16 de diciembre de 2002

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NÚMERO 239

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THE EXCHANGE RATE IN A BAND WITH LIMITED RESERVES

Abstract

A discrete time model of the exchange rate in a currency band is developed. Fundamental variables are assumed to follow a Gaussian random walk in order to consider explicitly how the size of the foreign reserve stock at the central bank can affect the behavior of the exchange rate within the band. The equilibrium relationship between the exchange rate and the fundamental at each reserves level is characterized. This implies that when reserves are low the inverse relationship between the interest differential and the exchange rate is reversed as the latter approaches the weaker end of the band. We test this hypothesis on the Mexican peso/dollar band, and on the French-franc/Deutsche-mark band finding no significant empirical contradiction.

Resumen

En el presente documento se desarrolla un modelo discreto del tipo de cambio dentro de una banda cambiaria. Se supone que los determinantes fundamentales de la demanda por dinero se comportan como una caminata al azar donde el choque aleatorio tiene una distribución Normal. Esto, con el objeto de considerar de manera explícita el efecto del tamaño del acervo de reservas internacionales en poder del banco central sobre el equilibrio del tipo de cambio dentro de la banda cambiaria. Lo anterior permite caracterizar a la relación entre el tipo de cambio y el fundamental para cada nivel de reserves internacionales. Se encuentra que la relación inversa entre el diferencial de interés y el tipo de cambio se revierte para niveles bajos de reservas internacionales. Esta hipótesis se analiza para el caso de la banda cambiaria del peso Mexicano con respecto al dólar, y del Franco Francés con respecto al Marco Alemán. No se encuentra una contradicción empírica significativa.

Introduction

Following Krugman (1991) models of exchange rate determination within a currency band have paid particular attention to the effect of the band on exchange rate expectations. The basic idea is that the band, as long as it is credible, has a stabilizing effect on the exchange rate because the central bank commits to control the money supply in order to contain the movements of the exchange rate as it reaches the band limits. These studies work in a continuous-time setting of regulated Brownian motion making the analysis algebraically more tractable than a discrete-time formulation. In particular, in continuous-time it is possible to obtain a closed form solution characterized by having the equilibrium exchange rate function of the fundamental tangent at the band limits.

When attempting to generalize the previous result to the case of an imperfectly credible band, Krugman and Rotemberg (1991) find that any velocity shock that takes the exchange rate to the upper limit of the band results in a speculative attack that collapses it. The only other possible equilibrium at the weaker end of the band is with reserves above some threshold and the tangency equilibrium, i.e. the perfect credibility case once again.

Casual observation, however, supports the view that the exchange rate appreciates back into the band after intervention at the weaker end. That is, reserves are depleted in steps and never in one shot.

In this study we introduce a model that shares the main features of Krugman's with the only variants being that the time dimension is discrete, and the fundamental follows a gaussian random walk. This allows us to consider explicitly how the size of the foreign reserve stock can affect the equilibrium relationship between the exchange rate and the fundamental in the band.

In section 1 we review how the equilibrium is characterized when the fundamental follows a Brownian motion and the band is perfectly credible. The aspect we want to highlight here is that the expected change in the fundamental is always zero when the exchange rate is within the limits of the band. That is, g(k), the function that characterizes the relationship between the exchange rate and the fundamental, depends linearly on its own first and second derivatives with respect to the fundamental. This fact allows g(k) to be pinned down explicitly whenever the exchange rate lies within the limits of the band. We also show how the expected change in the fundamental differs from zero only at the band limits, imiplying the need for an additional condition to prevent an infinite expected rate of return at the band's edges. The sufficient condition turns out to be Krugman´s "smooth-pasting" condition.

In section 2 we fix the time interval at a finite number and let the fundamental follow a gaussian random walk. This sole assumption turns the expected change in the fundamental into a function of the fundamental, making g(k) depend non-linearly on its own first and second derivatives with respect to the fundamental, preventing us from solving for it explicitly. We device a method to obtain a numerical approximation to g(k) and show how this converges to the continuous-time solution of Section 1 as the time interval is reduced to zero.

Noting that a gaussian random walk fundamental implies there must exist a minimum adverse shock that can deplete any bounded foreign reserve stock, in Section 3 we associate these minima with the level of reserves at the central bank. Therefore, we characterize the equilibrium relationship between the exchange rate and the fundamental at each reserves level, finding that the larger the reserve stock the more stable the exchange rate in the band.

In order to test the empirical relevance of this result, in section 4 we find out how the relationship between the interest differential (an observable) and the exchange rate varies with the level of reserves. We find that, when reserves are low the inverse relationship between the exchange rate and the interest differential, implicit in Krugman's model, is reversed as the exchange rate approaches the upper limit of the band. This result leads us to a straightforward test of the credibility of the band as a function of the reserve stock. In section 5 we test this hypothesis on the Mexican-peso/US-dollar band, and on the French-franc/Deutsche-mark band. In section 6 we conclude.

1 *Continuous Time*

Consider a log-linear monetary model of the exchange rate[1]. Money market equilibrium implies[2].

$$m_t - e_t = -\alpha E_t\left[\frac{\Delta e_{t+\Delta t}}{\Delta t}\right] - v_t \tag{1}$$

where e_t is the (log of) the spot price of foreign exchange, $m_t = log(B^G + R_t)$ is the (log of the) monetary base which consists of central bank holdings of government bonds B^G and foreign currency reserves R_t, v_t represents shocks to velocity, α is a positive parameter, and $E_t\left(\frac{\Delta e_{t+\Delta t}}{\Delta t}\right) \equiv E_t\left(\frac{e_{t+\Delta t} - e_t}{\Delta t}\right)$ is the expected change in the exchange rate from the perspective of period t.

From (1) the exchange rate at any point in time is given by

$$e_t = m_t + v_t + \alpha E_t\left[\frac{\Delta e_{t+\Delta t}}{\Delta t}\right] \tag{2}$$

So, the model has the exchange rate depending on its own expected rate of change and on current fundamentals. Domestic credit is assumed constant throughout, i.e. there is no time subscript in B^G. So the velocity shift term v_t is the only source of exogenous dynamics that is assumed to follow the stochastic process

$$\Delta v_{t+\Delta t} = \sigma \Delta z_{t+\Delta t} \tag{3}$$

[1] See Obstfeld and Rogoff (1996) chapter 8.

[2] (1) is obtained assuming PPP and uncovered interest parity, the foreign price level is fixed at 1 and the foreign interest rate at 0.

where the random variable z_t is a Wiener process, i.e. $\Delta z_{t+\Delta t} = \varepsilon\sqrt{\Delta t}$ and $\varepsilon \sim N(0,1)$.

Consider the equilibrium exchange rate under a free float. Propose the following solution to (2)

$$e_t = m + v_t \tag{4}$$

which implies

$$e_{t+\Delta t} - e_t = \Delta v_{t+\Delta t} \tag{5}$$

Taking expectations on both sides and substituting into (2) we obtain (4) back. So, under a free float the exchange rate behaves according to (5), which implies that it follows the same stochastic process as the fundamental.

Consider the exchange rate to remain within pre-specified limits $\bar{e}$ and $\underline{e}$, assumed for simplicity to be equidistant from the central parity at the origin. More specifically, the central bank is prepared to reduce (increase) foreign reserves in order to prevent the exchange rate from going over (below) $\bar{e}$ ($\underline{e}$). And, as long as the exchange rate lies inside the band the money supply remains unchanged. Throughout this section the reserve stock is assumed to be large, assumption that is relaxed later.

Let the fundamental be $k_t = m_t + v_t$. Since it follows a random walk within specified bounds, its current level summarizes all information about the future probability distribution of the fundamental. From (2) we have

$$e_t = k_t + \alpha E_t\left[\frac{\Delta e_{t+\Delta t}}{\Delta t}\right] \tag{6}$$

So, let $e_t = g(k_t)$ be a candidate solution to (6) with $g(.)$ at least twice differentiable. If this is a solution it must satisfy (6) implying that

$$g(k_t) = k_t + \alpha E_t\left[\frac{\Delta g(k_{t+\Delta t})}{\Delta t}\right] \tag{7}$$

Consider a second-order Taylor approximation to $E_t[g(k_t)]$ at k_t, we have

$$E_t[g(k_{t+\Delta t})] \cong E_t\left[g(k_t) + g'(k_t)\Delta k_{t+\Delta t} + \frac{1}{2}g''(k_t)(\Delta k_{t+\Delta t})^2\right] \tag{8}$$

From (3), and as long as there is yet no need for intervention[3], $E_t[g'(k_t)\Delta k_{t+\Delta t}] = g'(k_t)E_t[\Delta k_{t+\Delta t}] = 0$. Moreover, $E_t\left[(\Delta k_{t+\Delta t})^2\right] = \sigma^2\Delta t$. Thus, as long as k_t leaves the exchange rate within the band, we have the approximation

$$E_t\left[\frac{\Delta g(k_{t+\Delta t})}{\Delta t}\right] \cong \frac{\sigma^2}{2}g''(k_t)$$

which can be regarded as an equality for arbitrarily small[4] Δt. Plugging this last result into (7) we have that, as long as the exchange rate lies strictly within the band any candidate solution to (6) must satisfy

[3] This result follows simply because of the stochastic process followed by the fundamental.

[4] Notice that the higher-order terms in the infinite Taylor expansion are multiplied by powers of Δt equal to or higher than $(\Delta t)^{3/2}$ which go to zero more quickly than Δt does as the continuous-time limit is approached.

$$g(k_t) = k_t + \frac{\alpha\sigma^2}{2} g''(k_t). \tag{9}$$

This is a second-order stochastic differential equation describing the exchange rate's evolution as a function of the fundamental. A general solution to it is

$$e_t = k_t + A[exp(\rho k_t) + exp(-\rho k_t)] \tag{10}$$

where $\rho = \sqrt{\frac{2}{\alpha\sigma^2}}$. To describe the exchange rate's behavior in the band we still need to find the value for the constant A in (10). We do this by studying the behavior of the exchange rate as it reaches the band's limits.

