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BOTH A GARDEN OF EDEN ECONOMY AND THE SAMUELSON CASE ECONOMY CAN ACHIEVE THE GOLDEN RULE

Abstract

We show a very simple and standard overlapping generation model starting in the Garden of Eden, in which a pseudo-equilibrium optimal path not only converges to the Golden Rule but also reaches that intertemporally optimal equilibrium. This constitutes an alternative proposal to the "Samuelson Impossibility Theorem," conjectured in Samuelson (1958) and proven en Gale (1973). Samuelson also unfolded the "Infinity Paradox Revealed," facing the idea that in order to make sense of the Golden Rule, it was necessary that some young people be debtors. We then present here an alternative common sense interpretation, so that the Golden Rule is not paradoxical and has no need for governmental action, for both the Garden of Eden economy and the general case with arbitrary initial conditions. Thus, in the general Samuelson Case, there is a common sense pseudo-equilibrium optimal path that instead of converging to the no-trade equilibrium –as it was proven in Gale (1973)- which is not optimal, reaches the Golden Rule sooner or later.

Resumen

Exhibimos un modelo simple de generaciones solapadas que comienza en el Jardín del Edén, en el cual un pseudo-equilibrio no solamente converge a la Regla de Oro sino que la economía alcanza esa Regla de Oro en el primer período. Esto constituye una contrapropuesta del llamado "Samuelson Impossibility Theorem," conjeturado por Samuelson (1958) y probado por Gale (1973). Samuelson también sacó a relucir la llamada "Infinity Paradox Revealed," en la cual se afirma que para que la Regla de Oro tenga sentido económico era necesaria la presencia de jóvenes deudores. Presentamos nosotros aquí una interpretación alternativa de sentido común, de modo que la Regla de Oro deja de ser paradójica y no necesita de la acción gubernamental, tanto para el caso de la economía tipo Jardín del Edén como para el caso más general con condiciones iniciales arbitrarias. De esta manera, en el llamado Caso Samuelson, existe un pseudo-equilibrio que en vez de converger al equilibrio estacionario en el que no hay intercambio -tal como fue probado en Gale (1973)- el cual no es óptimo, alcanza la Regla de Oro tarde o temprano.

I. Introduction

In the paper by Samuelson (1958) an OLG economy with homogeneous agents endowed with one unit of a perishable good in the first period of their lives and nothing in the second period was considered, in order to present a general equilibrium explanation of positive interest rates. As is well known by now, Samuelson found that an equilibrium interest rate exists which is equal to the growth rate of the population whenever the economy either starts with young and old people or has no beginning. Furthermore, it was suggested that the economy in that equilibrium was optimal, formally established and proven in Starret (1972). Nevertheless, understanding whether this result had a common sense interpretation from an economic point of view was an issue, suggesting that the non-trade program, a non-optimal equilibrium, is the most reasonable result in this economy. In addition to this point in question, and profoundly connected with it, it was conjectured that if the economy starts with only young people then, the growth rate cannot be the equilibrium interest rate and the economy cannot converge to the Golden Rule, and the economy stays in the non-trade program forever. This last fact was formally proven in Gale (1973). Also, in this last paper, it is remarked that Samuelson's examples "have characteristics exactly opposite to those considered to be typical by Fisher and Böhm Bawerk. Instead of impatient people whose income is delayed, he considers people who receive income in the early periods of their lives but none at the end." Thus, apparently, due to Böhm's third reason of positive interest rates,[1] it was understood that negative interest rates were a natural consequence in that world, the "Samuelson Case," as Gale called it. And in fact, this was obtained in the three period example by Samuelson. Furthermore, Gale proved that in the more general two period "Samuelson Case" (an OLG economy starting with young and old people or with no beginning, such that the first period endowment is larger than the first period consumption in the Golden Rule program), the economy converges toward the no-trade steady state, and due to Böhm's third reason of positive interest rates and impatience to spend income, it is argued that Samuelson's world is probably not the one we live in.

