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MYOPIA, INTERNATIONAL COMPETITIVENESS AND POLITICAL CONSTRAINS

Abstract

In this paper we present a model that describes how historical political constraints by themselves, or in combination with a sufficient degree of impatience, may be the cause of bankruptcy in some industries when a closed economy is opened to foreign competition. The model assesses the behavior of two types of firms, impatient and patient, which may or may not adopt foreign technology. The costs involved are not only economic but also political. These political costs are, nonetheless, measured in monetary terms. At some moment, which depends on the political constraints, a third firm inters the market –the foreign one. Depending on the national firms' degree of impatience and the costs associated with political constraints, Nash equilibria, in which one or even both firms—in the medium or long run-- have to shut down, exist. All these strategies result to be subgame perfect equilibrium.

Resumen

En este trabajo presentamos un modelo que describe cómo restricciones de carácter político ocurridas en el pasado por sí mismas, o en combinación con un suficiente grado de miopía de parte de los empresarios, podrían ser la causa de bancarrota en algunas industrias cuando una economía cerrada se abre a la competencia internacional. El modelo considera dos tipos de firmas, una impaciente y otra paciente, las cuales tienen la opción de adoptar o no una tecnología extranjera, más eficiente. Los costos de esta nueva tecnología no son sólo económicos, sino también de carácter político. Estos costos políticos, sin embargo, son medidos en términos monetarios. El en algún momento futuro, que depende de las condiciones políticas, una tercera firma entra en el mercado a competir, la que posee la tecnología extranjera. Dependiendo del grado de miopía de las empresas y de todos los costos políticos y económicos, y en concreto del entorno que ambos provocan en relación a la adopción de esta nueva tecnología de parte de las empresas nacionales, existen equilibrios de Nash, en los cuales una o bien ambas empresas nacionales deciden no adoptar la nueva tecnología, y por lo tanto entran en bancarrota cuando la firma extrajera entra en el mercado. Todos estos equilibrios resultan ser perfectos en subjuegos.

Introduction

Historical evidence suggests that protectionist trade policies are often the result of a complex interaction between unions, firms, and the government. When a new labor-saving and cost reducing technology appears in the international scenario, these three actors may find themselves better off in the short run by maintaining the technology employed by the industry unchanged. This is the case when specific economic, financial, and political conditions, make them face as an alternative: unemployment, widespread bankruptcies, and social unrest. Yet every time the decision to change the technology and modernize the industry is postponed, the problem for the future worsens. If, at a given moment, the status quo was maintained for fear of unemployment and of firms' bankruptcies, as the gap between the technology used by the domestic industry and that in the industry's leaders elsewhere in the world widens, the danger of widespread unemployment and bankruptcies in the industry only increases. Thus, when the decision to modernize the industry and open up the economy is finally taken the industry is hard hit.

The history of the Mexican textile industry closely fits this description of events as is shown in Gómez-Galvarriato (2001). The comparison of production costs c. 1911 of one of the most modern and productive firms (the Compañía Industrial Veracruzana S.A.), with its international counterparts suggest that by that time the firm could compete with English cloth prices (although not with American cloth prices). Yet as time went by its competitive standing deteriorated as a result of legally binding industry wide collective contracts that hindered the firm from adopting new technology. The first "wage-list" was signed by firms' and workers' representatives in 1912. Yet it did not become legally binding until 1927 when as a result of the Convention of Workers and Industrialists of 1925-27, a collective contract was agreed with basically the same technical features as that of 1912. This collective contract fixed the maximum number of machines per worker and established specific wages-per-piece. Under these conditions, industrialists had no incentive to introduce better machinery because it would not enable them to reduce labor costs, since wages-per-piece and the workers-per-machine had to remain invariable. It set, for example, the maximum number of looms per weaver to 6, when using Nortrhop automatic looms a weaver could tend 20. It also required that the companies maintained fixed the number and type of jobs they provided. The 1925-27 Convention agreements may be understandable under the circumstances of worldwide depression in the textile industry. Nevertheless, the precepts adopted were ratified over and over again, without any changes until at least 1951, and until 1972 with few modifications. It was not until 1994, that the industry-wide collective contract in this industry was abolished. Company documents tell on the difficulties firms faced to install modern machinery, as a result of these regulations, making it many times simply impossible. These agreements were, of course, paralleled by rises in tariffs that the government carried out in order for the status quo to prevail. When tariffs were reduced after 1985 few of these firms survived.

Whereas the case of the Mexican spinning and weaving industry may be an extreme example of a sector institutionally tied down in order not to modernize, we believe this story is not exceptional, but a pattern experienced, in a lesser or greater degree, by several industries in many of the developing countries which have recently opened-up their economies. Ana Revenga's (1997) study of the Mexican manufacturing during 1984-90 period indicates that the 1985-87 trade liberalization episode affected firm-level employment and wages through several channels. It shifted down the industry product and labor demand. This in itself may have accounted for a 3%-4% decline in real wages on average (and for as much as 10%-14% decline in the more affected industries). Moreover, trade reform also reduced the rents available to be captured by firms and workers. This had an additional negative effect on firm-level employment and wages.

Several papers have addressed the question of why protectionist trade policies have failed to serve as an instrument to provide time and resources to firms to undertake cost-reducing investments that would eventually enable them to compete internationally. Their argument is based on the idea that governments are unable to credibly precommit to the unconditional elimination of protection, and thus protection generates a trade-off for the firm. "If during the program, the firm does not invest sufficiently in cost reductions, then it gains a renewal of future protection, and it saves the opportunity cost of capital. It loses, however, the benefits derived from cost reductions. If, at the margin, the gains are greater than the losses, then the firm will inevitably choose no to invest sufficiently" (Tornell, 1991). Temporary protectionist programs are thus "time inconsistent". Staiger and Tabellini (1987) have shown that an optimal trade policy may be time inconsistent, and that a suboptimal but time-consistent policy involves an excessive amount of protection, and that when protectionist policies are time inconsistent tariffs may dominate production subsidies. Matsuyama (1990) has also found dynamic inconsistency of optimal temporary protection by examining whether or not there exists a sequence of credible government threats to liberalize in the future which would induce the firms to invest as a sub-game perfect equilibrium. Although such an equilibrium exists, it fails to pass the "renegotiation-proof" criterion and thus time-inconsistency results. Tornell (1991) shows that "investment-contingent subsidies" do not eliminate time inconsistency in protectionist programs. Wright (1995) shows the time inconsistency persist even when the firm effort and costs are publicly observable. These papers suggest that a third party such as the GATT or an international treaty is necessary to make the government's threat credible and thus enable a temporary protection policy to be effective in terms of forcing the firms to invest in new technology.

In this paper we address the issue of why firms may choose not to invest in new technology even when the threat of liberalization is credible, and they have a perfect foresight of when they will face foreign competition. We suggest a theoretical approach, based on game theory, in order to describe how historical political constraints by themselves, or in combination with a sufficient degree of impatience, may be the cause of bankruptcy in some industries when a closed economy is opened to foreign competition.

The rest of the paper is organized as follows. Section II lays down the model. Section III discusses the results of the model. Finally Section IV concludes. All proofs are given in the Appendix.

The model

The general set-up.

Time is discrete and the horizon is infinite. In the economy, at the outset, there are two firms, one impatient and one patient, characterized by their discount factors $0 < \beta^I < \beta^P < 1$ respectively. These two firms are the players of the game; the rest of the actors in the economy will take no strategic decisions. At some moment, a third firm enters the market, the foreign one, with a discount factor β^F. All the firms produce a single good, and the inverse demand function of it is $P(Q) = a - Q$. There is complete and perfect information, so all the fundamentals of the market are common knowledge. The general structure of the economy is such that there are no credit opportunities and, at the end of each period, positive profits are given to the shareholders by means of dividends, so a firm can only face costs at each period by paying them from the corresponding period profits. This last hypothesis is a reasonable assumption to make when studying closed developing economies, which are the main subject of this paper.

The pay-off functions and strategies.

The firms compete à la Cournot to sell the good in each period according to the respective costs, in such a way that the pay-off functions are as follows. If the firms adopt sequences of costs $\{C_t^i\}_{t=0}^{\infty}$ where $i = I, P$, the pay-off function of firm i is given by

$$\Pi^i(\{C_t^i\}_{t=0}^{\infty}, \{C_t^j\}_{t=0}^{\infty}) = \sum_{t=0}^{\infty} (\beta^i)^t \pi^i(C_t^i, C_t^j) \tag{1}$$

with $i, j = I, P$, where $\pi^i(C_t^i, C_t^j)$ is the Cournot profit of firm i at time t, if the respective costs chosen for that period are C_t^i and C_t^j. [1] Therefore, a strategy of the firm $i = I, P$ is any sequence $\{C_t^i\}_{t=0}^{\infty}$.[2] These strategies, nonetheless, have to be affordable; that is, for a given firm $i = I, P$, a strategy $\{C_t^i\}_{t=0}^{\infty}$, given that the other firms' strategy is $\{C_t^j\}_{t=0}^{\infty}$, is affordable if for any $t \geq 0$ we have $\pi^i(C_t^i, C_t^j) > 0$. We assume the convention that if a strategy $\{C_t^j\}_{t=0}^{\infty}$ for a firm i is not affordable, then, according to this strategy, the corresponding firm shuts down at the first period at which profits are zero. Implicitly, there are fixed costs embedded in these extra political and economic

[1] Recall that we suppose that the foreign firm has no strategic behavior. Nevertheless, it will be straightforward to see that, under our assumptions listed below, the strategic behavior of the foreign firm is trivial. In any case, for these reasons, we drop, in the notation of the pay-off of the firms I and P, the participation of the foreign firm; that is, we simply write $\Pi^i((\{C_t^i\}_{t=0}^{\infty}), (\{C_t^j\}_{t=0}^{\infty}))$. However, for any t large enough (this expression will be precise once the model is completely formalized), $\pi^i(C_t^i, C_t^j)$ has to take into account that there exist another firm in the market whose constant marginal cost will be some number C^F (see '*The costs,*' below), that is, $\pi^i(C_t^i, C_t^j)$ is the cournot profit where there are three firms, the firm i ($i = I, P$), the firm j ($j = I, P$) and the foreign firm, whose corresponding marginal costs are C_t^i, C_t^j and C^F.