Let $S(k_t)$ be the solution with the correct value of the constant A in (10). We ask now what happens when the fundamental is at $\bar{k}$, with $S(\bar{k}) = \bar{e}$. (The argument for the band's lower limit is symmetric, $S(\underline{k}) = \underline{e}$). At $\bar{k}$ a small increase in the fundamental would push the exchange rate out of the band, an event that would automatically be prevented by central bank intervention. Notice that, from (3), before the fundamental hits $\bar{k}$ its expected change is always zero. But once the fundamental reaches $\bar{k}$ the exchange rate is at the top of the band, and monetary authorities will only allow the fundamental to go down, not up. Thus at the top of the band $E_t(\Delta k_{t+\Delta t})<0$, so that as $k \to \bar{k}$ the expected change in the fundamental jumps discontinuously from zero to a negative value.

At the top of the band the solution must also satisfy (6) so

$$S(\bar{k}) = \bar{k} + \alpha E_t\left[\frac{\Delta S(\bar{k} + \Delta k)}{\Delta t}\right] \tag{11}$$

where $\Delta S(\bar{k} + \Delta k) = S(\bar{k} + \Delta k) - S(\bar{k})$ and for simplicity we drop time subscripts. Consider now a second-order Taylor approximation to $E_t[S(k)]$ at $\bar{k}$, we have

$$E_t[S(\bar{k} + \Delta k) - S(\bar{k})] \cong S'(\bar{k})E_t[\Delta k] + \frac{1}{2}S''(\bar{k})E_t[(\Delta k)^2] \tag{12}$$

where, again, the expression would hold with equality as long as Δt is close enough to zero. Since Δk is distributed symmetrically around zero, $E_t[(\Delta k)^2] = \sigma^2\Delta t$ still. However, $E_t(\Delta k)<0$ because at $\bar{k}$ central bank intervention is sure to keep fundamentals from rising. Plugging this back into (12), and the result into (11) we have

$$S(\bar{k}) = \bar{k} + S'(\bar{k})\frac{E_t[\Delta k]}{\Delta t} + \frac{\alpha\sigma^2}{2}S''(\bar{k}) \tag{13}$$

Now, from the previous analysis, $S(k_t)$ must satisfy (6) whenever the exchange rate is strictly within the band, so we also have

$$S(k) = k + \frac{\alpha\sigma^2}{2}S''(k), \quad \underline{k} < k < \bar{k}$$

Since, in order to prevent a foreseeable infinite arbitrage opportunity at $k = \bar{k}$, the exchange rate cannot be expected to jump discontinuously, as $k \to \bar{k}$ it must be the case that $S(k) \to S(\bar{k})$. Since $E_t(\Delta k)<0$, then it must be the case that $S'(\bar{k})$ equal zero. This is the tangency condition, or "smooth-pasting" condition, invoked by Krugman (1991) to solve for A in (10).

From the previous analysis we not that the need for a tangency condition derives from a no-infinite arbitrage opportunity argument. That is, before the fundamental hits one of the band limits its expected change is zero (from (3)). But once it reaches say the top of the band, monetary authorities will only allow k_t to go down, not up[5]. Thus, at the top of the band the expected change in the fundamental changes discontinuously from zero to a negative value. If $S'(\bar{k})$ were not zero at $k = \bar{k}$, a discontinuous jump in $E_t(\Delta k)$ would imply a discontinuous jump in $E_t[\Delta e_{t+\Delta t}]$, implying an infinite expected rate of return at $\bar{k}$, because the time interval is infinitesimal. No equilibrium path can approach such a point since speculators would anticipate arbitrage profits at an infinite rate. So, we need $\Delta E(\Delta e)=0$ at $k = \bar{k}$.

Another way to see this argument is by noticing that from (3) $E_t(\Delta k)$ in (11) is equal to $\sigma\sqrt{\Delta t}E_t(\varepsilon)$, where $E_t(\varepsilon)$ is one sided at the top of the band since intervention will absorb adverse shocks. Therefore, as $\Delta t \to 0$ the expected change in the fundamental goes to zero at a rate that is proportional to $\sqrt{\Delta t}$. In other words when the process followed by the fundamental is as in (3), at $k = \bar{k}$ the numerator in the second term in (13) goes to zero at a rate proportional to $\sqrt{\Delta t}$, which is slower than the rate at which Δt goes to zero. So, as $\Delta t \to 0$ the quotient in the second term of (13) would explode implying that there exists no interest rate that would compensate investors for such an expected rate of return. To prevent this it is necessary to invoke an additional condition, this being the no-expected infinite arbitrage profits condition described above.

Finally, notice that $E_t[\Delta e_{t+\Delta t}]$ will be given by the third term in (13) independently of whether the fundamental has the exchange rate within the band or at a limit. This is so because if the former is the case $E_t(\Delta k) = 0$ in (13), and $S'(\bar{k})=0$ if the latter.

2 Discrete Time

Throughout the previous section credibility on the central bank's commitment to keep the exchange rate within the band was assumed independent of the size of its foreign reserve stock. This was so because intervention was always infinitesimal, and because the foreign currency reserve stock at the central bank was always large enough to support the necessary intervention to keep the exchange rate within the band.

In order to consider how the size of the reserve stock can alter the relationship between the exchange rate and the fundamental we must allow shocks of different size to provoke different size interventions. Krugman and Rotemberg's (1992) continuous time analysis of the limited reserves case results in there being only two possibilities for intervention at the upper limit of the band. One, with large enough

[5] And we have been assuming that monetary authorities hold enough foreign currency reserves to keep to their commitment of keeping the exchange rate in the band.

reserves, has the smooth-pasting result of the previous section. The only other possibility, where reserves are below some threshold, has the regime collapsing to a free float immediately after the exchange rate reaches the upper limit of the band.

In this section, we relax some of the assumptions of the previous section allowing the fundamental to follow a more elaborate stochastic process. We begin by relaxing the first assumption. Thus, by studying the case where the fundamental follows a gaussian random walk, but still assume that reserves are large enough so that intervention is insignificant. In the following section we relax the latter assumption.

Let time be discrete and the velocity shift term v_t follow, instead of (3), the gaussian random walk

$$v_{t+\Delta t} = v_t + u_{t+\Delta t} \tag{14}$$

where $u_{t+\Delta t} \sim N(0,\sigma_u^2)$.

Consider the equilibrium exchange rate under a free float. Just as before propose the solution

$$e_t = m + v_t \tag{15}$$

that implies

$$e_{t+\Delta t} - e_t = u_{t+\Delta t} \tag{16}$$

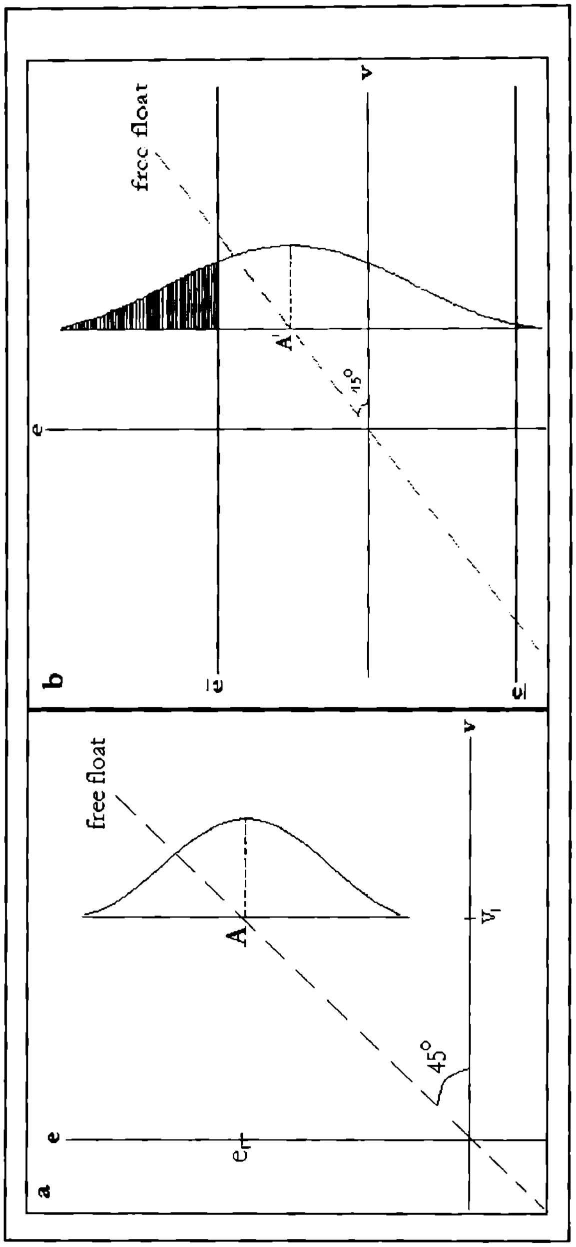
taking expectations on both sides of (16) and substituting into (2) we obtain (15) back. So, having v_t behave according to (14) does not alter the result when the exchange rate is floating freely. Notice, from (16), that the stochastic distribution of the change in the exchange rate is given by the distribution of the exogenous shock.

Given that under a free float there is no limit on the fundamental's, and therefore on the exchange rate's, range of variation, the expected change in the exchange rate will be given by the mean of the distribution of the exogenous shock. To illustrate we make use of figure 1a. Consider point **A** where the fundamental takes on a value v_1 above the central parity at zero. We draw over **A** the distribution of $e_{t+1} - e_t$. Since its mean is zero, (2) implies that the exchange rate must be equal to the fundamental, i.e. lies on the 45-degree line on (v,e) space.

Generalizing to every possible value of the fundamental we reach the previous conclusion that under a free float the exchange rate follows the same stochastic process as the fundamental, moving up and down the 45-degree line with every random shock.