In this paper, we present an OLG economy starting with only young people at time zero, which is sometimes referred to as starting with Adam and Eve, so that there exists a pseudo-equilibrium (following the denomination of Radner (1972)) that not only converges to the Golden Rule but also reaches that steady state from the initial period of the economy. In addition, we then present a common sense interpretation from an economic point of view so that the Golden Rule can be reached in a decentralized way, as an intertemporaly optimal stationary equilibrium. Furthermore, and most importantly, given that an economy starting in the Garden of Eden is not interesting in

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[1] Income is more available if one is willing to wait for it, or in other words, net productivity. Implicitly, it is assumed that income is scarce in the early periods and plentiful in later years. Apparently, due to the demand's low, present prices are high at first and fall as time goes on, and thus the contrary in the opposite case. This is not necessarily true in this model, but not controversial, as the present study shows.

itself,[2] this interpretation is trivially extended to the general two period "Samuelson Case," so that a pseudo-equilibrium exists that reaches the Golden Rule sooner or later as well.

The crucial motivations for these results are the following. First, we hold that a Garden of Eden OLG economy would never stay in the non-trade program, because if people consume zero in the second period, it would be a sort of collective suicide; although the non-trade program may be an equilibrium program for the economy, this is not a reasonable outcome.[3] Second, and more importantly, if one assumes that the Samuelson Case is the most reasonable assumption in order to face practical problems by means of a two period OLG model (we argue that this should be the case in general), it was proven that an equilibrium in the "Samuelson Case" converges to the non-optimal non-trade program. Once again, we believe that it is not reasonable to expect an OLG economy to converge to a program that represents a collective suicide.

In this sense, our paper can be seen as an argument against the impossibility theorem and an argument in support of Samuelson's point of view. We provide a common sense interpretation so that the Golden Rule program is the most reasonable outcome in both the Garden of the Eden and the more general two period "Samuelson Case." As will be clear from our description, Samuelson's point of view is not in total opposition to Fisher and Böhm Bawerk's point of view: Besides the impatience to spend income, which does not matter in order to describe positive interest rates, [4] net productivity in a broader sense is perfectly compatible with a Samuelson world.

On the other hand, if one is willing to simplify real life such that a two period OLG economy will be used to describe some of its facts, indeed the Samuelson case is the most reasonable assumption: Apart from the minimal proportion of people that are born rich (they do not need to work or save), the great majority have to work during their youth (this is the endowment in the first period), and have almost nothing or nothing at all for their old age because they are useless or almost useless for the market (zero endowment in the second period or almost zero).

Of course, in our interpretation, a reasonable optimal path will be a pseudo- equilibrium: We allow for the free disposal property to hold, that is, neither the consumer constraint condition nor the market clearing condition are required to hold with equality. Indeed, we do not think that, a priori, one should impose any of these conditions with equality. In this sense, although it is not our main objective, this paper can also be seen as an argument in favor of disequilibrium analysis.

[2] It is not our concern here to study whether the real world started in the Garden of Eden or not. For the sake of applications, it is more reasonable to consider young and old people at time zero (in a Samuelson world, we argue), the starting time of the problem under consideration. Nevertheless, if we consider the extreme case in which the endowment is $(1, 0)$, as we do and most of the papers listed in Section II do, at any time the economy is like in a Garden of Eden economy, because the non-trade program is a steady state (the old people consume nothing, so that the situation is as if they do not exist).

[3] Moreover, and perhaps an even stronger motivation for the results of this paper, is that in the most reasonables cases, the non-trade program cannot be an steady state equilibrium. See Section III, Remark 2.

[4] The equilibrium interest rate in both cases, the Samuelson Case and the Classical case, is totally independent of people's preferences.

People with common sense would prefer to throw away goods if in exchange they could survive during the second period and, assuming this, positive interest rates are not only a natural consequence but also very convenient and almost a necessity, if the population is growing.

Let us extend our argument by imagining what would happen in a real situation like our OLG economy. If we were indeed embedded in the first generation of an OLG economy like the one described above (let's suppose that we are Adam and Eve and we can work in the first period of our lives to obtain a unit of a perishable good, one banana so to speak, but when aged we cannot work our land anymore because we are too tired to do it) then we would try to invent a way to save in order to not starve during the second period. Clearly enough, if we do not find that way to save, we die just at the beginning of the second period, not because of natural death, but because of starvation. [5] Imagine one intelligent member of that first generation, Mr. Smart, tells the others: "I will create a bond that promises to pay one plus -the growth rate- bananas tomorrow per banana today to the holder if the holder gives me one banana today. In addition, an institution will be the intermediary between the holder and a new young agent of the next generation, in order to make the market credible for the next generation. The holder will then go to the institution when he is old, and will receive his savings."

