy to sell his/her unique cigarette since $p_2^{smoker}[1]=\frac{1}{2}$ and

$$v_1^{nons}[1]<u[y_1^{nons}]+v_1^{nons}[0]-\varepsilon[1] .$$

The difference $v_1^{nons}[1]-u[y_1^{nons}]-v_1^{nons}[0]\approx -0.159$.

Concluding Remarks

The examples that we have seen are the first material that I use to teach random matching models of money to undergraduates. The story of POW camps has been particularly useful. Moreover, the classical description of Radford [1945] of the simple economies of POW camps has guided me in the construction of my examples. For instance, I have chosen parameters so that smokers do not sell their cigarettes stocks. This seems to be simpler and not too different from the mention by Radford [1945] of ... "several cases of malnutrition reported among the more devoted smokers" ... (p. 199)

In this paper, the example called Economy A has been regarded as the starved-currency economy as opposed to the example called Economy B. Bearing in mind both economies, I make the following interpretation. An indivisible-currency shortage brings about deflation. In my model, currency supply is exogenous and deflation is a consequence of decreasing marginal utility of smoking and the bargaining rule. In turn, deflation may increase the set of small transactions which are not made if cigarettes are assumed to be indivisible. In Economy A, deflation is severe enough for cigarettes not to play the role of media of exchange. Consequently, there does not exist media of exchange in Economy A. Barter is the only form of exchange that survives. This seems to be analog to the description of Radford [1945], quoted already in the Introduction, that in deflationary periods, "trading declined in volume and became increasingly a matter or barter" (p. 195). Moreover, I regard a cigarette as something like the "smallest practical gold coin"[1] to try to understand currency shortages in economies with commodity standards.

The equilibria of my examples seem robust to small changes in parameters, explicit and implicit, and in the bargaining rule. For instance, we can give a little bit more bargaining power to the potential producer without changing the strategies in equilibrium. Notice that the optimality of the strategies is characterized by strict inequalities. Moreover, the result that deflation increases the set of small transactions which are not made with indivisible cigarettes may be obtained with some difficulty in more general settings, say in economies with three or more periods. Analysis of more general economies may be found in Renero [1998, in progress] which is more technically demanding.

[1] The phrase in quotation marks is in Conant [1903, p. 471]

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