CIDE

NÚMERO 240

Rodolfo Cermeño & Kevin Grier

CONDITIONAL HETEROSKEDASTICITY AND CROSS-SECTIONAL DEPENDENCE IN PANEL DATA: THEORY, SIMULATIONS AND EXAMPLES

Abstract

In this paper we propose and implement an estimator to account for conditional heteroskedasticity and cross-sectional dependence in panel data. We present simple tests based on OLS and LSDV residuals to determine whether conditional heteroskedasticity exists and to test for individual effects in the conditional variance. Estimation of the model is based on direct maximization of the log-likelihood function by numerical methods. Monte Carlo simulations are conducted in order to evaluate the performance of this MLE estimator. We also present 3 empirical applications. We show that investment in a panel of five large U.S. manufacturing firms, inflation in a panel of seven Latin American countries, and consumption growth in a panel of 21 countries all exhibit significant conditional heteroskedasticity and cross-sectional dependence.

Resumen

En este artículo proponemos e implementamos un estimador que tome en cuenta la heterocedasticidad condicional y la dependencia de corte transversal en modelos de panel. Para detectar posible heterocedasticidad condicional y efectos individuales en la varianza condicional proponemos pruebas simples basadas en residuales de los estimadores OLS y LSDV. La estimación del modelo se basa en la maximización directa de la función logarítmica de verosimilitud mediante métodos numéricos. El desempeño del estimador MLE es evaluado con simulaciones de Monte Carlo. También presentamos 3 aplicaciones empíricas donde mostramos que la inversión en un panel de 5 empresas manufactureras de EUA, la inflación en un panel de 7 países latinoamericanos, y el crecimiento del consumo en un panel de 21 países, exhiben heterocedasticidad condicional y dependencia de corte transversal significativos.

Introduction

A new development in panel data econometrics is the use of large T panels of financial or macroeconomic data. A recent search of ECONLIT using the keyword phrases "financial panel data" and "macroeconomic panel data" produced 352 and 210 hits respectively. While it is well known that such financial and macroeconomic time series data are conditionally heteroskedastic, rendering traditional estimators consistent, but inefficient, this rapidly growing literature has not yet addressed the issue.

In this paper we combine typical panel modeling assumptions with the assumption that the error terms are multivariate normal with a time varying conditional variance-covariance matrix to produce a Pooled Panel-GARCH (PP-GARCH) model. We show how to estimate the model via maximum likelihood, present a methodology for its practical application, show some simulation evidence regarding its small sample properties, and present three empirical examples of the method.

Of course, sophisticated multivariate GARCH models already are in wide use, but these models are simply not practical for most panel applications. For example consider a panel with a cross-sectional dimension (N) of 20. Even if we restrict ourselves to a GARCH(1,1) conditional covariance matrix, the diagonal VECH model (*Journal of Political Economy* 1988) would have 630 parameters in the conditional covariance matrix. The BEKK model (*Econometric Theory* 1995) would require the estimation of 1010 coefficients. Even the relatively simple constant correlation model (*Review of Economics & Statistics* 1990) would have 270 parameters. By contrast, the analogous model for the estimator we develop here would have no more than 25 coefficients.

The paper is organized as follows. In section 1 we derive our basic estimator under the assumption of total parameter homogeneity. Section 2 discusses several generalizations that relax some of the homogeneity assumptions. Section 3 reports some simulation evidence about the finite sample properties of our estimator. Section 4 describes a testing and estimation procedure to determine what type of model is appropriate for a given set of data. In section 5, we provide three empirical examples of our procedure in action, investigating whether investment in a panel of five large US manufacturing firms, inflation in a panel of seven countries, and consumption growth in a panel of 21 countries exhibit conditional heteroskedasticity or cross-sectional dependence. Finally, section 6 concludes by reviewing our contribution and making some suggestions for future work.

1. The Basic Pooled Panel-GARCH (PP-GARCH) Model

This section describes the specification and estimation of our basic panel data model with a time-varying conditional covariance matrix. At this stage we assume complete parameter homogeneity across units in the panel. In the next section this assumption is relaxed to allow for some forms of parameter heterogeneity. We consider the following general pooled regression model:

$$y_{it} = \mu + \phi y_{it-1} + \mathbf{x}_{it}\boldsymbol{\beta} + u_{it}, \quad i = 1,\dots,N, \quad t = 1,\dots T \tag{1}$$

where N and T are the number of cross sections and time periods in the panel respectively, y_{it} is the dependent variable, μ is the common intercept coefficient, $\mathbf{x}_{it}$ is a row vector of exogenous explanatory variables of dimension k, $\boldsymbol{\beta}$ is a k by 1 vector of coefficients, ϕ is the AR parameter. We assume that $|\phi| < 1$ and T is relatively large so that we can invoke the consistency of Least Squares estimators[1]. Under the assumption $\phi = 0$, the process given by equation (1) becomes static. The disturbance term, u_{it}, is assumed to follow a mean zero normal distribution with

$$\begin{aligned}
&\text{(i)}\ E[u_{it}u_{js}] = \sigma_{it}^2 && \text{for } i = j \text{ and } t = s \\
&\text{(ii)}\ E[u_{it}u_{js}] = \sigma_{ijt} && \text{for } i \neq j \text{ and } t = s \\
&\text{(iii)}\ E[u_{it}u_{js}] = 0 && \text{for } i = j \text{ and } t \neq s \\
&\text{(iv)}\ E[u_{it}u_{js}] = 0 && \text{for } i \neq j \text{ and } t \neq s
\end{aligned} \tag{2}$$

Assumption (iii) states that there is no autocorrelation while assumption (iv) disallows non-contemporaneous cross-sectional correlation. Assumptions (i) and (ii) define a very general time specific variance-covariance process; some structure has to be imposed in order to make this process tractable. We propose the following model for the variance-covariance process:[2]

$$\sigma_{it}^2 = \alpha + \sum_{n=1}^{p} \delta_n \sigma_{i,t-n}^2 + \sum_{m=1}^{q} \gamma_m u_{i,t-m}^2 \qquad \text{for } i = 1 \ldots N \tag{3}$$

$$\sigma_{ijt} = \eta + \sum_{n=1}^{p} \lambda_n \sigma_{ij,t-n} + \sum_{m=1}^{q} \rho_m u_{i,t-m} u_{j,t-m} \qquad \text{for } i \neq j \tag{4}$$

Hereafter, the model defined by equations (1) (conditional mean), (3) (conditional variance) and (4) (conditional covariance) will be referred to as Model A.

Modeling the conditional variance-covariance process in this way is quite convenient in a panel data context for several reasons. First, by imposing a common

[1] For dynamic models with fixed effects and *i.i.d.* errors, it is well known that the LSDV estimator is biased in small T samples. See for example Kiviet (1995).

[2] The model is an adaptation of the model in Bollerslev, Engle and Wooldrige (1988)

dynamics to the variance and covariance processes across individuals, the number of parameters is reduced dramatically. In this particular case there are $2(p+q)+2$ parameters, regardless of the cross-sectional dimension of the panel. Second, the model does not imply constant cross-sectional correlation over time.[3] Third, it can easily be shown that the conditions $\alpha > 0$, $(\sum_{n=1}^{p}\delta_n + \sum_{m=1}^{q}\gamma_m) < 1$, and $(\sum_{n=1}^{p}\lambda_n + \sum_{m=1}^{q}\rho_m) < 1$ are sufficient for the conditional variance-covariance matrix to be positive definite (at each point in time) and to converge over time to some fixed positive definite matrix. Thus, unconditionally Model A is nothing more than a pooled panel data model with cross-sectionally correlated disturbances.