That intelligent agent would continue explaining to the others (to the only partner, if you prefer) that: "This is possible because if we plan to have as many children as the bananas the bond promises to pay, our children at the end of their first period will be perfectly willing to forgo their bananas; if not they would starve without that piece of paper -the bond-. Our children will see that the system works since their parents can survive by means of these bonds."

The other agents of that first generation may ask: "Why do you need our bananas? or, What will you do with them?" Mr. Smart may answer: "Your bananas jointly with my banana is the creation cost of the bonds and the institution." On the other hand, the others may say: "But then how will the cost of the new bonds in the future be financed ?" Mr. Smart replies: "OK, you are right, I am lying. There is no such a cost. But in order to not starve we can do at least two things. First, we can throw away part of our bananas in order to make our children believe in the bonds. They will see that we go to the institution, forgo our bananas (these bananas will be gone), and hence obtain our bonds before they start working. Furthermore, they will see that by means of those bonds their parents can survive when aged. Second, we can lie by telling them that we have paid for those bonds and face the risk of our children's skepticism. I suggest the first option. But in any case, think of the following situation. We can try to solve our future by means of the creation of those bonds, even without being sure of its success, or we can eat all our bananas today and accept starvation." [6]

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[5] Of course, there is no way to think of a market for the goods "bananas today" and "bananas tomorrow," because all of them want to save, no one is willing to borrow. This fact was exactly what bothered Samuelson.

[6] Instead of bonds, we can think that Mr. Smart may invent another type of piece of paper: Money. We prefer bonds because in this context positive interest rates are better understood. Money cannot promise a positive interest rate and with money we may need to think of people as trading goods, a fact hardly justifiably in this model, as we remarked in the Footnote 3.

Some comments in relation to our fictitious story are in order. First, it is clear that we are introducing an additional decision: People choose whether to create the bonds and the institution or to die just after their youth. This decision-making problem is not initially in the model. We will not try to give a general theory to describe this decision. We will only present an example. Second, from analyzing our story, one may feel that some of its elements are not contemplated in the OLG model by Samuelson, the bonds and the institution - a Bank, let's say-. However, if we suppose that neither of these two elements imply a cost, as we do, there is no inconsistency. Indeed, from the outset, the model assumes the possibility of transferring value from the period of youth to the period of old age without cost. [7] Third, we have to note that the model considers homogeneous agents so we should assume that all the agents are able to reason as Mr. Smart does. However, further reasoning shows that this last consideration, instead of being dangerous to our argument, reinforces it. Finally, and deeply connected with the first three comments, it is important to note that we need to describe the agent's preferences on the two non consumption goods of our new economy: The bond (which implies throwing away bananas in principle) and death when young, so to speak.

Clearly enough, there are at least two possible outcomes in this economy: One, when people are not willing to believe that the bonds will work as a means of transferring value and prefer to die when they are young (the non-trade stationary equilibrium that Samuelson (1958) and Gale (1973) found);[8] and second, a common sense pseudo-equilibrium in which they prefer bonds over death such that in the first period people throw away bananas into the sea (only Adam and Eve) and from the outset the economy reaches the Golden Rule, as we will show in Section IV.

The rest of the paper is as follows. In Section II we comment on some related literature. Section III formally presents the model and in Section IV the two main results of this paper are shown. In Section V we add some final remarks.

II. Related Literature

As far as we know, there are no antecedents of studies dealing with the paradoxical fact of the non-convergence to the Golden Rule if the OLG economy starts with Adam and Eve. Gale (1973), instead of considering this fact a sort of collective suicide, provided a logical argument. On the other hand, the optimality property of the Golden Rule has been extensively studied and discussed. Starting with Samuelson (1958), the following series of papers have been written regarding this point: [9] Cass and Yaari (1966), Lerner (1959a, b), Meckling (1960a, b), Samuelson (1959, 1960), Phelps (1961), Thompson (1967) and Starret (1972). This topic, at least the formal statement, is clearly understood

[7] In any case it is clear that we can introduce these costs by thinking that people will work both their land and in the bank when young. Perhaps, producing less bananas than if they were working the land full-time, but creating the possibility to transfer value by means of the bonds and the institution. In this way, one can think that among these costs there is net profit for the bank, as there is in real life. The explicit introduction of these elements would complicate the model unnecessarily.

