CIDE

NÚMERO 195

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MODELING GARCH PROCESSES IN PANEL DATA: MONTE CARLO SIMULATIONS AND APPLICATIONS

Abstract

In this paper we propose and implement a methodology of testing and estimation in a panel data context with GARCH effects. In order to determine the presence of GARCH effects and poolability of the mean and variance equations, we propose simple tests based on OLS and LSDV residuals. The estimation of the model is based on direct maximization of the log-likelihood function by numerical methods. A Monte Carlo study is conducted in order to evaluate the performance of this MLE estimator for various relevant designs. As an illustration, we present two empirical applications. We investigate whether investment in a panel of five large U.S. manufacturing firms and inflation in a panel of four Latin American countries exhibit GARCH effects. The results strongly suggest that the error processes can indeed be modeled as conditionally heteroskedastic in both cases.

Resumen

En este articulo se propone e implementa una metodología para pruebas de hipótesis y estimación en un contexto con datos panel y efectos GARCH. Para determinar la presencia de efectos GARCH así como el agrupamiento (pooling) de las ecuaciones de la media y la varianza, se proponen pruebas simples basadas en los residuales OLS y LSDV. La estimación del modelo esta basado en la maximización directa de la función de verosimilitud por métodos numéricos. También se realiza un análisis de Monte Carlo con el objeto de evaluar el desempeño del estimador MLE para varias configuraciones relevantes de parámetros. Como ejemplo, se presentan dos aplicaciones empíricas. Se investiga si la inversión en un panel de cinco grandes empresas manufactureras de Estados Unidos y la inflación en un panel de cuatro países latinoamericanos presentan efectos GARCH. Los resultados sugieren fuertemente que en ambos casos los procesos de error pueden modelarse como procesos con heterocedasticidad condicional.

1. Introduction

GARCH modeling has proven to be one of the most useful new time series tools of the last 15 years. The ability to model the conditional variance of a stochastic process with ARMA methods has allowed for greatly improved testing of hypotheses about the real effects of risk/uncertainty. Further, while OLS estimator is still best linear unbiased in the presence of conditional heteroskedasticity, the non-linear GARCH estimator can provide large efficiency gains over OLS.

In this paper we consider GARCH estimation and testing in panels with moderate values of N (number of cross-sections) and T (number of time periods). The reasons to study GARCH effects in panels with country data are compelling. First, uncertainty is likely to have greater real effects in countries where insurance markets are not well developed, which is to say in developing countries. However, these countries rarely produce long time series of data for researchers to exploit. To study the real effects of risk/uncertainty in the developing world, we may need a panel GARCH model.

Second, recent results on co-integration testing in panels have made testing long run economic relationships via panel data more attractive. In some cases, results using panel tests differ from the older time series results. For example, several recent papers find evidence in favor of PPP using panel data, where previous time series studies show much less support. It is possible that the switch from a single long time series to several pooled shorter time series can reduce the efficiency of least squares relative to GARCH. It therefore would be valuable to be able to test panel regressions of financial data for GARCH effects and have a more efficient panel estimator available if the error term is found to be conditionally heteroskedastic.

In section 2 below we review the development of GARCH models and discuss how panel GARCH fits into the overall framework. Section 3 derives our basic panel GARCH estimator under the assumption of total parameter homogeneity. Section 4 discusses several generalizations that relax some of the homogeneity assumptions. Section 5 describes a testing procedure that could be used to determine exactly what type of panel GARCH model is appropriate for any given set of data. Section 6 presents the results of the Monte Carlo simulation on the finite sample performance of the MLE estimator. In section 7, we provide two simple empirical examples by investigating if investment in a panel of five large US manufacturing firms and inflation in a panel of four Latin American countries, exhibit GARCH effects. Finally, section 8 concludes by reviewing our contribution and making some suggestions for future work.

2.Arch and Garch Modeling

In many financial data, the phenomenon of volatility clustering is often observed. Volatility clustering simply means that the occurrence of large residuals is correlated over time. Engle's (1982) ARCH model attacks volatility clustering by assuming that the conditional variance of today's error term, given the previous errors, follows a moving average process. Engle shows that the efficiency gain from using ARCH estimation instead of least squares can be extremely large when the degree of conditional serial dependence in the error variance is severe.

The effects of ARCH estimation on empirical results can be dramatic. For example, as several studies report, the effect of money growth on interest rates is estimated to be positive in OLS regressions using data from the 1970s and 80s. This result had been referred to as the disappearing liquidity effect. However, Grier & Perry (1993) show that there is strong conditional heteroskedasticity in the data and using ARCH estimation reverses the sign of the liquidity effect back to the expected negative one.

Two important extensions of the Engle model are Bollerslev's (1986) GARCH model, and Engle, Lillien & Robbins' (1987) ARCH-M model, where the conditional variance is used as an explanatory variable in the equation under study. GARCH allows the conditional variance of the error term to follow an autoregressive-moving average (ARMA) process, and the GARCH(1,1) model has become the most commonly used specification in empirical applications. Bollerslev shows that any arbitrary ARCH model can be well approximated by the GARCH(1,1) specification.

ARCH-M allows the testing of economic hypotheses about the real effects of risk or uncertainty. Fluctuations in the conditional variance of the error term can be considered as fluctuations in the predictability of the process. A high conditional variance implies less predictability, or more risk/uncertainty. ARCH-M models are used to measure and test the significance of time varying risk premia in financial data.

Bollerslev, Engle & Wooldridge (1988) introduce the multivariate GARCH model where the conditional covariance matrix $\mathbf{H}$ at time t (for the GARCH(1,1) case) is given as:

$$vech(\mathbf{H}_t) = \mathbf{C} + \mathbf{A}vech(\varepsilon_{t-1}\varepsilon_{t-1}^{'}) + \mathbf{B}vech(\mathbf{H}_{t-1}) \tag{1}$$

Here *vech* refers to the column stacking operator of the lower portion of a symmetric matrix, ε_{t-1} is the vector of errors at time $t-1$, and $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{C}$ are coefficient matrices. In the three variable case, this covariance structure requires estimating 78 coefficients. To simplify, Bollerslev, Engle & Wooldridge assume that matrices $\mathbf{A}$ and $\mathbf{B}$ are diagonal, which in the tri-variate case reduces the number of coefficients to be estimated to 18. Bollerslev (1990) introduces a further simplification, the constant correlation model, further reducing the estimated parameters in the tri-variate case to 12. Given the large number of coefficients to be estimated, even with simplifying assumptions, to date multivariate GARCH models consider only a small number of variables.

While the majority of GARCH applications are in finance (see Bollerslev, Chou, and Kroner 1992 for a review), the technique is useful in macro and development economics as well. Recently, Grier and Perry (1996, 2000) use a multivariate GARCH-M model to test for the effects of inflation uncertainty on the dispersion of relative prices and on real output growth in the US.

Our goal is to extend the utility of GARCH modeling in economics by introducing a tractable methodology for GARCH estimation and testing in a panel data setting. The extension of GARCH modeling to panels is important for two reasons. First, the real effects of uncertainty should be greatest in countries where there is a lot of fluctuation in uncertainty and where it is difficult to use futures/insurance markets to hedge. Specifically, we have in mind developing countries, which rarely have long country-specific time series data available. To test for the real effects of uncertainty where they are likely to be most important requires pooling several countries into a single data set and then estimating the time varying conditional variances. Our panel GARCH estimator can accomplish exactly that.

Second, the recent switch from time series to panel approaches for testing economic theories may well exacerbate conditional heteroskedasticity problems in the data.

The shorter time series that typically are pooled to create a panel data set encompass more recent data, and more recent data tends to show greater volatility clustering. As older observations are discarded and the sample becomes more heavily weighted with more recent data, the strength of GARCH effects grows, as does the efficiency gain from using a GARCH estimator.

For example, suppose that the years 1981 – 85 contain a large amount of volatility clustering in most countries. For a 100 year, single country time series, volatility clustering occurs in 5% of the sample. For a 20 year, 10-country panel covering the 1970s and 1980s, volatility clustering occurs in 50% of the sample. To the extent that switching from a time series to a panel approach piles up more observations that are conditionally heteroskedastic, least squares becomes less and less efficient compared to GARCH estimation.

3. The Basic Model

This section describes the specification and estimation of a simple panel data model with a time varying conditional variance. At this stage we assume complete parameter homogeneity across individuals in the panel. In the next section this assumption is relaxed to allow for some forms of parameter heterogeneity. We consider the following general pooled regression model:

$$y_{it} = \mu + \phi y_{it-1} + \mathbf{x}_{it}\beta + u_{it}, \quad i = 1,\ldots,N, \quad t = 1,\ldots T \tag{2}$$

$$u_{it} = \sigma_{it}\varepsilon_{it}, \quad \varepsilon_{it} \sim NID(0,1), \tag{3}$$

where, N and T are the number of cross sections and time periods in the panel respectively, y_{it} is the dependent variable, μ is the common intercept coefficient, $\mathbf{x}_{it}$ is a row vector of explanatory variables of dimension k, β is a k by 1 vector of coefficients, u_{it} is the disturbance term, and ϕ is the AR parameter. We assume that $\phi < 1$. Under the assumption $\phi = 0$, the process given by equation (1) becomes static. A general process for σ_{it} is given by the following GARCH (p, q):

$$\sigma_{it}^2 = \alpha + \sum_{m=1}^{q} \gamma_m u_{i,t-m}^2 + \sum_{n=1}^{p} \delta_n \sigma_{i,t-n}^2, \tag{4}$$

which, can be expressed more compactly as

$$\sigma_{it}^2 = \alpha + A(L,\gamma)u_{it}^2 + B(L,\delta)\sigma_{it}^2, \tag{5}$$

where, α is a common intercept coefficient, γ and δ are column vectors of dimensions q and p respectively, and $A(L,\gamma)$, and $B(L,\delta)$ are polynomials in the lag operator L. The previous equations, are simply extensions of Bollerslev's (1986) GARCH process to each cross-section in the panel. Notice that, if $B(L,\delta) = 0$ we have an ARCH (q) process as in Engle (1982), and if $A(L,\gamma) = B(L,\delta) = 0$ we have conditionally homoskedastic disturbances. The model defined by equations (2) and (5) will be referred to as Model A. From theorem (1) in Bollerslev (1986), the condition $A(1) + B(1) < 1$ will assure that the GARCH (p, q) given by equation (5) be stationary for each cross-section in the panel. We assume that this condition holds although we should remark that the underlying non-negativity restrictions $\delta_j, \gamma_j \geq 0$ are sufficient but not necessary to ensure non-negativity of the conditional variance.

Engle (1982) has pointed out that in a pure time series context, each observation is conditionally normally distributed but the vector of T observations is not jointly normally distributed. In fact, the joint density is the product of the conditional densities for all T observations. The previous statement applies directly to each cross-sectional unit in the panel considered in Model A. Thus, extension to the panel data case is straightforward as long as the disturbances in the model are assumed to be cross-sectionally independent.

For observation (i, t), the conditional density is:

$$f(y_{it}/x_{it},\mu,\beta,\alpha,\gamma,\delta)=(2\pi\sigma_{it}^{2}(\alpha,\gamma,\delta))^{-1/2}\exp-\frac{(y_{it}-\mu-\phi y_{it-1}-\mathbf{x}_{it}\beta)^{2}}{2\sigma_{it}^{2}(\alpha,\gamma,\delta)} \quad (6)$$

which implies, under the previous cross-sectional independence assumption, the following log-likelihood function:

$$l=-\frac{NT}{2}\ln(2\pi)-\frac{1}{2}\sum_{i=1}^{N}\sum_{t=1}^{T}\ln(\sigma_{it}^{2}(\alpha,\gamma,\delta))-\frac{1}{2}\sum_{i=1}^{N}\sum_{t=1}^{T}\frac{(y_{it}-\mu-\phi y_{it-1}-\mathbf{x}_{it}\beta)^{2}}{\sigma_{it}^{2}(\alpha,\gamma,\delta)}, \quad (7)$$

Even though the OLS estimator in equation (2) is still consistent and the most efficient among the class of linear estimators, the MLE estimator based upon (7) is a more efficient non-linear estimator. In addition, by using MLE we can obtain the parameters of both the conditional mean (equation 2) and conditional variance (equation 5) simultaneously.

In fact, from MLE theory we know that under regularity conditions the MLE estimator of the parameter vector $\theta=(\mu,\beta',\alpha,\gamma',\delta')'$ is consistent, asymptotically efficient and asymptotically normally distributed. In spite of these general results, we investigate the finite sample performance of the MLE estimator relative its OLS counterpart by Monte Carlo simulations for a few designs. We present these results in Section 6 below.

In this paper we will pursue direct maximization of (7) by numerical methods as given in the Optimization module of the GAUSS computer program. The asymptotic covariance matrix of the MLE estimator will be approximated as the inverse of the outer product of the gradient vectors of l, evaluated at actual MLE estimates.

Finally, it is important to notice that the previous setting can be extended to the ARCH-M class of models in which the conditional variance enters into the conditional mean regression. In this case, equation (2) becomes:

$$y_{it}=\mu+\phi y_{it-1}+\mathbf{x}_{it}\beta+\rho\sigma_{it}+u_{it}, \quad i=1,\ldots,N, \quad t=1,\ldots T \quad (8)$$

4.Relaxing The Homogeneity Assumption

Model A can easily be modified to allow for different forms of parameter heterogeneity. In principle, it is possible to have heterogeneity in intercepts and/or slopes in both the mean and variance regressions. In fact there are 16 distinct combinations. In order to have a manageable number of cases we only allow for heterogeneity in intercepts in the mean and variance equations. In addition to Model A, we consider the following 3 models:

(i) Individual effects in the mean equation and full parameter homogeneity in the variance equation (Model B).
(ii) Individual effects in the variance equation and full parameter homogeneity in the mean equation (Model C)
(iii) Individual effects in both the mean and variance equations (Model D).

The mean and variance equations for Model D are given by

$$y_{it} = \mu_i + \phi y_{it-1} + \mathbf{x}_{it}\beta + u_{it}, \qquad i = 1,\ldots,N, \quad t = 1,\ldots T \tag{9}$$

$$\sigma_{it}^2 = \alpha_i + A(L,\gamma)u_{it}^2 + B(L,\delta)\sigma_{it}^2, \tag{10}$$

with μ_i and α_i representing the corresponding individual specific effects. In this case, the full parameter vector $\theta = (\mu', \beta', \alpha', \gamma', \delta')'$ has $(2N + k + p + q + 1)$ elements since μ and α are vectors of dimension N with typical elements μ_i and α_i respectively. If these individual-specific effects are treated as fixed, the basic model given in the previous section applies directly to this case with no modifications other than including dummy variables in both the mean and variance equations. With a relatively small number of cross-sections (N) and explanatory variables (k), and small orders for the GARCH process, estimation of Model D becomes feasible. For example a GARCH (1, 1) with 5 cross sections and 2 explanatory variables, implies estimating 15 parameters.

Model B considers individual effects in the mean equation and a common intercept coefficient in the variance equation $(\alpha_i = \alpha)$. In this case there are $(N + k + p + q + 2)$ parameters to be estimated. The same number of parameters would have to be estimated in the case of Model C. In the example given before, Models B and C require estimating a total of 11 parameters. Notice that Model A would require estimating 7 parameters only.

If the cross-section dimension of the sample (N) is not small, however, the number of parameters to estimate can become unusually large. In this situation it might be convenient to sweep out the individual effects if they are found significant in order to reduce the number of parameter to be estimated. For instance, consider N = 20 as might be the case of a sample of Latin American countries. In this case, Models B and D imply estimating a total of 26 and 45 parameters respectively and so initially removing individual effects in the mean would reduce those numbers by 20 in both cases.

5. Testing Some Relevant Hypotheses

Given that we have considered 4 possible model specifications and several possible ARCH or GARCH orders for the conditional variance process within each model, testing for individual effects and the order of the GARCH process simultaneously can become fairly complicated.

In this paper we propose the following methodology to identify the appropriate model. First, we test for the presence of individual effects in the mean equation. Next, we test for ARCH effects and determine if there are individual effects in the conditional variance process. Finally, after choosing among the models, we estimate a few ARCH and GARCH specifications by maximum likelihood and attempt to select the one that best fits the data.

5.1. Testing for individual effects in the mean equation.

Given that OLS and LSDV estimators are still the best linear estimators, we propose testing for the presence of individual effects in the mean equation using OLS and LSDV estimation results for this equation. We can use the standard *F*-test, with (N-1) and (TN-N-k-1) degrees of freedom in the numerator and denominator respectively. It should be remarked that this test is valid asymptotically irrespective of the process followed by the conditional variance.

5.2. Testing for ARCH effects

In the second step, we use LSDV or OLS (according to whether individual effects were found or not in the mean equation) squared residuals to test for ARCH effects. Specifically, we try to find the auto regressive pattern of the squared residuals on the basis of estimated partial auto correlation coefficients. We use the statistical significance of these coefficients, as a criterion to set a preliminary order for the ARCH process. Similarly, we can test the null of conditional homoskedasticity or ARCH (0), against ARCH (j) for a few relevant values of j. This can be done with *LM*-test statistics based on the previous squared residuals and referred to the $\chi^2_{(j)}$ distribution. Finally, we test for individual effects in the previous ARCH process by means of a usual *F*-test.