In vector notation we can express the static version of model (1) as:

$$\mathbf{y}_t = \mathbf{i}_N\mu + \mathbf{X}_t\boldsymbol{\beta} + \mathbf{u}_t; \qquad \mathbf{u}_t \approx N(\mathbf{0}, \boldsymbol{\Omega}_t); \qquad t = 1,\ldots,T \tag{5}$$

where $\mathbf{y}_t, \mathbf{u}_t$ are $Nx1$ vectors, $\mathbf{X}_t$ is a NxK matrix and $\mathbf{i}_N$ is an $Nx1$ vector of ones. It should be remarked that the N-dimensional vector of disturbances, $\mathbf{u}_t$, is distributed as a zero-mean multivariate normal. The variance-covariance matrix, $\boldsymbol{\Omega}_t$, is time dependent and its diagonal and off-diagonal elements are given by equations (3) and (4) respectively. The vector of observations $\mathbf{y}_t$ is therefore conditionally normally distributed with mean $\mathbf{i}_N\mu + \mathbf{X}\boldsymbol{\beta}_t$, and variance-covariance $\boldsymbol{\Omega}_t$. That is, $\mathbf{y}_t \approx N(\mathbf{i}_N\mu + \mathbf{X}_t\boldsymbol{\beta}, \boldsymbol{\Omega}_t)$, and its conditional density is given by:

$$f(\mathbf{y}_t / \mathbf{X}_t, \mu, \boldsymbol{\beta}, \varphi) = (2\pi)^{-\frac{N}{2}} |\boldsymbol{\Omega}_t|^{-\frac{1}{2}} \exp(\mathbf{y}_t - \mathbf{i}_N\mu - \mathbf{X}_t\boldsymbol{\beta})'\boldsymbol{\Omega}_t^{-1}(\mathbf{y}_t - \mathbf{i}_N\mu - \mathbf{X}_t\boldsymbol{\beta}) \tag{6}$$

where φ includes the parameters of the variance-covariance process given in (3) and (4). For the complete panel we will have the following log-likelihood function:

$$l = -(\frac{NT}{2})\ln(2\pi) - (\frac{1}{2})\sum_{t=1}^{T}\ln|\boldsymbol{\Omega}_t| - (\frac{1}{2})\sum_{t=1}^{T}(\mathbf{y}_t - \mathbf{i}_N\mu - \mathbf{X}_t\boldsymbol{\beta})'\boldsymbol{\Omega}_t^{-1}(\mathbf{y}_t - \mathbf{i}_N\mu - \mathbf{X}_t\boldsymbol{\beta}) \tag{7}$$

This likelihood function is similar to those derived in the context of prediction error decomposition models for multivariate time series.[4]

It is straightforward to show that if the disturbances are cross-sectionally independent the NxN matrix $\boldsymbol{\Omega}_t$ becomes diagonal and the log-likelihood function takes the simpler form:[5]

---

[3] Notice that a constant-correlation model would imply estimating $(N^2 - N)/2$ correlation parameters in addition to the conditional variance parameters, which is clearly unpractical even in panels with $N = 10$. On the other hand, assuming the same (cross) correlation coefficient for each pair of entities in the panel would be too restrictive.

[4] See for example Brockwell and Davis (1991) and Harvey (1990).

[5] Selecting between (7) and (8) is not as simple as it would appear. We discuss this issue in Section 5.

$$l = -\frac{NT}{2}\ln(2\pi) - \frac{1}{2}\sum_{i=1}^{N}\sum_{t=1}^{T}\ln(\sigma_{it}^2(\varphi)) - \frac{1}{2}\sum_{i=1}^{N}\sum_{t=1}^{T}\frac{(y_{it} - \mu - \phi - \mathbf{x}_{it}\beta)^2}{\sigma_{it}^2(\varphi)}, \qquad (8)$$

Even though the OLS estimator in equation (1) is still consistent and the most efficient among the class of linear estimators, the MLE estimator based upon (7) or (8) (depending on whether we have cross-sectionally correlated disturbances or not) is a more efficient non-linear estimator. In addition, by using the MLE estimator we can obtain the parameters of the conditional mean and conditional variance (covariance) equations simultaneously.[6]

From MLE theory we know that under regularity conditions the MLE estimator is consistent, asymptotically efficient and asymptotically normally distributed. We also know that these properties carry through when the observations are time dependent (Harvey (1990)). Therefore, we can assume that the MLE estimator in (7) or (8) is asymptotically normally distributed with mean equal to the true parameter vector and a covariance matrix equal to the inverse of the corresponding information matrix. These excellent asymptotic properties, however, do not directly speak to the properties of the estimator in sample sizes likely to be encountered in practice. We thus provide some evidence on the finite sample performance of this MLE PP-GARCH estimator relative its OLS counterpart by Monte Carlo simulations for a few designs. We present these results in Section 4 below, but first present some generalizations of the basic model.

2. Relaxing the Homogeneity Assumptions

Model A above can easily be modified to allow for some forms of parameter heterogeneity. In principle, it is possible to have heterogeneity in intercepts and/or slopes in both the mean and variance and covariance equations. In this present work, we only allow for heterogeneity in intercepts in the mean and variance (equations (1) and (3) respectively). In addition to Model A we consider the following 3 models:

(i) Individual effects in the mean equation and full parameter homogeneity in the covariance (Model B).

(ii) Individual effects in the variance equation and full parameter homogeneity in the mean equation and covariance equation (Model C)

(iii) Individual effects in both the mean and variance equations and full parameter homogeneity in the covariance equation (Model D).

[6] In this paper we will pursue direct maximization of (7) or (8) by numerical methods using the Optimization module of the GAUSS program. The asymptotic covariance matrix of this MLE estimator will be approximated by the inverse of the outer product of the gradient of l evaluated at MLE parameter estimates.

Explicitly, Model D is defined by:

$$y_{it} = \mu_i + \phi y_{it-1} + \mathbf{x}_{it}\boldsymbol{\beta} + u_{it}, \quad i = 1,\ldots,N, \quad t = 1,\ldots T \tag{9}$$

$$\sigma_{it}^2 = \alpha_i + \sum_{n=1}^{p} \delta_n \sigma_{i,t-n}^2 + \sum_{m=1}^{q} \gamma_m u_{i,t-m}^2 \qquad \text{for } i = 1\ldots N \tag{10}$$

$$\sigma_{ijt} = \eta + \sum_{n=1}^{p} \lambda_n \sigma_{ij,t-n} + \sum_{m=1}^{q} \rho_m u_{i,t-m} u_{j,t-m} \qquad \text{for } i \neq j \tag{11}$$

with μ_i and α_i representing the corresponding individual specific effects. In this case, the full parameter vector has $(2N + k + 2(p+q) + 1)$ elements. If the individual-specific effects are treated as fixed, the basic model given in the previous section applies directly to this case with no modifications other than including dummy variables in both the conditional mean and conditional variance equations.

Model B considers individual effects in the conditional mean equation and common intercept and slope coefficients in the conditional variance equation ($\alpha_i = \alpha$ in equation 10). In this case there are $(N + k + 2(p+q) + 2)$ parameters to be estimated. The same number of parameters would have to be estimated in the case of Model C.[7]

3. Finite Sample Performance of the PP-GARCH Estimator

It is well known that in the context of time series GARCH models, the (non-linear) MLE estimator not only has desirable asymptotic properties but also it is more efficient than the OLS estimator. Little is known, however, on the finite sample performance of the MLE estimator relative to its OLS counterpart in finite samples, particularly in panel data.

This section presents some Monte Carlo results on the performance of the MLE and OLS estimators in panels with conditionally heteroskedastic and cross-sectionally correlated errors. We study the bias and precision of the MLE and OLS estimators of the parameters of the conditional mean equation (equation 1) as well as the performance of the MLE estimator of the parameters in the conditional variance and covariance equations (equations 3 and 4).

We generate data according to equations (1), (3) and (4) and perform two sets of experiments. In the first one, we estimate the proper model by maximizing the log-likelihood function given by (7). In the second set of experiments, we incorrectly assume cross-sectional independence and estimate the model by

[7] We impose homogeneity on the conditional covariance process in all cases. Including individual effects in the covariance is possible in principle, but generally impractical since it would imply estimating $(N^2 - N)/2$ additional parameters.

maximizing the log-likelihood function given by (8). In this case we want to see how costly it is to assume cross-sectional independence when it is not true.