[2] Nevertheless, the numbers C_t^i will belong to a finite set $\{C^F, C^N\}$, whose elements are described below. Hence, the set of strategies will be a discrete (numerable) sub-set of $\prod_{t=1}^{\infty} \Re^t = \Re^{\infty}$, where $\Re$ is the set of the real numbers.

costs. According to the general structure of the economy, these fixed costs can only be paid at the end of the period if there are positive profits. Since we assume perfect and complete information, the assumption that fixed costs can be paid at the end of the period is a particular case of the rational expectation hypothesis.

The costs.

The firms may use the extant national technology, characterized by its constant marginal cost C^N in each period, or they may adopt the new foreign technology, characterized by C^F, which is the cost that the foreign firm that owns it has to face. If the national firms want to adopt the new technology, they still have to face not only economic costs but also some political costs, in addition to C^F, which are described below.

- *The economic costs.*
  The extra economic costs are exogenously given and defined by a decreasing finite sequence $C_0^e, C_1^e, ..., C_n^e$ ($C_t^e > C_{t+1}^e$ for all $0 \leq t \leq n-1$), where C_n^e is the permanent cost that the national firm adopting the new technology has to pay to the owner of said technology. In this way, we capture the idea that at the beginning the economic costs are high but decrease over time until stabilizing at the level C_n^e, which represents the royalty paid to the owner of the foreign technology.[3] Hence, if at $t = \bar{t}$ the new technology is adopted, the economic costs paid by the firm from that moment are $C_{\bar{t}+t} = C^F + C_t^e$ for all $0 \leq t \leq n$, and the firm faces $C^F + C_n^e$ from $t = \bar{t} + n$; that is, $C_t = C^F + C_n^e$ for all $t \geq \bar{t} + n$. In other words, if the new technology is adopted at $t = \bar{t}$, the sequence of costs that the firm faces is given by $\{C_t\}_{t=0}^{\infty}$, where $C_t = C^N$ for all $0 \leq t < \bar{t}$, $C_t = C^F + C_t^e$ for all $\bar{t} \leq t < \bar{t} + n - 1$, and $C_t = C^F + C_n^e$ for all $t \geq \bar{t} + n$.
  The time length n is the number of periods that a national firm needs to completely install the new technology. After this, the firm only has to pay the natural cost (C^F) plus the royalty (C_n^e). It is natural to think of these costs as decreasing, since normally installing a new technology causes some exceptional costs at the beginning.

- *The political costs.*

In this paper we do not model the political process that leads to protection, but include political costs political costs are exogenously given and defined by a possibly infinite sequence $\{C_t^p\}_{t=0}^{l}$ ($l \leq \infty$). Each C_t^p represents the extra cost that the firm has to pay if it adopts the new technology at time t, but once and for all, due to, for instance, the fact that the firm may have to dismiss some workers that are not useful anymore. These costs depend on negotiations between the firms, the government, and the trade unions. The more powerful the trade unions are, the larger these costs would be. It would be reasonable to assume that those costs are increasing because as the gap

[3] An alternative interpretation for the permanent cost C_n^e can be given: The owner of the technology is the person who produces it, and only this person. Therefore, C_n^e may represent his profits, if we understand that he is selling not the new technology but the strategic elements to use it. These elements cannot be produced by anyone but the owner; thus, the buyer cannot develop that new technology.

between the domestic and the foreign technology widens it is likely that more workers would be redundant when the foreign technology is adopted. Nonetheless, without that assumption, the model can be used to assess situations under which those costs can become constant or even decreasing —at least temporarily—, as it is the case in some countries in Europe, Spain, for example. [4]

The role of the government

The government decides the period at which the economy opens, although it is exogenously given. At that moment, the foreign firm may enter the market. As for the domestic firms, for simplicity, we assume that when the foreign firm enters, faces a constant marginal cost each period. Given this last assumption, without loss of generality, we set that cost at C^F. Let denote by t^g the period time at which the economy opens. As it has been expressed, the government also plays a role, together with the firms and the trade unions, as a party in the negotiations that determine the political costs firms face if they adopt the new technology.

Technical assumptions

Some fundamentals of the economy satisfy the following general conditions:

A1 $C^F < C^N < a$.

This is to account for the fact that the new technology is more efficient than the national one at some degree. It also says that, given the demand function, with both technologies is possible to earn positive Cournot profits.

A2 $\frac{C^F+a}{2} < C^N$.

This means that the foreign technology not only is more efficient than the national one but also that the national one is not competitive, in the sense that it only can earn zero profits. Therefore, given that there are no credit opportunities, the national firm shuts down.

A3 $C^N < C_t^e + C^F$ for all $0 \leq t \leq n-1$ and $C^N > C_n^e + C^F$

This assumption capture the following natural idea: The new technology is more costly than the national one at the beginning, but at some moment becomes more efficient. We set this special moment at $t = n$ just for simplicity. No results change without this simplification. Nevertheless, it is reasonable to think of that the new technology, from the point of view of the national firms, becomes more efficient at the moment the costs stabilize.

A4 $\frac{C^F+a}{2} > C_n^e + C^F$ and $\frac{C^F+a}{2} < C_{n-1}^e + C^F$.

This is to express the following trivial assumption: If both firms adopt the new technology , it is possible to produce positive quantities even when the economy opens, provided that the new technology is completely installed, that is, provided all the extra economic costs have been paid.

[4] In Spain, the labor market has been historically very rigid, being this, perhaps, the cause of very high rates of unemployment. In any case, lately, the labor market is more flexible than in the past, allowing for temporal job and, because of this, lowering the political cost, in our broad sense.

The Results

For the sake of clarity, we first give the intuition of a result and then we announce formally the corresponding theorem. In this section, no formal proofs are presented. All formal proofs are given in the Appendix.

The first result responds to the following intuition. If the economy opens too early, or if the political costs are too high at the beginning, both firms, independently of their degree of impatience, decide not to adopt the foreign technology, because they cannot afford the total costs. At the moment the foreign firm enters the market, both national firms have to shut down, and those decisions, if revised in the future, are not changed (there are no advantages in do it so), given that there are no credit market opportunities. Formally:

Theorem 1 *If (1) $t^g < n$ and $\frac{a-2C^F+C^N}{3} < C^e_{t^g}$ (the government opens the economy too early), or (2) $t^g \geq n$ but $\frac{a-2C^F+C^N}{2} < C^e_0 + C^p_t$ for all $t \leq t^g - 1$ (the political costs are too high at the beginning), then there is a subgame perfect Nash equilibrium, given by $(\{C_t(N)\}^{\infty}_{t=0}, \{C_t(N)\}^{\infty}_{t=0})$ where $C^i_t(N) = C^N$ for all $t \geq 0, i = I, P$. That is, both firms choose the same strategy $\{C_t(N)\}^{\infty}_{t=0}$. Therefore, both firms shut down at the moment the economy opens, that is, at $t = t^g$.*

The second result is in correspondence with the following intuition. Even if the political forces are in a minimal degree of coordination, in the sense that by themselves are not the cause of bankruptcy, a sufficient degree of impatience in a firm, makes the corresponding firm to ignore future possible profits, and then to decide not to adopt the new technology at the appropriate moment, so at the moment the foreign firm enters the market, the national firm shuts down. If that firm would like to adopt the new technology later, this technology is not affordable anymore, because of the presence of the foreign firm. On the other hand, if it were the case that one firm is patient enough and the other is sufficiently impatient, then the patient one adopts the new technology at the outset, and the other decides not to adopt the new technology. Also, if both firms are sufficiently impatient, both firms decide not to adopt the new technology, and both shut down at the moment the economy opens. Also, if these decisions are revised in future times, are not changed, because there are no advantages in do it so. Formally:

Theorem 2 *Suppose that $t^g \geq n$ and $C^p_0 < C^p_t$ for all $t \leq t^g - 1$, then: (1) If the firm I is sufficiently impatient (β^I is small enough) and the other firm P is sufficiently patient (β^P is large enough), then there is a subgame perfect Nash equilibrium in which the impatient firm chooses $\{C_t(N)\}^{\infty}_{t=0}$ (not to adopt the new technology) and the patient one chooses $\{C_t(F)\}^{\infty}_{t=0}$, where $C_t(F) = C^F$ for all $t \geq 0$ (to adopt the foreign tehcnology at $t = 0$); (2) If both firms are sufficiently impatient, there is a subgame perfect Nash equilibrium in which both firms choose not to adopt the new technology.*

The next result deeply reflect the role of the political cost at the beginning of the process, and the strong trade off to which the firms are faced.