5.3. Selecting the appropriate model for the conditional variance

After choosing an appropriate model specification, we can perform maximum likelihood estimation of a few relevant ARCH and GARCH specifications and determine the one that best fits the data. For nested hypothesis, the likelihood ratio test appears to be appropriate to discriminate among the different specifications for the conditional variance.

6.Finite Sample Performance Of The Garch MLE Estimator

It is well known that in the context of time series GARCH models, the (non-linear) MLE estimator not only has desirable asymptotic properties but also it is more efficient than the OLS estimator. Little is known, however, on the finite sample performance of the MLE estimator relative to its OLS counterpart in finite samples, particularly in panel data. In this section we present some Monte Carlo results that shred light on the performance of the MLE estimator in panels with GARCH errors. In particular, we study the bias and precision of the MLE and OLS estimators of the parameters of the conditional mean equation (equation 2). Also, we study the performance of the MLE estimator of the parameters in the variance equation (equation 4) for relatively small time dimensions. In all cases we try to establish the patterns of the bias and precision of the estimators as T increases for given N, and as the persistence in the variance processes increases given T and N.

6.1.Monte Carlo design

The data-generating model is defined by equation (2) and equation (4) given before. Notice that the error term in the mean equation is drawn from a normal distribution with mean zero and variance that changes over time according to equation (4). For practical purposes we limit the simulation study to the pooled regression model and to the cases of ARCH (1) and GARCH (1,1) errors.

For the conditional mean, we consider separately the static and dynamic cases. In the static case, only one exogenous regressor with coefficient $\beta = 1$ enters the mean equation. In the dynamic case, we consider a pure AR (1) process without exogenous regressors. In this case, the AR parameter ϕ takes the alternative values $\{0.5, 0.8\}$. In this way we represent moderate and high persistence of the AR (1) process respectively. We set the parameter values $\mu = \alpha = 1$ (the intercept of the mean and variance equations respectively) for all cases. For the ARCH (1) process, we consider that γ_1 (the coefficient on the term u_{t-1}^2) takes on the alternative values $\{0.2, 0.5, 0.9\}$, representing low, moderate and high persistence of the conditional variance process. For the GARCH (1,1) process, we set $\gamma_1 = 0.1$ (the coefficient on the term u_{t-1}^2) for all cases, but we allow δ_1 (the coefficient on the term σ_{t-1}^2) to take on the values $\{0.1, 0.4, 0.8\}$.

Finally, we have set the number of trials in each Monte Carlo experiment to 1000. Given that not always the program that solves the model numerically will achieve convergence, the final number of (valid) trials is less than 1000. This happens particularly in the GARCH (1,1) model. The results are presented in Tables A1 through A18 in the Appendix.

6.2.Panel ARCH results (static mean case)

In Tables A1 trough A3 we present the results for the static mean case. The first observation is that as T increases given N the OLS and MLE estimators of the intercept and slope coefficients in the mean equation improve on a mean squared error criterion. Although we do not observe a clear pattern in the bias of both estimators, their standard errors clearly diminish with T. Secondly, when comparing the OLS and MLE estimators, we find that the MLE outperforms the OLS estimator in terms of precision and mean squared error. In terms of bias, though, we do not find a clear superiority of one estimator over the other.

Regarding the MLE estimator of the variance coefficients α and γ_1 (intercept and ARCH (1) coefficient respectively), in both cases we observe improvements in precision and mean squared error as T increases. This last result is despite the biases do not show a clear pattern.

Given T and N we next explore some patterns as the persistence of the ARCH (1) process is increased. We should note that the biases of both coefficients are generally negative when the true γ_1 equals 0.2 (Table A1). For a true γ_1 of 0.9, the bias in the intercept remains negative while the bias in the ARCH (1) coefficient becomes positive (Table A3). We should also note that the biases increase as the persistence of the ARCH process is increased and that the bias in the ARCH (1) coefficient becomes much larger than that of the intercept. For example, for the case $\gamma_1 = 0.2$ (Table A2), the biases of the intercept and slope coefficients of the variance equation are 2% and 3.3% of their respective true parameter values in the smallest sample considered (N=5, T=*20*). For the biggest samples these biases are reduced substantially. For the case $\gamma_1 = 0.9$ (Table A3), those biases reach 1.9% and 9.4% respectively in the smallest sample (N=5, T=*20*) and 0.5% and 2.3% in the biggest sample (N=20, T=100).

Regarding precision, it is interesting to note that when $\gamma_1 = 0.2$ the MLE estimator of the intercept is more precise than that of the slope. However, as the persistence of the ARCH process is increased the standard error of the intercept coefficient tends to increase while that of the slope tends to decrease. When $\gamma_1 = 0.9$, we find that the MLE estimator of the ARCH parameter is generally more precise than that of the intercept. On a mean squared error criterion, except for the smallest sample size considered the MLE estimator of the variance coefficients appears to be quite acceptable in all cases.

Turning back to the mean equation, it is worth mentioning that although the MLE estimator is better than its OLS counterpart, the MLE estimator of the intercept coefficient shows quite large mean squared errors as percentage of the true parameter values. In fact, this estimator performs quite poorly in the smallest sample (N=5, T=20) where its mean squared error reaches 13.9%, 17.1% and 21.4% for the cases of $\gamma_1 = 0.2, 0.5, 0.9$ respectively.

6.3. Panel ARCH results (dynamic mean case)

Tables A4 trough A9 show the results for the ARCH (1) process in the dynamic mean case. We find that both the OLS and MLE estimators of the coefficients of the (dynamic) mean equation become more precise as T increases. Also the biases get smaller as T increases except in the cases of the smallest samples (N=5) and most persistent ARCH process ($\gamma_1 = 0.9$). Overall, both estimators improve as T increases showing smaller mean squared errors in all cases. Differently than in the static case, in this case we observe that the MLE estimator of the intercept and AR parameters in the mean equation clearly outperforms the OLS estimator in terms of both bias and precision.

Regarding the MLE estimators of the ARCH equation in the dynamic mean case, we find that their precision increases with the sample size although the biases do not show a well defined pattern. On a mean squared error criterion, we find improvement in the MLE estimators of both parameters (intercept and ARCH (1) coefficients) as the sample size increases in all cases. When looking at the performance of these estimators as the persistence of the ARCH process increases given the sample sizes, we observe that for true ARCH (1) coefficients $\gamma_1 = 0.2, 0.5$ the biases of the MLE estimators are generally negative. For $\gamma = 0.9$, the bias of the intercept coefficient becomes negative while the bias of the ARCH (1) coefficient becomes positive. Another pattern that we find is that as the persistence of the ARCH process is increased the standard error of the intercept coefficient of the variance equation generally increases while the standard error of the ARCH coefficient generally decreases.

Overall, on a mean squared error criterion, we find that the results for the MLE estimator of the variance equation are acceptable, except perhaps in the cases (N=5, T=20) were it ranges from about 4% to 11% of the true parameter values.

In the dynamic mean case, we also compare the performance of the MLE estimator of the coefficients of the mean and variance equations as the persistence in the mean process (parameter ϕ) is increased from 0.5 to 0.8. We find that the mean squared error of the variance coefficients does not change much. However, we observe noticeable changes in the performance of the estimators of the mean equation. In general, we find that the MLE estimator of the intercept of the mean equation becomes biased and less precise while the MLE estimator of the AR parameter (ϕ) becomes less biased and more precise. Interestingly, we find a similar pattern for the OLS estimator of both parameters. This result is different than in the case of dynamic panel data models with homoskedastic disturbances where as the process becomes more persistent the OLS estimator of the AR parameter becomes more biased and less precise.

6.4.Panel GARCH results (static mean case)

The Monte Carlo results for the static mean case with GARCH errors are presented in Tables A10 trough A12. We observe that the MLE and OLS estimators of the coefficients of the mean equation (μ and β) improve as T increases in terms of standard errors and mean squared errors. Also the MLE estimators of the previous coefficients are less biased that their OLS counterparts in most cases. In terms of precision, the MLE estimators outperform the OLS estimators except in a few cases. A similar result is found in terms of mean squared errors.

Concerning the MLE estimators of the coefficients in the variance equation ($\alpha, \gamma_1, \delta_1$), we find that when the true parameter of the GARCH coefficient (δ_1) takes the values 0.1 and 0.5, (Tables A10 and A11) their biases do not exhibit a clear pattern as T increases. Similar results are found for the standard error and mean squared error of the MLE estimator of δ_1. In contrast the MLE estimators of the intercept and ARCH parameters (α and γ_1) improve in terms of standard errors and mean squared errors as T increases. When the true parameter $\delta_1 = 0.8$, we observe that the MLE estimators of all variance coefficients become less biased and more precise. It should be pointed out, however, that the MLE estimator of the intercept and GARCH coefficient in the variance equation is particularly highly distorted in the cases $\delta_1 = 0.1, 0.4$. In fact, the bias of the GARCH coefficient could be as high as 50.1% of the true parameter value (Table A10, case N=20, T=50). Similarly, the standard error of this coefficient could be as high as or even higher than two times the true parameter value (Table A10). Only in the case $\delta_1 = 0.8$ and for the largest samples, the MLE estimator of the variance parameter seems to be reliable (Table A12).

6.5.Panel GARCH results (dynamic mean case)

The Monte Carlo results for the model with dynamic mean and GARCH (1,1) errors are presented in Tables A13 trough A18. In this case we find that the MLE and OLS estimators of the parameters in the mean equation (μ and ϕ) improve on a mean squared error criterion as T increases for given N. Also, in most cases both estimators become less biased and more precise as T increases. When comparing both estimators we find that except in a few cases the MLE estimator is less biased and more (or equally) precise than the OLS estimator.

For the MLE estimators of the variance coefficients, except in the case $\delta_1 = 0.1$, we find that they perform generally better (less biased and more precise) as T increases. More specifically, for the case $\delta_1 = 0.1$ (Tables A12 and A16) the MLE estimators of the intercept (α) and ARCH (γ_1) parameters generally improve on a mean squared error criterion as T increases. The MLE estimator of the GARCH parameter (δ_1) does not show the previous pattern except in the cases for N=20 which correspond to the largest samples studied. When looking at specific values of biases and standard errors we find that the MLE estimators of α and γ_1 perform reasonably well in the cases where the true GARCH coefficient $\delta_1 = 0.1$ (i.e. the true GARCH process is not highly persistent), particularly if N=10 or higher. This is not the case, however, for the MLE estimator of δ_1 which performs poorly in terms of bias and precision (Table A13). It is important to mention that in this case, the MLE estimators are more reliable for the mean equation than for the variance equation since in the former case we observe a better performance than in the later case in terms of mean squared error. We should also mention that the MLE estimator of the slope coefficient in the mean equation (ϕ) performs better than the MLE estimator of the intercept coefficient (μ).

When looking at more persistent GARCH processes, given T and N, we observe that the mean squared error of the MLE estimators of the slope in the mean equation (ϕ) and the ARCH coefficient in the variance equation (γ_1) remains at the same values or even decreases. For the intercepts in the mean equation (μ) and variance equation (α), we find that the mean squared error increases. In particular, it should be noticed that for the case $\delta_1 = 0.8$, the mean squared error of α could be as high as 11 times the true parameter value (Tables A15 and A18). On the other hand, it should be noticed that the performance of the MLE estimator of δ_1 improves as the persistence of the true GARCH process increases. The previous results show a trade off concerning the reliability of the MLE estimator of the different parameters in the variance equation. For a low level of persistence (i.e. $\delta_1 = 0.1$) the MLE estimator of this parameter is likely to be unreliable even if the sample is large. In this case, the bias of the intercept is negative while the bias of the GARCH coefficient is positive. For a medium level of persistence ($\delta_1 = 0.4$), the MLE estimator of δ_1 becomes more reliable (less biased and more precise) while the MLE estimator of the intercept (α) becomes more biased and less precise. It is important to note that in this case the sign of the bias changes compared with the previous case, appearing a positive bias for the intercept and a negative one for the GARCH coefficient. For $\delta_1 = 0.8$, we can get more reliable estimates in the case GARCH parameter but in most cases the MLE estimator of the intercept of the GARCH process will be unreliable. In this case, the intercept coefficient exhibits a much higher positive bias while the bias of the GARCH coefficient becomes less negative.

Finally we should note that as the persistence of the GARCH process increases, the MLE estimator of the intercept of the variance equation becomes less precise while the MLE estimator of the GARCH parameter becomes more precise.

7. Empirical Applications

In this section we illustrate the applicability of the panel GARCH estimation and testing methodology proposed in section 5. Using very simple frameworks, we investigate whether the uncertainty associated to investment in a panel of five large U.S. manufacturing firms, and inflation in a panel of four Latin American countries can in fact be captured with ARCH or GARCH models.

7.1. Investment in a panel of five large U.S manufacturing firms

Here we use the well-known Grunfeld investment data set.[1] This is a panel of 5 firms and 20 years. For each firm and for every year we have observations on gross investment (I), market value of the firm (F), and value of the stock of plant and equipment (C). The values of the variables F and C correspond to the end of the previous year. We believe that the uncertainty associated to the investment process is time dependent. The model is specified as follows:

$$I_{it} = \mu_i + \beta_1 F_{it} + \beta_2 C_{it} + u_{it}, \qquad i = 1,\ldots,5; t = 1,\ldots,20 \qquad (11)$$

$$\sigma_{it}^2 = \alpha_i + \sum_{n=1}^{p} \delta_n \sigma_{i,t-1}^2 + \sum_{m=1}^{q} \gamma_m u_{i,t-m}^2 \qquad (12)$$

Notice that we allow for heterogeneity only through individual effects in both the conditional mean and conditional variance equations. We begin by testing for individual effects in the mean equation. The computed statistics $F_{(4,93)} = 58.956$ and $\chi^2_{(4)} = 126.292$ are high enough as to reject the null hypothesis of no individual effects in the mean equation.[2] Next we attempt to identify ARCH effects using the squared residuals from LSDV estimation of the mean equation given the previous evidence against no individual effects. We compute the partial autocorrelation coefficients of the squared residuals (Table 1).

[1] These data are taken from Greene (1997, p. 650, Table 15.1).

[2] The regressions required to calculate these statistics are reported in the first two panels of Table 2.

TABLE 1:
Estimated partial auto-correlation coefficients on squared LSDV residuals
(investment data)

	Coefficient	t-ratio	p-value
PAC(1)	0.5225*	4.0785	0.0000
PAC(2)	-0.0925	-0.6633	0.7455
PAC(3)	0.0519	0.3542	0.3621
PAC(4)	0.0728	0.4862	0.3139
PAC(5)	0.2325°	1.5168	0.0664
PAC(6)	-0.1235	-0.7816	0.7817
PAC(7)	0.1293	0.7731	0.2207
PAC(8)	0.0482	0.2828	0.3889
PAC(9)	0.1245	0.6978	0.2435
PAC(10)	-0.1638	-0.9763	0.8342

LSDV estimated squared residuals are used since there is evidence of individual effects in the mean equation. The symbols *, ^ and ° indicate respectively 1%, 5% and 10% significance levels.

In these data, only the first partial autocorrelation coefficient is statistically significant at the 0.05 level. We thus consider that the conditional variance of the error process follows at most an ARCH(1). Next, we try to determine if this MA(1) in the squared residuals has individual effects. The computed statistics $F_{(4,94)} = 2.960$ and $\chi^2_{(4)} = 11.864$ do not provide compelling evidence against the null hypothesis of no individual effects in the ARCH process as they reject it at the 5% significance level but not at the 1%.

Our preliminary results suggest that we have individual effects in the mean equation. They also suggest that it is likely that the conditional variance follows an ARCH (1) with possible individual effects. Therefore we choose preliminarily Models B and D as the best possible specifications. In Table 2 we present maximum likelihood estimation results for the previous specifications. For comparison we also consider Model A, which corresponds to a pooled regression model whose conditional variance follows an ARCH(1) process, and present OLS and LSDV estimates of the mean equation.