3.1. Monte Carlo design

For both sets of simulations the data generating model is defined by equations (1), (3) and (4). Recall that the last equation defines the dynamics of the conditional covariance process for each pair of entities in the panel. For practical purposes we study only the static mean pooled regression model with ARCH (1) variance and covariance processes.[8] For the conditional mean we set $\mu = \beta = 1$ and for the conditional variance we set $\alpha = 1, \gamma_1 = 0.8$. For the conditional covariance process we assume $\eta = 0.5$, and we allow ρ_1 to take the alternative values $\{0.25, 0.5\}$. In this way we will be able to evaluate the performance of the OLS and MLE estimators as the persistence of conditional covariance process is increased. The results are presented in Tables A1 to A4. For both sets of experiments we have set the number of trials in each Monte Carlo experiment to 1000.

3.2. Performance of the theoretically correct Panel GARCH estimator

Tables A1 and A2 show the results of our first set of simulations. The first observation is that as *T* increases for a given *N*, the OLS and MLE estimators of the intercept and slope coefficients in the mean equation improve on a mean squared error criterion. Second, when comparing the OLS and MLE estimators (for the mean equation), we find that the MLE outperforms the OLS estimator in terms of bias, precision and mean squared error. In every sample, the MLE estimator has a MSE smaller than the OLS estimator by at least a factor of 4 when ρ_1 =.25 and at least a factor of 5 when ρ_1= .5.

Turning to the MLE estimator of the variance coefficients α and γ_1 (intercept and ARCH (1) coefficient respectively), in both cases we observe improvements in precision and mean squared error as *T* increases. However, there is no obvious pattern in the biases. On a mean squared error criterion, the MLE estimator of the variance coefficients appears to be quite acceptable. The same pattern is observed for the covariance coefficients η and ρ_1 although their mean squared errors are higher than those of the variance coefficients.

Another observation is that as the persistence of the covariance process is increased the performance of the OLS estimator in terms of precision and MSE worsens (compare the OLS panels of tables A1 and A2). For the MLE estimator, however, we do not observe a similar pattern. Overall, on a mean squared error

[8] We have performed other simulations with dynamic mean equations and with GARCH(1,1) conditional covariance processes with results qualitatively similar to those presented in the text. These additional simulations are available upon request.

criterion, the performance of the MLE is quite acceptable and regarding the mean coefficients this estimator significantly outperforms its OLS counterpart.

3.3. Performance of the miss-specified PP-GARCH estimator

In these simulations we ignore the fact that the data is cross sectionally correlated by assuming that the coefficients η and ρ_1 are zero (i.e. we incorrectly assume that there is cross-sectional independence). Now both the OLS and the PP-GARCH estimators are misspecified. Regarding the parameters in the mean equation, we observe that the performance of both estimators improves with the sample size and that the PP-GARCH estimator clearly outperforms the OLS estimator. In fact the MSEs for the PP-GARCH estimator are always smaller by at least a factor of 2.5. The main cost of ignoring the conditional covariance terms is a lack of precision in estimating the conditional variance terms. This can be seen by comparing the MSEs for α and γ_1 across the relevant entries in Tables A1 and A3, as well as in Tables A2 and A4. While even the misspecified PP-GARCH model dominates OLS, there is a clear benefit to modeling cross-sectional correlation when it is present.

4. Choosing the Correct PP-GARCH model

We propose the following methodology to identify the appropriate statistical model. First, test for the presence of individual effects in the mean equation. Second, test for ARCH effects using OLS or LSDV residuals depending on the results in the first step. Third, determine if there are individual effects in the conditional variance process. Finally, after choosing and estimating a model, check its residuals to ensure that there is no remaining conditional heteroskedasticity.

4.1. Testing for individual effects in the mean equation

We test for individual effects in the mean equation using the LSDV estimator with heteroskedasticity and autocorrelation consistent covariance matrix, along the lines of White (1980) and Newey and West (1987) estimators applied to panel.[9]

For models A and B, where the variance process is identical across units, the OLS and LSDV are still best linear estimators. However for models C and D the unconditional variance will be different across units and the previous estimators will no longer be efficient. Therefore, inference based upon them will not be valid.

[9] Arellano (1987) has extended the White's heteroskedasticity consistent covariance estimator to panel data but this estimator is not apropriate here since it has been formulated for small T and large N panels which is not our case.

Given that we do not know a priori which is the appropriate model and that we can have auto correlation problems in practice, it seems convenient to use a covariance matrix robust to heteroskedasticity and autocorrelation. Specifically we will test the null hypothesis $H_0 : \mu_1 = \mu_2 = \cdots = \mu_N$ by means of a Wald-test, which will follow a $\chi^2_{(N-1)}$ distribution asymptotically.

4.2. Testing for ARCH effects and individual effects in the conditional variance

The second step uses either LSDV or OLS squared residuals (according to whether individual effects were found or not in the mean equation) to test for ARCH effects. We can use the estimated autocorrelation and / or partial autocorrelation coefficients to determine the existence and possible order of ARCH effects. Alternatively, the null hypothesis of conditional homoskedasticity or ARCH (0), against ARCH (*j*) can be tested for a few relevant values of j. This can be done via *LM*-test statistics based on the previous squared residuals and referred to the $\chi^2_{(j)}$ distribution. In practice, rejecting ARCH(0) in favor of a large number of significant lags will lead to the estimation of a GARCH model. That is to say, we are testing for ARCH, but given that a GARCH(1,1) approximates quite well ARCH models of arbitrarily large orders, we are considering here as viable alternatives ARCH(1), ARCH(2), and GARCH(1,1).

Finally, we test for individual effects in the ARCH process in two ways. First we test whether the squared residuals have a constant mean across the cross-sectional units. Second, we regress the OLS/LSDV squared residuals on an appropriate number of lagged squared residuals with and without individual effects and compare the fits via an *F* or Chi-square test.

4.3. Selecting the final model

After initially choosing an appropriate conditional variance model, based on the steps described above, and estimating the full model via maximum likelihood, it is important to make sure that all conditional heteroskedasticty has been captured in the estimation. We accomplish this in two ways. First, we add additional auto-regressive or moving average terms and check their significance. Second, we test the squared normalized residuals for any autocorrelation pattern. If significant patterns remain, alternative specifications of the conditional variance should be estimated and checked. At this final stage it also seems appropriate to evaluate whether a model with cross-sectional independent disturbances (i.e. zero covariances) is plausible. If after estimating the full model by maximizing the log-likelihood given in (7), we find that all the parameters of equation (4) are not significantly different from zero we can conclude in favor of cross-sectional independence since the covariances are zero conditionally and unconditionally. In this case we can re-estimate the model using the simpler log-likelihood function given by (8). Otherwise, we keep the unrestricted results from maximization of (7). Alternatively, we can test the hypothesis of cross-sectional independence by means of a likelihood ratio test upon estimating (7) and (8).[10]

5. Examples

In this section we illustrate the applicability of the PP-GARCH model with three examples using real world data.

5.1. Investment in a panel of five large U.S manufacturing firms

Here we use the well-known Grunfeld investment data set.[11] This is a panel of 5 large U.S. firms over 20 years. For each firm and for every year we have observations on gross investment (I), the market value of the firm (F), and the value of the stock of plant and equipment (C). The values of the variables F and C correspond to the end of the previous year. We test whether the conditional variance of the investment process is time dependent.

The model is specified as follows:

$$I_{it} = \mu_i + \beta_1 F_{it} + \beta_2 C_{it} + u_{it}, \qquad i = 1,\ldots,5; t = 1,\ldots,20 \tag{12}$$

$$\sigma_{it}^2 = \alpha_i + \sum_{n=1}^{p} \delta_n \sigma_{i,t-n}^2 + \sum_{m=1}^{q} \gamma_m u_{i,t-m}^2 \tag{13}$$

$$\sigma_{ijt} = \eta + \sum_{n=1}^{p} \lambda_n \sigma_{ij,t-n} + \sum_{m=1}^{q} \rho_m u_{i,t-m} u_{j,t-m} \tag{14}$$

We begin by testing for individual effects in the mean equation. The computed Wald statistic (using a HAC covariance matrix with lag truncation equal to 2), is $\chi^2_{(4)} = 115.98$, which is large enough to clearly reject the null hypothesis of no individual effects in the mean equation. Next we attempt to identify ARCH effects using the squared residuals from LSDV estimation of the mean equation. The computed partial auto correlation coefficients of the squared residuals are displayed in Table 1.