Corollary 3 *Suppose that $t^g \geq n$. Then, there is a $\hat{\beta}^P$ such that for $\hat{\beta}^P < \beta^P$,*

$\{C_t(F)\}_{t=0}^{\infty}$ *(to adopt the new technology) is the best response for the firm P given that the firm I chooses* $\{C_t(N)\}_{t=0}^{\infty}$, *if and only if* $C_0^p < C_t^p$ *for all* $t \leq t^g - 1$.

The last result of this paper is a positive one, in the sense that if the economy 'works nicely at a minimal degree,' that is, if there is a minimal degree of coordination between the political forces, the government and trade unions, and the national firms are sufficiently patient, then both firms decide to adopt the new technology at the outset.

Theorem 4 *Suppose that* $t^g \geq n$, *and* $C_0^p < C_t^p$ *for all* $t \leq t^g - 1$. *Then, if both discount factors* β^I *and* β^P *are sufficiently large, the pair* $(\{C_t(F)\}_{t=0}^{\infty}, \{C_t(F)\}_{t=0}^{\infty})$ *is a subgame perfect Nash equilibrium.*

Conclusions

The model developed in this paper suggests that even when a government can credibly precommit to open-up the economy to foreign competition and firms have perfect foresight of when that will happen, they may choose not to invest in new technology. This is the case when unions are too strong, and thus the political costs firms face when they adopt the new technology are too high, or/and when the time-period given by the government for trade liberalization is too short, even when firms are sufficiently patient. The same result arises when firms are too impatient, regardless of political costs, or the length of time-period before trade liberalization. However, when firms are patient enough and there is political coordination in terms of the relation between political costs and the time-period given before trade liberalization national firms can adopt the new technology and successfully compete with the foreign firm.

This model raises the problem of path-dependency in the sense that if political costs are increasing, as a result of the widening of the technology gap between the national and the new technology, then it is important that firms choose to adopt the new technology early, otherwise they will not be able to do it later, and will close when the foreign firm enters. It also raises the importance of credit markets given that if there are no credit opportunities, the firms must close if the conditions that they face are adverse for the adoption of the new technology. Conversely, if there are credit opportunities the firms may survive once the foreign firm enters the market even if they had not invested in the new technology earlier.

In this paper we have not included the possibility that firms' decisions to invest or not to invest may affect the time-period the government defines before liberalization as Staiger and Tabellini (1987), Matsuyama (1990) and Tornell (1991) have done. This could be an interesting extension to this paper. Since we consider two domestic firms instead of a monopolist firm, as Matsuyama (1990) and Tornell (1991) do, it could show the necessity of coordination between firms to follow a similar strategy in order to generate the desired response from the government.

Appendix

First of all, we recall some well known results in relation to Cournot Competence.

Lemma 5 *Suppose that the inverse demandfunction is given by* $P(Q) = a - Q$. *Then a)If there are two firms facing constant marginal costs* C^1 *and* C^2 *that compete à la Cournot, then if* (q^1, q^2) *denotes the Nash equilibrium, we have*

$$q^i = \begin{cases} \frac{a-2C^i+C^j}{3} \text{ if } a - 2C^i + C^j \geq 0 \\ 0 \text{ if } a - 2C^i + C^j < 0 \end{cases}$$

for $i = 1, 2$; *the Cournot profits of the firm* i *are given by* $\pi^i(C^i, C^j) = (q^i)^2$ *; and b) If there are three firms facing constant marginal costs* C^i *with* $i = 1, 2$ *and* 3 *that compete á la Cournot, then*

$$q^i = \begin{cases} \frac{a-3C^i+\sum_{j\neq i} C^j}{4} \text{ if } a - 3C^i + \sum_{j\neq i} C^j \geq 0 \\ 0 \text{ if } a - 3C^i + \sum_{j\neq i} C^j < 0 \end{cases}$$

for $i = 1, 2$ *and* 3; *the Cournot profits of the firm* i *are given by* $\pi^i(C^i, C^{-i}) = (q^i)^2$.

Proof: Routine and omitted.

1 Proof of theorem 1.

Our proof's strategy, in all our theorems, is to show, by a direct comparison, that the corresponding strategies are best responses to the other's firm strategies, given the corresponding assumptions in each theorem. This proof's strategy, apart from some collateral technical tricks, is simple; however, it forces us to a difficult and delicate computation of all possible strategic alternatives. We cannot apply differentiability or any other type of first order conditions, given that the strategies' space is infinite and discrete.

1.1 The proof of (1).

Let denote by S the set of strategies, that is,

$$S = \left\{ \{C_t\}_{t=0}^{\infty} \in \Re^{\infty} \left| C_t \in \left\{C^N, C^F\right\} \text{ for all } t \geq 0 \right. \right\}.$$

For the sake of the exposition, we decompose S as follows:

$$S = \{\{\{C_t(N)\}_{t=0}^{\infty}\} \cup \{\{C_t(F)\}_{t=0}^{\infty}\} \cup A_N \cup A_F \cup B_{N,F} \cup B_{F,N}\}$$

where,

$$A_i = \left\{ \{C_t(a)\}_{t=0}^{\infty} \left| \begin{array}{c} C_t(a) = \left\{ \begin{array}{c} C^i \text{ if } t \leq t^a - 1 \\ C^j \text{ if } t \geq t^a \end{array} \right\} \\ \text{where } t^a \in \{1, 2, ..\} \end{array} \right. \right\}$$

with $i, j \in \{N, F\}, i \neq j$, and

$$B_{i,j} = \left\{ \{C_t(a)\}_{t=0}^{\infty} \;\middle|\; \begin{array}{l} C_t(a) = \left\{ \begin{array}{l} C^i \text{ if } t \in [a_l, a_{l+1}) \text{ with } l \text{ even} \\ C^j \text{ if } t \in [a_l, a_{l+1}) \text{ with } l \text{ odd} \end{array} \right\} \\ \text{where } \{a_l\}_{l \geq 0} \text{ is any} \\ \text{incresing sequence of integers} \\ \text{such that } a_0 = 0 \end{array} \right\}$$

with $i, j \in \{N, F\}$, $i \neq j$. Let denote by $\Pi^i(N, N)$ the pay-off of the firm $i = I$, P, when both firms decide not to adopt the new technology, that is $\Pi^i(N, N) = \Pi^i(\{C_t(N)\}_{t=0}^{\infty}, \{C_t(N)\}_{t=0}^{\infty})$. Hence, $\Pi^i(N, N) = \sum_{t=0}^{t^g-1} (\beta^i)^t \frac{(a-C^N)^2}{9}$, because at the $t = t^g$ the foreign firm enters the market and then for $t \geq t^g$ both national firms produce $q_t^i = 0$ (both firms shut down at $t = t^g$), given that $a - 2C^N + C^F < 0$ (using A2 and (b) in lemma 1). Now, set $\Pi^i(F, N) = \Pi^i(\{C_t(F)\}_{t=0}^{\infty}, \{C_t(N)\}_{t=0}^{\infty})$. We have then that $\Pi^i(F, N) = \frac{(a-2(C_0^e+C_0^p+C^F)+C^N)^2}{9} + \sum_{t=1}^{t^g-1} (\beta^i)^t \frac{(a-2(C_t^e+C^F)+C^N)^2}{9}$, since $\pi^i(C_{t^g}(F), C_{t^g}(N)) = 0$, due to that the foreign firm enters the market at $t = t^g$, and therefore $a-3(C_{t^g}^e+C^F)+C^N+C^F < 0$—which is equivalent to $\frac{a-2C^F+C^N}{3} < C_{t^g}^e$—, see (b) in the lemma 1. Clearly, due to A3 and $t^g < n$, we have

$$\Pi^i(N, N) > \Pi^i(F, N) \qquad (2)$$

Remark 1 *It is important to notice the following fact. Take any* $\{C_t(a)\}_{t=0}^{\infty} \in A_F$. *Therefore,* $\{C_t(a)\}_{t=0}^{\infty}$ *is either dominated by* $\{C_t(N)\}_{t=0}^{\infty}$ *or by* $\{C_t(F)\}_{t=0}^{\infty}$. *Indeed, if* $t^a \leq n-1$ *it is clear that* $\{C_t(N)\}_{t=0}^{\infty}$ *dominates* $\{C_t(a)\}_{t=0}^{\infty}$, *just because* $C^N < C_t^e + C^F$ *for all* $t \leq n-1$ *(A3), independently of the other firm's strategy ; also, if* $t^a \geq n$, *we have that* $\{C_t(F)\}_{t=0}^{\infty}$ *dominates* $\{C_t(a)\}_{t=0}^{\infty}$, *because* $C^N > C_n^e + C^F$ *(A3), independently of the other firm's strategy . Hence, the following statement also holds: If* $\{C_t(N)\}_{t=0}^{\infty}$ *dominates the strategy* $\{C_t(F)\}_{t=0}^{\infty}$, *then* $\{C_t(N)\}_{t=0}^{\infty}$ *dominates any strategy* $\{C_t(a)\}_{t=0}^{\infty} \in A_F$.

As a direct consequence of (2) and the previous remark, the following statement is proven:

$$\Pi^i(N, N) > \Pi^i(a, N), \text{ for any } \{C_t(a)\}_{t=0}^{\infty} \in A_F \qquad (3)$$

where $\Pi^i(a, N) = \Pi^i(\{C_t(a)\}_{t=0}^{\infty}, \{C_t(N)\}_{t=0}^{\infty})$.