TABLE 2:
Panel estimation results for Investment with ARCH effects

	Constant	μ_1	μ_2	μ_3	μ_4	μ_5	F	C	Log-likelihood
OLS estimates Mean equation	-48.0297 (-1.0854)						0.1051 (11.0661)*	0.3054 (7.0186)*	-624.9927
$\sigma^2 = 16194.677$									
LSDV estimates Mean equation		-76.0668 (-0.9342)	-29.3736 (-1.4408)°	-242.1708 (-5.7738)*	-57.8994 (-2.9636)*	92.5385 (2.3339)*	0.1060 (7.4728)*	0.3467 (11.5096)*	-561.8468
$\sigma^2 = 4777.2951$									
ARCH(1): Pooled	-37.4254 (-6.6876)*						0.1087 (40.3168)*	0.3358 (15.2096)*	-584.8165
Regression (Model A)									
$\sigma_t^2 = \underset{(1.5385)^{\circ}}{796.6344} + \underset{(2.9566)^{\wedge}}{1.5593}\,\hat{u}_{t-1}^2$									
ARCH(1): Individual		16.9425 (0.2239)	-32.2124 (-4.0328)*	-198.2478 (-10.6223)*	60.1395 (-8.0113)*	123.2295 (5.8153)*	0.1406 (12.7917)*	0.2991 (9.7756)*	-556.9984
Effects in mean only (Model B)									
$\sigma_t^2 = \underset{(1.0614)}{161.6394} + \underset{(4.0227)^{*}}{1.16011}\,\hat{u}_{t-1}^2$									
ARCH(1): Individual		242.3569 (6.1002)*	12.6154 (2.2186)^	-51.2771 (-2.3148)^	-9.8486 (-1.6494)^	231.1292 (6.1284)*	0.0691 (9.6805)*	0.0314 (1.2391)	-507.7838
Effects in mean and variance (Model D)									
$\sigma_t^2 = \underset{(1.3733)^{\circ}}{1676.0896} + \underset{(1.6793)^{\wedge}}{72.9596} + \underset{(1.9938)^{\wedge}}{676.5132} + \underset{(1.7674)^{\wedge}}{58.7863} + \underset{(1.6958)^{\wedge}}{5388.1957} + \underset{(3.3328)^{*}}{0.7618}\,\hat{u}_{t-1}^2$									

These results have been obtained by direct maximization of the log-likelihood function by numerical methods. For each model we show the mean coefficients followed by the estimated equation for the conditional variance process. Values in parenthesis are t-ratios and the symbols *, ^, °, indicate significance levels of 1%, 5% and 10% respectively. Standard errors for the OLS and LSDV estimates are robust to heteroskedasticity.

As noted above, the data reject the null hypothesis of no individual effects in the mean equation at the 0.01 level. This can be seen in Table 2 either by comparing either panels one and two (ols vs. lsdv) or by comparing panels three and four (ARCH(1) pooled vs. ARCH(1) with individual mean effects). Further, the data reject the null hypothesis of conditional homoskedasticity, also at the 0.01 level.

This can be seen either by comparing panels one and three (ols vs. ARCH(1) pooled), or panels two and four (lsdv vs. ARCH(1) with individual mean effects) in Table 2. Finally the data also reject the null hypothesis of no individual effects in the conditional variance equation at the 0.01 level (as seen by comparing panels 4 and 5 in Table 2). Our preferred model is thus the final estimation in Table 2: ARCH(1) with individual effects in both the mean and conditional variance equations.

From the reported results, we can see that modeling the conditional variance of the panel changes the values of the coefficients on the explanatory variables in the mean equation. The coefficient on C, the value of the firm's plant and equipment is 10 times smaller and much less significant in our preferred model than in the OLS, LSDV or ARCH(1) pooled specifications. The coefficient on F, the firm's market capitalization is about 40% smaller but still significant at the 0.01 level.[3] In the example, there is strong evidence of conditional heteroskedasticity, and modeling it changes the results of interest.

7.2.Inflation in a panel of four Latin American countries

Here we study inflation in 4 countries: México, Colombia, Venezuela, and Uruguay, with yearly observations on inflation rates (π) during the period 1950-1998.[4] As in the case for investment, we consider that inflation uncertainty can be well approximated by a GARCH process. The model for inflation is specified as a simple *AR(1)* process with time trend:

$$\pi_{it} = \mu_i + \theta t + \beta_1 \pi_{it-1} + u_{it}, \qquad i = 1,\ldots,4; t = 1,\ldots,39 \qquad (13)$$

$$\sigma_{it}^2 = \alpha_i + \sum_{n=1}^{p} \delta_n \sigma_{i,t-1}^2 + \sum_{m=1}^{q} \gamma_m u_{i,t-m}^2 \qquad (14)$$

As in the model for investment given in the previous section, we allow for heterogeneity only through individual effects in both the conditional mean and conditional variance equations. Testing for individual effects in the mean equation yields the computed statistics $F_{(3,186)} = 3.154$ and $\chi^2_{(3)} = 9.528$, which are significant at the 5% but not at the 1%. In this case, we do not find strong evidence against the null of no individual effects in the mean equation perhaps because the four countries considered in this study have similar inflationary patterns. For

[3] It is also interesting to note that including the individual effect in the conditional variance changes the model from possibly non-stationary (ARCH coefficient > 1.0) to clearly stationary (ARCH coefficient of 0.76).

[4] These data are compiled from the International Monetary Fund's (IMF) International Financial Statistics, Data Bases 1999.

convenience, we will proceed to estimate the mean equation as a pooled regression model and use OLS squared residuals to test for ARCH effects in the next step. However, given the results in the tests for individual effects, estimation of the relevant pooled ARCH / GARCH models will also be performed after removing individual effects in the mean equation.

As before, we compute partial auto-correlation coefficients on the squared residuals for the first 10 lags. We find that lags 4 and 6 are statistically significant at the 1%, while lags 2 and 8 are significant at 2.1% and 8.9% respectively. These results are shown in Table 3.

TABLE 3:
Estimated partial auto-correlation coefficients on squared OLS residuals
(inflation data)

	Coefficient	t-ratio	p-value
PAC(1)	-0.1240	-1.0052	0.1591
PAC(2)	0.3610^	2.0646	0.0214
PAC(3)	-0.1125	-0.5642	0.2872
PAC(4)	1.3782*	6.3942	0.0000
PAC(5)	-0.6196^	-2.2704	0.0132
PAC(6)	1.7393*	5.6468	0.0000
PAC(7)	-0.1473	-0.3957	0.3468
PAC(8)	-0.7549°	-1.3611	0.0890
PAC(9)	-0.0912	-0.1677	0.4337
PAC(10)	-0.1106	-0.1952	0.4229

OLS estimated squared residuals are used since there is evidence of no individual effects in the mean equation. The symbols *, ^ and ° indicate respectively 1%, 5% and 10% significance levels.

We find much more persistence in the squared errors in this example than in the previous case, and will account for it by using GARCH(1,1) model of the conditional variance. We also pre-test for individual effects in the conditional variance in a variety of models without ever finding any evidence against the hypothesis of homogeneity. We thus restrict the analysis to homogeneous conditional variance equations. However, given that we do obtain at least a weak rejection of the hypothesis of no individual effects in the mean equation, we will estimate our models to the data both before and after removing individual specific effects. We consider the ARCH (1) and GARCH (1,1) cases. Table 4 presents the maximum likelihood estimation results.

Whether individuals effects are removed from the mean equation or not, the data reject conditional homoskedasticity in favor of an ARCH(1) model, and also reject the ARCH(1) model in favor of the GARCH(1,1) model. Since in the GARCH(1,1) context, pre-sweeping any individual effects has virtually no impact on the maximized value of the likelihood function, our preferred model is given in panel 5 of Table 4, which is a GARCH(1,1) conditional variance with no individual effects either in the mean or conditional variance equation.

The model shows moderate persistence in both the mean of inflation (lagged inflation has a coefficient of .82) and in its conditional variance (ARCH coefficient of .55 and GARCH coefficient of .40). Relative to the least squares estimates in panel 1, the GARCH(1,1) estimates show a slightly larger (.82 compared to .75) coefficient on lagged inflation and a smaller and less significant coefficient on the time trend. While much more work needs to be done before any strong statements can be made, these results suggest that modeling conditional heteroskedasticity in panel data may affect the outcome of panel unit root tests.

Another interesting result here is that there is significant time-varying inflation uncertainty in these data and that the effect of an inflation shock on the conditional variance of inflation is sizeable, lasting for several years.

TABLE 4:
Panel estimation results for Inflation with ARCH and GARCH effects

	constant	π_{t-1}	θ	Log-likelihood
OLS estimates Mean equation +	2.2616 (0.9044)	0.7587 (15.5362)*	0.1670 (1.7019)°	-809.5858
		$\sigma^2 = 273.4331$		
OLS estimates Mean equation -	0.0000 (0.0000)	0.6682 (11.9525)*	0.2541 (2.5468)*	-804.8217
		$\sigma^2 = 260.1947$		
ARCH(1): Pooled regression (Model A+)	-2.8399 (-1.6972)^	0.9776 (28.4543)*	0.2277 (2.4925)*	-787.9334
		$\sigma_t^2 = \underset{(4.2384)^*}{95.2079} + \underset{(3.4747)^*}{1.1882}\,\hat{u}_{t-1}^2$		
ARCH(1): Pooled regression (Model A-)	1.9490 (2.4791)*	1.0235 (26.0731)*	0.1717 (2.2192)*	-787.2574
		$\sigma_t^2 = \underset{(5.0673)^*}{95.2152} + \underset{(4.0929)^*}{1.1682}\,\hat{u}_{t-1}^2$		
GARCH(1,1): Pooled regression (Model A+)	0.6619 (0.6373)	0.8240 (16.3027)*	0.0653 (1.2958)	-748.6255
		$\sigma_t^2 = \underset{(2.2908)^\wedge}{9.5079} + \underset{(8.3754)^*}{0.5560}\,\hat{\sigma}_{t-1}^2 + \underset{(6.1140)^*}{0.4052}\,\hat{u}_{t-1}^2$		
GARCH(1,1): Pooled regression (Model A-)	-1.7896 (-2.3765)^	0.7499 (11.8703)*	0.1261 (2.1306)*	-748.3421
		$\sigma_t^2 = \underset{(2.3528)^*}{11.9058} + \underset{(7.6565)^*}{0.5544}\,\hat{\sigma}_{t-1}^2 + \underset{(5.5673)^*}{0.3965}\,\hat{u}_{t-1}^2$		

These results have been obtained by direct maximization of the log-likelihood function by numerical methods. For each model we show the mean coefficients followed by the estimated equation for the conditional variance process. Values in parenthesis are t-ratios and the symbols *, ^, °, indicate significance levels of 1%, 5% and 10% respectively. The symbols + and – in the first column indicate that estimation is performed before and after removing individual effects in the mean equation.

8. Conclusion

In this paper we have proposed and implemented a methodology for the estimation of, and testing for, GARCH effects in panel data sets. Our method consists of the following steps: (i) testing for individual effects in the mean equation. (ii) Testing for ARCH effects using squared LSDV or OLS residuals (depending on whether individual effects are found or not in the previous stage), and testing for the presence of individual effects in the ARCH process. (iii) Estimating a few relevant GARCH specifications by maximum likelihood and comparing them by means of *LR* tests if they are nested.

The results of the Monte Carlo study suggest that the MLE estimator of the coefficients of the mean equation is generally less biased and more precise relative to their OLS counterpart. We also find that the patterns of bias and precision in panels with GARCH errors can be quite different from standard results in dynamic panels with non-correlated disturbances and that getting reliable estimates of the parameters of the GARCH process can be problematic in some cases.

The results of the empirical applications strongly suggest that the uncertainty associated with investment decisions (in the panel of 5 large U.S. manufacturing firms) as well as inflation (in the panel of 4 Latin American countries), can be well approximated by a pooled conditionally heteroskedastic error process. Our results show that accounting for this volatility clustering in the data may materially change the estimated effects of variables of interest.

Straightforward, but important, extensions of this work include panel GARCH-M models where the effects of uncertainty on the conditional mean can be tested, and panel GARCH models with exogenous variables in the conditional variance equation.

TABLE 1A: Monte Carlo results for static mean model and ARCH (1) errors ($\gamma_1 = 0.2$)

			OLS						MLE					
Sample	Coeff.	Value	Bias	(%)	Std. Dev.	(%)	MSE	(%)	Bias	(%)	Std. Dev.	(%)	MSE	(%)
N = 5	μ	1	-0.0048	-0.5	0.2261	22.6	0.0512	5.1	-0.0044	-0.4	0.2163	21.6	0.0468	4.7
T = 20	β	1	0.0086	0.9	0.3939	39.4	0.1553	15.5	0.0053	0.5	0.3731	37.3	0.1392	13.9
	α	1							-0.0195	-2.0	0.2001	20.0	0.0404	4.0
	γ_1	0.2							-0.0066	-3.3	0.1556	77.8	0.0243	12.2
N = 5	μ	1	-0.0092	-0.9	0.1439	14.4	0.0208	2.1	-0.0077	-0.8	0.1379	13.8	0.0191	1.9
T = 50	β	1	0.0212	2.1	0.2420	24.2	0.0590	5.9	0.0176	1.8	0.2304	23.0	0.0534	5.3
	α	1							-0.0028	-0.3	0.1275	12.8	0.0163	1.6
	γ_1	0.2							-0.0057	-2.9	0.0996	49.8	0.0100	5.0
N = 5	μ	1	0.0016	0.2	0.1051	10.5	0.0110	1.1	0.0009	0.1	0.0999	10.0	0.0100	1.0
T = 100	β	1	-0.0013	-0.1	0.1796	18.0	0.0323	3.2	-0.0006	-0.1	0.1711	17.1	0.0293	2.9
	α	1							0.0057	0.6	0.0952	9.5	0.0091	0.9
	γ_1	0.2							-0.0036	-1.8	0.0696	34.8	0.0049	2.5
N = 10	μ	1	-0.0017	-0.2	0.1582	15.8	0.0250	2.5	-0.0031	-0.3	0.1529	15.3	0.0234	2.3
T = 20	β	1	0.0033	0.3	0.28115	28.1	0.0791	7.9	0.0075	0.8	0.2711	27.1	0.0736	7.4
	α	1							0.0005	0.1	0.1480	14.8	0.0219	2.2
	γ_1	0.2							-0.0026	-1.3	0.1104	55.2	0.0122	6.1
N = 10	μ	1	0.0032	0.3	0.0980	9.8	0.0096	1.0	0.0032	0.3	0.0934	9.3	0.0087	0.9
T = 50	β	1	-0.0006	-0.1	0.1742	17.4	0.0303	3.0	-0.0007	-0.1	0.1678	16.8	0.0281	2.8
	α	1							-0.0005	-0.1	0.0883	8.8	0.0078	0.8
	γ_1	0.2							-0.0030	-1.5	0.0685	34.3	0.0047	2.4
N = 10	μ	1	0.0016	0.2	0.0720	7.2	0.0052	0.5	0.0019	0.2	0.0693	6.9	0.0048	0.5
T = 100	β	1	-0.0005	-0.1	0.1224	12.2	0.0150	1.5	-0.0009	-0.1	0.1175	11.8	0.0138	1.4
	α	1							-0.0017	-0.2	0.0634	6.3	0.0040	0.4
	γ_1	0.2							-0.0021	-1.1	0.0496	24.8	0.0025	1.3
N = 20	μ	1	0.0051	0.5	0.1178	11.8	0.0139	1.4	0.0058	0.6	0.1131	11.3	0.0128	1.3
T = 20	β	1	-0.0038	-0.4	0.1978	19.8	0.0391	3.9	-0.0049	-0.5	0.1894	18.9	0.0359	3.6
	α	1							-0.0040	-0.4	0.1009	10.1	0.0102	1.0
	γ_1	0.2							-0.0025	-1.3	0.0774	38.7	0.0060	3.0
N = 20	μ	1	-0.0013	-0.1	0.0697	7.0	0.0049	0.5	-0.0021	-0.2	0.0665	6.7	0.0044	0.4
T = 50	β	1	0.0072	0.7	0.1224	12.2	0.0150	1.5	0.0072	0.7	0.1182	11.8	0.0140	1.4
	α	1							-0.0034	-0.3	0.0596	6.0	0.0036	0.4
	γ_1	0.2							0.0012	0.6	0.0479	24.0	0.0023	1.2
N = 20	μ	1	0.0033	0.3	0.0505	5.1	0.0026	0.3	0.0033	0.3	0.0487	4.9	0.0024	0.2
T = 100	β	1	-0.0058	-0.6	0.0878	8.8	0.0077	0.8	-0.0059	-0.6	0.0852	8.5	0.0073	0.7
	α	1							-0.0001	0.0	0.0442	4.4	0.0020	0.2
	γ_1	0.2							-0.0027	-1.4	0.0343	17.2	0.0012	0.6

TABLE 2A: Monte Carlo results for static mean model and ARCH (1) errors ($\gamma_1 = 0.5$)