[8] Although, as we have already noted, this equilibrium may not exist. See Section III, Remark 2.

[9] Of course, we do not pretend to present a complete account of this literature.

by now. Nevertheless, we present a very transparent and intuitive interpretation for a class of utility functions to which our example belongs, showed in Section V. Finally, regarding the "infinity paradox revealed," apart from Samuelson (1958), this problem is clearly highlighted and considered in Cass and Yaari (1966). In their opinion, it does not have a solution, that is, it is not possible to imagine any type of trade in this economy. Actually, they consider the possibility of a financial intermediary, but they reject it on the basis of the following argument: "The outlook seems rosy until one takes a brief look at the balance sheet of our financial intermediary: The balance sheet as of the end of period t shows zero assets and liabilities of $s(1+n)^t$.[10] where s is the (stationary) saving ratio of people in their first period of life. This means that at the end of period t the net worth of the intermediary is given by $-s(1+n)^t$. Now by not doing anything (that is, by shutting down) the intermediary can guarantee itself a net worth of zero and so one might argue that it will never choose to engage in the aforementioned transactions. " (Cass and Yaari (1966), page 360). We offer another panorama of the situation which considers two important points.

First, we must see if this business is profitable for the banker. It seems to us that this is exactly what bothered Cass and Yaari. In effect, until the banker is able to receive the new assets, the net worth of the bank is negative. However, it is important to consider that he is not forced to pay the liabilities before that moment. Nevertheless, at the time the obligations have to be paid, he will be receiving the new assets and thus, will be able to pay them . Therefore, one could think that the bank is making zero profits, as we do. Indeed, this is a one period business, and to analyze if it is profitable, one has to wait until the business is mature, in this case, until the new generation comes and buys new bonds.[11]

Second, we must wonder if such a financial institution would be considered healthy by a monetary authority. The introduction of a monetary authority in this economy may seem artificial, given that money has no natural interpretation here due to that, once again, there is no trade between the goods "the good today" and "the good tomorrow." True, but in any event, if one insists on the existence of an institution in charge of the supervision of the bank's behavior, [12]the situation is not dangerous for society. In any financial institution in real life, one may find particular moments when its net worth is negative, that is, without the ability to cover the deposits. What matters is the bank's ability to pay the debt at the correct time. To the contrary, the monetary authority would force the bank to contract a new debt at a very high interest rate or, in the worst case, would force the institution to shut down. During the day, banks may have a negative net worth, but at the end of the day, if they are in debt, they can go to the overnight market and cover it, or they may even accept being in debt, provided that they have no debt at the end of the natural period of the economy -28 days in many countries in real

[10] $(1+n)$ is the gross population growth.

[11] Indeed, it is a business as many business in real life. Think of working the land. The business man may need funds to begin with. Imagine that he decides to issue bonds, which promise an interest rate for the next year. This business man is in debt during the whole year, but at the moment the interest promised by the bonds has to be paid, if the harvest is good enough, he will pay the debt and the business goes on.

[12] In real life, banks and financial institutions are heavily regulated.

life-. In our economy something similar is happening. Until the bank receives the new assets, it is in debt, but at the moment the return of the bonds has to be paid, the bank can paid the debt. Period by period -this is the natural period in our economy and the natural period of the bank's business- the bank is contracting new debts, but period by period is paying the old debt, no consumer would be disappointed. Thus the monetary authority, if it exists, would consider the bank healthy.

III. The Model

Time is discrete, $t = 0, 1, 2, \ldots$ At each t, a generation of agents is born and lives for two periods. There is only one good in each period which is perishable. In the first period of their life, agents receive a fixed endowment or they work for it, which we normalize to 1. The second period they receive nothing.

Agents have an intertemporal utility function

$$u(c_0, c_1) = u(c_0) + \beta u(c_1)$$

where c_0, c_1 are the consumption levels in the first and second period of their lives and β is a positive real number.

We take the instantaneous utility to be $u(c) = \ln(c)$, which means the intertemporal utility is of the Cobb-Douglas type.

For each generation t, we denote their consumption decisions by $c(t) = (c_0(t), c_1(t+1))$.

The size of generation t is N_t. There is an exogenous gross population growth rate $\gamma > 0$, that is, $N_{t+1} = \gamma N_t \; \forall \, t \geq 0$.