Take any strategy $\{C_t(a)\}_{t=0}^{\infty} \in B_{F,N}$, that is, there is an increasing sequence $\{a_l\}_{l \geq 0}$ such that (with $a_0 = 0$) $C_t(a) = \left\{ \begin{array}{l} C^F \text{ if } t \in [a_l, a_{l+1}) \text{ with } l \text{ even} \\ C^N \text{ if } t \in [a_l, a_{l+1}) \text{ with } l \text{ odd} \end{array} \right.$. Now, if for every l even, the cardinality of the set $\{t \in \{1, 2, 3..\} \,|t \in [a_l, a_{l+1})\}$ is lower than n (that is, $|\{t \in \{1, 2, 3..\} \,|t \in [a_l, a_{l+1})\}| < n$), the strategy $\{C_t(a)\}_{t=0}^{\infty}$ is dominated by $\{C_t(N)\}_{t=0}^{\infty}$, provided that $C^N < C_t^e + C^F$ for all $t \leq n-1$ (A3), independently of the other firm's strategy. On the other hand, if there is a j even such that the cardinality of the set $\{t \in \{1, 2, 3..\} \,|t \in [a_j, a_{j+1})\}$ is as large as n (that is,

$|\{t\in\{1,2,3..\}\,|t\in[a_j,a_{j+1})\}|\geq n$), then the strategy $\{C_t(a)\}_{t=0}^{\infty}$ is dominated by the strategy $\left\{\tilde{C}_t(a)\right\}_{t=0}^{\infty}$, where

$$\tilde{C}_t(a)=\begin{cases} C^N \text{ if } t\in[a_0,a_{\tilde{j}-1}) \\ C^F \text{ if } t\geq a_{\tilde{j}}\end{cases}$$

with $\tilde{j}=\min\{j \text{ even}\,||\{t\in\{1,2,3..\}\,|t\in[a_j,a_{j+1})\}|\geq n\}$, because $C^N>C_n^e+C^F$ (A3), independently of the other firm's strategy.

Remark 2 *First, observe that the argument used above to show that any strategy $\{C_t(a)\}_{t=0}^{\infty}\in B_{F,N}$, is either dominated by $\{C_t(N)\}_{t=0}^{\infty}$ or by a strategy $\left\{\tilde{C}_t(a)\right\}_{t=0}^{\infty}\in A_N$, does not depend on the other firm's strategy. Second and consequently, if we prove that $\{C_t(N)\}_{t=0}^{\infty}$ dominates any strategy $\{C_t(a)\}_{t=0}^{\infty}\in A_N$ we can conclude that $\{C_t(N)\}_{t=0}^{\infty}$ dominates any strategy $\{C_t(a)\}_{t=0}^{\infty}\in B_{F,N}$. That is, as a by-product, we have proven the following statement: If $\{C_t(N)\}_{t=0}^{\infty}$ dominates any strategy $\{C_t(a)\}_{t=0}^{\infty}\in A_N$, then $\{C_t(N)\}_{t=0}^{\infty}$ dominates any strategy $\{C_t(a)\}_{t=0}^{\infty}\in B_{F,N}$.*

Take then any $\{C_t(a)\}_{t=0}^{\infty}\in A_N$, and consider $\Pi^i(a,N)=\Pi^i(\{C_t(a)\}_{t=0}^{\infty},\{C_t(N)\}_{t=0}^{\infty})$. We have two situations, $t^a\leq t^g-1$ and $t^a\geq t^g$.

1.1.1 Suppose $t^a\leq t^g-1$.

In this case, it can be shown that

$$\Pi^i(a,N)=\left\{\begin{array}{l}\sum_{t=0}^{t^a-1}(\beta^i)^t\frac{(a-C^N)^2}{9}+\frac{1}{9}(a-2(C_0^e+C_{t^a}^p+C^N)+C^N)^2+ \\ \left\{\begin{array}{l}\sum_{t=t^a+1}^{t^g-1}(\beta^i)^t\frac{(a-2(C_{t-t^a}^e+C^N+)+C^N)^2}{9} \text{ if } t^a<t^g-1 \\ 0 \text{ if } t^a=t^g-1\end{array}\right\}\end{array}\right\}$$

because at $t=t^g$ we have $\pi^i(C_{t^g}(a),C_{t^g}(N))=0$ (since the foreign firm enters the market at $t=t^g$). Therefore, using precisely the same reasoning as in (2), the following statement is proven:

$$\begin{array}{rcl}\Pi^i(N,N) & > & \Pi^i(a,N) \\ \text{for any } \{C_t(a)\}_{t=0}^{\infty} & \in & A_N \text{ such that } t^a\leq t^g-1\end{array}\qquad(4)$$

1.1.2 Suppose $t^a\geq t^g$.

In this case, it is straightforward to show that $\Pi^i(a)=\sum_{t=0}^{t^g-1}(\beta^i)^t\frac{(a-C^N)^2}{9}$, given that at $t=t^g$ the foreign firm enters the market and therefore, once again, $\pi^i(C_{t^g}(a),C_{t^g}(N))=0$, and therefore the firm i shuts down.

Consequently, the following statement is proven:

$$\Pi^i(N,N)=\Pi^i(a,N) \text{ for any } \{C_t(a)\}_{t=0}^{\infty}\in A_N \text{ such that } t^a\geq t^g.\qquad(5)$$

On the other hand, take any strategy $\{C_t(a)\}_{t=0}^{\infty}\in B_{N,F}$, that is, there is an increasing sequence $\{a_l\}_{l\geq 0}$ such that $C_t(a) = \begin{cases} C^N \text{ if } t \in [a_l, a_{l+1}) \text{ with } l \text{ even} \\ C^F \text{ if } t \in [a_l, a_{l+1}) \text{ with } l \text{ odd} \end{cases}$. Now, if for every l odd, the cardinality of the set $\{t \in \{1,2,3..\} \,|t \in [a_l, a_{l+1})\}$ is lower than n (that is, $|\{t \in \{1,2,3..\} \,|t \in [a_l, a_{l+1})\}| < n$), the strategy $\{C_t(a)\}_{t=0}^{\infty}$ is dominated by $\{C_t(N)\}_{t=0}^{\infty}$, provided that $C^N < C_t^e + C^F$ for all $t \leq n-1$ (A3). On the contrary, if there is a j odd such that the cardinality of the set $\{t \in \{1,2,3..\} \,|t \in [a_j, a_{j+1})\}$ is as large as n (that is, $|\{t \in \{1,2,3..\} \,|t \in [a_j, a_{j+1})\}| \geq n$), then the strategy $\{C_t(a)\}_{t=0}^{\infty}$ is dominated by the strategy $\left\{\tilde{C}_t(a)\right\}_{t=0}^{\infty}$, where

$$\tilde{C}_t(a) = \begin{cases} C^N \text{ if } t \in [a_0, a_{\tilde{j}}) \\ C^F \text{ if } t \geq a_{\tilde{j}} \end{cases} \tag{6}$$

with $\tilde{j} = \min\{j \text{ odd} \,||\{t \in \{1,2,3..\} \,|t \in [a_j, a_{j+1})\}| \geq n\}$, because $C^N > C_n^e + C^F$ (A3).

Remark 3 *Notice that the reasoning used here to show that for any firm i, any strategy $\{C_t(a)\}_{t=0}^{\infty} \in B_{N,F}$ adopted by that firm, is either dominated by the corresponding strategy $\left\{\tilde{C}_t(a)\right\}_{t=0}^{\infty} \in A_N$, given in (6) or by $\{C_t(N)\}_{t=0}^{\infty}$, does not depend on the other firms' strategies. Therefore, the following statement holds: If $\{C_t(N)\}_{t=0}^{\infty}$ dominates any strategy $\{C_t(a)\}_{t=0}^{\infty} \in A_N$, then $\{C_t(N)\}_{t=0}^{\infty}$ dominates any strategy $\{C_t(a)\}_{t=0}^{\infty} \in B_{N,F}$.*

Consequently, taking into account (2), (3), remark 2, (4), (5), and remark 3, the strategy $\{C_t(N)\}_{t=0}^{\infty}$ is the best response for the firm i if the firm j chooses $\{C_t(N)\}_{t=0}^{\infty}$. Hence, the pair $(\{C_t(N)\}_{t=0}^{\infty}, \{C_t(N)\}_{t=0}^{\infty})$ is a Nash equilibrium

By a similar argumentation, it is possible to show that $(\{C_t(N)\}_{t=0}^{\infty}, \{C_t(N)\}_{t=0}^{\infty})$ is also a subgame perfect equilibrium.

1.2 The proof of (2).

First, notice that, due to remarks 1,2 and 3, it suffices to show that

$$\Pi^i(N,N) \geq \Pi^i(F,N), \tag{7}$$

and that

$$\Pi^i(N,N) \geq \Pi^i(a,N) \text{ for any } \{C_t(a)\}_{t=0}^{\infty} \in A_N \tag{8}$$

We will prove (8) first. Consider then any strategy $\{C_t(a)\}_{t=0}^{\infty} \in A_N$, and suppose that the firm i chooses $\{C_t(a)\}_{t=0}^{\infty}$ instead of $\{C_t(N)\}_{t=0}^{\infty}$. We have two possibilities, $t^a \leq t^g - 1$, or $t^a \geq t^g$.