[10] We want to remark that in a panel GARCH context the issue of cross-sectional independence can not be evaluated with usual tests for cross-sectional correlation since they are focused on determining whether or not the unconditional covariances (and cross-correlations) are significantly different from zero. By examining equation (5) we can easily see that if the intercept is equal to zero but the slope coefficients are not, we still have non-zero conditional covariances and therefore cross-sectional dependence even though unconditionally we will have zero covariances and cross-sectional independence.

[11] These data are taken from Greene (1997, p. 650, Table 15.1).

In these data, only the first partial autocorrelation coefficient is statistically significant at the 0.05 level. It thus appears that the conditional variance of the error process follows an ARCH(1) process. Next, we try to determine if the conditional variance equation has individual effects by regressing the LSDV squared residuals on their first lag and a set of firm specific intercepts and then testing whether the intercepts share a common coefficient.

The computed statistics $F_{(4,94)} = 2.960$ and $\chi^2_{(4)} = 11.864$ reject the null hypothesis of no individual effects in the conditional variance at the 5% significance level. The model selection process thus suggests that there are individual effects in the mean equation, and that the conditional variance follows an ARCH (1) with individual effects, which is our model D in section 3 above.

Table 2 presents maximum likelihood estimates of this model in the sixth row of the table. For comparison we also present five other models, which are restricted versions of Model D. As noted above, the data reject the null hypothesis of no individual effects in the mean equation at the 0.01 level. This can be seen in Table 2 either by comparing either rows one and two (OLS vs. LSDV) or by comparing rows three and four (ARCH(1) pooled vs. ARCH(1) with individual mean effects). The data also reject the null hypothesis of conditional homoskedasticity, also at the 0.01 level. This can be seen either by comparing rows one and three (OLS vs. ARCH(1) pooled), or rows two and four (LSDV vs. ARCH(1) with individual mean effects) in Table 2. Finally the data reject the null hypothesis of no individual effects in the conditional variance equation at the 0.01 level (as seen by comparing rows 4 and 5 in Table 2). The final preferred model is still Model D, the final estimation in Table 2, which can be described as an ARCH(1) diagonal covariance with individual effects in both the mean and conditional variance equations. We do not find evidence of any significant auto-correlation in the normalized squared residuals from Model D, indicating that this specification is probably adequate.

From the reported results, we see that accounting for the conditional heteroskedasticity and cross-sectional dependence in these data notably changes the values of the coefficients on the explanatory variables in the mean equation. The coefficient on C (value of the firm's plant and equipment) falls from around .11 using LSDV to .04 using our PP-GARCH estimator, while remaining significant at the 0.01 level. The coefficient on F (the firm's market capitalization) falls from .35 using LSDV to .09 with PP-GARCH and loses its statistical significance as well.[12] In sum we find that this well-known panel, contains significant conditional heteroskedasticity and cross-sectional correlation. Further, modeling these phenomena materially affects the results of interest.

[12] It is also interesting to note that including the individual effect in the conditional variance changes the model from possibly non-stationary (ARCH coefficient > 1.0) to stationary (ARCH coefficient of 0.90).

5.2. Inflation in a panel of Latin American countries

Here we study inflation in 7 countries (Argentina, Brasil, Chile, Colombia, México, Peru and Venezuela) using quarterly observations on inflation rates (π) from 1991.1 to 1999.4.[13] Many papers have used a time-varying error variance as a measure of time-varying inflation uncertainty. In this application, we investigate whether such uncertainty still exists in Latin American countries in the relatively tranquil 1990's.

The model is given in equations 15-17. Note that the mean of inflation is specified as a simple *AR(1)* process:

$$\pi_{it} = \mu_i + \beta_1 \pi_{it-1} + u_{it}, \qquad i = 1,...,7; t = 1,...,36 \tag{15}$$

$$\sigma_{it}^2 = \alpha_i + \sum_{n=1}^{p} \delta_n \sigma_{i,t-n}^2 + \sum_{m=1}^{q} \gamma_m u_{i,t-m}^2 \tag{16}$$

$$\sigma_{ijt} = \eta + \sum_{n=1}^{p} \lambda_n \sigma_{ij,t-n} + \sum_{m=1}^{q} \rho_m u_{i,t-1} u_{j,t-m} \tag{17}$$

Again, we allow for heterogeneity only through individual effects in the conditional mean and conditional variance equations. Testing for individual effects in the mean equation yields the computed Wald statistic (using a HAC covariance matrix with lag truncation equal to 2,) of $\chi_{(6)}^2 = 7.82$, which is insignificant at any conventional level. In this case, there is no evidence against the null of no individual effects in the mean equation.

Table 3 presents the computed partial auto-correlation coefficients of the squared OLS residuals for the first 10 lags. Only the first autocorrelation is statistically significant, leading again to the preliminary choice of an ARCH(1) model for the conditional variance of inflation in this panel.

To look for individual effects in the conditional variance equation, we tested the null hypothesis of equality of the average squared residual across the seven countries and tested the significance of country specific intercepts in a regression of the squared residual on its first lag. In neither case was there any evidence found in favor of individual effects in the conditional variance.

The model selection procedure here picks an ARCH(1) covariance model with full parameter homogeneity in both equations (Model A). The three panels in Table 4 show the estimated inflation process using pooled OLS, then using Model A with the covariance coefficients constrained to be zero, then our preferred model for these data, Model A.

We find a strong degree of conditional heteroskedasticity in these data, with a highly significant estimated moving average coefficient of around .66 in the last panel. A likelihood ratio tests rejects the null hypothesis of cross-sectional independence at the 0.05 level. Relative to OLS, the MLE estimator finds a larger

[13] These data are compiled from the International Monetary Fund's (IMF) International Financial Statistics CD dated March 2000.

(1.9 vs. 1) and more significant intercept term and a smaller (.79 vs. .91) AR(1) term in the mean equation.

Even in the 1990s, Latin American inflation exhibits strong, though not very persistent volatility clustering and cross sectional dependence, indicating that there is still substantial inflation uncertainty and interdependence in the region.

5.3. Lack of consumption risk sharing in a panel of 21 countries

One of the empirical puzzles in international economics is that consumption fluctuations are highly correlated with idiosyncratic income fluctuations, while the theory predicts that these fluctuations should only be correlated with aggregate fluctuations. Following Lewis (1996) and Driscoll & Kraay (1998) we estimate cross-national panel regressions of the form:

$$C_{jt} = \theta + \beta X_{jt} + \varepsilon_{jt} \qquad (18)$$

Where C_{jt} is the growth rate of consumption in country j at time t, and X_{jt} is the deviation of GDP growth in country j at time t from the world average GDP growth at time t. Under perfect consumption risk sharing, β should not be significantly different from zero. For the sample, we chose the 21 countries in the Penn World Tables with data quality of C+ or better and no missing observations between 1950 and 1992.[14] Our panel dimensions are N=21 and T=42.

In the first column of Table 5, we present OLS estimates of β along with the OLS standard error and White's heteroskedasticity consistent standard error, which is about 26% larger. The second column of the Table reports the analogous coefficient and standard error resulting from applying our PP-GARCH estimator to the data assuming that the conditional variance and covariance equations each follow an ARCH(1) process. While the estimated coefficient is very similar (a 2.5% difference), the estimated standard error is roughly 29% smaller than the OLS standard error. There is significant conditional heteroskedasticity and cross sectional correlation in this panel as can be seen by examining the estimated conditional variance and covariance equations presented in the bottom half of Table 5.

[14] The countries are: Australia, Austria, Belgium, Brazil, Canada, Denmark, Finland, France, Germany, Ireland, Italy, Japan, Mexico, Netherlands, Norway, New Zealand, Spain, Sweden, Switzerland, UK, and USA.

6. Conclusions

In this paper we present an estimator designed to deal with issues of conditional heteroskedasticity and cross-sectional correlation in panel data. The estimator is especially relevant due to the following 4 factors: (1) The rapid growth of research using large T panels, (2) The ubiquity of conditional heteroskedasticity in macroeconomic and financial data, (3) the potential extreme inefficiency of estimators that fail to account for these phenomena, and (4) the impracticality of using existing multivariate GARCH models in a panel setting.