Sample	Coeff.	Value	OLS						MLE					
			Bias	(%)	Std. Dev.	(%)	MSE	(%)	Bias	(%)	Std. Dev.	(%)	MSE	(%)
N = 5	μ	1	0.0067	0.7	0.2787	27.9	0.0777	7.8	0.0166	1.7	0.2266	22.7	0.0516	5.2
T = 20	β	1	-0.0019	-0.2	0.5038	50.4	0.2539	25.4	-0.0252	-2.5	0.4130	41.3	0.1712	17.1
	α	1							0.0070	0.7	0.2483	24.8	0.0617	6.2
	γ_1	0.5							-0.0223	-4.5	0.2065	41.3	0.0432	8.6
N = 5	μ	1	-0.0020	-0.2	0.1704	17.0	0.0290	2.9	-0.0005	-0.1	0.1301	13.0	0.0169	1.7
T = 50	β	1	0.0045	0.5	0.3130	31.3	0.0980	9.8	0.0000	0.0	0.2363	23.6	0.0559	5.6
	α	1							0.0090	0.9	0.1489	14.9	0.0223	2.2
	γ_1	0.5							-0.0029	-0.6	0.1245	24.9	0.0155	3.1
N = 5	μ	1	0.0039	0.4	0.1292	12.9	0.0167	1.7	0.0035	0.4	0.1024	10.2	0.0105	1.1
T = 100	β	1	-0.0024	-0.2	0.2211	22.1	0.0489	4.9	-0.0032	-0.3	0.1717	17.2	0.0295	3.0
	α	1							-0.0030	-0.3	0.0982	9.8	0.0096	1.0
	γ_1	0.5							-0.0005	-0.1	0.0867	17.3	0.0075	1.5
N = 10	μ	1	-0.0147	-1.5	0.2010	20.1	0.0406	4.1	-0.0078	-0.8	0.1617	16.2	0.0262	2.6
T = 20	β	1	0.0342	3.4	0.3554	35.5	0.1275	12.8	0.0225	2.3	0.2896	29.0	0.0844	8.4
	α	1							-0.0124	-1.2	0.1673	16.7	0.0281	2.8
	γ_1	0.5							-0.0017	-0.3	0.1392	27.8	0.0193	3.9
N = 10	μ	1	0.0077	0.8	0.1231	12.3	0.0152	1.5	0.0057	0.6	0.0955	9.6	0.0092	0.9
T = 50	β	1	-0.0062	-0.6	0.2228	22.3	0.0497	5.0	-0.0036	-0.4	0.1708	17.1	0.0292	2.9
	α	1							0.0041	0.4	0.1016	10.2	0.0103	1.0
	γ_1	0.5							-0.0076	-1.5	0.0883	17.7	0.0079	1.6
N = 10	μ	1	0.0042	0.4	0.0891	8.9	0.0079	0.8	0.0010	0.1	0.0724	7.2	0.0052	0.5
T = 100	β	1	-0.0032	-0.3	0.1559	15.6	0.0243	2.4	0.0007	0.1	0.1241	12.4	0.0154	1.5
	α	1							-0.0014	-0.1	0.0695	7.0	0.0048	0.5
	γ_1	0.5							-0.0011	-0.2	0.0619	12.4	0.0038	0.8
N = 20	μ	1	0.0072	0.7	0.1345	13.5	0.0181	1.8	0.0075	0.8	0.1062	10.6	0.0113	1.1
T = 20	β	1	-0.0064	-0.6	0.2367	23.7	0.0561	5.6	-0.0113	-1.1	0.1873	18.7	0.0352	3.5
	α	1							0.0004	0.0	0.1162	11.6	0.0135	1.4
	γ_1	0.5							-0.0047	-0.9	0.0964	19.3	0.0093	1.9
N = 20	μ	1	0.0042	0.4	0.0956	9.6	0.0092	0.9	0.0036	0.4	0.0732	7.3	0.0054	0.5
T = 50	β	1	-0.0041	-0.4	0.1576	15.8	0.0249	2.5	-0.0032	-0.3	0.1224	12.2	0.0150	1.5
	α	1							-0.0030	-0.3	0.0691	6.9	0.0048	0.5
	γ_1	0.5							-0.0035	-0.7	0.0602	12.0	0.0036	0.7
N = 20	μ	1	0.0024	0.2	0.0628	6.3	0.0030	0.3	0.0008	0.1	0.0487	4.9	0.0024	0.2
T = 100	β	1	-0.0041	-0.4	0.1112	11.1	0.0124	1.2	-0.0017	-0.2	0.0841	8.4	0.0071	0.7
	α	1							-0.0015	-0.2	0.0488	4.9	0.0024	0.2
	γ_1	0.5							0.0013	0.3	0.0437	8.7	0.0019	0.4

TABLE 3A Monte Carlo results for static mean model and ARCH (1) errors ($\gamma_1 = 0.9$)

			OLS						MLE					
Sample	Coeff.	Value	Bias	(%)	Std. Dev.	(%)	MSE	(%)	Bias	(%)	Std. Dev.	(%)	MSE	(%)
N = 5 T = 20	μ	1	-0.0271	-2.7	0.7242	72.4	0.5253	52.5	-0.0056	-0.6	0.2489	24.9	0.0610	6.1
	β	1	0.0338	3.4	1.2232	122.3	1.4974	149.7	0.0123	1.2	0.4626	46.3	0.2141	21.4
	α	1							-0.0187	-1.9	0.2951	29.5	0.0875	8.8
	γ_1	0.9							0.0845	9.4	0.2315	25.7	0.0607	6.7
N = 5 T = 50	μ	1	-0.0083	-0.8	0.3396	34.0	0.1154	11.5	-0.0037	-0.4	0.1431	14.3	0.0205	2.1
	β	1	0.0234	2.3	0.5930	59.3	0.3522	35.2	0.0131	1.3	0.2468	24.7	0.0611	6.1
	α	1							-0.0165	-1.7	0.1705	17.1	0.0293	2.9
	γ_1	0.9							0.0367	4.1	0.1519	16.9	0.0244	2.7
N = 5 T = 100	μ	1	0.0030	0.3	0.2369	23.7	0.0562	5.6	0.0022	0.2	0.1008	10.1	0.0102	1.0
	β	1	0.0095	1.0	0.4063	40.6	0.1656	16.6	-0.0013	-0.1	0.1719	17.2	0.0296	3.0
	α	1							-0.0008	-0.1	0.1189	11.9	0.0141	1.4
	γ_1	0.9							0.0127	1.4	0.1072	11.9	0.0116	1.3
N = 10 T = 20	μ	1	0.0018	0.2	0.4282	42.8	0.1834	18.3	-0.0043	-0.4	0.1648	16.5	0.0272	2.7
	β	1	0.0197	2.0	0.8894	88.9	0.7914	79.1	0.0070	0.7	0.3024	30.2	0.0915	9.2
	α	1							-0.0270	-2.7	0.1908	19.1	0.0372	3.7
	γ_1	0.9							0.0901	10.0	0.1561	17.3	0.0325	3.6
N = 10 T = 50	μ	1	0.0003	0.0	0.2569	25.7	0.0660	6.6	-0.0004	0.0	0.1027	10.3	0.0105	1.1
	β	1	0.0090	0.9	0.4365	43.7	0.1906	19.1	0.0030	0.3	0.1745	17.5	0.0305	3.1
	α	1							-0.0087	-0.9	0.1144	11.4	0.0132	1.3
	γ_1	0.9							0.0330	3.7	0.1002	11.1	0.0111	1.2
N = 10 T = 100	μ	1	-0.0005	-0.1	0.1836	18.4	0.0337	3.4	0.0049	0.5	0.0679	6.8	0.0046	0.5
	β	1	0.0064	0.6	0.3083	30.8	0.0951	9.5	-0.0044	-0.4	0.1188	11.9	0.0141	1.4
	α	1							-0.0057	-0.6	0.0815	8.2	0.0067	0.7
	γ_1	0.9							0.0136	1.5	0.0762	8.5	0.0060	0.7
N = 20 T = 20	μ	1	-0.0038	-0.4	0.2757	27.6	0.0760	7.6	0.0053	0.5	0.1229	12.3	0.0151	1.5
	β	1	0.0139	1.4	0.4968	49.7	0.2470	24.7	-0.0052	-0.5	0.2147	21.5	0.0461	4.6
	α	1							-0.0171	-1.7	0.1342	13.4	0.0183	1.8
	γ_1	0.9							0.0767	8.5	0.1086	12.1	0.0177	2.0
N = 20 T = 50	μ	1	0.0073	0.7	0.1881	18.8	0.0354	3.5	0.0024	0.2	0.0732	7.3	0.0054	0.5
	β	1	-0.0196	-2.0	0.3272	32.7	0.1074	10.7	-0.0024	-0.2	0.1252	12.5	0.0157	1.6
	α	1							0.0006	0.1	0.0831	8.3	0.0069	0.7
	γ_1	0.9							0.0312	3.5	0.0730	8.1	0.0063	0.7
N = 20 T = 100	μ	1	-0.0018	-0.2	0.1218	12.2	0.0148	1.5	-0.0004	0.0	0.0484	4.8	0.0023	0.2
	β	1	0.0023	0.2	0.2042	20.4	0.0417	4.2	0.0014	0.1	0.0847	8.5	0.0072	0.7
	α	1							-0.0053	-0.5	0.0574	5.7	0.0033	0.3
	γ_1	0.9							0.0205	2.3	0.0540	6.0	0.0033	0.4

TABLE 4AMonte Carlo results for dynamic mean model ($\phi = 0.5$) and ARCH (1) errors ($\gamma_1 = 0.2$)

Sample	Coeff.	Value	OLS Bias	(%)	OLS Std. Dev.	(%)	OLS MSE	(%)	MLE Bias	(%)	MLE Std. Dev.	(%)	MLE MSE	(%)
N = 5	μ	1	0.0561	5.6	0.2222	22.2	0.0525	5.3	0.0493	4.9	0.2112	21.1	0.0470	4.7
T = 20	ϕ	0.5	-0.0276	-5.5	0.0940	18.8	0.0096	1.9	-0.0250	-5.0	0.0895	17.9	0.0086	1.7
	α	1							-0.0142	-1.4	0.2017	20.2	0.0409	4.1
	γ_1	0.2							-0.0114	-5.7	0.1505	75.3	0.0228	11.4
N = 5	μ	1	0.0237	2.4	0.1394	13.9	0.0200	2.0	0.0206	2.1	0.1329	13.3	0.0181	1.8
T = 50	ϕ	0.5	-0.0108	-2.2	0.0605	12.1	0.0038	0.8	-0.0095	-1.9	0.0573	11.5	0.0034	0.7
	α	1							0.0015	0.2	0.1247	12.5	0.0155	1.6
	γ_1	0.2							-0.0107	-5.4	0.1016	50.8	0.0104	5.2
N = 5	μ	1	0.0134	1.3	0.1008	10.1	0.0103	1.0	0.0116	1.2	0.0948	9.5	0.0091	0.9
T = 100	ϕ	0.5	-0.0061	-1.2	0.0444	8.9	0.0020	0.4	-0.0053	-1.1	0.0410	8.2	0.0017	0.3
	α	1							0.0064	0.6	0.0914	9.1	0.0084	0.8
	γ_1	0.2							-0.0046	-2.3	0.0681	34.1	0.0047	2.4
N = 10	μ	1	0.0218	2.2	0.1608	16.1	0.0263	2.6	0.0181	1.8	0.1535	15.4	0.0239	2.4
T = 20	ϕ	0.5	-0.0110	-2.2	0.0687	13.7	0.0048	1.0	-0.0088	-1.8	0.0659	13.2	0.0044	0.9
	α	1							-0.0011	-0.1	0.1524	15.2	0.0232	2.3
	γ_1	0.2							-0.0049	-2.5	0.1135	56.8	0.0129	6.5
N = 10	μ	1	0.0173	1.7	0.1039	10.4	0.0111	1.1	0.0155	1.6	0.0947	9.5	0.0092	0.9
T = 50	ϕ	0.5	-0.0073	-1.5	0.0450	9.0	0.0021	0.4	-0.0065	-1.3	0.0408	8.2	0.0017	0.3
	α	1							0.0016	0.2	0.0900	9.0	0.0081	0.8
	γ_1	0.2							-0.0047	-2.4	0.0703	35.2	0.0050	2.5
N = 10	μ	1	0.0092	0.9	0.0694	6.9	0.0049	0.5	0.0083	0.8	0.0648	6.5	0.0043	0.4
T = 100	ϕ	0.5	-0.0040	-0.8	0.0302	6.0	0.0009	0.2	-0.0035	-0.7	0.0276	5.5	0.0008	0.2
	α	1							-0.0001	0.0	0.0622	6.2	0.0039	0.4
	γ_1	0.2							-0.0033	-1.7	0.0488	24.4	0.0024	1.2
N = 20	μ	1	0.0153	1.5	0.1077	10.8	0.0118	1.2	0.0126	1.3	0.1014	10.1	0.0104	1.0
T = 20	ϕ	0.5	-0.0051	-1.0	0.0469	9.4	0.0022	0.4	-0.0039	-0.8	0.0437	8.7	0.0019	0.4
	α	1							-0.0053	-0.5	0.0986	9.9	0.0097	1.0
	γ_1	0.2							-0.0011	-0.6	0.0764	38.2	0.0058	2.9
N = 20	μ	1	0.0071	0.7	0.0691	6.9	0.0048	0.5	0.0060	0.6	0.0657	6.6	0.0044	0.4
T = 50	ϕ	0.5	-0.0024	-0.5	0.0294	5.9	0.0009	0.2	-0.0022	-0.4	0.0274	5.5	0.0008	0.2
	α	1							-0.0053	-0.5	0.0592	5.9	0.0035	0.4
	γ_1	0.2							0.0018	0.9	0.0481	24.1	0.0023	1.2
N = 20	μ	1	0.0030	0.3	0.0524	5.2	0.0028	0.3	0.0022	0.2	0.0494	4.9	0.0024	0.2
T = 100	ϕ	0.5	-0.0014	-0.3	0.0227	4.5	0.0005	0.1	-0.0010	-0.2	0.0212	4.2	0.0005	0.1
	α	1							0.0002	0.0	0.0441	4.4	0.0019	0.2
	γ_1	0.2							-0.0032	-1.6	0.0356	17.8	0.0013	0.7

TABLE 5A Monte Carlo results for dynamic mean model ($\phi = 0.5$) and ARCH (1) errors ($\gamma_1 = 0.5$)

Sample	Coeff.	Value	OLS Bias	(%)	OLS Std. Dev.	(%)	OLS MSE	(%)	MLE Bias	(%)	MLE Std. Dev.	(%)	MLE MSE	(%)
N = 5	μ	1	0.0546	5.5	0.2769	27.7	0.0797	8.0	0.0377	3.8	0.2112	21.1	0.0460	4.6
T = 20	ϕ	0.5	-0.0243	-4.9	0.1160	23.2	0.0141	2.8	-0.0168	-3.4	0.0870	17.4	0.0078	1.6
	α	1							0.0076	0.8	0.2518	25.2	0.0635	6.4
	γ_1	0.5							-0.0306	-6.1	0.2126	42.5	0.0461	9.2
N = 5	μ	1	0.0292	2.9	0.1838	18.4	0.0347	3.5	0.0170	1.7	0.1283	12.8	0.0167	1.7
T = 50	ϕ	0.5	-0.0149	-3.0	0.0813	16.3	0.0068	1.4	-0.0090	-1.8	0.0533	10.7	0.0029	0.6
	α	1							0.0099	1.0	0.1506	15.1	0.0228	2.3
	γ_1	0.5							-0.0067	-1.3	0.1274	25.5	0.0163	3.3
N = 5	μ	1	0.0124	1.2	0.1306	13.1	0.0172	1.7	0.0078	0.8	0.0913	9.1	0.0084	0.8
T = 100	ϕ	0.5	-0.0049	-1.0	0.0587	11.7	0.0035	0.7	-0.0031	-0.6	0.0377	7.5	0.0014	0.3
	α	1							-0.0023	-0.2	0.0995	10.0	0.0099	1.0
	γ_1	0.5							-0.0014	-0.3	0.0873	17.5	0.0076	1.5
N = 10	μ	1	0.0333	3.3	0.2029	20.3	0.0423	4.2	0.0261	2.6	0.1442	14.4	0.0215	2.2
T = 20	ϕ	0.5	-0.0149	-3.0	0.0864	17.3	0.0077	1.5	-0.0109	-2.2	0.0589	11.8	0.0036	0.7
	α	1							-0.0118	-1.2	0.1670	16.7	0.0280	2.8
	γ_1	0.5							-0.0049	-1.0	0.1459	29.2	0.0213	4.3
N = 10	μ	1	0.0237	2.4	0.1324	13.2	0.0181	1.8	0.0140	1.4	0.0890	8.9	0.0081	0.8
T = 50	ϕ	0.5	-0.0095	-1.9	0.0584	11.7	0.0035	0.7	-0.0047	-0.9	0.0375	7.5	0.0014	0.3
	α	1							0.0025	0.3	0.1016	10.2	0.0103	1.0
	γ_1	0.5							-0.0072	-1.4	0.0867	17.3	0.0076	1.5
N = 10	μ	1	0.0112	1.1	0.1012	10.1	0.0104	1.0	0.0065	0.7	0.0645	6.5	0.0042	0.4
T = 100	ϕ	0.5	-0.0043	-0.9	0.0455	9.1	0.0021	0.4	-0.0024	-0.5	0.0265	5.3	0.0007	0.1
	α	1							-0.0016	-0.2	0.0698	7.0	0.0049	0.5
	γ_1	0.5							-0.0019	-0.4	0.0629	12.6	0.0040	0.8
N = 20	μ	1	0.0163	1.6	0.1472	14.7	0.0219	2.2	0.0078	0.8	0.1033	10.3	0.0107	1.1
T = 20	ϕ	0.5	-0.0065	-1.3	0.0629	12.6	0.0040	0.8	-0.0023	-0.5	0.0419	8.4	0.0018	0.4
	α	1							-0.0020	-0.2	0.1174	11.7	0.0138	1.4
	γ_1	0.5							-0.0073	-1.5	0.0957	19.1	0.0092	1.8
N = 20	μ	1	0.0083	0.8	0.0999	10.0	0.0100	1.0	0.0061	0.6	0.0648	6.5	0.0042	0.4
T = 50	ϕ	0.5	-0.0029	-0.6	0.0443	8.9	0.0020	0.4	-0.0018	-0.4	0.0271	5.4	0.0007	0.1
	α	1							-0.0044	-0.4	0.0698	7.0	0.0049	0.5
	γ_1	0.5							-0.0030	-0.6	0.0610	12.2	0.0037	0.7
N = 20	μ	1	0.0031	0.3	0.0763	7.6	0.0058	0.6	0.0002	0.0	0.0435	4.4	0.0019	0.2
T = 100	ϕ	0.5	-0.0012	-0.2	0.0355	7.1	0.0013	0.3	-0.0001	0.0	0.0185	3.7	0.0003	0.1
	α	1							-0.0006	-0.1	0.0493	4.9	0.0024	0.2
	γ_1	0.5							0.0009	0.2	0.0433	8.7	0.0019	0.4