There is a bond that promises to pay $1 + r_t$ units of the good tomorrow per unit of the good today, so that the price of the bond is $\frac{1}{1+r_t}$, in terms of the good today. Agents born at t can save part of their endowment this period through the bond market and get it repaid next period. The interest rate then between period t and period $t+1$ is given by r_t, and we denote $R_t = 1 + r_t$.

Agents face the following optimization problem:

$$\left.\begin{array}{ll} \max & u(c_0(t), c_1(t+1)) \\ s.t. & R_t(1 - c_0(t)) - c_1(t+1) \geq 0 \end{array}\right\} \qquad (1)$$

The feasibility condition for the economy at time $t \geq 1$ is:

$$N_t \geq N_t c_0(t) + N_{t-1} c_1(t)$$

Dividing by N_{t-1},

$$\gamma \geq \gamma c_0(t) + c_1(t) \qquad (2)$$

At time 0, there are only young agents in the population and hence the feasibility condition is

$$N_0 \geq N_0 c_0(0)$$

that is (dividing by N_0),

$$1 \geq c_0(0) \qquad (3)$$

Following the denomination of Radner (1972), we define:

Definition 1 *A sequence* $((c(t), R_t)_{t=0}^{\infty}$ *is a pseudo-equilibrium if for all* $t \geq 0$, $c(t)$ *solves (1), (2) is satisfied for all* $t \geq 0$ *and (3) is satisfied at* $t = 0$.

IV. The results

The Garden of Eden Economy

For the given utility function, provided that $R_t > 0$, the first order conditions are necessary and sufficient for an optimum of the consumer's problem (1). We obtain:

$$c_1(t+1) = \beta R_t c_0(t)$$

and the restriction. Substituting, we obtain

$$c_0(t) = \frac{1}{(1+\beta)}, \quad c_1(t+1) = \frac{\beta R_t}{(1+\beta)} \tag{4}$$

From the feasibility condition at time zero (3) we obtain

$$1 > \frac{1}{(1+\beta)} \tag{5}$$

which is trivially satisfied. Note that since $0 < \beta$, this last condition is satisfied with inequality not depending on the value of the interest rate, consequently resources are wasted.

From the feasibility condition for all $t \geq 1$(2), it follows:

$$\gamma \geq \gamma \frac{1}{(1+\beta)} + \frac{\beta R_{t-1}}{(1+\beta)} \tag{6}$$

which is satisfied for all stationary gross interest rates $R \leq \gamma$, and in particular for $R = \gamma$.

Result 1: *If* $R_t = \gamma$ *for* $t \geq 0$, *the corresponding pseudo- equilibrium program from* $t \geq 0$ *is exactly the Golden Rule. Thus, by throwing away resources, the Golden Rule is achieved without delay.*

It is pertinent to point out some technical details, although they have a very intuitive economic sense.

Remark 1 First, notice that if $R_t < 0$, the consumer's problem has no solution because he will consume an infinite quantity of the good today and tomorrow, and hence there is no equilibrium in this case. However, $R_t \leq 0$ corresponds to the case when $\gamma \leq 0$. On the other hand, from the definition of γ, we have that $\gamma \geq 0$. Now, if $\gamma = 0$ then people are not having children and therefore the whole economy dies at time 1 and equilibrium should not be found.

Remark 2 Second, observe that even when $R_t > 0$ (or $\gamma > 0$) for all $t \geq 0$, there is never an equilibrium if the agents avoid starvation and we impose the feasibility condition with equality in a Garden of the Eden economy. More formally, if the agents' utility function is such that zero consumption at the second period of life can never be an optimal solution, then there is no equilibrium. The reason is simple. The condition (3) imposed with equality implies zero consumption at time one for Adam and Eve, and this, by assumption, cannot be optimal. Here we have a simple although subtle detail which was neither taken into account in the paper by Samuelson (1958) nor in the paper by Gale (1973).