Consider now the case where the central bank commits to keep the exchange rate within pre-specified limits $\bar{e}$ and $\underline{e}$. Again, just to illustrate, consider the distribution of the change in the exchange rate at a point like **A** in figure 1b, where the fundamental takes on a value v_1 with the exchange rate above the central parity at zero. Since the exchange rate can only take on values below $\bar{e}$ and above $\underline{e}$, when the exchange rate is above the central parity $e_{t+1} - e_t$ can take on fewer positive values than negative, and for every positive value that it can take on there is a negative counterpart that occurs with the same probability. (The opposite is true for the case where the fundamental lies below the central parity.) So, the distribution of the change in the exchange rate will be censored on both sides, as illustrated in the figure. Thus for any value of the fundamental above (below) the central parity the mean of the distribution of the change in the exchange rate must be lower (larger) than zero. Moreover, for values of the fundamental further away from the origin the mean of the censored distribution will be even more negative, since more of the right, and less of the left, tail is being censored.

Figure 1



Now, notice that although Δt is not close to zero here we can still propose a candidate solution g(.). This is so because even when time evolves discretely the equilibrium relationship between e_t and k_t can still be portrayed by a continuous function. So, go back to equation (6) in the previous section. For simplicity, consider the case where the fundamental is above the central parity (the situation for the fundamental below the central parity is the mirror image).

When v_t follows (14), it must be the case that the equilibrium eventually reaches the band limits[6]. To see why this is so consider money market equilibrium at a value of the fundamental that has the exchange rate inside the band. From (6) we have

$$k_t = e_t - \alpha E_t\left[\frac{\Delta e_{t+\Delta t}}{\Delta t}\right] \tag{17}$$

Consider k_t taking on larger and larger values. Since the exchange rate is inside the band m_t is fixed. On the right-hand side of (17) we have e_t approaching $\bar{e}$ compensating for part of the rise in k_t. As e_t approaches $\bar{e}$ the expected change in the exchange rate, $E_t[\Delta e_{t+\Delta t}]$, becomes more and more negative compensating too for the rise in k_t. However, because of the assumption that the band is perfectly credible the fall in $E_t[\Delta e_{t+\Delta t}]$ must be bounded. That is, the exchange rate can never be expected to appreciate (or depreciate) more than the difference between the two band limits. So, in (17) since e_t is bounded by $\bar{e}$, and since $E_t[\Delta e_{t+\Delta t}]$ is bounded too, then it must be the case that k_t eventually reaches a limit, say, $\bar{k}$.

From the previous argument the distribution of Δk at k_t is the distribution of the shock but censored, and $E_t(\Delta k_{t+\Delta t}) < 0$ (>0) when the exchange rate is above (below) the central parity at zero. So, going back to (6), and dividing both sides by $\Delta t = 1$ we have[7]

$$E_t[g(k_{t+\Delta t}) - g(k_t)] \cong g'(k_t)E_t(\Delta k_{t+\Delta t}) + \frac{1}{2}g''(k_t)E_t(\Delta k_{t+\Delta t})^2 \tag{18}$$

The first term on the right-hand side of (18) is negative (positive) when the fundamental is above (below) the central parity. Moreover, since g(k) must be concave (convex) for k above (below) the central parity, $E_t[\Delta e_{t+\Delta t}]<0$ (>0) whenever the fundamental is above (below) the central parity. So, even when the fundamental follows a gaussian random walk the equilibrium relationship between the exchange rate and the fundamental can still be illustrated using the familiar S-shape curve characteristic of the continuous-time case.

The Equilibrium Exchange Rate

In order to solve for the equilibrium exchange rate in terms of the fundamental we would need to substitute (18) into (7) and solve for e_t. But notice that in (18) the expected change in the fundamental is now a function of its position within the band.

[6] Recall that we are assuming that the central bank owns an UN-limited reserve stock.

[7] Since the higher moments of the normal distribution can always be expressed as powers of the mean and variance, the higher-orders terms in the infinite Taylor expansion will be a function of the mean and variance of u, and will be of second-order.

In the continuous-time case this was only true at the band's limits, thus the need for an additional condition only at the edges of the band[8].

So, the fact that the expected change in the fundamental is a function of its position within the band will not permit us to obtain a closed form solution in this case[9].

In what follows we illustrate a method to approximate the equilibrium numerically. Once we obtain such an approximation we let the time interval go to zero to be able to compare it with the continuous-time solution of section 1.

Therefore, in the perfect credibility case the equilibrium exchange rate at any point in time must have expectations at t consistent with the fundamental following a gaussian random walk with next period's shock u_{t+1} lying possibly anywhere on the real line, and intervention at the band limits being independent of future intervention[10].

We propose the following solution:

The equilibrium exchange rate at any point in time t is given by the fixed point of the sequence $\{g_i\}$ $\quad i = 1,2,...$ *where*

$$g_i(k_t) = k_t + \alpha E_t^i(e_{t+1} - e_t) \tag{19}$$

with the expected change in the exchange rate at iteration i given by

$$\mathrm{E}_t^i(e_{t+1} - e_t) = \sum_{j=-\infty}^{\infty} \left[g_i(k_t + u_j) - g_i(k_t)\right] f(u_j) \qquad where$$

$$g_i(k_t + u_j) = \begin{cases} \bar{e}, & \bar{k}_i - k_t \le u_j \\ g_i(k_t + u_j), & \underline{k}_i - k_t < u_j < \bar{k}_i - k_t \\ \underline{e}, & u_j \le \underline{k}_i - k_t \end{cases}$$

where f(.) is the shock's p.d.f. given by the normal distribution. The initial conditions to this program are

$$g_0(k_t + u_j) = \begin{cases} \bar{e}, & \bar{k}_0 - k_t \le u_j \\ k_t + u_j, & \underline{k}_0 - k_t < u_j < \bar{k}_0 - k_t \\ \underline{e}, & u_j \le \underline{k}_0 - k_t \end{cases}$$

Equation (19) is nothing but a recipe to obtain the numerical approximation to the equilibrium, and consists of the following steps. Define a grid over the possible values that v can take on [11]. Next, define a prior to start the iterative process; this is

[8] That is, the expected change in the fundamental was zero everywhere and negative (positive) at the band' s upper (lower) limit.

[9] Inspecting (18) a little closer we can see that the formula for the conditional expectation of the change in the exchange rate is quite elaborate, not allowing us to solve for the exchange rate when substituting back in (7).

[10] Why this is relevant will become clearer when we treat the imperfect credibility case. In that case intervention will be a function of future intervention, which will complicate things considerably.

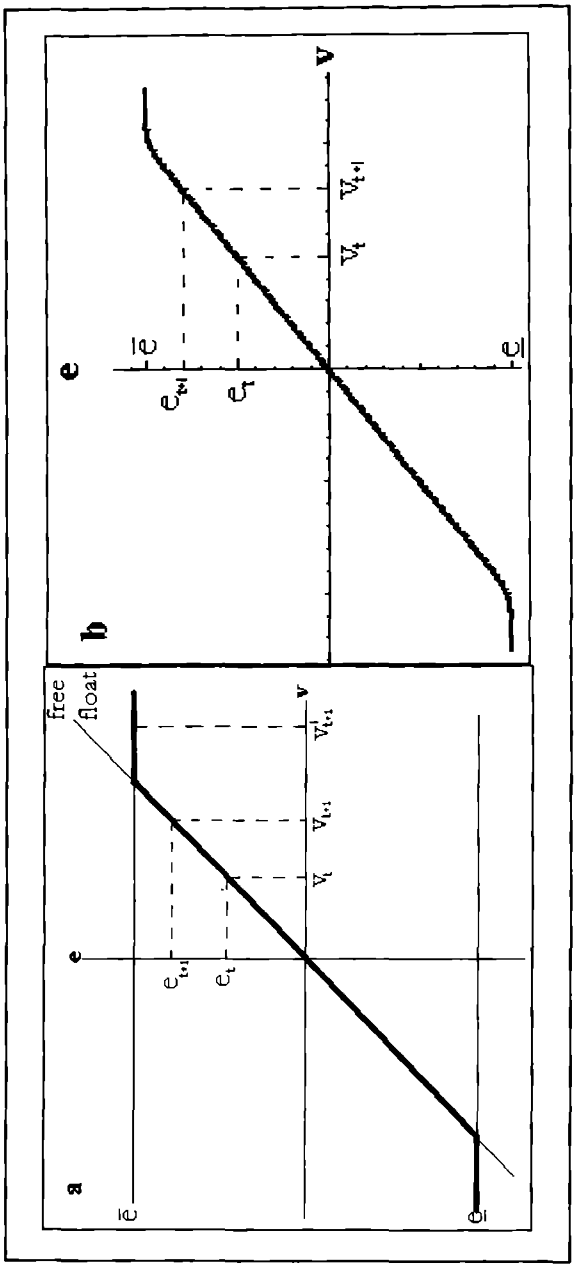
[11] I defined $1.x10^{-2}$ which proved sufficient to achieve a good enough approximation. There is also an endpoint problem; that is, it is necessary to define a finite range fir the values that v can take on. I defined [-5,5].

[11] Start from $(v, e)_1$ and suppose that the fundamental is initially at v_t with the exchange rate at e_t in figure 2a. If, for example, a shock takes the fundamental from v_t to $v_{t+\Delta t}$, then the probability of this event (probabilities given by the distribution of u) is multiplied by the magnitude $e_{t+\Delta t} - e_t$ and the result added to the calculation of $E_t[\Delta e_{t+\Delta t}]$. Or, if a shock takes the fundamental from v_t to $v_{t+\Delta t}$' in the figure, then the probability of such an event is multiplied by the magnitude $\bar{e} - e_t$ and the result added to the calculation of $E_t[\Delta e_{t+\Delta t}]$. We do this for every possible shock. We repeat the process throughout the range of possible values taken by the fundamental, to obtain a value of $E_t[\Delta e_{t+\Delta t}]$ at each v_t.

the program's initial condition. Our prior has the exchange rate in the interior of the band responding one to one to changes in the fundamental[12].

[12] Note that this will only be the case at the first iteration.