We show how to estimate the model via maximum likelihood and present simulations to shed light on its small sample properties and to illustrate the consequences of modeling the conditional variance but ignoring the cross-sectional correlation. We outline a methodology for model selection, then give 3 examples of real world panels that contain significant conditional heteroskedasticity and cross sectional correlation. We believe our results strongly indicate that inferences drawn from real world panels that ignore these phenomena may well be in error.

Future work on this topic could include extending the model to permit the conditional variance to influence the conditional mean (PP-GARCH-M), allowing for exogenous variables in the conditional variance equation, and perhaps employing more sophisticated models of the covariance matrix without losing the ability to estimate the model on real world panels.

TABLE 1:
Estimated partial auto-correlation coefficients on squared LSDV residuals
(Grunfeld investment data)

	Coefficient	**t-ratio**	**p-value**
PAC(1)	0.5225*	4.0785	0.0000
PAC(2)	-0.0925	-0.6633	0.7455
PAC(3)	0.0519	0.3542	0.3621
PAC(4)	0.0728	0.4862	0.3139
PAC(5)	0.2325°	1.5168	0.0664
PAC(6)	-0.1235	-0.7816	0.7817
PAC(7)	0.1293	0.7731	0.2207
PAC(8)	0.0482	0.2828	0.3889
PAC(9)	0.1245	0.6978	0.2435
PAC(10)	-0.1638	-0.9763	0.8342

LSDV estimated squared residuals are used since there is evidence of individual effects in the mean equation. The symbols *, ^ and ° indicate respectively 1%, 5% and 10% significance levels.

TABLE 2:
Panel estimation results for Investment with conditional heteroskedasticity and cross-sectional correlation

	Constant	μ_1	μ_2	μ_3	μ_4	μ_5	F	C	Log-likelihood
OLS estimates Mean equation	-48.0297 (-2.136)^						0.1051 (8.2780)*	0.3054 (3.8407)*	-624.99
		$\sigma^2 = 16194.677$							
LSDV estimates Mean equation		-76.0668 (-0.803)	-29.3736 (-1.7878)°	-242.1708 (-4.9985)*	-57.8994 (-3.375)*	92.5385 (1.7413)°	0.1060 (4.8109)*	0.3467 (7.1722)*	-561.8468
		$\sigma^2 = 4777.2951$							
ARCH(1): Pooled Regression (Model A with cross-sectional independence)	-37.4254 (-6.6876)*						0.1087 (40.3168)*	0.3358 (15.2096)*	-584.8165
		$\sigma_t^2 = \underset{(1.5385)^\circ}{796.6344} + \underset{(2.9566)^\wedge}{1.5593}\,\hat{u}_{t-1}^2$							
ARCH(1): Individual Effects in mean only (Model B with cross-sectional independence)		222.2649 (11.6958)*	20.6421 (4.9555)*	-82.6617 (-6.4023)*	-4.8258 (-0.9006)	230.9331 (21.7843)*	0.0502 (10.4699)*	0.1699 (20.0284)*	-510.6109
		$\sigma_t^2 = \underset{(3.0355)^*}{109.4899} + \underset{(5.3285)^*}{2.1890}\,\hat{u}_{t-1}^2$							
ARCH(1): Individual Effects in mean and variance (Model D with cross-sectional independence)		256.4222 (8.3794)*	24.7232 (3.5760)*	-51.7389 (-1.9547)^	-0.2614 (-0.0368)	275.3949 (7.8697)*	0.0457 (3.7677)*	0.1518 (3.0724)*	-503.6508
		$\sigma_t^2 = \underset{(0.9555)}{2434.819} + \underset{(1.8053)^\wedge}{124.2918} + \underset{(2.5735)^*}{594.4582} + \underset{(2.1779)^\wedge}{74.3458} + \underset{(2.0791)^\wedge}{5852.0537} + \underset{(2.8303)^*}{0.9004}\,\hat{u}_{t-1}^2$							
ARCH(1): Individual Effects in mean and variance (Model D: Pooled diagonal VECH))		280.5919 (4.8889)*	31.2229 (3.8360)*	-18.9448 (-0.7788)	4.0096 (0.6342)	225.0933 (7.0838)*	0.0444 (4.1186)*	0.0889 (1.4966)^	-492.3286
		$\sigma_{iit}^2 = \underset{(1.8685)^\circ}{4177.46} + \underset{(1.6978)^\circ}{231.4123} + \underset{(2.7945)^*}{406.8610} + \underset{(1.9248)^\circ}{67.2430} + \underset{(1.5856)}{3442.7053} + \underset{(2.9523)^*}{0.9085}\,\hat{u}_{iit-1}^2$ $\sigma_{ijt}^2 = \underset{(1.6500)^\circ}{76.1522} + \underset{(2.3668)^\wedge}{0.7254}\,\hat{u}_{ijt-1}^2$							

Panel ARCH estimates have been obtained by direct maximization of the log-likelihood function by numerical methods. For each model we show the mean coefficients followed by the estimated equation for the conditional variance process. Values in parenthesis are t-ratios and the symbols *, ^, °, indicate significance levels of 1%, 5% and 10% respectively. The t-ratios for OLS and LSDV estimates are based on HAC standard errors with a lag truncation of 2.

TABLE 3:
Estimated partial auto-correlation coefficients on squared OLS residuals
(inflation data)

	Coefficient	t-ratio	p-value
PAC(1)	0.3637**	5.6554	0.0000
PAC(2)	-0.0143	-0.2083	0.4176
PAC(3)	0.0386	0.5640	0.2866
PAC(4)	0.0323	0.4720	0.3187
PAC(5)	-0.0216	-0.3147	0.3766
PAC(6)	0.0210	0.3060	0.3799
PAC(7)	0.0107	0.1566	0.4379
PAC(8)	-0.0041	-0.0601	0.4761
PAC(9)	-0.0182	-0.2661	0.3952
PAC(10)	0.0577	0.8969	0.1853

OLS estimated squared residuals are used since there is evidence of no individual effects in the mean equation. The symbol ** indicate 1% significance levels.

TABLE 4:
Panel estimation results for Inflation with conditional heteroskedasticity and cross-sectional correlation

	Constant	π_{t-1}	Log-likelihood
OLS estimates Mean equation	1.0392 (0.5547)	0.9065 (9.7488)*	-1156.3338
		$\sigma^2 = 570.974$	
ARCH(1): Pooled regression (Model A with cross sectional independence)	1.8356 (3.3807)*	0.8195 (37.2695)*	-926.1028
		$\sigma_t^2 = \underset{(8.1457)^*}{36.2354} + \underset{(6.1209)^*}{0.8297}\,\hat{u}_{t-1}^2$	
ARCH(1): Pooled VECH Regression (Model A)	1.9016 (3.6363)*	0.7996 (35.8912)*	-920.8791
		$\sigma_{iit}^2 = \underset{(8.5355)^*}{35.4813} + \underset{(5.9655)^*}{0.6626}\,\hat{u}_{iit-1}^2$	
		$\sigma_{ijt}^2 = -\underset{(0.7703)}{0.5490} - \underset{(-3.0812)^*}{0.1229}\,\hat{u}_{ijt-1}^2$	

Panel ARCH estimates have been obtained by direct maximization of the log-likelihood function by numerical methods. For each model we show the mean coefficients followed by the estimated variance or the estimated equation for the conditional variance process. Values in parenthesis are t-ratios and the symbol (*) indicates significance level of 1%. The t-ratios for OLS estimates are based on HAC standard errors with a lag truncation of 3.

TABLE 5:
Panel estimation results for Consumption Growth with heteroskedasticity and cross-sectional correlation

A. Estimated Conditional mean equation: $C_{jt} = \theta + \beta X_{jt} + \varepsilon_{jt}$

$\hat{\beta}_{OLS}$	0.803	$\hat{\beta}_{MLE}$	0.7829
Standard error $\hat{\beta}_{OLS}$	0.5225*	Standard error $\hat{\beta}_{MLE}$ (Panel GARCH)	0.0198
White std. error $\hat{\beta}_{OLS}$	0.0352		0.3621

B. Estimated Conditional variance and covariance equations (Panel GARCH)

$$\sigma_{iit} = \underset{7.5632^{*}}{4.0621} + \underset{5.4319^{*}}{0.2368}\,\varepsilon^2_{iit-1}$$

$$\sigma_{ijt} = \underset{4.0674^{*}}{2.1220} + \underset{1.7643^{\circ}}{0.0457}\,\varepsilon_{it-1}\varepsilon_{jt-1}$$

Values in parenthesis for the conditional variance and covariance equations are *t*-ratios. The symbols * and ° indicate respectively 1%, and 10% significance levels.