The General Samuelson Case

For the sake of transparency, we decided to present the Garden of Eden economy first and with detail assuming Cobb-Douglas utility functions. Nonetheless, as already mentioned, the most reasonable framework is to assume arbitrary utility functions in a Samuelson world (that is, when young people's optimal consumption in the Golden Rule program is lower than the first period endowment) and that the economy comes from the past, taking at time zero the consumption of the old people as given. Formally, we assume that at time zero there are N_0 old people consuming $c_1(0)$ and from $t \geq 0$ the population grows at a rate $\gamma - 1$, so that at time one there are $N_0\gamma$ young people. Every generation from $t \geq 0$ face the following optimization problem:

$$\left.\begin{array}{ll} \max & u\left(c_0(t), c_1(t+1)\right) \\ s.t. & R_t\left(1 - c_0(t)\right) + \left(e - c_1(t+1)\right) \geq 0 \end{array}\right\},$$

where e is the endowment in the second period, and $u(c_0, c_1)$ is now an arbitrary utility function.

The Golden Rule is defined as a constant program $c(t) = (c_0^{GR}, c_1^{GR})$ for $t > 0$ such that (c_0^{GR}, c_1^{GR}) solves

$$\left.\begin{array}{ll} \max & u\left(c_0, c_1\right) \\ s.t. & \gamma\left(1 - c_0\right) + \left(e - c_1\right) \geq 0 \end{array}\right\}. \tag{7}$$

The Samuelson case then is formally defined when $c_0^{GR} < 1$.

The feasibility condition for $t \geq 0$ in this context is as follows:

$$\gamma + e \geq \gamma c_0(t) + c_1(t). \tag{8}$$

The definition of a pseudo-equilibrium in this case is totally analogue.

Now we assume that u is such that $c_0^{GR} < 1$ and that the economy is in an equilibrium $c^E(t) = (c_0^E(t), c_1^E(t+1))$ from $t \geq 0$ such that $c_0^E(0) = (\gamma + e - c_1(0))/\gamma$, where $c_1(0)$ is the consumption of the old people at time zero. Notice that we are assuming the feasibility condition (8) with equality. Now we apply the result proven in Gale (1973), that is, we assume that $c_1(0)$ is close enough to e so that local stability holds, that is: $c^E(t) \to (1, e)$. Hence, there exists a T such that for all $t > T$, we have $c_0^{GR} < c_0^E(t)$. Take one such a t, let's say $T+1$. Now notice that the feasibility condition for the young generation at $T+1$ is satisfied (not with equality) if the young people choose c_0^{GR} instead of $c_0^E(T+1)$, thus the same reasoning as in the Garden of Economy can be applied here, so that from $T+1$ the economy reaches the Golden Rule program.

Result 2: *For any OLG economy in the Samuelson Case such that* $c_1(0)$ *is close enough to* e *, it is possible to construct a pseudo-equilibrium that reaches the Golden*

Rule sooner or later.

Remark 3 Notice that the result applies to any situation in which there exists an equilibrium such that there exists a T with the property that $c_0^{GR} < c_0^E(T)$. It is fair then to wonder if it possible to find OLG economies for which $c_0^{GR} > c_0^E(t)$ for all $t \geq 0$ and such that for all $t > 0$, $c_1(t)$ is never close enough to e.

Remark 4 In Geanakoplos (1987) the differences between the Arrow-Debreu model and OLG models are clearly highlighted. For the two period models that we consider here, the most important differences are in relation to the indeterminacy and the non-optimality of the equilibrium, because here money may have a positive value but, as already remarked, makes little sense in view of the fact that there is no trade of goods in our model. One of the questions was: Why may the equilibrium not be optimal in OLG models? In general, it does not seem to be easy to find a simple answer. Nevertheless, at least for the examples studied here, our point of view suggests the possibility of a simple answer: If we allow for the free disposal property to hold, optimality can be obtained. [13] Thus, the only point to take into account is the feasibility condition.

Remark 5 Notice that whenever the price effect of the good c_1 on the demand of good c_0 is zero and both c_0 and c_1 are desirable (so that in equilibrium we must have $R_t(1 - c_0^E(t+1)) + e - c_1(t) = 0$ for all $t \geq 0$), and agents avoid starvation, as all Cobb-Douglas utility functions, the condition $R_t \leq \gamma$ for all $t \geq 0$ is always true in any two period OLG economy with endowment $(1, 0)$ in every period. Following Cass and Yaari (1966), if $((c_0^E(t), c_1^E(t+1)), R_t)_{t\geq 0}$ denotes an equilibrium or a pseudo-equilibrium in a two period OLG economy , then we have:

$$c_0^E(t-1) - c_0^E(t) + c_1^E(t)(\frac{1}{R_{t-1}} - \frac{1}{\gamma}) \geq 0$$