Figure 2



Departing from the prior, Figure 2a, calculate $E_t[\Delta e_{t+\Delta t}]$ standing at a given value of the fundamental. Since the distribution of the exogenous shock is normal we must account for every possible value of the shock at[13] $t + \Delta t$. Once we have a value for

[13] We take into account only those values that occur with probability higher than $1.x10^{-15}$

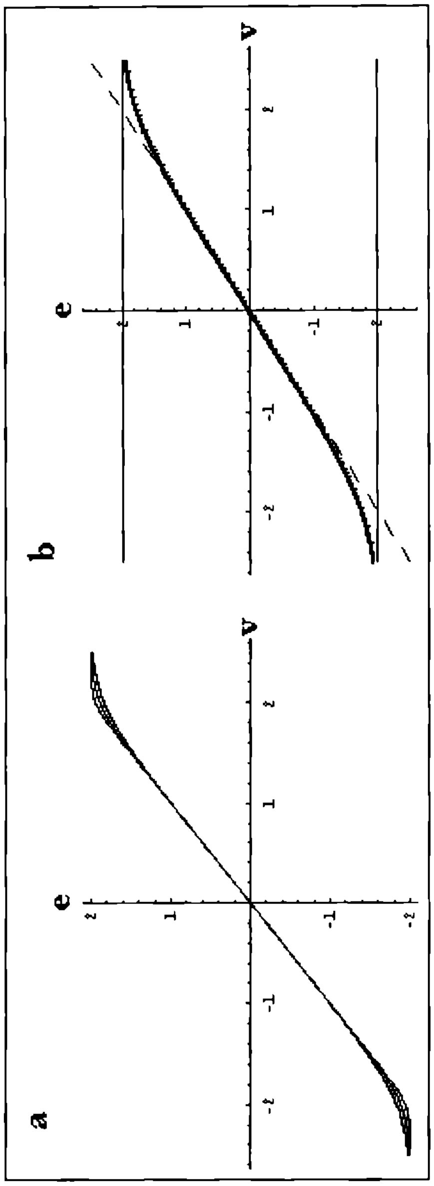
the expected change in the exchange rate[14] at each k_t, we substitute it back into (2), the expression for the exchange rate, to obtain a new curve $g_i(k_t)$ together with $\bar{k}_i$ and $\underline{k}_i$. The latter are the values of the fundamental at which the new curve touches the band limits. Thus, we obtain a sequence of curves $\{g_i(k_t)\}$ $i = 1,2,\ldots$ Let $d_i = max|g_i(k_t) - g_{i-1}(k_t)|$ be the distance between two successive curves. We look for the sequence $\{d_i\}$ to converge to zero.

Why would this iterative process converge at all? First note that the edges of the curve $g_i(k_t)$ are smoothed with each successive iteration[15]. Now, concentrate on one value of the fundamental k_t. As the curve is smoothed away the change in the exchange rate resulting from a given shock at each k_t, the amount $g_i(k_t + u_j) - g_i(k_t)$, falls after each iteration. That is, smoother curves result in lower exchange rate changes at k_t. And since the expected change in the exchange rate is made up of the weighted average of these, the former falls with each successive iteration. Moreover, since for a given k_t the difference between each subsequent curve is nothing but the difference between the values taken by the expression in (19), then this difference is going to zero with each successive iteration. Thus $\{d_i\}$ must converge.

For parameter values $\bar{e}$=1, $\underline{e}$=-1, and α=0.5 after the 20-th iteration we obtain the curve illustrated in Figure 3b.

[15] See figure 3a.

Figure 3



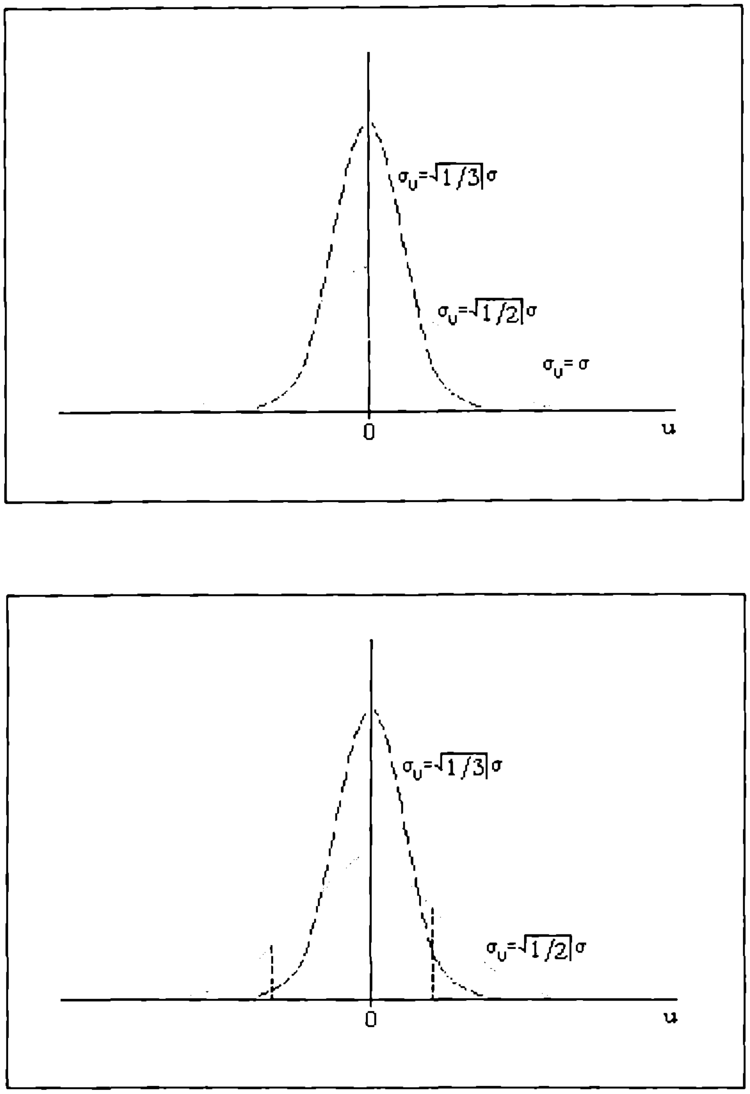
Continuous versus Discrete-time

Once we have our numerical approximation to the equilibrium between the exchange rate and the fundamental where the latter follows a gaussian random walk, we would like to establish a link between this and the standard case where the fundamental follows a Brownian motion.

Since $\sigma_u^2 = \sigma^2 \Delta t$, each one of the curves we converge to in the previous section is associated with Δt, which we have been assuming equal to 1 up to here. Consider now how the variance of the distribution of the shock is affected by a change in the size of the time interval. This is illustrated in Figure 4a. Notice in the figure how the mean of the distribution does not change as the variance is reduced. However, as this is done the probability for the values around the mean rises. Since the distribution of Δk at k_t is the distribution of the shock censored on both sides, as $\Delta t \to 0$ the expected change in the fundamental goes to zero. So, in figure 4b, the reduction in the variance of the shock makes the censorship less and less significant, provoking that the mean of the censored distribution go to zero. That is, as $\Delta t \to 0$ the mean of the censored distribution at k_t concentrates more and more on what happens in the vicinity of k_t, and we have $E_t[\Delta e_{t+\Delta t}] \to 0$ at each value of the fundamental[16].

[16] Recall that in the Brownian motion case the expected change in the fundamental equals zero whenever the exchange rate lies within the band.

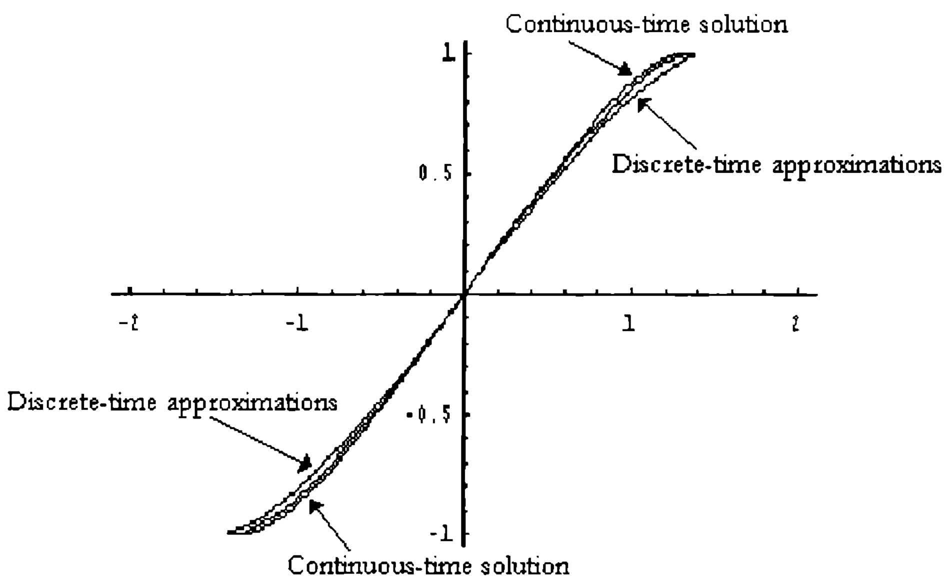
Figure 4



In Figure 5 we illustrate how the equilibrium relationship between the exchange rate and the fundamental is affected by consequent reductions in the time interval. Each curve is associated with a given Δt and represents in (k,e) space the values taken by $g_i(k_t)$. As we move from right to left the time interval is being reduced. Notice that each $g_i(k_t)$ reaches the upper limit of the band at an angle. We want to concentrate on two things: 1) whether the point at which the discrete-time approximation hits the upper limit of the band approaches $\bar{k}$, the point of tangency for the continuous-time solution, and 2) how the slope of the curve changes in the vicinity of $\bar{k}$ as $\Delta t \to 0$.

To look at the behavior of the curve in the vicinity of $\bar{k}$ we fix the value taken by k_t. We calculate the difference between the change in the exchange rate caused by a change in k_t that takes it to $\bar{k}$. This is simply an approximation to the slope of the curve right at the point where it reaches the upper limit of the band. As we reduce the size of the time interval we obtain lower values each time corroborating that the slope of $g_i(k_t)$ in the vicinity of $\bar{k}$ becomes flatter with each fall in the time interval. In figure 5 we present the curves that result from the iteration process of the previous section. $\Delta t = \frac{1}{N}$ for the first curve N=8, and for the second N=32. We can see how, in the limit, as $\Delta t \to 0$ the slope at $\bar{k}$ approaches zero. Moreover, it is also not hard to see how the value at which the discrete-time approximation reaches the upper limit of the band falls as the time limit is reduced, approaching $\bar{k}$ with each successive reduction in the time interval.