TABLE 6A: Monte Carlo results for dynamic mean model ($\phi = 0.5$) and ARCH (1) errors ($\gamma_1 = 0.9$)

			OLS						MLE					
Sample	Coeff.	Value	Bias	(%)	Std. Dev.	(%)	MSE	(%)	Bias	(%)	Std. Dev.	(%)	MSE	(%)
N = 5	μ	1	0.0710	7.1	0.4415	44.2	0.2000	20.0	0.0249	2.5	0.1956	19.6	0.0389	3.9
T = 20	ϕ	0.5	-0.0379	-7.6	0.1610	32.2	0.0273	5.5	-0.0124	-2.5	0.0706	14.1	0.0051	1.0
	α	1							-0.0025	-0.3	0.2927	29.3	0.0857	8.6
	γ_1	0.9							0.0386	4.3	0.2523	28.0	0.0651	7.2
N = 5	μ	1	0.0518	5.2	0.3038	30.4	0.0950	9.5	0.0192	1.9	0.1194	11.9	0.0146	1.5
T = 50	ϕ	0.5	-0.0234	-4.7	0.1238	24.8	0.0159	3.2	-0.0077	-1.5	0.0445	8.9	0.0020	0.4
	α	1							-0.0109	-1.1	0.1650	16.5	0.0273	2.7
	γ_1	0.9							0.0183	2.0	0.1577	17.5	0.0252	2.8
N = 5	μ	1	0.0597	6.0	0.2719	27.2	0.0775	7.8	0.0086	0.9	0.0788	7.9	0.0063	0.6
T = 100	ϕ	0.5	-0.0267	-5.3	0.1212	24.2	0.0154	3.1	-0.0033	-0.7	0.0298	6.0	0.0009	0.2
	α	1							-0.0012	-0.1	0.1146	11.5	0.0131	1.3
	γ_1	0.9							0.0093	1.0	0.1084	12.0	0.0118	1.3
N = 10	μ	1	0.0661	6.6	0.3571	35.7	0.1319	13.2	0.0088	0.9	0.1262	12.6	0.0160	1.6
T = 20	ϕ	0.5	-0.0309	-6.2	0.1479	29.6	0.0228	4.6	-0.0039	-0.8	0.0465	9.3	0.0022	0.4
	α	1							-0.0139	-1.4	0.1922	19.2	0.0371	3.7
	γ_1	0.9							0.0448	5.0	0.1632	18.1	0.0286	3.2
N = 10	μ	1	0.0481	4.8	0.2508	25.1	0.0652	6.5	0.0069	0.7	0.0758	7.6	0.0058	0.6
T = 50	ϕ	0.5	-0.0212	-4.2	0.1111	22.2	0.0128	2.6	-0.0037	-0.7	0.0291	5.8	0.0009	0.2
	α	1							-0.0077	-0.8	0.1167	11.7	0.0137	1.4
	γ_1	0.9							0.0242	2.7	0.1088	12.1	0.0124	1.4
N = 10	μ	1	0.0185	1.9	0.2133	21.3	0.0459	4.6	0.0035	0.4	0.0578	5.8	0.0034	0.3
T = 100	ϕ	0.5	-0.0081	-1.6	0.1009	20.2	0.0102	2.0	-0.0004	-0.1	0.0212	4.2	0.0005	0.1
	α	1							-0.0045	-0.5	0.0801	8.0	0.0064	0.6
	γ_1	0.9							0.0092	1.0	0.0780	8.7	0.0062	0.7
N = 20	μ	1	0.0325	3.3	0.2806	28.1	0.0798	8.0	0.0087	0.9	0.0869	8.7	0.0076	0.8
T = 20	ϕ	0.5	-0.0158	-3.2	0.1162	23.2	0.0138	2.8	-0.0041	-0.8	0.0312	6.2	0.0010	0.2
	α	1							-0.0201	-2.0	0.1274	12.7	0.0166	1.7
	γ_1	0.9							0.0503	5.6	0.1152	12.8	0.0158	1.8
N = 20	μ	1	0.0273	2.7	0.2295	23.0	0.0534	5.3	0.0043	0.4	0.0569	5.7	0.0033	0.3
T = 50	ϕ	0.5	-0.0145	-2.9	0.1072	21.4	0.0117	2.3	-0.0015	-0.3	0.0210	4.2	0.0004	0.1
	α	1							-0.0013	-0.1	0.0817	8.2	0.0067	0.7
	γ_1	0.9							0.0230	2.6	0.0752	8.4	0.0062	0.7
N = 20	μ	1	0.0217	2.2	0.2119	21.2	0.0454	4.5	0.0018	0.2	0.0391	3.9	0.0015	0.2
T = 100	ϕ	0.5	-0.0107	-2.1	0.1024	20.5	0.0106	2.1	-0.0007	-0.1	0.0146	2.9	0.0002	0.0
	α	1							-0.0064	-0.6	0.0575	5.8	0.0033	0.3
	γ_1	0.9							0.0164	1.8	0.0556	6.2	0.0034	0.4

TABLE 7A: Monte Carlo results for dynamic mean model ($\phi = 0.8$) and ARCH (1) errors ($\gamma_1 = 0.2$)

			OLS						MLE					
Sample	Coeff.	Value	Bias	(%)	Std. Dev.	(%)	MSE	(%)	Bias	(%)	Std. Dev.	(%)	MSE	(%)
N = 5	μ	1	0.0965	9.7	0.2735	27.4	0.0841	8.4	0.0829	8.3	0.2620	26.2	0.0755	7.6
T = 20	ϕ	0.8	-0.0195	-2.4	0.0499	6.2	0.0029	0.4	-0.0170	-2.1	0.0478	6.0	0.0026	0.3
	α	1							-0.0176	-1.8	0.1996	20.0	0.0401	4.0
	γ_1	0.2							-0.0083	-4.2	0.1497	74.9	0.0225	11.3
N = 5	μ	1	0.0459	4.6	0.1889	18.9	0.0378	3.8	0.0395	4.0	0.1827	18.3	0.0350	3.5
T = 50	ϕ	0.8	-0.0087	-1.1	0.0352	4.4	0.0013	0.2	-0.0075	-0.9	0.0339	4.2	0.0012	0.2
	α	1							-0.0006	-0.1	0.1249	12.5	0.0156	1.6
	γ_1	0.2							-0.0086	-4.3	0.1020	51.0	0.0105	5.3
N = 5	μ	1												
T = 100	ϕ	0.8												
	α	1												
	γ_1	0.2												
N = 10	μ	1	0.0376	3.8	0.1938	19.4	0.0390	3.9	0.0312	3.1	0.1854	18.5	0.0353	3.5
T = 20	ϕ	0.8	-0.0075	-0.9	0.0349	4.4	0.0013	0.2	-0.0060	-0.8	0.0335	4.2	0.0012	0.2
	α	1							-0.0038	-0.4	0.1516	15.2	0.0230	2.3
	γ_1	0.2							-0.0024	-1.2	0.1134	56.7	0.0129	6.5
N = 10	μ	1	0.0304	3.0	0.1367	13.7	0.0196	2.0	0.0260	2.6	0.1258	12.6	0.0165	1.7
T = 50	ϕ	0.8	-0.0055	-0.7	0.0253	3.2	0.0007	0.1	-0.0047	-0.6	0.0234	2.9	0.0006	0.1
	α	1							0.0008	0.1	0.0899	9.0	0.0081	0.8
	γ_1	0.2							-0.0037	-1.9	0.0701	35.1	0.0049	2.5
N = 10	μ	1												
T = 100	ϕ	0.8												
	α	1												
	γ_1	0.2												
N = 20	μ	1	0.0204	2.0	0.1251	12.5	0.0161	1.6	0.0173	1.7	0.1188	11.9	0.0144	1.4
T = 20	ϕ	0.8	-0.0031	-0.4	0.0229	2.9	0.0005	0.1	-0.0025	-0.3	0.0217	2.7	0.0005	0.1
	α	1							-0.0065	-0.7	0.0984	9.8	0.0097	1.0
	γ_1	0.2							0.0001	0.1	0.0760	38.0	0.0058	2.9
N = 20	μ	1	0.0125	1.3	0.0936	9.4	0.0089	0.9	0.0111	1.1	0.0897	9.0	0.0082	0.8
T = 50	ϕ	0.8	-0.0020	-0.3	0.0170	2.1	0.0003	0.0	-0.0019	-0.2	0.0162	2.0	0.0003	0.0
	α	1							-0.0057	-0.6	0.0593	5.9	0.0035	0.4
	γ_1	0.2							0.0022	1.1	0.0482	24.1	0.0023	1.2
N = 20	μ	1												
T = 100	ϕ	0.8												
	α	1												
	γ_1	0.2												

TABLE 8A: Monte Carlo results for dynamic mean model ($\phi = 0.8$) and ARCH (1) errors ($\gamma_1 = 0.5$)

Sample	Coeff.	Value	OLS Bias	(%)	OLS Std. Dev.	(%)	OLS MSE	(%)	MLE Bias	(%)	MLE Std. Dev.	(%)	MLE MSE	(%)
N = 5 T = 20	μ	1	0.0760	7.6	0.3176	31.8	0.1066	10.7	0.0507	5.1	0.2455	24.6	0.0628	6.3
	ϕ	0.8	-0.0144	-1.8	0.0557	7.0	0.0033	0.4	-0.0096	-1.2	0.0427	5.3	0.0019	0.2
	α	1							0.0024	0.2	0.2499	25.0	0.0625	6.3
	γ_1	0.5							-0.0255	-5.1	0.2091	41.8	0.0444	8.9
N = 5 T = 50	μ	1	0.0630	6.3	0.2305	23.1	0.0571	5.7	0.0353	3.5	0.1675	16.8	0.0293	2.9
	ϕ	0.8	-0.0129	-1.6	0.0433	5.4	0.0020	0.3	-0.0073	-0.9	0.0306	3.8	0.0010	0.1
	α	1							0.0075	0.8	0.1506	15.1	0.0227	2.3
	γ_1	0.5							-0.0049	-1.0	0.1273	25.5	0.0162	3.2
N = 5 T = 100	μ	1												
	ϕ	0.8												
	α	1												
	γ_1	0.5												
N = 10 T = 20	μ	1	0.0522	5.2	0.2150	21.5	0.0490	4.9	0.0367	3.7	0.1600	16.0	0.0269	2.7
	ϕ	0.8	-0.0098	-1.2	0.0379	4.7	0.0015	0.2	-0.0065	-0.8	0.0276	3.5	0.0008	0.1
	α	1							-0.0135	-1.4	0.1662	16.6	0.0278	2.8
	γ_1	0.5							-0.0030	-0.6	0.1445	28.9	0.0209	4.2
N = 10 T = 50	μ	1	0.0363	3.6	0.1630	16.3	0.0279	2.8	0.0242	2.4	0.1176	11.8	0.0144	1.4
	ϕ	0.8	-0.0063	-0.8	0.0298	3.7	0.0009	0.1	-0.0039	-0.5	0.0211	2.6	0.0005	0.1
	α	1							0.0016	0.2	0.1011	10.1	0.0102	1.0
	γ_1	0.5							-0.0066	-1.3	0.0865	17.3	0.0075	1.5
N = 10 T = 100	μ	1												
	ϕ	0.8												
	α	1												
	γ_1	0.5												
N = 20 T = 20	μ	1	0.0225	2.3	0.1616	16.2	0.0266	2.7	0.0103	1.0	0.1194	11.9	0.0144	1.4
	ϕ	0.8	-0.0038	-0.5	0.0287	3.6	0.0008	0.1	-0.0014	-0.2	0.0204	2.6	0.0004	0.1
	α	1							-0.0029	-0.3	0.1170	11.7	0.0137	1.4
	γ_1	0.5							-0.0067	-1.3	0.0958	19.2	0.0092	1.8
N = 20 T = 50	μ	1	0.0176	1.8	0.1194	11.9	0.0146	1.5	0.0130	1.3	0.0811	8.1	0.0067	0.7
	ϕ	0.8	-0.0030	-0.4	0.0220	2.8	0.0005	0.1	-0.0021	-0.3	0.0145	1.8	0.0002	0.0
	α	1							-0.0046	-0.5	0.0698	7.0	0.0049	0.5
	γ_1	0.5							-0.0028	-0.6	0.0609	12.2	0.0037	0.7
N = 20 T = 100	μ	1												
	ϕ	0.8												
	α	1												
	γ_1	0.5												

TABLE 9A: Monte Carlo results for dynamic mean model ($\phi = 0.8$) and ARCH (1) errors ($\gamma_1 = 0.9$)

			OLS						MLE					
Sample	Coeff.	Value	Bias	(%)	Std. Dev.	(%)	MSE	(%)	Bias	(%)	Std. Dev.	(%)	MSE	(%)
N = 5	μ	1	0.0992	9.9	0.4816	48.2	0.2418	24.2	0.0295	3.0	0.2007	20.1	0.0411	4.1
T = 20	ϕ	0.8	-0.0204	-2.6	0.0750	9.4	0.0060	0.8	-0.0063	-0.8	0.0308	3.9	0.0010	0.1
	α	1							-0.0078	-0.8	0.2937	29.4	0.0863	8.6
	γ_1	0.9							0.0419	4.7	0.2540	28.2	0.0663	7.4
N = 5	μ	1	0.0927	9.3	0.3566	35.7	0.1357	13.6	0.0277	2.8	0.1319	13.2	0.0182	1.8
T = 50	ϕ	0.8	-0.0177	-2.2	0.0594	7.4	0.0038	0.5	-0.0048	-0.6	0.0208	2.6	0.0005	0.1
	α	1							-0.0115	-1.2	0.1653	16.5	0.0274	2.7
	γ_1	0.9							0.0189	2.1	0.1578	17.5	0.0252	2.8
N = 5	μ	1												
T = 100	ϕ	0.8												
	α	1												
	γ_1	0.9												
N = 10	μ	1	0.0882	8.8	0.4096	41.0	0.1755	17.6	0.0146	1.5	0.1264	12.6	0.0162	1.6
T = 20	ϕ	0.8	-0.0170	-2.1	0.0734	9.2	0.0057	0.7	-0.0027	-0.3	0.0183	2.3	0.0003	0.0
	α	1							-0.0168	-1.7	0.1915	19.2	0.0370	3.7
	γ_1	0.9							0.0481	5.3	0.1639	18.2	0.0292	3.2
N = 10	μ	1	0.0700	7.0	0.2809	28.1	0.0838	8.4	0.0078	0.8	0.0841	8.4	0.0071	0.7
T = 50	ϕ	0.8	-0.0128	-1.6	0.0508	6.4	0.0027	0.3	-0.0016	-0.2	0.0135	1.7	0.0002	0.0
	α	1							-0.0083	-0.8	0.1165	11.7	0.0137	1.4
	γ_1	0.9							0.0247	2.7	0.1091	12.1	0.0125	1.4
N = 10	μ	1												
T = 100	ϕ	0.8												
	α	1												
	γ_1	0.9												
N = 20	μ	1	0.0350	3.5	0.2707	27.1	0.0745	7.5	0.0084	0.8	0.0830	8.3	0.0070	0.7
T = 20	ϕ	0.8	-0.0070	-0.9	0.0442	5.5	0.0020	0.3	-0.0016	-0.2	0.0115	1.4	0.0001	0.0
	α	1							-0.0209	-2.1	0.1278	12.8	0.0168	1.7
	γ_1	0.9							0.0514	5.7	0.1156	12.8	0.0160	1.8
N = 20	μ	1	0.0417	4.2	0.2742	27.4	0.0769	7.7	0.0043	0.4	0.0616	6.2	0.0038	0.4
T = 50	ϕ	0.8	-0.0088	-1.1	0.0520	6.5	0.0028	0.4	-0.0006	-0.1	0.0093	1.2	0.0001	0.0
	α	1							-0.0014	-0.1	0.0814	8.1	0.0066	0.7
	γ_1	0.9							0.0230	2.6	0.0750	8.3	0.0062	0.7
N = 20	μ	1												
T = 100	ϕ	0.8												
	α	1												
	γ_1	0.9												

TABLE 10A.Monte Carlo results for static mean model and GARCH (1,1) errors ($\gamma_1 = 0.1$, $\delta_1 = 0.1$)