APPENDIX

TABLE A1:
Monte Carlo results for static mean pool VECH model with cross-sectional correlation and ARCH (1) errors in variance and covariance ($\gamma_1 = 0.8, \rho_1 = 0.25$)

		OLS						MLE					
Sample	Coeff.	Bias	(%)	Std. Dev.	(%)	MSE	(%)	Bias	(%)	Std. Dev.	(%)	MSE	(%)
N = 5 T = 20	μ	-0.011	-1.1	0.4809	48.1	0.2314	23.1	0.0003	0.0	0.2290	23	0.0524	5.2
	β	0.0211	2.1	0.7728	77.3	0.5977	59.8	-0.004	-0.3	0.3385	34	0.1146	12
	α							-0.018	-1.8	0.2782	28	0.0777	7.8
	γ_1							-0.034	-4.3	0.1303	16	0.0181	2.3
	η							-0.022	-4.3	0.2429	49	0.0595	12
	ρ_1							-0.015	-6.0	0.1568	63	0.0248	9.9
N = 5 T = 50	μ	-0.013	-1.3	0.2836	28.4	0.0806	8.1	-0.002	-0.2	0.1423	14	0.0202	2.0
	β	0.0308	3.1	0.4451	44.5	0.1991	19.9	0.0094	0.9	0.2085	21	0.0436	4.4
	α							-0.003	-0.3	0.1698	17	0.0288	2.9
	γ_1							-0.021	-2.7	0.0956	12	0.0096	1.2
	η							-0.007	-1.3	0.1533	31	0.0235	4.7
	ρ_1							-0.014	-5.7	0.1049	42	0.0112	4.5
N = 5 T = 100	μ	-0.001	-0.1	0.2299	23.0	0.0529	5.3	0.0029	0.3	0.0952	9.5	0.0091	0.9
	β	-0.000	0.0	0.3617	36.2	0.1308	13.1	-0.006	-0.6	0.1481	15	0.0220	2.2
	α							0.0066	0.7	0.1238	12	0.0154	1.5
	γ_1							-0.007	-0.9	0.0749	9.4	0.0057	0.7
	η							0.0051	1.0	0.1114	22	0.0124	2.5
	ρ_1							-0.003	-1.1	0.0696	28	0.0048	1.9
N = 10 T = 20	μ	0.0048	0.5	0.3447	34.5	0.1189	11.9	0.0044	0.4	0.1488	15	0.0222	2.2
	β	-0.007	-0.6	0.5212	52.1	0.2716	27.2	-0.002	-0.2	0.2240	22	0.0502	5.0
	α							-0.006	-0.6	0.2190	22	0.0480	4.8
	γ_1							-0.014	-1.8	0.0860	11	0.0076	1.0
	η							0.0011	0.2	0.1959	39	0.0384	7.7
	ρ_1							-0.011	-4.3	0.1001	40	0.0101	4.1
N = 10 T = 50	μ	0.0103	1.0	0.2242	22.4	0.0504	5.0	0.0019	0.2	0.0978	9.8	0.0096	1.0
	β	-0.016	-1.6	0.3364	33.6	0.1134	11.3	-0.002	-0.2	0.1408	14	0.0198	2.0
	α							-0.002	-0.2	0.1432	14	0.0205	2.1
	γ_1							-0.006	-0.8	0.0634	7.9	0.0041	0.5
	η							-0.004	-0.8	0.1294	26	0.0168	3.4
	ρ_1							-0.004	-1.6	0.0653	26	0.0043	1.7
N = 10 T = 100	μ	0.0009	0.1	0.1702	17.0	0.0290	2.9	-0.001	-0.1	0.0686	6.9	0.0047	0.5
	β	0.0058	0.6	0.2466	24.7	0.0608	6.1	0.0046	0.5	0.0967	9.7	0.0094	0.9
	α							-0.004	-0.4	0.0945	9.5	0.0090	0.9
	γ_1							-0.003	-0.4	0.0548	6.8	0.0030	0.4
	η							-0.006	-1.1	0.0868	17	0.0076	1.5
	ρ_1							-0.003	-1.0	0.0480	19	0.0023	0.9

TABLE A1 (Continued):
Monte Carlo results for static mean pool VECH model with cross-sectional correlation and ARCH (1) errors in variance and covariance ($\gamma_1 = 0.8, \rho_1 = 0.25$)

		OLS						MLE					
Sample	Coeff.	Bias	(%)	Std. Dev.	(%)	MSE	(%)	Bias	(%)	Std. Dev.	(%)	MSE	(%)
N = 20 T = 20	μ	-0.010	-1.0	0.2869	28.7	0.0824	8.2	-0.001	0.0	0.1144	11	0.0131	1.3
	β	0.0119	1.2	0.3710	37.1	0.1378	13.8	0.0047	0.5	0.1464	15	0.0214	2.1
	α							0.0018	0.2	0.1914	19	0.0367	3.7
	γ_1							-0.009	-1.1	0.0604	7.5	0.0037	0.5
	η							-0.001	-0.1	0.1766	35	0.0312	6.2
	ρ_1							-0.008	-3.1	0.0764	31	0.0059	2.4
N = 20 T = 50	μ	0.0009	0.1	0.1819	18.2	0.0331	3.3	-0.002	-0.2	0.0684	6.8	0.0047	0.5
	β	0.0036	0.4	0.2353	23.5	0.0554	5.5	0.0069	0.7	0.0925	9.3	0.0086	0.9
	α							-0.003	-0.3	0.1198	12	0.0144	1.4
	γ_1							-0.007	-0.9	0.0510	6.4	0.0027	0.3
	η							-0.003	-0.5	0.1122	22	0.0126	2.5
	ρ_1							-0.010	-3.8	0.0493	20	0.0025	1.0
N = 20 T = 100	μ	0.0039	0.4	0.1314	13.1	0.0173	1.7	0.0015	0.1	0.0500	5.0	0.0025	0.2
	β	0.0010	0.1	0.1790	17.9	0.0321	3.2	-0.001	-0.1	0.0684	6.8	0.0047	0.5
	α							-0.000	0.0	0.0831	8.3	0.0069	0.7
	γ_1							-0.005	-0.6	0.0412	5.2	0.0017	0.2
	η							-0.000	-0.1	0.0781	16	0.0061	1.2
	ρ_1							-0.006	-2.3	0.0390	16	0.0016	0.6

TABLE A2:
Monte Carlo results for static mean pool VECH model with cross-sectional correlation and ARCH (1) errors in variance and covariance ($\gamma_1 = 0.8, \rho_1 = 0.5$)