Figure 5



So, as $\Delta t \to 0$ the equilibrium exchange rate in a band where the fundamental follows a gaussian random walk approximates the equilibrium exchange rate where the fundamental follows a Brownian motion. Therefore, the continuous-time solution can be viewed as the limit as $\Delta t \to 0$ of a discrete-time model where the fundamental's stochastic process is a gaussian random walk[17].

[17] Other studies have been able to obtain the continuos-time tangency condition endogenously, See for example Flood and Garber (1992).

3 Limited Reserves

The results so far hinged on the fact that the central bank's reserve stock was, for all practical matters, infinite. When this is not the case the central bank will not be as able to sustain the weaker end of the band. This must alter the shape of the equilibrium relationship between the exchange rate and the fundamental. In this section we make use of the results so far to determine the equilibrium under these new conditions.

To study exchange rate behavior in a regime with non-zero probability of collapsing it is important to consider what happens when it does, we assume the simplest case: the regime goes back to a free float.

Before proceeding to the more formal analysis it helps to recall Figure 2b and illustrate the effect of a limited reserve stock on the exchange rate in the band. We saw there that when reserves are infinite shocks that take the exchange rate away from the boundaries must be censored. When reserves are finite, however, it follows that there exists a non-empty set of adverse shocks capable of depleting the rest of reserves. Therefore, with positive probability, the change in the exchange rate, $e_{t+\Delta t} - e_t$ can take on values that were impossible under infinite reserves. Consider the fundamental at **A**. Since shocks that can deplete the reserve stock occur with positive probability, some mass must be replenished on the extreme right-hand side of the distribution of $e_{t+\Delta t} - e_t$. Adding probability mass at the right-hand end of the censored distribution has the effect of raising the mean, and, from (2), this results in the exchange rate moving up towards the free float line.

It is not hard to see that the amount of mass to be replenished will be inversely proportional to the size of the reserve stock at t, R_t. So, when reserves are limited $E_t(e_{t+\Delta t} - e_t)$ will be a function of the fundamental and the reserve stock at the central bank. How $E_t(e_{t+\Delta t} - e_t)$ depends on R_t is key to determine not only the equilibrium exchange rate, but also the rate at which reserves are lost after an adverse shock that leads to intervention, and finally the conditions under which the regime collapses to the free float.

So, in the present case once the exchange rate reaches $\bar{e}$ there exist a set of adverse shocks that can deplete the reserve stock. Let $\underline{u}(R_t)$ be the minimum of this set, that is, the smallest shock that can deplete reserve stock R_t.

In the limited reserves case the equilibrium exchange rate at any point in time

$$e_t = k_t + \alpha E_t\left(e_{t+1} - e_t \middle| k_t, R_t\right)$$

must have expectations at t consistent with:

a) the fundamental following a gaussian random walk with next period's shock u_{t+1} lying possibly anywhere on the real line, and

b) intervention at t+1 depending on the shock u_{t+1} as well as on future intervention[15] .

We propose the following solution[16] :

[15] Notice that this condition was not present in the previous case.

The equilibrium exchange rate at any point in time t is given by the fixed point of the sequence $\{g_i\}$ $\quad i = 1,2,\dots$ *where*

$$g_i(k_t,R_t) = k_t + \alpha E_t^i\left(e_{t+1} - e_t \middle| k_t,R_t\right) \tag{20}$$

with the expected change in the exchange rate at iteration i given by

$$\mathrm{E}_t^i\left(e_{t+1} - e_t \middle| k_t,R_t\right) = \sum_{j=-\infty}^{\infty}\left[g_i\left(k_t + u_j \middle| R_{t+1}\right) - g_i\left(k_t \middle| R_t\right)\right] f(u_j) \qquad \text{where}$$

$$g_i\left(k_t + u_j \middle| R_{t+1}\right) = \begin{cases} g_i\left(\bar{k}_i - k_t + u_j \middle| 0\right), & \bar{k}_i + \underline{u}(R_t) - k_t \le u_j \\ \bar{e}, & \bar{k}_i - k_t < u_j < \bar{k}_i + \underline{u}(R_t) - k_t \\ g_i\left(k_t + u_j \middle| R_{t+1}\right), & u_j \le \bar{k}_i - k_t \end{cases}$$

and R_{t+1} the convergence point of the sequence $\{R_{t+1}^m\}$ $\quad m = 1,2,\dots$ $\quad$ *where*

$$\log(1+R_t) - \log(1+R_{t+1}^m) = u_j - (\bar{k}_i - k_t) - \alpha\Delta E_t^{i-1}(e_{t+1} - e_t)$$

with expectations revision $\Delta E_t^{i-1}(e_{t+1} - e_t) = E_t^{i-1}\left(e_{t+1} - e_t \middle| \bar{k}_i, R_t\right) - E_t^{i-1}\left(e_{t+1} - e_t \middle| \bar{k}_i, R_{t+1}^m\right)$.

Finally, initial conditions to this program are

$$\mathrm{E}_t^0\left(e_{t+1} - e_t \middle| k_t,R_{t+1}^0\right) = \sum_{j=-\infty}^{\infty}\left[g_i\left(k_t + u_j \middle| R_{t+1}^0\right) - k_t\right] f(u_j) \qquad \text{where}$$

$$g_0\left(k_t + u_j \middle| R_{t+1}^0\right) = \begin{cases} \bar{k}_0 - k_t + u_j - \underline{u}(R_t), & \bar{k}_0 + \underline{u}(R_t) - k_t \le u_j \\ \bar{e}, & \bar{k}_0 - k_t < u_j < \bar{k}_0 + \underline{u}(R_t) - k_t \\ k_t + u_j, & u_j \le \bar{k}_0 - k_t \end{cases}$$

and

$$\log(1+R_t) - \log(1+R_{t+1}^0) = u_j - (\bar{k}_i - k_t)$$

To corroborate that $g(k_t,R_t)$, where $g_i(k_t,R_t) \to g(k_t,R_t)$, is consistent with conditions a and b, we use it to evaluate

$$\mathrm{E}_t\left(e_{t+1} - e_t \middle| k_t,R_t\right) = \sum_{j=-\infty}^{\infty}\left[g\left(k_t + u_j \middle| R_{t+1}\right) - g\left(k_t \middle| R_t\right)\right] f(u_j)$$

where

$$g\left(k_t + u_j \middle| R_{t+1}\right) = \begin{cases} g\left(\bar{k}(R_t) - k_t + u_j \middle| 0\right), & \bar{k}(R_t) + \underline{u}(R_t) - k_t \le u_j \\ \bar{e}, & \bar{k}(R_t) - k_t < u_j < \bar{k}(R_t) + \underline{u}(R_t) - k_t \\ g\left(k_t + u_j \middle| R_{t+1}\right), & u_j \le \bar{k}(R_t) - k_t \end{cases}$$

and R_{t+1} results from the equilibrium intervention rule

[16] Since we are interested in how the size of the reserves stock can affect sustainability of the upper limit of the band, in the limit reserves case we study only the simpler case of a unilateral band. The generalization to a bilateral band would be straightforward.

$$log(1+R_t) - log(1+R_{t+1}) = u_j - (\bar{k}(R_t) - k_t) - \alpha\Delta E_t(e_{t+1} - e_t)$$

with expectations revision $\Delta E_t(e_{t+1} - e_t) = E_t[e_{t+1} - e_t | \bar{k}(R_t), R_t] - E_t[e_{t+1} - e_t | \bar{k}(R_{t+1}), R_{t+1}]$, where $\bar{k}(R_t)$ is the value of the fundamental at which the equilibrium reaches $\bar{e}$ at reserves level R_t. Finally, substitute the result into (20) to obtain $e_t = g(k_t, R_t)$ back.

Notice that equilibrium expected change in the exchange rate is consistent with condition a and an intervention rule that, not only follows from money market equilibrium but, takes into account expectations revision after an adverse shock. Recall that in the continuous-time case the smooth-pasting condition was necessary because of the process followed by the fundamental. In the present case the equilibrium must also be consistent with the process followed by the fundamental, which means that whenever there is a shock the equilibrium relationship must be explicit about how money market equilibrium is regained after the shock. This includes the consideration that every time intervention is called for the solution take into account the fact that intervention depends on expectations revision and vice-versa.

To describe the nature of the equilibrium we use Figures 6 and 7. Figure 6a represents the relationship between the fundamental and the exchange rate at different reserves levels. The latter variable falls as we go from 6.1 to 6.4. Notice, first of all, that in all cases the equilibrium reaches $\bar{e}$ at an angle. Moreover, as the fundamental gets closer to $\bar{k}(R_t)$ the exchange rate becomes less responsive to changes in the fundamental. This effect is stronger the larger the reserves level. Therefore, when the economy starts from a smaller reserves level the high responsiveness of the exchange rate to changes in the fundamental makes it reach $\bar{e}$ faster. So, we reach our first conclusion: <u>when reserves losses are significant, a larger reserve stock stabilizes the exchange rate within the band</u>.

Once k_t is at $\bar{k}(R_t)$ an adverse shock leads to reserves losses. The relationship between intervention to defend $\bar{e}$ and the level of the fundamental is illustrated in Figure 7. Notice first that for small shocks reserves loss, $R_{t+1} - R_t$, is almost proportional to the size of the shock. For larger shocks it is more than proportional, with the proportion rising as the size of the intervention gets closer to the ex-ante reserve stock. Thus our second conclusion is: <u>when reserves losses are significant, if intervention after an adverse shock approaches the prevailing reserve stock, reserves losses are more than proportional to the size of the shock</u>. This is so because of the revision of expectations that takes place as the economy moves from equilibrium to another. That is, the movement to the new equilibrium takes into account the fact that after the adverse shock there will be a smaller reserves level and a larger likelihood that these be depleted by subsequent shocks.