			OLS						MLE					
Sample	Coeff.	Value	Bias	(%)	Std. Dev.	(%)	MSE	(%)	Bias	(%)	Std. Dev.	(%)	MSE	(%)
N = 5 T = 20	μ	1	-0.0049	-0.5	0.2258	22.6	0.0510	5.1	-0.0056	-0.6	0.2253	22.5	0.0508	5.1
	β	1	0.0081	0.8	0.3933	39.3	0.1547	15.5	0.0080	0.8	0.3884	38.8	0.1509	15.1
	α	1							-0.0632	-6.3	0.2776	27.8	0.0810	8.1
	γ_1	0.1							0.0171	17.1	0.1240	124.0	0.0157	15.7
	δ_1	0.1							0.0147	14.7	0.1770	177.0	0.0315	31.5
N = 5 T = 50	μ	1	-0.0091	-0.9	0.1439	14.4	0.0208	2.1	-0.0099	-1.0	0.1428	14.3	0.0205	2.1
	β	1	0.0210	2.1	0.2418	24.2	0.0589	5.9	0.0224	2.2	0.2402	24.0	0.0582	5.8
	α	1							-0.0499	-5.0	0.2661	26.6	0.0733	7.3
	γ_1	0.1							0.0038	3.8	0.0795	79.5	0.0063	6.3
	δ_1	0.1							0.0296	29.6	0.1962	196.2	0.0394	39.4
N = 5 T = 100	μ	1	0.0017	0.2	0.1052	10.5	0.0111	1.1	0.0027	0.3	0.1046	10.5	0.0110	1.1
	β	1	-0.0017	-0.2	0.1799	18.0	0.0324	3.2	-0.0032	-0.3	0.1793	17.9	0.0322	3.2
	α	1							-0.0527	-5.3	0.2578	25.8	0.0692	6.9
	γ_1	0.1							0.0007	0.7	0.0570	57.0	0.0033	3.3
	δ_1	0.1							0.0430	43.0	0.2018	201.8	0.0426	42.6
N = 10 T = 20	μ	1	0.0008	0.1	0.1528	15.3	0.0233	2.3	0.0003	0.0	0.1540	15.4	0.0237	2.4
	β	1	0.0026	0.3	0.2605	26.1	0.0679	6.8	0.0029	0.3	0.2622	26.2	0.0688	6.9
	α	1							-0.0480	-4.8	0.2662	26.6	0.0732	7.3
	γ_1	0.1							0.0049	4.9	0.0848	84.8	0.0072	7.2
	δ_1	0.1							0.0241	24.1	0.1911	191.1	0.0371	37.1
N = 10 T = 50	μ	1	0.0021	0.2	0.1054	10.5	0.0111	1.1	0.0019	0.2	0.1034	10.3	0.0107	1.1
	β	1	-0.0026	-0.3	0.1800	18.0	0.0324	3.2	-0.0022	-0.2	0.1776	17.8	0.0315	3.2
	α	1							-0.0512	-5.1	0.2561	25.6	0.0682	6.8
	γ_1	0.1							-0.0007	-0.7	0.0550	55.0	0.0030	3.0
	δ_1	0.1							0.0426	42.6	0.2009	200.9	0.0422	42.2
N = 10 T = 100	μ	1	-0.0015	-0.2	0.0735	7.4	0.0054	0.5	-0.0009	-0.1	0.0725	7.3	0.0053	0.5
	β	1	0.0058	0.6	0.1231	12.3	0.0152	1.5	0.0047	0.5	0.1216	12.2	0.0148	1.5
	α	1							-0.0646	-6.5	0.2389	23.9	0.0612	6.1
	γ_1	0.1							-0.0002	-0.2	0.0401	40.1	0.0016	1.6
	δ_1	0.1							0.0530	53.0	0.1948	194.8	0.0407	40.7
N = 20 T = 20	μ	1	-0.0018	-0.2	0.1101	11.0	0.0121	1.2	-0.0035	-0.4	0.1093	10.9	0.0120	1.2
	β	1	0.0045	0.5	0.1842	18.4	0.0339	3.4	0.0065	0.7	0.1831	18.3	0.0336	3.4
	α	1							-0.0607	-6.1	0.2679	26.8	0.0755	7.6
	γ_1	0.1							0.0027	2.7	0.0651	65.1	0.0042	4.2
	δ_1	0.1							0.0476	47.6	0.2052	205.2	0.0444	44.4
N = 20 T 50	μ	1	-0.0015	-0.2	0.0733	7.3	0.0054	0.5	-0.0008	-0.1	0.0722	7.2	0.0052	0.5
	β	1	0.0055	0.6	0.1229	12.3	0.0151	1.5	0.0048	0.5	0.1207	12.1	0.0146	1.5
	α	1							-0.0609	-6.1	0.2410	24.1	0.0618	6.2
	γ_1	0.1							0.0003	0.3	0.0413	41.3	0.0017	1.7
	δ_1	0.1							0.0501	50.1	0.1946	194.6	0.0404	40.4
N = 20 T = 100	μ	1	0.0026	0.3	0.0550	5.5	0.0030	0.3	0.0024	0.2	0.0539	5.4	0.0029	0.3
	β	1	-0.0016	-0.2	0.0924	9.2	0.0085	0.9	-0.0014	-0.1	0.0907	9.1	0.0082	0.8
	α	1							-0.0357	-3.6	0.2014	20.1	0.0418	4.2
	γ_1	0.1							-0.0015	-1.5	0.0292	29.2	0.0009	0.9
	δ_1	0.1							0.0302	30.2	0.1657	165.7	0.0284	28.4

TABLE 11A:Monte Carlo results for static mean model and GARCH (1,1) errors ($\gamma_1 = 0.1$, $\delta_1 = 0.4$

			OLS						MLE					
Sample	Coeff.	Value	Bias	(%)	Std. Dev.	(%)	MSE	(%)	Bias	(%)	Std. Dev.	(%)	MSE	(%)
N = 5	μ	1	0.0125	1.3	0.2812	28.1	0.0792	7.9	0.0209	2.1	0.2855	28.6	0.0819	8.2
T = 20	β	1	-0.0117	-1.2	0.5095	51.0	0.2597	26.0	-0.0289	-2.9	0.5195	52.0	0.2707	27.1
	α	1							0.2796	28.0	0.5733	57.3	0.4069	40.7
	γ_1	0.1							0.0251	25.1	0.1223	122.3	0.0156	15.6
	δ_1	0.4							-0.1767	-44.2	0.2682	67.1	0.1031	25.8
N = 5	μ	1	-0.0016	-0.2	0.1678	16.8	0.0282	2.8	-0.0022	-0.2	0.1623	16.2	0.0263	2.6
T = 50	β	1	0.0035	0.4	0.3096	31.0	0.0959	9.6	0.0039	0.4	0.3017	30.2	0.0910	9.1
	α	1							0.1848	18.5	0.5515	55.2	0.3384	33.8
	γ_1	0.1							0.0098	9.8	0.0739	73.9	0.0056	5.6
	δ_1	0.4							-0.1001	-25.0	0.2754	68.9	0.0858	21.5
N = 5	μ	1	0.0038	0.4	0.1293	12.9	0.0167	1.7	0.0043	0.4	0.1285	12.9	0.0165	1.7
T = 100	β	1	-0.0024	-0.2	0.2195	22.0	0.0482	4.8	-0.0043	-0.4	0.2184	21.8	0.0477	4.8
	α	1							0.1025	10.3	0.5042	50.4	0.2647	26.5
	γ_1	0.1							0.0065	6.5	0.0533	53.3	0.0029	2.9
	δ_1	0.4							-0.0596	-14.9	0.2651	66.3	0.0738	18.5
N = 10	μ	1	-0.0006	-0.1	0.2024	20.2	0.0409	4.1	-0.0017	-0.2	0.2010	20.1	0.0404	4.0
T = 20	β	1	-0.0014	-0.1	0.3717	37.2	0.1382	13.8	-0.0002	0.0	0.3687	36.9	0.1359	13.6
	α	1							0.2022	20.2	0.5551	55.5	0.3490	34.9
	γ_1	0.1							0.0188	18.8	0.0841	84.1	0.0074	7.4
	δ_1	0.4							-0.1198	-30.0	0.2792	69.8	0.0923	23.1
N = 10	μ	1	0.0041	0.4	0.1292	12.9	0.0167	1.7	0.0030	0.3	0.1284	12.8	0.0165	1.7
T = 50	β	1	-0.0026	-0.3	0.2194	21.9	0.0481	4.8	-0.0019	-0.2	0.2194	21.9	0.0481	4.8
	α	1							0.1173	11.7	0.4809	48.1	0.2450	24.5
	γ_1	0.1							0.0085	8.5	0.0567	56.7	0.0033	3.3
	δ_1	0.4							-0.0691	-17.3	0.2508	62.7	0.0677	16.9
N = 10	μ	1	0.0066	0.7	0.0885	8.9	0.0079	0.8	0.0065	0.7	0.0874	8.7	0.0077	0.8
T = 100	β	1	-0.0071	-0.7	0.1526	15.3	0.0234	2.3	-0.0064	-0.6	0.1510	15.1	0.0228	2.3
	α	1							0.0919	9.2	0.4377	43.8	0.2000	20.0
	γ_1	0.1							-0.0007	-0.7	0.0396	39.6	0.0016	1.6
	δ_1	0.4							-0.0475	-11.9	0.2300	57.5	0.0552	13.8
N = 20	μ	1	0.0033	0.3	0.1441	14.4	0.0208	2.1	0.0054	0.5	0.1413	14.1	0.0200	2.0
T = 20	β	1	-0.0021	-0.2	0.2423	24.2	0.0587	5.9	-0.0052	-0.5	0.2403	24.0	0.0578	5.8
	α	1							0.1212	12.1	0.5295	53.0	0.2951	29.5
	γ_1	0.1							0.0147	14.7	0.0612	61.2	0.0040	4.0
	δ_1	0.4							-0.0755	-18.9	0.2702	67.6	0.0787	19.7
N = 20	μ	1	0.0065	0.7	0.0884	8.8	0.0079	0.8	0.0061	0.6	0.0875	8.8	0.0077	0.8
T = 50	β	1	-0.0070	-0.7	0.1528	15.3	0.0234	2.3	-0.0073	-0.7	0.1513	15.1	0.0229	2.3
	α	1							0.0841	8.4	0.4516	45.2	0.2110	21.1
	γ_1	0.1							-0.0004	-0.4	0.0389	38.9	0.0015	1.5
	δ_1	0.4							-0.0447	-11.2	0.2392	59.8	0.0592	14.8
N = 20	μ	1	0.0060	0.6	0.0652	6.5	0.0043	0.4	0.0067	0.7	0.0638	6.4	0.0041	0.4
T = 100	β	1	-0.0072	-0.7	0.1137	11.4	0.0130	1.3	-0.0091	-0.9	0.1117	11.2	0.0126	1.3
	α	1							0.0410	4.1	0.3358	33.6	0.1145	11.5
	γ_1	0.1							0.0021	2.1	0.0266	26.6	0.0007	0.7
	δ_1	0.4							-0.0239	-6.0	0.1773	44.3	0.0320	8.0

TABLE 12A.Monte Carlo results for static mean model and GARCH (1,1) errors ($\gamma_1 = 0.1$, $\delta_1 = 0.8$)

Sample	Coeff.	Value	OLS Bias	(%)	Std. Dev.	(%)	MSE	(%)	MLE Bias	(%)	Std. Dev.	(%)	MSE	(%)
N = 5	μ	1	-0.0206	-2.1	0.6286	62.9	0.3956	39.6	0.0327	3.3	0.5866	58.7	0.3452	34.5
T = 20	β	1	0.0399	4.0	1.1784	117.8	1.3903	139.0	-0.0601	-6.0	1.1212	112.1	1.2607	126.1
	α	1							2.2554	225.5	2.4110	241.1	10.8999	1090.0
	γ_1	0.1							0.0238	23.8	0.1102	110.2	0.0127	12.7
	δ_1	0.8							-0.2958	-37.0	0.2861	35.8	0.1694	21.2
N = 5	μ	1	-0.0133	-1.3	0.3979	39.8	0.1585	15.9	-0.0202	-2.0	0.3746	37.5	0.1407	14.1
T = 50	β	1	0.0386	3.9	0.6886	68.9	0.4756	47.6	0.0468	4.7	0.6470	64.7	0.4208	42.1
	α	1							1.1121	111.2	1.9224	192.2	4.9323	493.2
	γ_1	0.1							0.0164	16.4	0.0652	65.2	0.0045	4.5
	δ_1	0.8							-0.1395	-17.4	0.2267	28.3	0.0708	8.9
N = 5	μ	1	0.0094	0.9	0.2795	28.0	0.0782	7.8	0.0155	1.6	0.2671	26.7	0.0716	7.2
T = 100	β	1	0.0021	0.2	0.4773	47.7	0.2279	22.8	-0.0120	-1.2	0.4553	45.5	0.2075	20.8
	α	1							0.4304	43.0	1.1743	117.4	1.5642	156.4
	γ_1	0.1							0.0054	5.4	0.0441	44.1	0.0020	2.0
	δ_1	0.8							-0.0517	-6.5	0.1464	18.3	0.0241	3.0
N = 10	μ	1	-0.0014	-0.1	0.4847	48.5	0.2350	23.5	-0.0099	-1.0	0.4606	46.1	0.2122	21.2
T = 20	β	1	0.0083	0.8	0.8069	80.7	0.6512	65.1	0.0158	1.6	0.7672	76.7	0.5889	58.9
	α	1							1.3767	137.7	2.1804	218.0	6.6494	664.9
	γ_1	0.1							0.0287	28.7	0.0798	79.8	0.0072	7.2
	δ_1	0.8							-0.1805	-22.6	0.2570	32.1	0.0986	12.3
N = 10	μ	1	0.0104	1.0	0.2812	28.1	0.0792	7.9	0.0118	1.2	0.2697	27.0	0.0728	7.3
T = 50	β	1	-0.0005	-0.1	0.4808	48.1	0.2312	23.1	-0.0031	-0.3	0.4591	45.9	0.2108	21.1
	α	1							0.4002	40.0	1.1495	115.0	1.4815	148.2
	γ_1	0.1							0.0051	5.1	0.0403	40.3	0.0017	1.7
	δ_1	0.8							-0.0493	-6.2	0.1443	18.0	0.0233	2.9
N = 10	μ	1	0.0024	0.2	0.2093	20.9	0.0438	4.4	0.0022	0.2	0.2017	20.2	0.0407	4.1
T = 100	β	1	0.0065	0.7	0.3604	36.0	0.1299	13.0	0.0036	0.4	0.3478	34.8	0.1210	12.1
	α	1							0.1725	17.3	0.6912	69.1	0.5075	50.8
	γ_1	0.1							0.0037	3.7	0.0301	30.1	0.0009	0.9
	δ_1	0.8							-0.0218	-2.7	0.0905	11.3	0.0087	1.1
N = 20	μ	1	-0.0084	-0.8	0.3091	30.9	0.0956	9.6	0.0014	0.1	0.2959	29.6	0.0876	8.8
T = 20	β	1	0.0312	3.1	0.5502	55.0	0.3037	30.4	0.0079	0.8	0.5262	52.6	0.2769	27.7
	α	1							0.6662	66.6	1.6466	164.7	3.1551	315.5
	γ_1	0.1							0.0151	15.1	0.0496	49.6	0.0027	2.7
	δ_1	0.8							-0.0859	-10.7	0.1884	23.6	0.0429	5.4
N = 20	μ	1	0.0019	0.2	0.2096	21.0	0.0439	4.4	0.0018	0.2	0.1992	19.9	0.0397	4.0
T = 50	β	1	0.0064	0.6	0.3625	36.3	0.1315	13.2	0.0048	0.5	0.3436	34.4	0.1181	11.8
	α	1							0.1847	18.5	0.6845	68.5	0.5026	50.3
	γ_1	0.1							0.0041	4.1	0.0294	29.4	0.0009	0.9
	δ_1	0.8							-0.0236	-3.0	0.0873	10.9	0.0082	1.0
N = 20	μ	1	0.0048	0.5	0.1342	13.4	0.0180	1.8	0.0020	0.2	0.1259	12.6	0.0159	1.6
T = 100	β	1	-0.0050	-0.5	0.2393	23.9	0.0573	5.7	-0.0005	-0.1	0.2254	22.5	0.0508	5.1
	α	1							0.0799	8.0	0.3623	36.2	0.1376	13.8
	γ_1	0.1							-0.0005	-0.5	0.0206	20.6	0.0004	0.4
	δ_1	0.8							-0.0084	-1.1	0.0507	6.3	0.0026	0.3

TABLE 13A: Monte Carlo results for dynamic mean model ($\phi = 0.5$) and GARCH (1,1) errors ($\gamma_1 = 0.1$, $\delta_1 = 0.1$)