1.2.1 Suppose $t^a \leq t^g - 1$.

The pay-off of the firm i in this case, if the firm j chooses $\{C_t(N)\}_{t=0}^{\infty}$ is given by $\Pi^i(a,N) = \sum_{t=0}^{t^a-1} (\beta^i)^t \frac{(a-C^N)^2}{9}$ because, whenever $t^a \leq t^g - 1$, we have $(a - 2(C_0^e + C^F + C_{t^a}^p) + C^N) < 0$, due to that $\frac{a-2C^F+C^N}{2} < C_0^e + C_t^p$ for all $t \leq t^g - 1$, by assumption, and hence the firm i shuts down at $t = t^a$ (see (a) in Lemma 1). Now, as $t^a \leq t^g - 1$, we have $\Pi(N,N) = \sum_{t=0}^{t^g-1} (\beta^i)^t \frac{(a-C^N)^2}{9}$ because the firm i shuts down at

$t = t^a$. Now, clearly $\Pi(N, N) = \sum_{t=0}^{t^g-1} (\beta^i)^t \frac{(a-C^N)^2}{9} > \sum_{t=0}^{t^a-1} (\beta^i)^t \frac{(a-C^N)^2}{9} = \Pi^i(a, N)$, and therefore $\{C_t(a)\}_{t=0}^{\infty}$ is dominated by $\{C_t(N)\}_{t=0}^{\infty}$.

1.2.2 Suppose $t^a \geq t^g$.

Then, $\Pi^i(a, N) = \sum_{t=1}^{t^g-1} (\beta^i)^t \frac{(a-C^N)^2}{9}$. Indeed, if $t^a = t^g$, at t^g we have that $(a - 3(C^p_{t^g} + C^e_0 + C^F) + C^N + C^F) < (a - 3C^N + C^N + C^F) = a - 2C^N + C^F < 0$ (due to A3 and A2), and the firm i shuts down at $t = t^g$ (by (b) in Lemma 1); if $t^a > t^g$, at $t = t^g$ we have that $(a - 3C^N + C^N + C^F) = a - 2C^N + C^F < 0$ (A2), and the firm i shuts down at time $t = t^g$, by the same reason as in the case $t^a = t^g$ (once again, by (b) in Lemma 1). Therefore, $\Pi^i(N, N) = \Pi^i(a, N)$, and consequently the strategy strategy $\{C_t(a)\}_{t=0}^{\infty}$ is weakly dominated by $\{C_t(N)\}_{t=0}^{\infty}$. The proof of (8) is done.

To prove (7), observe that we can apply the same argument as in (1.2.1).

Therefore, we can conclude that $(\{C_t(N)\}_{t=0}^{\infty}, \{C_t(N)\}_{t=0}^{\infty})$ is a Nash equilibrium.

Finally, it is possible to show that $(\{C_t(N)\}_{t=0}^{\infty}, \{C_t(N)\}_{t=0}^{\infty})$ is also a subgame perfect equilibrium.

The proof of theorem 1 is done. ■

2 Proof of theorem 2.

For the sake of the exposition, we present the following result:

Lemma 6 *Given any profile of strategies* $(\{C_t^i\}_{t=0}^{\infty}, \{C_t^j\}_{t=0}^{\infty})$ *where* $i = I, P$, *and any integer* $\bar{t} > 0$, *we have that* $\Pi^i((\{C_t^i\}_{t=0}^{\infty}), (\{C_t^j\}_{t=0}^{\infty}))$ *is given by*

$$\sum_{t=0}^{\bar{t}-1} (\beta^i)^t \pi^i(C_t^i, C_t^j) + (\beta^i)^{\bar{t}} \Pi^i((\{C_t^i\}_{t=\bar{t}}^{\infty}), (\{C_t^j\}_{t=\bar{t}}^{\infty})) \tag{9}$$

where $\Pi^i((\{C_t^i\}_{t=\bar{t}}^{\infty}), (\{C_t^j\}_{t=\bar{t}}^{\infty})) = \sum_{t=\bar{t}}^{\infty} (\beta^i)^{t-\bar{t}} \pi^i(C_t^i, C_t^j)$. *Furthermore,*

$\lim_{\beta^i \to 0} \sum_{t=\bar{t}}^{\infty} (\beta^i)^{t-\bar{t}} \pi^i(C_t^i, C_t^j) = \pi^i(C_{\bar{t}}^i, C_{\bar{t}}^j)$.

Proof: It follows at once from inspection.

2.1 Proof of part (1).

2.1.I Consider the firm I.

Notice that, in order to show that $\{C_t(N)\}_{t=0}^{\infty}$ is the best response of the firm I if the firm P chooses $\{C_t(F)\}_{t=0}^{\infty}$ for β^I small enough, due to the remarks 1,2, and 3, it suffices to show that there is $\hat{\beta}^I$ such that for any $\beta^I < \hat{\beta}^I$ we have

$$\Pi^I(N, F) \geq \Pi^I(F, F) \tag{10}$$

and that

$$\Pi^I(N, F) \geq \Pi^I(a, F) \text{ for any } \{C_t(a)\}_{t=0}^{\infty} \in A_N \tag{11}$$

where $\Pi^I(N,F) = \Pi^I(\{C_t(N)\}_{t=0}^{\infty}, \{C_t(F)\}_{t=0}^{\infty}), \Pi^I(F,F) = \Pi^I(\{C_t(F)\}_{t=0}^{\infty}, \{C_t(F)\}_{t=0}^{\infty})$ and $\Pi^I(a,F) = \Pi^I(\{C_t(a)\}_{t=0}^{\infty}, \{C_t(F)\}_{t=0}^{\infty})$.

We prove first that there is a β_1^I such that if $\beta^I < \beta_1^I$, then (10) holds. Now, it follows at once from the lemma 2 that

$$\Pi^I(N,F) = \frac{1}{9}(a - 2C^N + (C_0^e + C_0^p + C^F))^2 + \beta^I \Pi^I(\{C_t(N)\}_{t=1}^{\infty}, \{C_t(F)\}_{t=1}^{\infty})$$

and

$$\Pi^I(F,F) = \frac{1}{9}(a - 2(C_0^e + C_0^p + C^F))^2 + \beta^I \Pi^I(\{C_t(F)\}_{t=1}^{\infty}, \{C_t(F)\}_{t=1}^{\infty}).^{5}$$

Clearly, $\lim_{\beta^I \to 0} \Pi^I(N,F) = \frac{1}{9}(a - 2C^N + (C_0^e + C_0^p + C^F))^2$, $\lim_{\beta^I \to 0} \Pi^I(F,F) = \frac{1}{9}(a - 2(C_0^e + C_0^p + C^F))^2$, and therefore $\Pi^I(N,F) > \Pi^i(F,F)$ for β^I small enough, due to that $C^N < C_0^e + C^F < C_0^e + C_0^p + C^F$ (see A3). For the sake of the exposition, we formalize this result in the following

Proposition 7 *There is a $\beta_1^I \in (0,1)$, such that if $\beta^I < \beta_1^I$, then $\Pi^I(N,F) \geq \Pi^I(F,F)$.*

Now we will prove that there is a $\beta_2^I \in (0,1)$, such that for $\beta^I < \beta_2^I$ (11) holds. Take then any $\{C_t(a)\}_{t=0}^{\infty} \in A_N$. We have two cases $t^a \leq t^g - 1$ and $t^a \geq t^g$.

2.1.I.1 The case $t^a \leq t^g - 1$. Therefore, we have two possibilities, $t^a \leq n - 1$ or $t^a \geq n$.

2.1.I.1.1 Consider then, the situation when $t^a \leq n - 1$. Therefore,

$$\begin{aligned} \Pi^I(N,F) &= \frac{1}{9}(a - 2C^N + (C_0^e + C_0^p + C^F))^2 + \\ &\quad \frac{1}{9}\sum_{t=1}^{t^a - 1}(\beta^I)^t (a - 2C^N + (C_t^e + C^F))^2 + \\ &\quad (\beta^I)^{t^a}\, \Pi^I((\{C_t(N)\}_{t=t^a}^{\infty}), (\{C_t(F)\}_{t=t^a}^{\infty})) \end{aligned} \tag{12}$$

and

$$\lim_{\beta^i \to 0} \Pi^I((\{C_t(N)\}_{t=t^a}^{\infty}), (\{C_t(F)\}_{t=t^a}^{\infty})) = \pi^I(C_{t^a}^I(N), C_{t^a}^P(F)),$$

(see Lemma 2), where

$$\pi^I(C_{t^a}^I(N), C_{t^a}^P(F)) = \frac{1}{9}(a - 2C^N + (C_{t^a}^e + C^F))^2$$

(see Lemma 1, part (a)), since $(C_{t^a}^I(N), C_{t^a}^P(F)) = (C^N, (C_{t^a}^e + C^F)$ given that $t^a \leq n - 1$. Similarly, we have that

$$\Pi^I(a,F) = \frac{1}{9}(a - 2C^N + (C_0^e + C_0^p + C^F))^2 +$$

[5] Notice that $\beta^I \Pi^I(\{C_t(N)\}_{t=1}^{\infty}, \{C_t(F)\}_{t=1}^{\infty})$ need not to be larger than $\beta^I \Pi^I(\{C_t(F)\}_{t=1}^{\infty}, \{C_t(F)\}_{t=1}^{\infty})$ for every β^I. There is here, indeed, a strong trade-off between the two alternatives, depending on the value of β^I. This comment is not only pertinent in this case, but it is also pertinent in all the statements presented henceforth.