Another relevant feature of the equilibrium $g(k_t, R_t)$ that we can observe by intervention after large shocks is much larger than it would be if the economy started from a large reserve stock. That is, expectations revision is more significant the smaller the reserves level the economy starts with. So, our final proposition is: <u>when reserves losses are significant, a larger the ex-ante reserves level reduces the responsiveness of intervention to a given adverse shock.</u>

The Effect of Reserves Losses on the Equilibrium Exchange Rate

There is still another way to describe the nature of the equilibrium $g(k_t, R_t)$. This is by studying how it behaves after an adverse shock that calls on the central bank to defend $\bar{e}$. Consider money market equilibrium at $\bar{k}(R_t) = log(1 + R_t) + \bar{v}$, we have

$$log(1 + R_t) - \bar{e} = -\bar{v} - \alpha E_t(\Delta e) \qquad (21)$$

If the economy draws an adverse shock $u_{t+1} > 0$ then intervention will be called for taking the equilibrium to

$$log(1 + R_t) - \bar{e} = -\bar{v} - u_{t+1} - \alpha E_{t+1}(\Delta e) \qquad (22)$$

Subtracting (22) from (21) we have

$$log(1 + R_t) - log(1 + R_{t+1}) = u_{t+1} - \alpha \Delta E(\Delta e) \qquad (23)$$

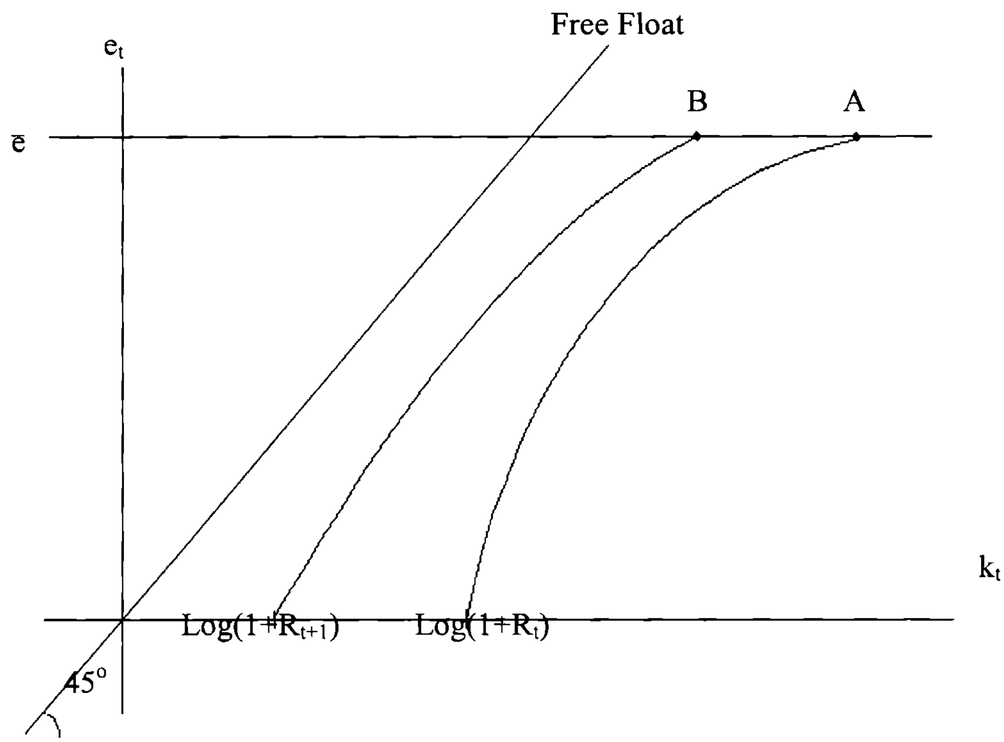
with expectations revision $\Delta E(\Delta e) = E_t(e_{t+1} - e_t) - E_{t+1}(e_{t+2} - e_{t+1})$.

We know that $u_{t+1}>0$ and that the left-hand side of (23) is positive, the question is how has the expected change in the exchange rate changed? Only a rise in $E_t\left[e_{t+1} - e_t \middle| \bar{k}(R_t)\right]$ is consistent with $g(k_t, R_t)$. Moreover, this means that adverse shocks at $\bar{e}$ are followed by more than proportional reserves losses, since money demand falls not only because of the rise in velocity but also because expectations adjust on the way to the new equilibrium[17].

[17] Notice that this argument is consistent with the literature of speculative attacks on fixed exchange rates.

Figure 8

So, consider the equilibrium in (k,e) space, illustrated in Figure 8. Notice that[18]



$m+v = log(1+R_t)$ at $v=0$. With the fundamental at $\bar{k}(R_t)$, an adverse shock leaves the exchange rate still at $\bar{e}$ and takes the equilibrium to a curve that intersects the horizontal axis at a point that is closer to the origin. This is so because reserves have been lost. Now, $v_{t+1} = v_t + u_{t+1}$ has risen and $m_t = log(1+R_t)$ has fallen, but what has happened to $m+v$? Since $E_t[e_{t+1} - e_t | \bar{k}(R_{t+1})]$ is higher than $E_t[e_{t+1} - e_t | \bar{k}(R_t)]$, on the way to the new equilibrium the expected change in the exchange rate has risen. And since at equilibrium

$$m_t + v_t = \bar{e} - \alpha E_t(e_{t+1} - e_t)$$

it must be the case that $m+v$ falls. So, we have $\bar{k}(R_{t+1}) < \bar{k}(R_t)$.

When does this process stop? Or, in other words, how do we obtain the value of the fundamental at which reserves are depleted? It is not hard to see that if more adverse shocks keep coming reserves will be depleted and the economy will be taken to the free float equilibrium over the 45-degree line with expected depreciation reaching zero. So, what is the value of $\underline{u}(R_t)$? Go back to equilibrium at **A** in figure 9. At collapse we have

$$0 - \bar{e} = -\bar{v} - \underline{u}(R_t) - 0 \qquad (24)$$

[18] For simplicity we assume $B^G = 1$.

Subtracting (24) from (21) we get

$$\underline{u}(R_t) = log(1+R_t) - E_t\left[e_{t+1} - e_t \middle| \bar{k}(R_t)\right] \qquad (25)$$

which gives us the exact relationship between the level of reserves and $\underline{u}(R_t)$. Given that $E_t\left[e_{t+1} - e_t \middle| \bar{k}(R_t)\right]$ is negative this expression only says that the smallest shock necessary to collapse the regime is lower than the size of the reserve stock R_t. This is so because when the exchange rate is at $\bar{e}$ any adverse shock reduces money demand more than proportionally leading to more than proportional reserve losses.

Krugman and Rotemberg (1992) derive the equilibrium exchange rate in a unilateral band with a fundamental that follows a Brownian motion with drift. Their model has also the feature that the equilibrium reaches $\bar{e}$ at an angle. In their model, however, whenever reserves are lower than some threshold, the instant $\bar{e}$ is reached there is a speculative attack and the regime collapses to a free float. Moreover, the only other possible equilibrium at $\bar{e}$ is when reserves are above the threshold with the band being perfectly credible and the equilibrium tangent at[19] $\bar{e}$. In our model here, unless there are not enough reserves to face the shock, once $\bar{e}$ is reached the economy remains at that point cutting $\bar{e}$ at an angle. This difference between the two models is significant, and in the following section we study its implications for the relationship between the exchange rate and the interest differential. In the final section we contrast our model's propositions and argue that they are not inconsistent with the data.

4 The Interest Differential and the Exchange Rate in a Band with Limited Reserves

We have seen how the equilibrium relationship between the exchange rate and the fundamental can be altered by the size of the reserve stock at the central bank. Now, in order to derive a testable hypothesis from our propositions so far it is still necessary to translate them in terms of relations between observable variables. Since the fundamental driving process is not an observable, in this section we look at how the relationship between the exchange rate and the interest differential is affected by the size of the reserve stock.

Under the assumption of uncovered interest parity the expected rate of change in the exchange rate, $E_t(e_{t+1} - e_t)$, is simply equal to the difference between the domestic and the foreign interest rate. So, we want to look at how the relationship between the exchange rate and the interest differential is altered as reserves change.

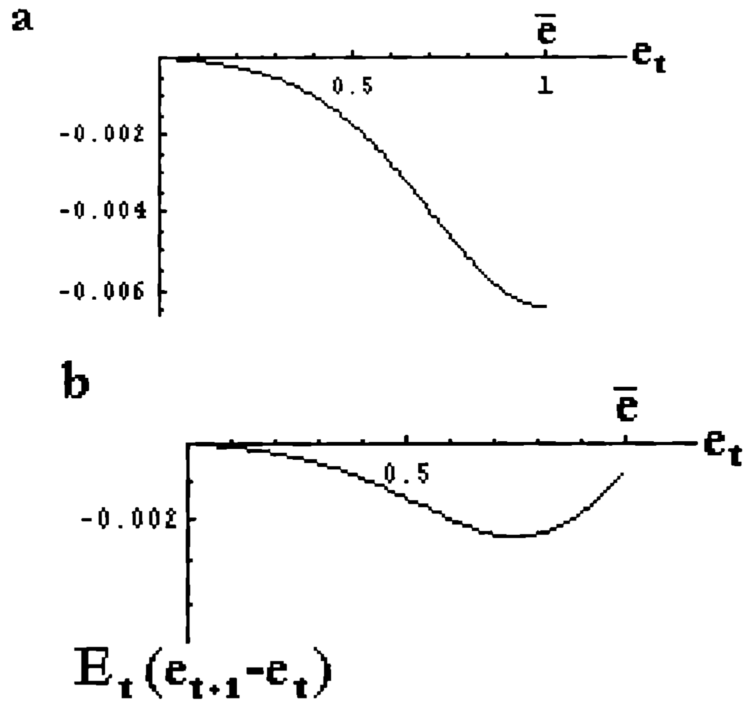
Recall that to get to the equilibrium our iterative process relied on the calculation of the expected change in the exchange rate at each value of the fundamental. So, once we have an expression for the equilibrium, $g(k_t, R_t)$, the relationship between $E_t(e_{t+1} - e_t)$ and the exchange rate is obtained as a by-product.