			OLS						MLE					
Sample	Coeff.	Value	Bias	(%)	Std. Dev.	(%)	MSE	(%)	Bias	(%)	Std. Dev.	(%)	MSE	(%)
N = 5 T = 20	μ	1	0.0527	5.3	0.2132	21.3	0.0482	4.8	0.0486	4.9	0.2099	21.0	0.0464	4.6
	ϕ	0.5	-0.0261	-5.2	0.0885	17.7	0.0085	1.7	-0.0242	-4.8	0.0871	17.4	0.0082	1.6
	α	1							-0.0485	-4.9	0.2783	27.8	0.0798	8.0
	γ_1	0.1							0.0084	8.4	0.1185	118.5	0.0141	14.1
	δ_1	0.1							0.0113	11.3	0.1687	168.7	0.0286	28.6
N = 5 T = 50	μ	1	0.0221	2.2	0.1325	13.3	0.0181	1.8	0.0206	2.1	0.1335	13.4	0.0182	1.8
	ϕ	0.5	-0.0100	-2.0	0.0566	11.3	0.0033	0.7	-0.0092	-1.8	0.0566	11.3	0.0033	0.7
	α	1							-0.0434	-4.3	0.2599	26.0	0.0694	6.9
	γ_1	0.1							-0.0005	-0.5	0.0807	80.7	0.0065	6.5
	δ_1	0.1							0.0278	27.8	0.1948	194.8	0.0387	38.7
N = 5 T = 100	μ	1	0.0122	1.2	0.0960	9.6	0.0094	0.9	0.0110	1.1	0.0957	9.6	0.0093	0.9
	ϕ	0.5	-0.0055	-1.1	0.0414	8.3	0.0017	0.3	-0.0048	-1.0	0.0410	8.2	0.0017	0.3
	α	1							-0.0556	-5.6	0.2587	25.9	0.0700	7.0
	γ_1	0.1							-0.0013	-1.3	0.0565	56.5	0.0032	3.2
	δ_1	0.1							0.0474	47.4	0.2017	201.7	0.0429	42.9
N = 10 T = 20	μ	1	0.0214	2.1	0.1479	14.8	0.0223	2.2	0.0193	1.9	0.1467	14.7	0.0219	2.2
	ϕ	0.5	-0.0096	-1.9	0.0622	12.4	0.0040	0.8	-0.0085	-1.7	0.0621	12.4	0.0039	0.8
	α	1							-0.0512	-5.1	0.2698	27.0	0.0754	7.5
	γ_1	0.1							0.0020	2.0	0.0870	87.0	0.0076	7.6
	δ_1	0.1							0.0282	28.2	0.1958	195.8	0.0391	39.1
N = 10 T = 50	μ	1	0.0080	0.8	0.0943	9.4	0.0090	0.9	0.0066	0.7	0.0927	9.3	0.0086	0.9
	ϕ	0.5	-0.0037	-0.7	0.0404	8.1	0.0016	0.3	-0.0029	-0.6	0.0398	8.0	0.0016	0.3
	α	1							-0.0439	-4.4	0.2517	25.2	0.0653	6.5
	γ_1	0.1							0.0003	0.3	0.0567	56.7	0.0032	3.2
	δ_1	0.1							0.0362	36.2	0.1948	194.8	0.0393	39.3
N = 10 T = 100	μ	1	0.0068	0.7	0.0709	7.1	0.0051	0.5	0.0068	0.7	0.0690	6.9	0.0048	0.5
	ϕ	0.5	-0.0025	-0.5	0.0291	5.8	0.0009	0.2	-0.0024	-0.5	0.0284	5.7	0.0008	0.2
	α	1							-0.0561	-5.6	0.2327	23.3	0.0573	5.7
	γ_1	0.1							0.0000	0.0	0.0395	39.5	0.0016	1.6
	δ_1	0.1							0.0456	45.6	0.1889	188.9	0.0378	37.8
N = 20 T = 20	μ	1	0.0088	0.9	0.1063	10.6	0.0114	1.1	0.0110	1.1	0.1039	10.4	0.0109	1.1
	ϕ	0.5	-0.0043	-0.9	0.0446	8.9	0.0020	0.4	-0.0050	-1.0	0.0436	8.7	0.0019	0.4
	α	1							-0.0633	-6.3	0.2712	27.1	0.0775	7.8
	γ_1	0.1							0.0017	1.7	0.0652	65.2	0.0042	4.2
	δ_1	0.1							0.0494	49.4	0.2069	206.9	0.0453	45.3
N = 20 T = 50	μ	1	0.0062	0.6	0.0706	7.1	0.0050	0.5	0.0058	0.6	0.0689	6.9	0.0048	0.5
	ϕ	0.5	-0.0024	-0.5	0.0298	6.0	0.0009	0.2	-0.0024	-0.5	0.0290	5.8	0.0008	0.2
	α	1							-0.0546	-5.5	0.2374	23.7	0.0594	5.9
	γ_1	0.1							0.0001	0.1	0.0423	42.3	0.0018	1.8
	δ_1	0.1							0.0454	45.4	0.1903	190.3	0.0383	38.3
N = 20 T = 100	μ	1	0.0040	0.4	0.0494	4.9	0.0025	0.3	0.0042	0.4	0.0475	4.8	0.0023	0.2
	ϕ	0.5	-0.0009	-0.2	0.0209	4.2	0.0004	0.1	-0.0009	-0.2	0.0199	4.0	0.0004	0.1
	α	1							-0.0280	-2.8	0.1949	19.5	0.0388	3.9
	γ_1	0.1							-0.0018	-1.8	0.0295	29.5	0.0009	0.9
	δ_1	0.1							0.0245	24.5	0.1604	160.4	0.0263	26.3

TABLE 14A: Monte Carlo results for dynamic mean model ($\phi = 0.5$) and GARCH (1,1) errors ($\gamma_1 = 0.1$, $\delta_1 = 0.4$)

Sample	Coeff.	Value	OLS Bias	(%)	OLS Std. Dev.	(%)	OLS MSE	(%)	MLE Bias	(%)	MLE Std. Dev.	(%)	MLE MSE	(%)
N = 5 T = 20	μ	1	0.0456	4.6	0.2326	23.3	0.0562	5.6	0.0403	4.0	0.2299	23.0	0.0545	5.5
	ϕ	0.5	-0.0194	-3.9	0.0885	17.7	0.0082	1.6	-0.0173	-3.5	0.0896	17.9	0.0083	1.7
	α	1							0.2911	29.1	0.5493	54.9	0.3864	38.6
	γ_1	0.1							0.0220	22.0	0.1233	123.3	0.0157	15.7
	δ_1	0.4							-0.1834	-45.9	0.2608	65.2	0.1016	25.4
N = 5 T = 50	μ	1	0.0206	2.1	0.1498	15.0	0.0229	2.3	0.0209	2.1	0.1480	14.8	0.0223	2.2
	ϕ	0.5	-0.0104	-2.1	0.0591	11.8	0.0036	0.7	-0.0104	-2.1	0.0576	11.5	0.0034	0.7
	α	1							0.1882	18.8	0.5531	55.3	0.3414	34.1
	γ_1	0.1							0.0079	7.9	0.0758	75.8	0.0058	5.8
	δ_1	0.4							-0.1017	-25.4	0.2767	69.2	0.0869	21.7
N = 5 T = 100	μ	1	0.0128	1.3	0.1041	10.4	0.0110	1.1	0.0133	1.3	0.1036	10.4	0.0109	1.1
	ϕ	0.5	-0.0057	-1.1	0.0418	8.4	0.0018	0.4	-0.0053	-1.1	0.0414	8.3	0.0017	0.3
	α	1							0.1281	12.8	0.5005	50.1	0.2669	26.7
	γ_1	0.1							0.0037	3.7	0.0541	54.1	0.0029	2.9
	δ_1	0.4							-0.0667	-16.7	0.2587	64.7	0.0714	17.9
N = 10 T = 20	μ	1	0.0204	2.0	0.1640	16.4	0.0273	2.7	0.0138	1.4	0.1631	16.3	0.0268	2.7
	ϕ	0.5	-0.0113	-2.3	0.0640	12.8	0.0042	0.8	-0.0090	-1.8	0.0636	12.7	0.0041	0.8
	α	1							0.2272	22.7	0.5496	55.0	0.3537	35.4
	γ_1	0.1							0.0171	17.1	0.0855	85.5	0.0076	7.6
	δ_1	0.4							-0.1310	-32.8	0.2707	67.7	0.0905	22.6
N = 10 T = 50	μ	1	0.0080	0.8	0.1024	10.2	0.0105	1.1	0.0068	0.7	0.0999	10.0	0.0100	1.0
	ϕ	0.5	-0.0036	-0.7	0.0407	8.1	0.0017	0.3	-0.0032	-0.6	0.0398	8.0	0.0016	0.3
	α	1							0.1614	16.1	0.5084	50.8	0.2845	28.5
	γ_1	0.1							0.0040	4.0	0.0545	54.5	0.0030	3.0
	δ_1	0.4							-0.0847	-21.2	0.2623	65.6	0.0760	19.0
N = 10 T = 100	μ	1	0.0073	0.7	0.0773	7.7	0.0060	0.6	0.0063	0.6	0.0756	7.6	0.0058	0.6
	ϕ	0.5	-0.0025	-0.5	0.0294	5.9	0.0009	0.2	-0.0021	-0.4	0.0287	5.7	0.0008	0.2
	α	1							0.0924	9.2	0.4252	42.5	0.1893	18.9
	γ_1	0.1							0.0027	2.7	0.0387	38.7	0.0015	1.5
	δ_1	0.4							-0.0493	-12.3	0.2261	56.5	0.0536	13.4
N = 20 T = 20	μ	1	0.0129	1.3	0.1151	11.5	0.0134	1.3	0.0095	1.0	0.1121	11.2	0.0127	1.3
	ϕ	0.5	-0.0054	-1.1	0.0451	9.0	0.0021	0.4	-0.0041	-0.8	0.0440	8.8	0.0019	0.4
	α	1							0.1517	15.2	0.5168	51.7	0.2901	29.0
	γ_1	0.1							0.0135	13.5	0.0645	64.5	0.0043	4.3
	δ_1	0.4							-0.0915	-22.9	0.2657	66.4	0.0789	19.7
N = 20 T = 50	μ	1	0.0077	0.8	0.0722	7.2	0.0053	0.5	0.0070	0.7	0.0713	7.1	0.0051	0.5
	ϕ	0.5	-0.0023	-0.5	0.0288	5.8	0.0008	0.2	-0.0018	-0.4	0.0281	5.6	0.0008	0.2
	α	1							0.0908	9.1	0.4477	44.8	0.2087	20.9
	γ_1	0.1							-0.0006	-0.6	0.0388	38.8	0.0015	1.5
	δ_1	0.4							-0.0481	-12.0	0.2374	59.4	0.0587	14.7
N = 20 T = 100	μ	1	0.0045	0.5	0.0535	5.4	0.0029	0.3	0.0045	0.5	0.0518	5.2	0.0027	0.3
	ϕ	0.5	-0.0009	-0.2	0.0211	4.2	0.0004	0.1	-0.0008	-0.2	0.0202	4.0	0.0004	0.1
	α	1							0.0688	6.9	0.3556	35.6	0.1312	13.1
	γ_1	0.1							-0.0004	-0.4	0.0286	28.6	0.0008	0.8
	δ_1	0.4							-0.0345	-8.6	0.1887	47.2	0.0368	9.2

TABLE 15A:Monte Carlo results for dynamic mean model ($\phi = 0.5$) and GARCH (1,1) errors ($\gamma_1 = 0.1$, $\delta_1 = 0.8$)

Sample	Coeff.	Value	OLS Bias	(%)	OLS Std. Dev.	(%)	OLS MSE	(%)	MLE Bias	(%)	MLE Std. Dev.	(%)	MLE MSE	(%)
N = 5 T = 20	μ	1	0.0386	3.9	0.3727	37.3	0.1404	14.0	0.0402	4.0	0.3520	35.2	0.1256	12.6
	ϕ	0.5	-0.0215	-4.3	0.0889	17.8	0.0084	1.7	-0.0223	-4.5	0.0861	17.2	0.0079	1.6
	α	1							2.3415	234.2	2.4830	248.3	11.6476	1164.8
	γ_1	0.1							0.0255	25.5	0.1142	114.2	0.0137	13.7
	δ_1	0.8							-0.3054	-38.2	0.2930	36.6	0.1792	22.4
N = 5 T = 50	μ	1	0.0284	2.8	0.2432	24.3	0.0599	6.0	0.0245	2.5	0.2302	23.0	0.0536	5.4
	ϕ	0.5	-0.0107	-2.1	0.0614	12.3	0.0039	0.8	-0.0100	-2.0	0.0574	11.5	0.0034	0.7
	α	1							1.0790	107.9	1.9828	198.3	5.0958	509.6
	γ_1	0.1							0.0138	13.8	0.0634	63.4	0.0042	4.2
	δ_1	0.8							-0.1334	-16.7	0.2278	28.5	0.0697	8.7
N = 5 T = 100	μ	1	0.0150	1.5	0.1649	16.5	0.0274	2.7	0.0142	1.4	0.1591	15.9	0.0255	2.6
	ϕ	0.5	-0.0046	-0.9	0.0417	8.3	0.0018	0.4	-0.0041	-0.8	0.0400	8.0	0.0016	0.3
	α	1							0.5017	50.2	1.2984	129.8	1.9375	193.8
	γ_1	0.1							0.0072	7.2	0.0428	42.8	0.0019	1.9
	δ_1	0.8							-0.0608	-7.6	0.1567	19.6	0.0282	3.5
N = 10 T = 20	μ	1	0.0306	3.1	0.2722	27.2	0.0750	7.5	0.0148	1.5	0.2582	25.8	0.0669	6.7
	ϕ	0.5	-0.0139	-2.8	0.0645	12.9	0.0043	0.9	-0.0111	-2.2	0.0616	12.3	0.0039	0.8
	α	1							1.4001	140.0	2.1211	212.1	6.4596	646.0
	γ_1	0.1							0.0313	31.3	0.0838	83.8	0.0080	8.0
	δ_1	0.8							-0.1863	-23.3	0.2572	32.2	0.1009	12.6
N = 10 T = 50	μ	1	0.0081	0.8	0.1652	16.5	0.0273	2.7	0.0073	0.7	0.1539	15.4	0.0237	2.4
	ϕ	0.5	-0.0035	-0.7	0.0424	8.5	0.0018	0.4	-0.0024	-0.5	0.0396	7.9	0.0016	0.3
	α	1							0.4630	46.3	1.3161	131.6	1.9465	194.7
	γ_1	0.1							0.0051	5.1	0.0426	42.6	0.0018	1.8
	δ_1	0.8							-0.0536	-6.7	0.1564	19.6	0.0273	3.4
N = 10 T = 100	μ	1	0.0100	1.0	0.1241	12.4	0.0155	1.6	0.0079	0.8	0.1166	11.7	0.0136	1.4
	ϕ	0.5	-0.0026	-0.5	0.0308	6.2	0.0010	0.2	-0.0019	-0.4	0.0285	5.7	0.0008	0.2
	α	1							0.1895	19.0	0.6673	66.7	0.4812	48.1
	γ_1	0.1							0.0020	2.0	0.0287	28.7	0.0008	0.8
	δ_1	0.8							-0.0213	-2.7	0.0837	10.5	0.0075	0.9
N = 20 T = 20	μ	1	0.0197	2.0	0.1807	18.1	0.0330	3.3	0.0241	2.4	0.1757	17.6	0.0315	3.2
	ϕ	0.5	-0.0046	-0.9	0.0461	9.2	0.0021	0.4	-0.0054	-1.1	0.0447	8.9	0.0020	0.4
	α	1							0.6058	60.6	1.5930	159.3	2.9046	290.5
	γ_1	0.1							0.0155	15.5	0.0509	50.9	0.0028	2.8
	δ_1	0.8							-0.0808	-10.1	0.1873	23.4	0.0416	5.2
N = 20 T = 50	μ	1	0.0130	1.3	0.1185	11.9	0.0142	1.4	0.0116	1.2	0.1129	11.3	0.0129	1.3
	ϕ	0.5	-0.0036	-0.7	0.0293	5.9	0.0009	0.2	-0.0032	-0.6	0.0278	5.6	0.0008	0.2
	α	1							0.1922	19.2	0.7300	73.0	0.5698	57.0
	γ_1	0.1							0.0047	4.7	0.0299	29.9	0.0009	0.9
	δ_1	0.8							-0.0248	-3.1	0.0932	11.7	0.0093	1.2
N = 20 T = 100	μ	1	0.0074	0.7	0.0847	8.5	0.0072	0.7	0.0077	0.8	0.0797	8.0	0.0064	0.6
	ϕ	0.5	-0.0009	-0.2	0.0221	4.4	0.0005	0.1	-0.0008	-0.2	0.0200	4.0	0.0004	0.1
	α	1							0.0608	6.1	0.3435	34.4	0.1217	12.2
	γ_1	0.1							-0.0004	-0.4	0.0203	20.3	0.0004	0.4
	δ_1	0.8							-0.0060	-0.8	0.0468	5.9	0.0022	0.3

TABLE 16A: Monte Carlo results for dynamic mean model ($\phi = 0.8$) and GARCH (1,1) errors ($\gamma_1 = 0.1$, $\delta_1 = 0.1$)