$$\frac{1}{9}\sum_{t=1}^{t^a-1}(\beta^I)^t(a-2C^N+(C_t^e+C^F))^2+$$
$$(\beta^I)^{t^a}\,\Pi^I((\{C_t(a)\}_{t=t^a}^{\infty}),(\{C_t(F)\}_{t=t^a}^{\infty})) \qquad (13)$$

and

$$\lim_{\beta^i\to 0}\Pi^I((\{C_t(a)\}_{t=t^a}^{\infty}),(\{C_t(F)\}_{t=t^a}^{\infty}))=\pi^I(C_{t^a}^I(a),C_{t^a}^P(F))$$

where,

$$\pi^I(C_{t^a}^I(a),C_{t^a}^P(F))=\frac{1}{9}(a-2(C_0^e+C_{t^a}^p+C^F)+(C_{t^a}^e+C^F))^2$$

(see Lemma 1, part (a)), because $(C_{t^a}^I(a),C_{t^a}^P(F))=((C_0^e+C_{t^a}^p+C^F),(C_{t^a}^e+C^F))$. Now, Noting that $(a-2C^N+(C_{t^a}^e+C^F))>a-2(C_0^e+C_{t^a}^p+C^F)+(C_{t^a}^e+C^F)$, because $C_0^e+C_{t^a}^p+C^F>C^N$ (by A3), we have that $\pi^I(C_{t^a}^I(a),C_{t^a}^P(F))<\pi^I(C_{t^a}^I(N),C_{t^a}^P(F))$ and hence, $(\beta^I)^{t^a}\pi^I(C_{t^a}^I(a),C_{t^a}^P(F))<(\beta^I)^{t^a}\pi^I(C_{t^a}^I(N),C_{t^a}^P(F))$; consequently, $(\beta^I)^{t^a}\,\Pi^I((\{C_t(a)\}_{t=t^a}^{\infty}),(\{C_t(F)\}_{t=t^a}^{\infty}))<(\beta^I)^{t^a}\,\Pi^I((\{C_t(N)\}_{t=t^a}^{\infty}),(\{C_t(F)\}_{t=t^a}^{\infty}))$ for all β^I small enough. *Ergo*, taking into account the equalities (12) and (13), we have proven the following statement:

$$\left\{\begin{array}{c}\text{If } \{C_t(a)\}_{t=0}^{\infty}\in A_N \text{ is such that } t^a \text{ satisfies } t^a\le n-1\\ \text{then, there is a } \beta_{t^a}^I<1 \text{ such that, if } \beta^I<\beta_{t^a}^I\text{, we have}\\ \Pi^I(N,F)\ge\Pi^I(a,F)\end{array}\right\} \qquad (14)$$

2.1.I.1.2 Now, we consider the case $n\le t^a\le t^g-1$. Proceeding in the same way as in (2.1.I.1.1), we see that

$$\begin{aligned}\Pi^I(N,F) \;=\;& \frac{1}{9}(a-2C^N+(C_0^e+C_0^p+C^F))^2+\\ & \frac{1}{9}\sum_{t=1}^{n-1}(\beta^I)^t(a-2C^N+(C_t^e+C^F))^2+\\ & \left\{\begin{array}{c}0 \text{ if } t^a=n\\ \frac{1}{9}\sum\limits_{t=n}^{t^a-1}(\beta^I)^t(a-2C^N+(C_n^e+C^F))^2 \text{ if } t^a>n\end{array}\right\}\\ & +(\beta^I)^{t^a}\,\Pi^I((\{C_t(N)\}_{t=t^a}^{\infty}),(\{C_t(F)\}_{t=t^a}^{\infty}))\end{aligned} \qquad (15)$$

and

$$\lim_{\beta^i\to 0}\Pi^I((\{C_t(N)\}_{t=t^a}^{\infty}),(\{C_t(F)\}_{t=t^a}^{\infty}))=\pi^I(C_{t^a}^I(N),C_{t^a}^P(F))$$

(see Lemma 2), where

$$\pi^I(C_{t^a}^I(N),C_{t^a}^P(F))=\frac{1}{9}(a-2C^N+(C_n^e+C^F))^2 \qquad (16)$$

(see Lemma 1, part (a)), since $(C_{t^a}^I(N),C_{t^a}^P(F))=(C^N,(C_n^e+C^F))$, due that $n\le t^a\le t^g-1$. Similarly,

$$\Pi^I(a,F) \;=\; \frac{1}{9}(a-2C^N+(C_0^e+C_0^p+C^F))^2+$$

$$\frac{1}{9}\sum_{t=1}^{n-1}(\beta^I)^t(a-2C^N+(C_t^e+C^F))^2+$$
$$\left\{\begin{array}{c} 0 \text{ if } t^a=n \\ \frac{1}{9}\sum_{t=n}^{t^a-1}(\beta^I)^t(a-2C^N+(C_n^e+C^F))^2 \text{ if } t^a>n \end{array}\right\}$$
$$+(\beta^I)^{t^a}\,\Pi^I((\{C_t(a)\}_{t=t^a}^{\infty}),(\{C_t(F)\}_{t=t^a}^{\infty})), \quad (17)$$

and

$$\lim_{\beta^i\to 0}\Pi^I((\{C_t(a)\}_{t=t^a}^{\infty}),(\{C_t(F)\}_{t=t^a}^{\infty}))=\pi^I(C_{t^a}^I(a),C_{t^a}^P(F)),$$

where,

$$\pi^I(C_{t^a}^I(a),C_{t^a}^P(F)=\frac{1}{9}\left(a-2(C_0^e+C_{t^a}^p+C^F)+(C^F+C_n^e)\right)^2. \quad (18)$$

once again, given that $n\le t^a\le t^g-1$ and $(C_{t^a}^I(a),C_{t^a}^P(F)=((C_0^e+C_{t^a}^p+C^F),(C^F+C_n^e))$. Now, we have $\pi^I(C_{t^a}^I(a),C_{t^a}^P(F)<\pi^I(C_{t^a}^I(N),C_{t^a}^P(F))$, since $(C_0^e+C_{t^a}^p+C^F)>C^N$ (A3 and $t^a\le t^g-1$). Therefore, taking into account (15), , (18), (17)and (16), and applying the same argument as in (2.1.I.1), it is proven the following statement:

$$\left\{\begin{array}{c} \text{If } \{C_t(a)\}_{t=0}^{\infty}\in A_N \text{ wiht } t^a\text{satisfying } n\le t^a\le t^g-1, \\ \text{there is a } \beta_{t^a}^I<1, \text{ such that,} \\ \text{if } \beta^I<\beta_{t^a}^I, \text{ then} \\ \Pi^I(N,F)>\Pi^I(a,F) \end{array}\right\} \quad (19)$$

2.1.I.2 It rests now to consider the case $t^a\ge t^g$. In this situation , we have that the following statement holds:

$$\left\{\begin{array}{c} \text{For every } \beta^I\in[0,1] \text{ and for every} \\ \{C_t(a)\}_{t=0}^{\infty}\in A_N \text{ with } t^a\ge t^g \\ \Pi^I(N,F)=\Pi^I(a,F)= \\ \frac{1}{9}(a-2C^N+(C_0^e+C_0^p+C^F))^2+ \\ \left\{\begin{array}{l} \left\{\frac{1}{9}\sum_{t=1}^{t^g-1}(\beta^I)^t(a-2C^N+(C_{t^g-1}^e+C^F))^2\right\} \text{ if } n=t^g \\ \left\{\begin{array}{l}\frac{1}{9}\sum_{t=1}^{n-1}(\beta^I)^t(a-2C^N+(C_t^e+C^F))^2+ \\ \frac{1}{9}\sum_{t=n}^{t^g-1}(\beta^I)^t(a-2C^N+(C_n^e+C^F))^2\end{array}\right\} \text{ if } n<t^g \end{array}\right\} \end{array}\right\} \quad (20)$$

This is because, at $t=t^g$, the foreign firm enters the market, $C_{t^g}^P(N)=C^N$, and $C_{t^g}(a)=\left\{\begin{array}{c}(C_0^e+C_{t^g}^p+C^F) \text{ if } t^a=t^g \\ C^N \text{ if } t^a>t^g\end{array}\right\}$; indeed, in any case, we have that the firm I shuts down at $t=t^g$, since $(a-3(C_0^e+C_{t^g}^p+C^F)+(C_n^e+C^F)+C^F<a-3C^N+(C_n^e+C^F)+C^F$ (due to that $(C_0^e+C_{t^g}^p+C^F)>C^N$ (A3)); at the same time, $a-3C^N+(C_n^e+C^F)+C^F<a-3C^N+C^N+C^F$ (due to that $C_n^e+C^F<C^N$,A3 again), and finally, as $a-3C^N+C^N+C^F<0$ (A2), we have that $\Pi^I((\{C_t(a)\}_{t=t^g}^{\infty}),(\{C_t(N)\}_{t=t^g}^{\infty}))=0$, since $\pi^I(C_{t^g}^I(a),C_{t^g}^P(F))=0$, in virtue of Lemmata 1 and 2. Therefore, our statement in is proven.

For the sake of the exposition, we announce the following

Proposition 8 *There is a* $\hat{\beta}_2^I \in (0,1)$, *such that if* $\beta^I < \beta_2^I$, *then for every* $\{C_t(a)\} \in A_N$, *we have* $\Pi^I((\{C_t(N)\}_{t=0}^{\infty}),(\{C_t(F)\}_{t=0}^{\infty})) \geq \Pi^I((\{C_t(a)\}_{t=0}^{\infty}),(\{C_t(F)\}_{t=0}^{\infty}))$.