In Figure 9 the interest differential at each value of the exchange rate is displayed for different reserves levels. Figure 9a displays how when reserves are large there is

[19] The latter is also the case if reserves are larger than zero and there is no drift in the fundamental.

an inverse relationship between the interest differential and the exchange rate. We can see in figure 9b how when reserves are low this inverse relationship is broken as the exchange rate approaches $\bar{e}$. So, a final characteristic of the equilibrium $g(k_t, R_t)$ is that <u>when reserves are low, the inverse relationship between the exchange rate and the interest differential is reverted at an exchange rate close enough to the weaker end of the band</u>.

Figure 9



This result gives us a way to test the empirical validity of the model that differs significantly from previous work. It remains relevant to look for a non-linearity in the relationship between the exchange rate and the interest differential, however, the relationship is no longer an inverse relationship across the band, given that when reserves are low we expect it to be reverted as the exchange rate approaches $\bar{e}$.

5 A Simple Credibility Test of the Central Bank's Commitment to a Currency Band

The previous result gives us a straightforward method for testing how credible is the central bank's commitment to keep the exchange rate within the band[18]. Here we implement this test on both the Mexican-peso/US-dollar currency band, in place from Jan 1992 until the Dec-1994 crisis, and the French- franc/Deutsche-mark band, from March-1979 to July-1993.

The central bank of Mexico committed to keep the exchange rate within a currency band defining both an official band and within it an intervention band, the limits of

[18] Rose A. & Svensson (1994) estimate realignment expectations for members of the European Exchange Rate Mechanism. Their perceived credibility measures are based on interest differentials adjusted for expected exchange rate drift. This method does not take into account agents' knowledge of the band when estimating expected exchange rate depreciation inside the band. More recent tests can be seen in Girardin and Marimoutou (1997).

which were announced at the start of each trading day[19]. Intervention in the foreign exchange market was carried out only when the exchange rate reached either limit of the intervention band (e.g. there were no "intramarginal interventions" within the intervention band). The upper limit of the intervention band was realigned after "too many" reserves were lost defending it. The limits of the official band were realigned only once[20].

In order to test the hypothesis of the previous section we constructed the following variables: 1) POS, the position of the exchange rate within the band relative to the central parity $\frac{e_t - c_t}{\bar{e} - c_t}$, where c_t is the central parity (the arithmetic mean of the two band limits). 2) an estimate of expected exchange rate depreciation, EXPDEV, calculated as the difference between the return on foreign currency denominated Mexican government bonds *Tesobonos* and domestic currency denominated Mexican government bonds *Cetes*. 3) the value of the reserve stock at the central bank in dollars, RES. So, we have an observation for POS, EXPDEV, and RES for each week between the first week of January 1992 and the third week of December 1994[21].

The hypothesis of the previous section implies that we should see EXPDEV rise as RES falls when POS is close to 1. In order to test it we grouped the data into two sets discriminating according to whether the exchange rate lies above or below the central parity.

In table 1 we present the result of running a linear regression between EXPDEV and RES for both cases: when the exchange rate is above and below central parity. As can be observed in regression 1 there is a statistically significant inverse relationship between EXPDEV and RES when the exchange rate is positioned above the central parity. The outcome in regression 2 implies that there seems to be no statistically significant relationship between these variables when the exchange rate is positioned below the central parity.

In the previous section we also illustrated how the behavior of the interest differential (equal to expected depreciation under the uncovered interest parity hypothesis) as the exchange rate moves across the band is affected by the level of reserves. We observed there an inverse relationship between the two variables when reserves are high, and that at low reserves this inverse relationship is reverted at an exchange rate close enough to the upper limit of the band. Therefore, our next test consists on finding out whether, when the exchange rate is above central parity, EXPDEV and POS behave as hypothesized. That is, we expect to find a monotonic inverse relationship between these two variables when reserves are high, and a non-linear bowl-like shape when reserves are low.

We ordered the data with respect to the level of reserves and again grouped it into two sets: high reserves, for observations that have reserves levels above sample mean; and low reserves, below sample mean. In figures 10 and 11 we illustrate the pattern observed in the data. Notice the accentuated bowl-like shape when the exchange rate

[19] For a description of Mexico's currency band see Sanchez (1997).

[20] In November 12, 1991 the official band's upper and lower limits were established at 3096.6 and 3051.2 old pesos per dollar respectively, and the upper limit was left to crawl initially at 0.2 old pesos a day. The rate of crawl was increased to 0.4 pesos a day from October 19, 1992 on. Here we do not consider a constant rate of crawl as a realignment, so the latter is the only realignment of the official band's limits in the sample.

[21] The week the band was abandoned and the exchange rate was left to float freely.

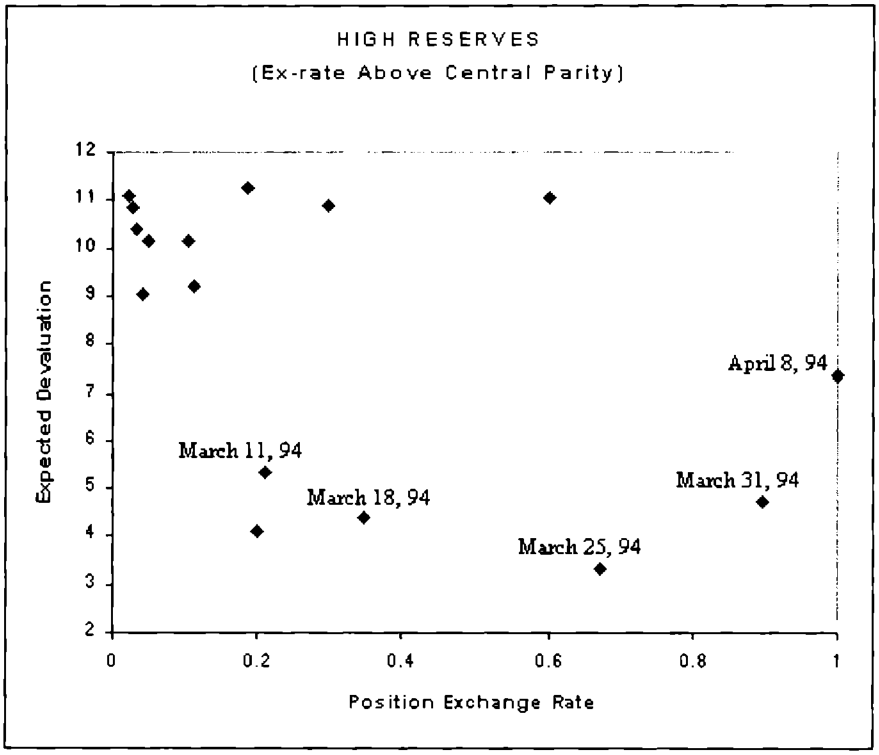
is above central parity and reserves are low (Figure 11). Since this could also be argued for the case of high reserves (Figure 10), we now try to test formally.

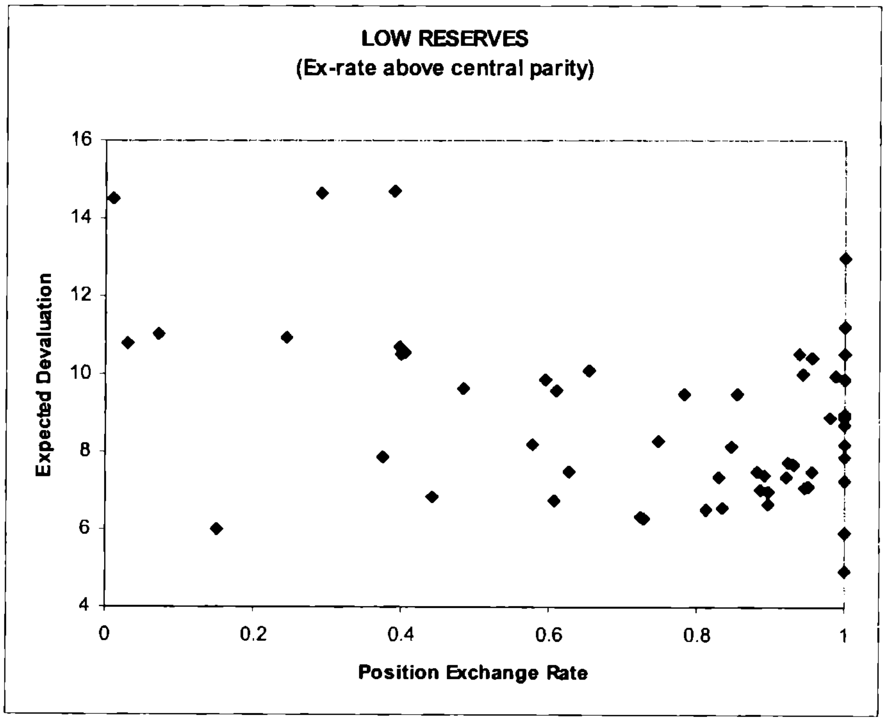
Table 1
Dependent Variable is EXPDEV

	Mexican-peso/US-dollar Band					French-franc/Deutsch-mark Band			
	1	2	3	4	4*	1	2	3	4
C	12.5 (8.72)	16.9 (7.55)	13.7 (12.82)	10.4 (10.23)	10.7 (8.95)	9.3 (26.3)	5.6 (28.2)	7.7 (8.3)	3.2 (8.0)
RES	-0.0001 (-2.56)	0.0003 (0.47)	--	--	--	-0.45 (-16.9)	-0.12 (-5.6)	--	--
POS	--	--	-12.3 (-3.10)	-13.7 (-2.28)	-14.2 (-1.08)	--	--	-11.5 (-1.87)	-3.9 (-2.11)
POS**2	--	--	7.2 (2.21)	10.9 (1.9)	25.2 (1.18)	--	--	22.8 (3.28)	3.4 (1.67)
R **2	0.18	0.13	0.33	0.26	0.15	0.4	0.09	0.22	0.02

1.- Exchange rate above central parity.
2.- Exchange rate always below central parity.
3.- Low reserves.
4.- High reserves.
4*.- High reserves without Colosio episode.
t-statistics in parenthesis.