		OLS						MLE					
Sample	Coeff.	Bias	(%)	Std. Dev.	(%)	MSE	(%)	Bias	(%)	Std. Dev.	(%)	MSE	(%)
N = 5 T = 20	μ	-0.012	-1.2	0.5327	53.3	0.2839	28.4	-0.004	-0.4	0.2190	22	0.0480	4.8
	β	0.0171	1.7	0.7868	78.7	0.6193	61.9	0.0076	0.8	0.3087	31	0.0954	9.5
	α							-0.025	-2.5	0.2640	26	0.0704	7.0
	γ_1							-0.037	-4.6	0.1272	16	0.0176	2.2
	η							-0.018	-3.7	0.2391	48	0.0575	12
	ρ_1							-0.038	-7.6	0.1567	31	0.0260	5.2
N = 5 T = 50	μ	-0.004	-0.4	0.3198	32.0	0.1023	10.2	0.0016	0.2	0.1290	13	0.0167	1.7
	β	0.0064	0.6	0.5046	50.5	0.2546	25.5	-0.002	-0.2	0.1934	19	0.0374	3.7
	α							0.0045	0.4	0.1685	17	0.0284	2.8
	γ_1							-0.017	-2.1	0.0960	12	0.0095	1.2
	η							0.0048	1.0	0.1557	31	0.0243	4.9
	ρ_1							-0.016	-3.1	0.1020	20	0.0106	2.1
N = 5 T = 100	μ	0.0013	0.1	0.2343	23.4	0.0549	5.5	0.0011	0.1	0.0970	9.7	0.0094	0.9
	β	-0.004	-0.4	0.3666	36.7	0.1344	13.4	-0.004	-0.4	0.1356	14	0.0184	1.8
	α							-0.007	-0.7	0.1151	12	0.0133	1.3
	γ_1							-0.009	-1.1	0.0745	9.3	0.0056	0.7
	η							-0.005	-1.1	0.1030	21	0.0106	2.1
	ρ_1							-0.008	-1.5	0.0789	16	0.0063	1.3
N = 10 T = 20	μ	-0.006	-0.5	0.3805	38.1	0.1448	14.5	-0.001	-0.1	0.1661	17	0.0276	2.8
	β	-0.007	-0.7	0.5991	59.9	0.3590	35.9	0.0050	0.5	0.2231	22	0.0498	5.0
	α							-0.020	-2.0	0.2189	22	0.0483	4.8
	γ_1							-0.020	-2.5	0.0926	12	0.0090	1.1
	η							-0.017	-3.4	0.2054	41	0.0425	8.5
	ρ_1							-0.028	-5.5	0.1090	22	0.0126	2.5
N = 10 T = 50	μ	-0.004	-0.4	0.2433	24.3	0.0592	5.9	0.0029	0.3	0.0975	9.7	0.0095	1.0
	β	0.0076	0.8	0.3447	34.5	0.1189	11.9	-0.000	0.0	0.1267	13	0.0160	1.6
	α							-0.007	-0.7	0.1347	14	0.0182	1.8
	γ_1							-0.016	-2.0	0.0731	9.1	0.0056	0.7
	η							-0.006	-1.3	0.1208	24	0.0146	2.9
	ρ_1							-0.015	-2.9	0.0771	15	0.0062	1.2
N = 10 T = 100	μ	0.0002	0.0	0.1864	18.6	0.0347	3.5	-0.001	-0.1	0.0687	6.9	0.0047	0.5
	β	0.0104	1.0	0.2641	26.4	0.0698	7.0	0.0047	0.5	0.0891	8.9	0.0080	0.8
	α							-0.004	-0.4	0.0922	9.2	0.0085	0.9
	γ_1							-0.008	-0.9	0.0596	7.5	0.0036	0.5
	η							-0.005	-1.1	0.0859	17	0.0074	1.5
	ρ_1							-0.008	-1.6	0.0590	12	0.0036	0.7

TABLE A2 (Continued):
Monte Carlo results for static mean pool VECH model with cross-sectional correlation and ARCH (1) errors in variance and covariance ($\gamma_1 = 0.8, \rho_1 = 0.5$),

		OLS						MLE					
Sample	Coeff.	Bias	(%)	Std. Dev.	(%)	MSE	(%)	Bias	(%)	Std. Dev.	(%)	MSE	(%)
N = 20 T = 20	μ	-0.005	-0.5	0.3035	30.3	0.0921	9.2	-0.002	-0.2	0.1216	12	0.0148	1.5
	β	0.0060	0.6	0.3550	35.5	0.1261	12.6	0.0047	0.5	0.1342	13	0.0180	1.8
	α							-0.002	-0.1	0.1842	18	0.0339	3.4
	γ_1							-0.012	-1.5	0.0646	8.1	0.0043	0.5
	η							-0.004	-0.7	0.1752	35	0.0307	6.1
	ρ_1							-0.013	-2.5	0.0785	16	0.0063	1.3
N = 20 T = 50	μ	0.0063	0.6	0.2190	21.9	0.0480	4.8	0.0000	0.0	0.0725	7.2	0.0053	0.5
	β	0.0013	0.1	0.2498	25.0	0.0624	6.2	0.0058	0.6	0.0831	8.3	0.0069	0.7
	α							-0.004	-0.4	0.1178	12	0.0139	1.4
	γ_1							-0.011	-1.4	0.0550	6.9	0.0032	0.4
	η							-0.004	-0.9	0.1126	23	0.0127	2.5
	ρ_1							-0.014	-2.7	0.0579	12	0.0035	0.7
N = 20 T = 100	μ	0.0082	0.8	0.1633	16.3	0.0267	2.7	0.0021	0.2	0.0519	5.2	0.0027	0.3
	β	-0.002	-0.2	0.1955	19.5	0.0382	3.8	-0.001	-0.1	0.0619	6.2	0.0038	0.4
	α							-0.001	-0.1	0.0818	8.2	0.0067	0.7
	γ_1							-0.009	-1.1	0.0474	5.9	0.0023	0.3
	η							-0.001	-0.2	0.0783	16	0.0061	1.2
	ρ_1							-0.010	-1.9	0.0494	9.9	0.0025	0.5

TABLE A3:
Monte Carlo results for static mean pool VECH model with cross-sectional correlation and ARCH (1) errors in variance and covariance when cross-sectional correlation is ignored
$(\gamma_1 = 0.8, \rho_1 = 0.25)$

		OLS						MLE					
Sample	Coeff.	Bias	(%)	Std. Dev.	(%)	MSE	(%)	Bias	(%)	Std. Dev.	(%)	MSE	(%)
N = 5	μ	-0.001	-0.1	0.4330	43.3	0.1875	18.8	0.0152	1.5	0.2769	28	0.0769	7.7
T = 20	β	0.0193	1.9	0.6801	68.0	0.4630	46.3	-0.013	-1.3	0.3982	40	0.1587	16
	α							0.0048	0.5	0.4462	45	0.1991	20
	γ_1							0.0093	1.2	0.2674	33	0.0716	8.9
N = 5	μ	-0.006	-0.6	0.3158	31.6	0.0998	10.0	0.0020	0.2	0.1708	17	0.0292	2.9
T = 50	β	0.0054	0.5	0.5009	50.1	0.2509	25.1	-0.001	-0.1	0.2463	25	0.0607	6.1
	α							0.0142	1.4	0.2741	27	0.0754	7.5
	γ_1							0.0150	1.9	0.1775	22	0.0317	4.0
N = 5	μ	0.0005	0.1	0.2329	23.3	0.0543	5.4	0.0034	0.3	0.1340	13	0.0180	1.8
T = 100	β	0.0038	0.4	0.3531	35.3	0.1247	12.5	-0.007	-0.7	0.1812	18	0.0329	3.3
	α							-0.002	-0.2	0.1885	19	0.0355	3.6
	γ_1							0.0040	0.5	0.1244	16	0.0155	1.9
N = 10	μ	-0.013	-1.3	0.4062	40.6	0.1651	16.5	0.0004	0.0	0.2429	24	0.0590	5.9
T = 20	β	0.0338	3.4	0.5623	56.2	0.3173	31.7	0.0146	1.5	0.2987	30	0.0894	8.9
	α							-0.003	-0.3	0.4120	41	0.1697	17
	γ_1							0.0256	3.2	0.2152	27	0.0469	5.9
N = 10	μ	0.0137	1.4	0.2469	24.7	0.0611	6.1	0.0066	0.7	0.1449	15	0.0210	2.1
T = 50	β	-0.006	-0.6	0.3480	34.8	0.1211	12.1	-0.001	-0.1	0.1709	17	0.0292	2.9
	α							0.0050	0.5	0.2479	25	0.0615	6.1
	γ_1							0.0079	1.0	0.1378	17	0.0190	2.4
N = 10	μ	0.0052	0.5	0.1688	16.9	0.0285	2.9	0.0015	0.2	0.1047	11	0.0110	1.1
T = 100	β	-0.009	-0.9	0.2355	23.6	0.0556	5.6	-0.006	-0.6	0.1242	12	0.0155	1.5
	α							0.0004	0.0	0.1710	17	0.0293	2.9
	γ_1							0.0033	0.4	0.1026	13	0.0105	1.3
N = 20	μ	0.0008	0.1	0.3260	32.6	0.1063	10.6	0.0039	0.4	0.2126	21	0.0452	4.5
T = 20	β	0.0112	1.1	0.3640	36.4	0.1326	13.3	-0.006	-0.6	0.1965	20	0.0387	3.9
	α							0.0019	0.2	0.4157	42	0.1728	17
	γ_1							0.0301	3.8	0.1811	23	0.0337	4.2
N = 20	μ	0.0091	0.9	0.2228	22.3	0.0497	5.0	0.0037	0.4	0.1375	14	0.0189	1.9
T = 50	β	-0.007	-0.7	0.2344	23.4	0.0550	5.5	-0.001	-0.1	0.1171	12	0.0137	1.4
	α							0.0079	0.8	0.2545	26	0.0649	6.5
	γ_1							0.0055	0.7	0.1252	16	0.0157	2.0
N = 20	μ	0.0103	1.0	0.1455	14.5	0.0213	2.1	0.0074	0.7	0.0915	9.1	0.0084	0.8
T = 100	β	-0.001	-0.1	0.1834	18.3	0.0336	3.4	0.0017	0.2	0.0939	9.4	0.0088	0.9
	α							-0.006	-0.6	0.1644	16	0.0271	2.7
	γ_1							-0.001	-0.1	0.0851	11	0.0072	0.9