Proof: It follows at once from (14), and (19), taking into acount(20) and taking $\beta_2^I = \min\left\{\beta_{t^a}^I \;|\beta_{t^a}^I \text{ is given in (14) or (19)}\right\}$.

Finally, we have the following

Proposition 9 *There is a* $\hat{\beta}^I \in (0,1)$, *such that if* $\beta^I < \hat{\beta}^I$, *then for every* $\{C_t(a)\} \in S$, *we have* $\Pi^I(N,F) \geq \Pi^I(a,F)$.

Proof: Take $\hat{\beta}^I = \min\left\{\hat{\beta}_1^I, \hat{\beta}_2^I\right\}$ where $\hat{\beta}_1^I$ and $\hat{\beta}_2^I$ are given in the propositions 1 and 2. Now, apply remarks 1,2 and 3, as argumented in the beginning of (2.1.I.)

2.1.P Consider the firm P.

In order to prove that for β^P large enough, $\{C_t(F)\}_{t=0}^{\infty}$ dominates any strategy $\{C_t(a)\} \in S$, we will need the following result, which is a simple consequence of what we have done so far. However, it notably simplifies the proofs.

Remark 4 *It can be easily seen that, due to the remarks 1,2, and 3, in order to prove that for* β^P *large enough,* $\{C_t(F)\}_{t=0}^{\infty}$ *dominates any strategy* $\{C_t(a)\} \in S$, *given that the firm I chooses* $\{C_t(N)\}_{t=0}^{\infty}$, *it suffices to show that for* β^P *large enough,* $\{C_t(F)\}_{t=0}^{\infty}$ *dominates any* $\{C_t(a)\} \in A_N$ *and dominates* $\{C_t(N)\}_{t=0}^{\infty}$, *given that the firm I chooses* $\{C_t(N)\}_{t=0}^{\infty}$. *Indeed, from the remark 1, if* $\{C_t(a)\} \in A_F$, $\{C_t(a)\}$ *is either dominated* $\{C_t(N)\}_{t=0}^{\infty}$ *or* $\{C_t(F)\}_{t=0}^{\infty}$, *hence, if* $\{C_t(F)\}_{t=0}^{\infty}$ *dominates* $\{C_t(N)\}_{t=0}^{\infty}$, *therefore* $\{C_t(F)\}_{t=0}^{\infty}$ *dominates any* $\{C_t(a)\} \in A_F$; *from the remark 2, if* $\{C_t(a)\} \in B_{F,N}$, *then* $\{C_t(a)\}$ *is either dominated by* $\{C_t(N)\}_{t=0}^{\infty}$ *or by a strategy* $\{C_t(a)\} \in A_N$ *and, consequently, if* $\{C_t(F)\}_{t=0}^{\infty}$ *dominates* $\{C_t(N)\}_{t=0}^{\infty}$ *and any* $\{C_t(a)\} \in A_N$, *then* $\{C_t(F)\}_{t=0}^{\infty}$ *dominates any* $\{C_t(a)\} \in B_{F,N}$; *finally, from the remark 3, if* $\{C_t(a)\} \in B_{N,F}$, *then* $\{C_t(a)\}$ *is either dominated by a strategy* $\{C_t(a)\} \in A_N$ *or by* $\{C_t(N)\}_{t=0}^{\infty}$ *and, hence, if* $\{C_t(F)\}_{t=0}^{\infty}$ *dominates* $\{C_t(N)\}_{t=0}^{\infty}$ *and dominates any* $\{C_t(a)\} \in A_N$, *it dominates any* $\{C_t(a)\} \in B_{N,F}$.

Therefore, we have to prove that

$$\Pi^P(F,N) \geq \Pi^P(N,N). \tag{21}$$

and that

$$\Pi^P(F,N) \geq \Pi^P(a,N) \text{ for any } \{C_t(a)\}_{t=0}^{\infty} \in A_N \tag{22}$$

where $\Pi^P(F,N) = \Pi^P(\{C_t(F)\}_{t=0}^{\infty}, \{C_t(N)\}_{t=0}^{\infty})$, $\Pi^P(N,N) = \Pi^P(\{C_t(N)\}_{t=0}^{\infty}, \{C_t(N)\}_{t=0}^{\infty})$ and $\Pi^P(a,N) = \Pi^P(\{C_t(a)\}_{t=0}^{\infty}, \{C_t(N)\}_{t=0}^{\infty})$.

We proceed then to prove (21) first. By direct inspection, we see that

$$\Pi^P(N,N) = \sum_{t=0}^{t^g-1} (\beta^P)^t \pi^P(C_t(N), C_t(N))$$

and therefore

$$\left\{\begin{array}{c}\text{for any } \beta^P \in [0,1]\text{, we have} \\ \Pi^P(N,N) \leq \lim_{\beta^P \to 1} \Pi^P(N,N) = \sum_{t=0}^{t^g-1} \pi^P(C_t(N), C_t(N)) < \infty \end{array}\right\} \quad (23)$$

On the other hand,

$$\Pi^P(F,N) = \sum_{t=0}^{t^g-1} (\beta^P)^t \pi^P(C_t(F), C_t(N)) + \frac{(\beta^P)^{t^g}}{1-\beta^P} \pi^P(F,N)$$

where $\pi^P(F,N) = \pi^P((C_n^e + C^F), C^N) = \left(\frac{a-3(C_n^e+C^F)+C^F+C^N}{4}\right)^2$, [6]since $t^g \geq n$ and therefore $C_t(F) = C_n^e + C^F$ for all $t \geq t^g$. Now, we have $\pi^P(F,N) > 0$, due to that $a-3(C_n^e+C^F)+C^F+C^N > a-2(C_n^e+C^F)+C^F > 0$ in virtue of A3. Consequently,

$$\lim_{\beta^P \to 1} \Pi^P(F,N) = \infty \quad (24)$$

Hence, we can announce the following result

$$\left\{\begin{array}{l}\text{There is a } \beta_0^P \in (0,1) \text{ such that,} \\ \text{if } \beta_0^P < \beta^P\text{, then } \Pi^P(F,N) > \Pi^P(N,N)\end{array}\right\} \quad (25)$$

Proof: It follows at once from (23) and (24).

We proceed now to prove (22)

There are two possible situations, $t^a \leq t^g - 1$ and $t^a \geq t^g$.

2.1.P.1 Take $t^a \leq t^g - 1$. Therefore, we have two possibilities $t^g < t^a + n$, or $t^g \geq t^a + n$.

2.1.P.1.1 Consider then, the situation when $t^g \geq t^a + n$. Then, we may have $t^a \leq n-1$ or $t^a \geq n$.

2.1.P.1.1.1 Suppose that $t^a \leq n-1$. It is not difficult to see that,

$$\Pi^P(F,N) - \Pi^P(a,N) = \frac{1}{9} \sum_{t=0}^{t^a+n-1} (\beta^P)^t (\pi^P(C_t(F), C_t(N)) - \pi^P(C_t(a), C_t(N)))$$

since $C_t(F)$ and $C_t(a)$ coincide after $t^a + n$ (both strategies prescribe that the foreign technology is completely installed at $t = t^a + n$). Hence,

$$\begin{aligned}\Pi^P(F,N) - \Pi^P(a,N) \;=\; & \frac{1}{9}\pi^P((C_0^e + C_0^P + C^F), C^N) - \pi^P(C^N, C^N) + \\ & \left\{\begin{array}{c} \frac{1}{9}\sum_{t=1}^{t^a-1} (\beta^P)^t \left[\pi^P((C_t^e + C^F), C^N) - \pi^P(C^N, C^N)\right] \text{ if } t^a > 1 \\ 0 \text{ if } t^a = 1 \end{array}\right\} \\ & + \left\{\begin{array}{c} \frac{1}{9}(\beta^P)(\pi^P((C_{t^a}^e + C^F), C^N) - \\ \pi^P((C_0^e + C_{t^a}^P + C^F), C^N)) \end{array}\right\} +\end{aligned}$$

[6] Recall the footnote 1 and that at t^g the foreign firm enters the market.

$$\left\{ \begin{array}{c} \left\{ \begin{array}{l} \frac{1}{9}\sum_{t=t^a+1}^{n-1}(\beta^P)^t\pi^P((C_t^e+C^F),C^N)- \\ \frac{1}{9}\sum_{t=t^a+1}^{n-1}(\beta^P)^t\pi^P((C_{t-t^a}^e+C^F),C^N) \end{array} \right\} \text{if } t^a<n-1 \\ 0 \text{ if } t^a=n-1 \end{array} \right\}$$

$$+\left\{ \begin{array}{l} \frac{1}{9}\sum_{t=n}^{t^a+n-1}(\beta^P)^t\pi^P((C_n^e+C^F),C^N)- \\ \frac{1}{9}\sum_{t=n}^{t^a+n-1}(\beta^P)^t\pi^P((C_{t-t^a}^e+C^F),C^N) \end{array} \right\}$$