Sample	Coeff.	Value	OLS Bias	(%)	OLS Std. Dev.	(%)	OLS MSE	(%)	MLE Bias	(%)	MLE Std. Dev.	(%)	MLE MSE	(%)
N = 5	μ	1	0.0933	9.3	0.2679	26.8	0.0804	8.0	0.0857	8.6	0.2634	26.3	0.0767	7.7
T = 20	ϕ	0.8	-0.0189	-2.4	0.0487	6.1	0.0027	0.3	-0.0175	-2.2	0.0482	6.0	0.0026	0.3
	α	1							-0.0692	-6.9	0.2956	29.6	0.0922	9.2
	γ_1	0.1							0.0134	13.4	0.1188	118.8	0.0143	14.3
	δ_1	0.1							0.0249	24.9	0.1890	189.0	0.0364	36.4
N = 5	μ	1	0.0438	4.4	0.1831	18.3	0.0354	3.5	0.0420	4.2	0.1852	18.5	0.0361	3.6
T = 50	ϕ	0.8	-0.0083	-1.0	0.0340	4.3	0.0012	0.2	-0.0080	-1.0	0.0343	4.3	0.0012	0.2
	α	1							-0.0425	-4.3	0.2537	25.4	0.0662	6.6
	γ_1	0.1							0.0015	1.5	0.0812	81.2	0.0066	6.6
	δ_1	0.1							0.0257	25.7	0.1888	188.8	0.0363	36.3
N = 5	μ	1	0.0249	2.5	0.1357	13.6	0.0190	1.9	0.0243	2.4	0.1333	13.3	0.0184	1.8
T = 100	ϕ	0.8	-0.0047	-0.6	0.0254	3.2	0.0007	0.1	-0.0045	-0.6	0.0248	3.1	0.0006	0.1
	α	1							-0.0626	-6.3	0.2628	26.3	0.0730	7.3
	γ_1	0.1							-0.0003	-0.3	0.0557	55.7	0.0031	3.1
	δ_1	0.1							0.0521	52.1	0.2057	205.7	0.0450	45.0
N = 10	μ	1	0.0377	3.8	0.1825	18.3	0.0347	3.5	0.0359	3.6	0.1802	18.0	0.0338	3.4
T = 20	ϕ	0.8	-0.0071	-0.9	0.0331	4.1	0.0011	0.1	-0.0067	-0.8	0.0329	4.1	0.0011	0.1
	α	1							-0.0474	-4.7	0.2633	26.3	0.0716	7.2
	γ_1	0.1							0.0040	4.0	0.0876	87.6	0.0077	7.7
	δ_1	0.1							0.0235	23.5	0.1896	189.6	0.0365	36.5
N = 10	μ	1	0.0189	1.9	0.1304	13.0	0.0174	1.7	0.0182	1.8	0.1284	12.8	0.0168	1.7
T = 50	ϕ	0.8	-0.0036	-0.5	0.0238	3.0	0.0006	0.1	-0.0035	-0.4	0.0236	3.0	0.0006	0.1
	α	1							-0.0492	-4.9	0.2574	25.7	0.0687	6.9
	γ_1	0.1							0.0002	0.2	0.0566	56.6	0.0032	3.2
	δ_1	0.1							0.0406	40.6	0.2012	201.2	0.0421	42.1
N = 10	μ	1	0.0154	1.5	0.1005	10.1	0.0103	1.0	0.0130	1.3	0.0983	9.8	0.0098	1.0
T = 100	ϕ	0.8	-0.0027	-0.3	0.0179	2.2	0.0003	0.0	-0.0022	-0.3	0.0175	2.2	0.0003	0.0
	α	1							-0.0551	-5.5	0.2310	23.1	0.0564	5.6
	γ_1	0.1							0.0005	0.5	0.0395	39.5	0.0016	1.6
	δ_1	0.1							0.0445	44.5	0.1880	188.0	0.0373	37.3
N = 20	μ	1	0.0185	1.9	0.1256	12.6	0.0161	1.6	0.0195	2.0	0.1236	12.4	0.0156	1.6
T = 20	ϕ	0.8	-0.0037	-0.5	0.0221	2.8	0.0005	0.1	-0.0039	-0.5	0.0218	2.7	0.0005	0.1
	α	1							-0.0544	-5.4	0.2589	25.9	0.0700	7.0
	γ_1	0.1							0.0029	2.9	0.0649	64.9	0.0042	4.2
	δ_1	0.1							0.0413	41.3	0.1958	195.8	0.0400	40.0
N = 20	μ	1	0.0144	1.4	0.0945	9.5	0.0091	0.9	0.0147	1.5	0.0927	9.3	0.0088	0.9
T = 50	ϕ	0.8	-0.0025	-0.3	0.0170	2.1	0.0003	0.0	-0.0026	-0.3	0.0167	2.1	0.0003	0.0
	α	1							-0.0578	-5.8	0.2402	24.0	0.0610	6.1
	γ_1	0.1							0.0002	0.2	0.0424	42.4	0.0018	1.8
	δ_1	0.1							0.0479	47.9	0.1933	193.3	0.0397	39.7
N = 20	μ	1	0.0079	0.8	0.0706	7.1	0.0050	0.5	0.0076	0.8	0.0682	6.8	0.0047	0.5
T = 100	ϕ	0.8	-0.0011	-0.1	0.0130	1.6	0.0002	0.0	-0.0011	-0.1	0.0126	1.6	0.0002	0.0
	α	1							-0.0279	-2.8	0.1928	19.3	0.0380	3.8
	γ_1	0.1							-0.0016	-1.6	0.0293	29.3	0.0009	0.9
	δ_1	0.1							0.0243	24.3	0.1582	158.2	0.0256	25.6

TABLE 17A:Monte Carlo results for dynamic mean model ($\phi = 0.8$) and GARCH (1,1) errors ($\gamma_1 = 0.1$, $\delta_1 = 0.4$)

			OLS						MLE					
Sample	Coeff.	Value	Bias	(%)	Std. Dev.	(%)	MSE	(%)	Bias	(%)	Std. Dev.	(%)	MSE	(%)
N = 5	μ	1	0.0696	7.0	0.2855	28.6	0.0864	8.6	0.0613	6.1	0.2793	27.9	0.0818	8.2
T = 20	ϕ	0.8	-0.0129	-1.6	0.0482	6.0	0.0025	0.3	-0.0114	-1.4	0.0477	6.0	0.0024	0.3
	α	1							0.2827	28.3	0.5570	55.7	0.3902	39.0
	γ_1	0.1							0.0252	25.2	0.1228	122.8	0.0157	15.7
	δ_1	0.4							-0.1808	-45.2	0.2623	65.6	0.1015	25.4
N = 5	μ	1	0.0501	5.0	0.1986	19.9	0.0419	4.2	0.0438	4.4	0.1982	19.8	0.0412	4.1
T = 50	ϕ	0.8	-0.0101	-1.3	0.0352	4.4	0.0013	0.2	-0.0091	-1.1	0.0352	4.4	0.0013	0.2
	α	1							0.1846	18.5	0.5498	55.0	0.3364	33.6
	γ_1	0.1							0.0109	10.9	0.0762	76.2	0.0059	5.9
	δ_1	0.4							-0.1027	-25.7	0.2747	68.7	0.0860	21.5
N = 5	μ	1	0.0318	3.2	0.1455	14.6	0.0222	2.2	0.0339	3.4	0.1446	14.5	0.0221	2.2
T = 100	ϕ	0.8	-0.0058	-0.7	0.0259	3.2	0.0007	0.1	-0.0064	-0.8	0.0255	3.2	0.0007	0.1
	α	1							0.1200	12.0	0.5062	50.6	0.2706	27.1
	γ_1	0.1							0.0058	5.8	0.0544	54.4	0.0030	3.0
	δ_1	0.4							-0.0689	-17.2	0.2636	65.9	0.0743	18.6
N = 10	μ	1	0.0365	3.7	0.1950	19.5	0.0394	3.9	0.0305	3.1	0.1956	19.6	0.0392	3.9
T = 20	ϕ	0.8	-0.0076	-1.0	0.0327	4.1	0.0011	0.1	-0.0067	-0.8	0.0326	4.1	0.0011	0.1
	α	1							0.2334	23.3	0.5388	53.9	0.3448	34.5
	γ_1	0.1							0.0201	20.1	0.0865	86.5	0.0079	7.9
	δ_1	0.4							-0.1368	-34.2	0.2660	66.5	0.0895	22.4
N = 10	μ	1	0.0189	1.9	0.1377	13.8	0.0193	1.9	0.0171	1.7	0.1324	13.2	0.0178	1.8
T = 50	ϕ	0.8	-0.0036	-0.5	0.0241	3.0	0.0006	0.1	-0.0032	-0.4	0.0233	2.9	0.0006	0.1
	α	1							0.1620	16.2	0.5079	50.8	0.2842	28.4
	γ_1	0.1							0.0050	5.0	0.0547	54.7	0.0030	3.0
	δ_1	0.4							-0.0860	-21.5	0.2634	65.9	0.0768	19.2
N = 10	μ	1	0.0160	1.6	0.1064	10.6	0.0116	1.2	0.0147	1.5	0.1040	10.4	0.0110	1.1
T = 100	ϕ	0.8	-0.0027	-0.3	0.0182	2.3	0.0003	0.0	-0.0025	-0.3	0.0177	2.2	0.0003	0.0
	α	1							0.0931	9.3	0.4326	43.3	0.1958	19.6
	γ_1	0.1							0.0030	3.0	0.0388	38.8	0.0015	1.5
	δ_1	0.4							-0.0498	-12.5	0.2303	57.6	0.0555	13.9
N = 20	μ	1	0.0201	2.0	0.1352	13.5	0.0187	1.9	0.0172	1.7	0.1332	13.3	0.0180	1.8
T = 20	ϕ	0.8	-0.0036	-0.5	0.0224	2.8	0.0005	0.1	-0.0032	-0.4	0.0223	2.8	0.0005	0.1
	α	1							0.1588	15.9	0.5142	51.4	0.2896	29.0
	γ_1	0.1							0.0148	14.8	0.0647	64.7	0.0044	4.4
	δ_1	0.4							-0.0960	-24.0	0.2635	65.9	0.0786	19.7
N = 20	μ	1	0.0128	1.3	0.0945	9.5	0.0091	0.9	0.0119	1.2	0.0937	9.4	0.0089	0.9
T = 50	ϕ	0.8	-0.0019	-0.2	0.0165	2.1	0.0003	0.0	-0.0017	-0.2	0.0163	2.0	0.0003	0.0
	α	1							0.0975	9.8	0.4417	44.2	0.2046	20.5
	γ_1	0.1							0.0004	0.4	0.0384	38.4	0.0015	1.5
	δ_1	0.4							-0.0523	-13.1	0.2336	58.4	0.0573	14.3
N = 20	μ	1	0.0085	0.9	0.0742	7.4	0.0056	0.6	0.0083	0.8	0.0711	7.1	0.0051	0.5
T = 100	ϕ	0.8	-0.0012	-0.2	0.0132	1.7	0.0002	0.0	-0.0011	-0.1	0.0126	1.6	0.0002	0.0
	α	1							0.0678	6.8	0.3555	35.6	0.1310	13.1
	γ_1	0.1							-0.0002	-0.2	0.0286	28.6	0.0008	0.8
	δ_1	0.4							-0.0340	-8.5	0.1889	47.2	0.0368	9.2

TABLE 18A: Monte Carlo results for dynamic mean model ($\phi = 0.8$) and GARCH (1,1) errors ($\gamma_1 = 0.1$, $\delta_1 = 0.8$)

			OLS						MLE					
Sample	Coeff.	Value	Bias	(%)	Std. Dev.	(%)	MSE	(%)	Bias	(%)	Std. Dev.	(%)	MSE	(%)
N = 5 T = 20	μ	1	0.0767	7.7	0.4185	41.9	0.1810	18.1	0.0580	5.8	0.3900	39.0	0.1555	15.6
	ϕ	0.8	-0.0160	-2.0	0.0496	6.2	0.0027	0.3	-0.0149	-1.9	0.0481	6.0	0.0025	0.3
	α	1							2.3853	238.5	2.4567	245.7	11.7255	1172.6
	γ_1	0.1							0.0278	27.8	0.1104	110.4	0.0130	13.0
	δ_1	0.8							-0.3128	-39.1	0.2912	36.4	0.1827	22.8
N = 5 T = 50	μ	1	0.0621	6.2	0.2921	29.2	0.0892	8.9	0.0518	5.2	0.2737	27.4	0.0776	7.8
	ϕ	0.8	-0.0107	-1.3	0.0367	4.6	0.0015	0.2	-0.0097	-1.2	0.0344	4.3	0.0013	0.2
	α	1							1.1091	110.9	1.9913	199.1	5.1956	519.6
	γ_1	0.1							0.0147	14.7	0.0633	63.3	0.0042	4.2
	δ_1	0.8							-0.1387	-17.3	0.2319	29.0	0.0730	9.1
N = 5 T = 100	μ	1	0.0471	4.7	0.1964	19.6	0.0408	4.1	0.0409	4.1	0.1856	18.6	0.0361	3.6
	ϕ	0.8	-0.0074	-0.9	0.0277	3.5	0.0008	0.1	-0.0062	-0.8	0.0259	3.2	0.0007	0.1
	α	1							0.4302	43.0	1.1119	111.2	1.4213	142.1
	γ_1	0.1							0.0045	4.5	0.0415	41.5	0.0017	1.7
	δ_1	0.8							-0.0509	-6.4	0.1373	17.2	0.0215	2.7
N = 10 T = 20	μ	1	0.0469	4.7	0.2954	29.5	0.0895	9.0	0.0298	3.0	0.2807	28.1	0.0797	8.0
	ϕ	0.8	-0.0085	-1.1	0.0340	4.3	0.0012	0.2	-0.0070	-0.9	0.0329	4.1	0.0011	0.1
	α	1							1.4252	142.5	2.1623	216.2	6.7067	670.7
	γ_1	0.1							0.0310	31.0	0.0828	82.8	0.0078	7.8
	δ_1	0.8							-0.1888	-23.6	0.2599	32.5	0.1032	12.9
N = 10 T = 50	μ	1	0.0199	2.0	0.1939	19.4	0.0380	3.8	0.0181	1.8	0.1832	18.3	0.0339	3.4
	β	0.8	-0.0037	-0.5	0.0253	3.2	0.0007	0.1	-0.0030	-0.4	0.0239	3.0	0.0006	0.1
	α	1							0.4922	49.2	1.3344	133.4	2.0228	202.3
	γ_1	0.1							0.0060	6.0	0.0428	42.8	0.0019	1.9
	δ_1	0.8							-0.0575	-7.2	0.1577	19.7	0.0282	3.5
N = 10 T = 100	μ	1	0.0196	2.0	0.1488	14.9	0.0225	2.3	0.0160	1.6	0.1396	14.0	0.0198	2.0
	ϕ	0.8	-0.0028	-0.4	0.0193	2.4	0.0004	0.1	-0.0022	-0.3	0.0178	2.2	0.0003	0.0
	α	1							0.1900	19.0	0.6639	66.4	0.4769	47.7
	γ_1	0.1							0.0021	2.1	0.0286	28.6	0.0008	0.8
	δ_1	0.8							-0.0215	-2.7	0.0832	10.4	0.0074	0.9
N = 20 T = 20	μ	1	0.0246	2.5	0.1969	19.7	0.0394	3.9	0.0220	2.2	0.1884	18.8	0.0360	3.6
	ϕ	0.8	-0.0027	-0.3	0.0239	3.0	0.0006	0.1	-0.0020	-0.3	0.0226	2.8	0.0005	0.1
	α	1							0.5926	59.3	1.5927	159.3	2.8880	288.8
	γ_1	0.1							0.0159	15.9	0.0508	50.8	0.0028	2.8
	δ_1	0.8							-0.0802	-10.0	0.1900	23.8	0.0426	5.3
N = 20 T = 50	μ	1	0.0193	1.9	0.1388	13.9	0.0196	2.0	0.0183	1.8	0.1329	13.3	0.0180	1.8
	ϕ	0.8	-0.0027	-0.3	0.0178	2.2	0.0003	0.0	-0.0025	-0.3	0.0169	2.1	0.0003	0.0
	α	1							0.2033	20.3	0.7252	72.5	0.5672	56.7
	γ_1	0.1							0.0052	5.2	0.0300	30.0	0.0009	0.9
	δ_1	0.8							-0.0264	-3.3	0.0926	11.6	0.0093	1.2
N = 20 T = 100	μ	1	0.0118	1.2	0.1017	10.2	0.0105	1.1	0.0115	1.2	0.0949	9.5	0.0091	0.9
	ϕ	0.8	-0.0012	-0.2	0.0139	1.7	0.0002	0.0	-0.0011	-0.1	0.0128	1.6	0.0002	0.0
	α	1							0.0639	6.4	0.3441	34.4	0.1225	12.3
	γ_1	0.1							-0.0002	-0.2	0.0203	20.3	0.0004	0.4
	δ_1	0.8							-0.0064	-0.8	0.0468	5.9	0.0022	0.3

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