for all $t \geq 1$. Now, under the assumption that the price effect of the good c_1 on the demand of good c_0 is zero, it is true that $c_0^E(t) = c_0^E(t-1)$ for all $t \geq 1$. Therefore, $R_t \leq \gamma$ for all $t \geq 0$, provided that $c_1^E(t) > 0$ for all $t \geq 1$, which is the case if agents avoid starvation. Observe that the following inequality

$$\gamma(1 - c_0^E(t)) - R_{t-1}(1 - c_0^E(t-1)) \geq 0,$$

also holds for all $t \geq 1$, even with endowment $(1, e)$, so that for any equilibrium such that $c_0^E(t) < 1$ for all $t \geq 1$, we also obtain $R_t \leq \gamma$ for all $t \geq 0$. It is pertinent then to ask under what conditions an equilibrium is such that $c_0^E(t) < 1$ for all $t \geq 1$ even with endowment $(1, e) \neq (1, 0)$. With Cobb-Douglas utility functions the condition is that e be small enough. We conjecture that, for more general utility functions (satisfying that the price effect of the good c_1 on the demand of good c_0 is zero and both c_0 and c_1 are desirable), the condition $c_0^E(t) < 1$ for all $t \geq 1$ is also obtained provided that e is small enough.

[13] Intertemporal optimality, in the sense of Starret (1973). The fisrt generation always can be better off.

V. Final Comments

At first glance, the argument suggesting that a Garden of Eden economy cannot converge to the Golden Rule seems convincing. Nevertheless, when one tries to imagine what would happen if we were indeed in that economy, a very bad feeling comes over us: Is there no way out of this sort of suicide? Our answer was yes, there should be a way out, and indeed *the answer is yes, there is a way out,* providing we do prefer the bonds and throw away goods instead of dying when young. It comes down to the question: Do we indeed prefer to die when young just because we have to throw away goods? Our guess is that people with common sense would create the bonds.

Another fact that should be pointed out is the following. From the feasibility condition for $t \geq 1$ (6), it is clear that not only is there a great deal of stationary pseudo-equilibria but there is also a great deal of non-stationary psuedo-equilibria. In fact, any sequence $\{R_t\}_{t=0}^{\infty}$ satisfying (6) for $t \geq 0$ is a pseudo-equilibrium. Nevertheless, it is important to note how clear it is that the only intertemporal optimum pseudo-equilibrium in the sense of Starret (1972) is the one whose interest rate is equal to the growth rate. Indeed, any gross interest rate lower than the growth rate would waste resources. Of the possible rates, the larger is the growth rate, and people optimize assuming the growth rate as the interest rate of the bond market, hence there is no way to improve any generation. Observe that this interpretation can be applied to the class of OLG economies with utility functions satisfying that the price effect of the good c_1 on the demand of good c_0 is zero and both c_0 and c_1 are desirable (see Remark 5).

Another problem stemming from the previous comment is the indeterminacy of the equilibrium. Clearly, this indeterminacy comes from the fact that neither the consumer constraint nor the feasibility constraints are imposed with equality. However, we insist here on the fact that these assumptions are quite reasonable from an economic point of view: In real life, markets may clear or may not clear, this is the only thing that we know for sure. It is not the purpose of this paper to present a general theory to solve this controversy. But, in our examples, that indeterminacy can be overcome by assuming that there are many people like Mr. Smart that may want to engage in the business, so that any one of them offering a lower interest rate than the others will lose all his demand. Thus, it is obvious that all of them will choose to offer the growth rate of the population, the largest one, as the interest rate. In this way, the market for bonds would be a competitive market in which managers cannot manipulate prices. This last interpretation has a very good consequence, that is, we obtain *an answer* for the existence of positive interest rates: Positive interest rates exist because *Business Men* are able to guess that a business may be profitable and hence they engage in an enterprise and offer positive interest rates, in conjuction with people's necessity to save (in the extreme situation where people have nothing during the second period, time preference or impatience to spend income, obviously, does not matter in order to explain positive interest rates). Mr. Smart is just a *Business Man* who sees that a business can work and decides to become a manager, and offers positive interest rates -exactly the growth rate of population- just because otherwise he would lose his demand and no business would be possible for him.

Some other remarks related to the issue of explaining positive interest rates, and to the significance of modeling real macroeconomic questions through OLG models (or through any kind of general equilibrium macroeconomic model using microeconomic foundations), are in order.