Figures 10 and 11





For each data set (high and low reserves) we chose only those values of POS that were above zero (implying that the exchange rate is above central parity). We ran the following regression for both cases (high and low reserves):

$$EXPDEV = \beta_0 + \beta_1 POS + \beta_2 POS^2$$

where we expect β_1 negative and β_2 positive for low reserves, and β_1 negative and β_2 not different from zero when reserves are high. The results are presented in regressions 3 and 4 of table 1. As can be observed for the low reserves case (regression 3) both β_1 and β_2 come out of the right sign and highly significant. Moreover, the overall regression performs very well. For the high reserves case, however, β_1 comes out of the right sign and β_2 is significantly different from zero only at the 10% level, with the regression performing not as well.

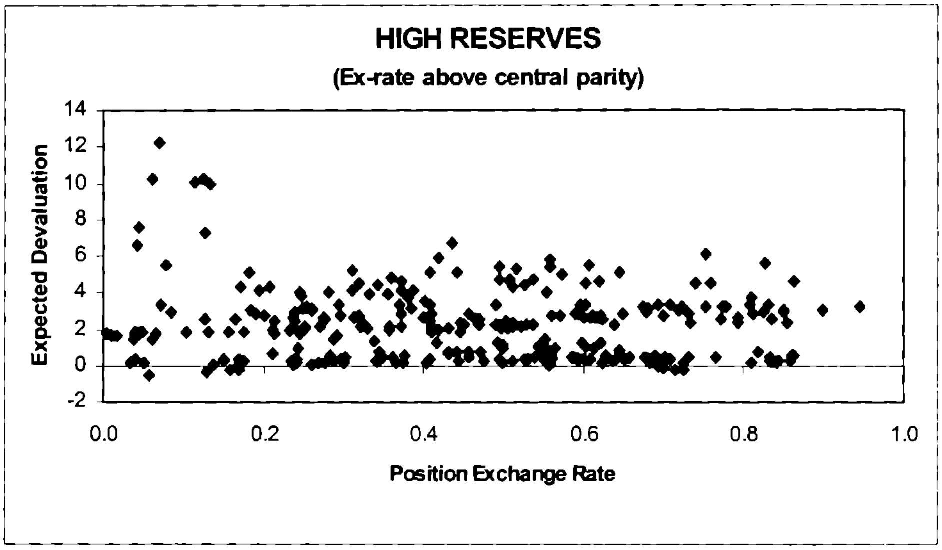
The fact that β_2 comes out somewhat significant even in the high reserves case could be due to including a low credibility episode in the high reserves set. Consider the evolution of the reserve stock at the central bank of Mexico during the first quarter of 1994. Because of the passage of the NAFTA through the US Congress in November 1993 Mexico experienced then a favorable shock that provoked a reserves increase taking the stock to US $29bn, a level unheard of since the currency band was established. However, an even more sizable adverse political shock hit the country at the end of the first quarter --the assassination of the ruling party's presidential candidate in March 23. Moreover, this event was preceded by a rise in the discount rate by the US Federal Reserve in late February. The story for the variables relevant here is summarized by the sequence of dated observations in Figure 10. There we see that on the weeks prior to the week of March 23 the interest differential and the exchange rate's position within the band respond to the adverse foreign interest rate shock in accordance with the credible band case. With the latter approaching the upper limit of the band and the former tilting against Mexico. However, right after the assassination the exchange rate shot to the upper limit with the interest differential rising with it, just as predicted by the imperfect credibility case. That is, after such a large adverse political shock not even the sizeable reserves stock that prevailed at the time reassured investors that the central bank could hold on to its commitment of keeping the exchange rate within the band.

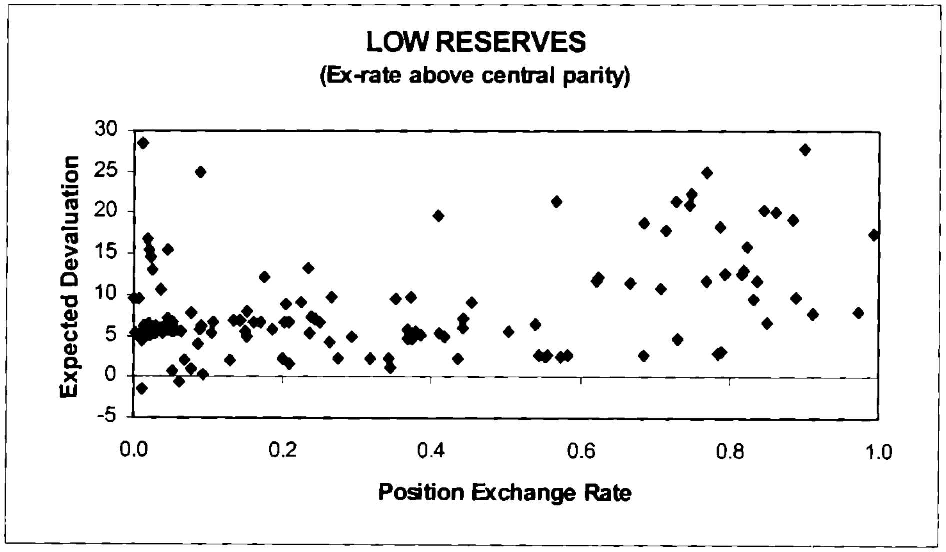
In regression 5 of Table 1 we test whether the results in the previous regression prevail after taking the Colosio episode out of the sample. We see there that the statistical significance of β_2 is lost. So, even when the significance of β_1 is also hurt this regression does contrast with regression 3, the low reserves case.

We also performed these tests on the French-franc/Deutsche-mark band. Regressions 1 and 2 of table 1 imply that, also for the French-franc/Deutsche-mark band, there is evidence that EXPDEV and RES move inversely both when the exchange rate is above the central parity. The coefficient is also negative and significant when the exchange rate is below the central parity. Nonetheless, in the former case the coefficient is four times as large, and much more significant, than in the latter.

Figures 12 and 13

As to our second test in Figures 12 and 13 we illustrate the behavior of EXPDEV





against RES when the exchange rate is above the central parity for both high and low reserves respectively. We can see how when reserves are high the interest differential tends to fall as the exchange rate approaches the upper limit of the band. When reserves are low, however, as the exchange rate approaches this limit a lot of the times the interest differential has to rise. In regression 3 we find evidence that in the low reserves case both parameters come out statistically significant at the 1-% confidence level and of the expected sign. When reserves are high β_2 comes out also negative but significant only at the 10% level, as can be read from regression 4.

6 Conclusions

In the present study we analyzed how the equilibrium exchange rate in a currency band is affected when credibility is less than perfect. In doing so we found it necessary to depart from the standard methodology of Krugman (1991) and work in a discrete-time framework. We observe that when the fundamental follows a gaussian random walk the distribution of the expected change in the exchange rate is censored normal. Using numerical methods we are able to characterize the equilibrium exchange rate both when foreign reserves at the central bank are plenty, and when they are not. We were also able to characterize the behavior of the expected change in the exchange rate at different reserve levels as the exchange rate moves within the band. The main implications of the model are that a large enough reserve stock stabilizes the exchange rate within the band. However, this is not the case when reserves are low. Moreover, this implies that as the exchange rate approaches the upper limit of the band the inverse relationship between the exchange rate and the interest differential is reversed.

The latter result led us to a straightforward test of the credibility of the band as a function of the central bank foreign reserve stock. We apply this test on the Mexican-peso/US-dollar band and the French-franc/Deutsche-mark band. The empirical results do not seem to contradict the hypothesis.

In the literature there are studies that develop models consistent with a direct relationship between the interest differential and the exchange rate in the band (See Bertola and Caballero (1992), and Bertola and Svensson (1993)). Moreover, other studies find that data for EMS countries do not contradict this hypothesis (Rose and Svensson (1995)). To our knowledge, however, no previous study has been able to link the level of the foreign reserve stock at the central bank with the sign of the relationship between the interest differential and the exchange rate in the band.

References.-

Bartolini, L. and Bodnar, G.M. (1992) "Target Zones and Forward Rates in a Model with Repeated Realignments", *Journal of Monetary Economics* 30 (3).

Bertola G. and Caballero R. J. (1992) "Target Zones and Realignments", *American Economic Review*, June, Vol 82, No.3.

Bertola G. and Svensson L.E.O. (1993) "Stochastic Devaluation Risk and the Empirical Fit of Target Zone Models", *Review of Economic Studies*.

Flood, R.P. and Garber, P.M. (1991) "The Linkage Between Speculative Attack and Target Zone Models of the Exchange Rate: some extended results" in *Exchange Rate Targets and Currency Bands*, Krugman P. and Miller M. eds., CEPR and NBER.

Girardin E. and Marimoutou V. (1997) "Estimating the credibility of an exchange rate target zone", *Journal of International Money and Finance*, Vol. 16, No.6.

Krugman P. (1991) "Target Zones and Exchange Rate Dynamics", *The Quarterly Journal of Economics*, Vol. CVI (3). August.

Krugman P. and Rotemberg J. (1991) "Speculative Attacks on Target Zones", *Exchange Rate Targets and Currency Bands*, Krugman P. and Miller M. eds., CEPR and NBER.

Obstfeld M. and Rogoff K. (1996) *Foundations of International Macroeconomics*, MIT Press.

Pesaran M.H. and Saimei H. (1995) "Limited-dependent Rational Expectations Models with Future Expectations" *Journal of Economic Dynamics and Control*, 19.

Rose, A. K., and Svensson, L.E.O. (1995) "Expected and Predicted Realignments: The FF/DM Exchange Rate during the EMS, 1979-1983, *Scandinavian Journal of Economics* 97 (2).