TABLE A4:
Monte Carlo results for static mean pool VECH model with cross-sectionalcorrelation and ARCH (1) errors in variance and covariance when cross-sectional correlation is ignored
($\gamma_1 = 0.8, \rho_1 = 0.5$)

		OLS						MLE					
Sample	Coeff.	Bias	(%)	Std. Dev.	(%)	MSE	(%)	Bias	(%)	Std. Dev.	(%)	MSE	(%)
N = 5 T = 20	μ	-0.003	-0.3	0.5083	50.8	0.2583	25.8	-0.012	-1.2	0.2979	30	0.0889	8.9
	β	0.0203	2.0	0.8010	80.1	0.6420	64.2	0.0333	3.3	0.4078	41	0.1674	17
	α							0.0279	2.8	0.4978	50	0.2485	25
	γ_1							-0.003	-0.4	0.3239	41	0.1049	13
N = 5 T = 50	μ	0.0018	0.2	0.3340	33.4	0.1115	11.2	-0.007	-0.7	0.1778	18	0.0316	3.2
	β	0.0052	0.5	0.4939	49.4	0.2440	24.4	0.0087	0.9	0.2473	25	0.0613	6.1
	α							-0.012	-1.2	0.2833	28	0.0804	8.0
	γ_1							-0.004	-0.5	0.2059	26	0.0424	5.3
N = 5 T = 100	μ	0.0119	1.2	0.2333	23.3	0.0546	5.5	0.0057	0.6	0.1222	12	0.0150	1.5
	β	-0.014	-1.4	0.3332	33.3	0.1113	11.1	-0.001	-0.1	0.1608	16	0.0259	2.6
	α							0.0052	0.5	0.1955	20	0.0382	3.8
	γ_1							-0.006	-0.7	0.1458	18	0.0213	2.7
N = 10 T = 20	μ	0.0197	2.0	0.4193	41.9	0.1762	17.6	0.0045	0.4	0.2533	25	0.0642	6.4
	β	-0.021	-2.1	0.5354	53.5	0.2871	28.7	-0.018	-1.8	0.2742	27	0.0755	7.6
	α							0.0096	1.0	0.4523	45	0.2047	21
	γ_1							0.0236	2.9	0.2700	34	0.0735	9.2
N = 10 T = 50	μ	-0.003	-0.3	0.2724	27.2	0.0742	7.4	-0.003	-0.2	0.1550	16	0.0240	2.4
	β	0.0068	0.7	0.3336	33.4	0.1114	11.1	0.0103	1.0	0.1750	18	0.0307	3.1
	α							-0.003	-0.3	0.2562	26	0.0656	6.6
	γ_1							-0.008	-1.0	0.1845	23	0.0341	4.3
N = 10 T = 100	μ	0.0090	0.9	0.1891	18.9	0.0358	3.6	-0.002	-0.2	0.1103	11	0.0122	1.2
	β	-0.011	-1.1	0.2450	24.5	0.0602	6.0	-0.002	-0.2	0.1263	13	0.0160	1.6
	α							-0.011	-1.1	0.1892	19	0.0359	3.6
	γ_1							-0.000	0.0	0.1343	17	0.0180	2.3
N = 20 T = 20	μ	0.0194	1.9	0.3523	35.2	0.1245	12.4	0.0146	1.5	0.2222	22	0.0496	5.0
	β	-0.002	-0.2	0.3856	38.6	0.1487	14.9	-0.006	-0.6	0.1830	18	0.0335	3.4
	α							0.0053	0.5	0.4112	41	0.1691	17
	γ_1							-0.001	-0.1	0.2490	31	0.0620	7.8
N = 20 T = 50	μ	0.0068	0.7	0.2313	23.1	0.0535	5.4	0.0059	0.6	0.1429	14	0.0205	2.0
	β	-0.004	-0.3	0.2361	23.6	0.0558	5.6	-0.005	-0.5	0.1170	12	0.0137	1.4
	α							0.0018	0.2	0.2597	26	0.0674	6.7
	γ_1							-0.002	-0.3	0.1667	21	0.0278	3.5
N = 20 T = 100	μ	0.0024	0.2	0.1691	16.9	0.0286	2.9	0.0009	0.1	0.1021	10	0.0104	1.0
	β	-0.002	-0.2	0.1705	17.1	0.0291	2.9	-0.001	-0.1	0.0852	9	0.0073	0.7
	α							0.0035	0.3	0.1860	19	0.0346	3.5
	γ_1							-0.008	-1.0	0.1270	16	0.0162	2.0

References

Arellano, M., 1987, "Computing Robust Standard Errors for Within-Groups Estimators," *Oxford Bulletin of Economics and Statistics*, 49, 431-434,

Bollerslev, Tim, 1986, "Generalized Autoregressive Conditional Heteroskedasticity," *Journal of Econometrics* 31, 307-327.

Bollerslev, Tim, 1990, "Modeling the Coherence in Short-Run Nominal Exchange Rates: A Multivariate Generalized ARCH Model," *Rev. of Economics and Statistics* 72, 498-505.

Bollerslev, Tim, R.Y. Chou, and Kenneth Kroner, 1992, "ARCH Modeling in Finance: A Review of the Theory and Empirical Evidence", *Journal of Econometrics*, 52, 5-59.

Bollerslev, Tim, Robert Engle, and Jeffrey Wooldridge, 1988, "A Capital Asset Pricing Model with Time Varying Conditional Covariances", *Journal of Political Economy*, 96, 116-131.

Brockwell, P.J. and R.A. Davis, 1991, *Time Series: Theory and Methods*, Second Edition, Springer-Verlag.

Davidson, R. and J.G. Mckinnon, 1993, *Estimation and Inference in Econometrics*, Oxford University Press.

Driscoll, John and Art Kraay, 1998, "Consistent Covariance Matrix Estimation with Spatially Dependent Panel Data", *Rev. of Economics & Stat.,* 80, 549–600.

Engle, Robert, 1982, "Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of UK Inflation", *Econometrica*, 50, 987-1007.

Engle, Robert, David Lillien and Russell Robbins, 1987, "Estimating Time Varying Risk Premia in the Term Structure: The ARCH-M Model," *Econometrica* 55, 391-407.

Greene, William, 1997, *Econometric Analysis*, 3rd Edition, Prentice Hall.

Harvey, A., 1990, *The Econometric Analysis of Time Series*, Second Edition, The MIT Press.

Kiviet, J. F., 1995, "On Bias, Inconsistency, and Efficiency of Various Estimators in Dynamic Panel Data Models," *Journal of Econometrics*, 68, 53-78.

Lewis, Karen, 1996, "What Can Explain the Apparent Lack of International Consumption Risk Sharing?", *Journal of Political Economy*, 104, 267-297.

Newey, W. and K. West, 1987, "A Simple Positive Semi-Definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix," *Econometrica*, 55, 703-708.

Vilasuso, Jon, 2002, Causality Tests and Conditional Heteroskedasticity, *Journal Of Econometrics*, 101, 25-35

White, H., 1980, "A Heteroskedasticity-Consistent Covariance Estimator and a Direct Test for Heteroskedasticity," *Econometrica*, 48, 817-838.