First, notice that in the paper by Gale (1973) it is argued that the most interesting case from the point of view of economic theory is "The Classical Case," in which resources are scarce in the earlier periods of the economy and plentiful in the later, in order to obtain some insight in relation to the theory of economic growth, in the sense of improving standards of living. [14] On the other hand, from our example, it is clear that the "Samuelson Case" has at least one common sense interpretation in which indeed people are at the Golden Rule from the very beginning (with endowment (1, 0)), and hence even faster than in a typical "Classical Case." [15] On the other hand, even in the general Samuelson case, as we showed in Section IV, there are common sense psudo-equilibria the reach the Golden Rule sooner or later, improving profoundly the standard of living, given that the non-trade equilibrium is not optimal.

Second, given our interpretation, we think that in order to give a theoretical description of positive interest rates, the Samuelson Case would be one of the most appropriate *regarding some circumstances*. As we have clearly shown in our example, in a Samuelson world, the growth rate would be not only something very convenient, but also the most reasonable interest rate. Notwithstanding, we stress here that our opinion is that neither the Samuelson Case nor the Classical case can provide a general answer to the existence of positive interest rates, because both are too simple. For a two period OLG model, the Samuelson Case is the most reasonable assumption, and it provides very good intuitive insight. The crucial point then is to identify precisely the circumstances to which a model can be applied.

Third, remember that due to Böhm's third reason for positive interest rates when applied to the Samuelson case, one apparently only finds negative interest rates, but we find positive interest rates, which is, to our understanding, the most reasonable result in a real Samuelson world. This begs the question: Is the low of the supply and demand failing in our example? *It may be failing*, and the reason for this failure is that people avoid starvation and decide to throw away goods. Another reason, which we consider *the answer*, is that the model as it is, cannot be considered a model in which people are trading goods today and goods tomorrow, so that a demand's low is an empty concept in relation to the goods or endowments. Of course, people trade, but they trade bonds, and in this market, as already shown, a demand's low applies. However, in relation to Böhm's third reason of positive interest rates, we would still like to add one more comment. The argument of net productivity was understood in the narrow sense of endowments. Indeed, one can interpret the population growing as net productivity in a broader sense. De facto, our business man understood that there was some net productivity in the so-

[14] Nevertheless, as Gale showed in the Classical Case, the non-trade equilibrium is optimal so that there are no incentives to trade at all, no incentives to deviate from the initial endowments, and the economy stays on this path forever.

[15] Obviously, only when the non-trade program is not optimal, as it is in general.

ciety. From this point of view, there is no contradiction between Böhm's argument and Samuelson's argument.

Fourth, one should realize the following fact. All the discussions and controversial results are exclusively due to the fact that people impose the feasibility condition with equality. On the other hand, following our point of view, the entire class of functions that display aversion to starvation shows that the equilibrium outcome is not a reasonable one, whereas our pseudo-equilibrium provides a very intuitive outcome and is not in contradiction with a competitive interpretation of it. In this way, our paper can be used as an argument in favor of disequilibrium analysis.

There is still one more comment in this respect. The model admits from the very beginning a way of transferring value. Thus, when people say "if we introduce money in the model," or "if we introduce bonds in the model", or even when they say "if we introduce a social security system," this can only be a way of speaking. The model admits all these interpretations, so that, strictly speaking, it assumes any of these. The point is that equilibrium is a kind of dynamic steady state, from which, once reached, there are no incentives to deviate. But the model as it is, describes neither how the equilibrium is reached nor why it is reached. Therefore, any assertion in relation to how or why the equilibrium is reached, is outside of the model and not a logical consequence of it.

Finally, we would like to emphasize the fact that from our interpretation there is no need for governmental action and the result is not paradoxical, just the market, a free market for bonds is sufficient. There is no contradiction with a laissez-faire economy and nothing mistaken about the mathematical solution of the model. Of course, if we insist on thinking that Samuelson's model is a pure exchange model, then it is paradoxical because all young people are creditors and hence there are no debtors. However, following from our interpretation, the debtors are represented by an institution, the bank. On the other hand, in real life, for the sake of savings, most debtors are chiefly banks and financial institutions, and in fact they offer positive interest rates because they have invested in another business that promises to pay them a higher interest rate.

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