Therefore

$$\begin{aligned} \lim_{\beta^P\to 1}\Pi^P(F,N)-\Pi^P(a,N) &= \frac{1}{9}\pi^P((C_0^e+C_0^P+C^F),C^N)-\pi^P(C^N,C^N)+ \\ &\left\{ \begin{array}{c} \frac{1}{9}\sum_{t=1}^{t^a-1}\left[\pi^P((C_t^e+C^F),C^N)-\pi^P(C^N,C^N)\right] \text{ if } t^a>1 \\ 0 \text{ if } t^a=1 \end{array} \right\} \\ &+\left\{ \begin{array}{c} \frac{1}{9}(\pi^P((C_{t^a}^e+C^F),C^N)- \\ \pi^P((C_0^e+C_{t^a}^P+C^F),C^N)) \end{array} \right\}+ \\ &\left\{ \begin{array}{c} \left\{ \begin{array}{l} \frac{1}{9}\sum_{t=t^a+1}^{n-1}\pi^P((C_t^e+C^F),C^N)- \\ \frac{1}{9}\sum_{t=t^a+1}^{n-1}\pi^P((C_{t-t^a}^e+C^F),C^N) \end{array} \right\} \text{if } t^a<n-1 \\ 0 \text{ if } t^a=n-1 \end{array} \right\} \\ &+\left\{ \begin{array}{l} \frac{1}{9}\sum_{t=n}^{t^a+n-1}\pi^P((C_n^e+C^F),C^N)- \\ \frac{1}{9}\sum_{t=n}^{t^a+n-1}\pi^P((C_{t-t^a}^e+C^F),C^N) \end{array} \right\} \end{aligned}$$

Consequently

$$\begin{aligned} \lim_{\beta^P\to 1}\Pi^P(F,N)-\Pi^P(a,N) &= t^a(\pi^P((C_n^e+C^F),C^N)-\pi^P(C^N,C^N))+ \\ &\pi^P((C_0^e+C_0^P+C^F),C^N)-\pi^P((C_0^e+C_{t^a}^P+C^F),C^N) \end{aligned}$$

because the rest of the terms cancel out each other. Now, recall that $\pi^P((C_n^e+C^F),C^N)-\pi^P(C^N,C^N)>0$ (A3), and since $C_{t^a}^P\geq C_0^P$ for all $t^a\leq t^g-1$ (by assumption), we have that $\lim_{\beta^P\to 1}\Pi^P(F,N)-\Pi^P(a,N)>0$, and then, we can state the following result

$$\left\{ \begin{array}{c} \text{There is a } \beta_1^P\in(0,1) \text{ such that} \\ \text{for every } \beta^P>\beta_1^P \text{ we have that,} \\ \text{if } \{C_t(a)\}\in A_N \text{ with} \\ t^a\leq t^g-1,\ t^g\geq t^a+n, \\ \text{and } t^a\leq n-1, \text{ we have} \\ \Pi^P(F,N)>\Pi^P(a,N) \end{array} \right\} \qquad (26)$$

2.1.P.1.1.2 On the other hand, if $t^a \geq n$, we can show that

$$\begin{aligned}
\Pi^P(F,N) - \Pi^P(a,N) \;=\; & \frac{1}{9}\pi^P((C_0^e + C_0^P + C^F), C^N) - \pi^P(C^N, C^N) + \\
& \frac{1}{9}\sum_{t=1}^{n-1}(\beta^P)^t \left[\pi^P((C_t^e + C^F), C^N) - \pi^P(C^N, C^N)\right] + \\
& \left\{ \begin{array}{c} \left\{ \begin{array}{c} \left\{\frac{1}{9}\sum_{t=n}^{t^a-1}(\beta^P)^t \left[\pi^P((C_n^e + C^F), C^N) - \pi^P(C^N, C^N)\right]\right\} \\ \text{if } n < t^a - 1 \end{array} \right\} \\ 0 \text{ if } n = t^a - 1 \end{array} \right\} \\
& +(\beta^P)^{t^a}\left[\pi^P((C_n^e + C^F), C^N) - \pi^P(C_0^e + C_{t^a}^p + C^F, C^N)\right] \\
& +\frac{1}{9}\sum_{t=t^a+1}^{t^a+n-1}(\beta^P)^t \left[\pi^P((C_n^e + C^F), C^N) - \pi^P(C_{t-t^a} + C^F, C^N)\right]
\end{aligned}$$

Then

$$\begin{aligned}
\lim_{\beta^P \to 1} \Pi^P(F,N) - \Pi^P(a,N) \;=\; & \frac{1}{9}\pi^P((C_0^e + C_0^P + C^F), C^N) - \pi^P(C^N, C^N) + \\
& \frac{1}{9}\sum_{t=1}^{n-1}\left[\pi^P((C_t^e + C^F), C^N) - \pi^P(C^N, C^N)\right] + \\
& \left\{ \begin{array}{c} \left\{ \begin{array}{c} \left\{\frac{1}{9}\sum_{t=n}^{t^a-1}\left[\pi^P((C_n^e + C^F), C^N) - \pi^P(C^N, C^N)\right]\right\} \\ \text{if } n \leq t^a - 1 \end{array} \right\} \\ 0 \text{ if } n = t^a \end{array} \right\} \\
& +\left[\pi^P((C_n^e + C^F), C^N) - \pi^P(C_0^e + C_{t^a}^p + C^F, C^N)\right] \\
& +\frac{1}{9}\sum_{t=t^a+1}^{t^a+n-1}\left[\pi^P((C_n^e + C^F), C^N) - \pi^P(C_{t-t^a} + C^F, C^N)\right]
\end{aligned}$$

and therefore

$$\begin{aligned}
\lim_{\beta^P \to 1} \Pi^P(F,N) - \Pi^P(a,N) \;=\; & \frac{1}{9}\left[(\pi^P((C_0^e + C_0^P + C^F), C^N) - n\pi^P(C^N, C^N)) - \right. \\
& \pi^P(C_0^e + C_{t^a}^p + C^F, C^N) + \\
& \left\{ \begin{array}{c} \left\{ \begin{array}{c} \left\{(t^a - n)(\pi^P((C_n^e + C^F), C^N) - \pi^P(C^N, C^N))\right\} \\ \text{if } n \leq t^a - 1 \end{array} \right\} \\ 0 \text{ if } n = t^a \end{array} \right\} \\
& \left. +n\pi^P((C_n^e + C^F), C^N))\right]
\end{aligned}$$

or

$$\lim_{\beta^P \to 1} \Pi^P(F,N) - \Pi^P(a,N) \;=\; \frac{1}{9}\left[\begin{array}{c} (\pi^P((C_0^e + C_0^P + C^F), C^N) - \pi^P(C_0^e + C_{t^a}^p + C^F, C^N) + \\ n(\pi^P((C_n^e + C^F), C^N) - (\pi^P(C^N, C^N)))+ \end{array} \right.$$

2.1.P.1.1.2 On the other hand, if $t^a \geq n$, we can show that

$$\begin{aligned}
\Pi^P(F,N) - \Pi^P(a,N) &= \frac{1}{9}\pi^P((C_0^e + C_0^P + C^F), C^N) - \pi^P(C^N, C^N) + \\
&\frac{1}{9}\sum_{t=1}^{n-1}(\beta^P)^t \left[\pi^P((C_t^e + C^F), C^N) - \pi^P(C^N, C^N)\right] + \\
&\left\{ \left\{ \begin{array}{c} \left\{\frac{1}{9}\sum_{t=n}^{t^a-1}(\beta^P)^t \left[\pi^P((C_n^e + C^F), C^N) - \pi^P(C^N, C^N)\right]\right\} \\ \text{if } n < t^a - 1 \end{array} \right\} \atop 0 \text{ if } n = t^a - 1 \right\} \\
&+(\beta^P)^{t^a}\left[\pi^P((C_n^e + C^F), C^N) - \pi^P(C_0^e + C_{t^a}^p + C^F, C^N)\right] \\
&+\frac{1}{9}\sum_{t=t^a+1}^{t^a+n-1}(\beta^P)^t \left[\pi^P((C_n^e + C^F), C^N) - \pi^P(C_{t-t^a} + C^F, C^N)\right]
\end{aligned}$$

Then

$$\begin{aligned}
\lim_{\beta^P \to 1} \Pi^P(F,N) - \Pi^P(a,N) &= \frac{1}{9}\pi^P((C_0^e + C_0^P + C^F), C^N) - \pi^P(C^N, C^N) + \\
&\frac{1}{9}\sum_{t=1}^{n-1}\left[\pi^P((C_t^e + C^F), C^N) - \pi^P(C^N, C^N)\right] + \\
&\left\{ \left\{ \begin{array}{c} \left\{\frac{1}{9}\sum_{t=n}^{t^a-1}\left[\pi^P((C_n^e + C^F), C^N) - \pi^P(C^N, C^N)\right]\right\} \\ \text{if } n \leq t^a - 1 \end{array} \right\} \atop 0 \text{ if } n = t^a \right\} \\
&+\left[\pi^P((C_n^e + C^F), C^N) - \pi^P(C_0^e + C_{t^a}^p + C^F, C^N)\right] \\
&+\frac{1}{9}\sum_{t=t^a+1}^{t^a+n-1}\left[\pi^P((C_n^e + C^F), C^N) - \pi^P(C_{t-t^a} + C^F, C^N)\right]
\end{aligned}$$

and therefore

$$\begin{aligned}
\lim_{\beta^P \to 1} \Pi^P(F,N) - \Pi^P(a,N) &= \frac{1}{9}\left[(\pi^P((C_0^e + C_0^P + C^F), C^N) - n\pi^P(C^N, C^N)) - \right. \\
&\pi^P(C_0^e + C_{t^a}^p + C^F, C^N) + \\
&\left\{ \left\{ \begin{array}{c} \left\{(t^a - n)(\pi^P((C_n^e + C^F), C^N) - \pi^P(C^N, C^N))\right\} \\ \text{if } n \leq t^a - 1 \end{array} \right\} \atop 0 \text{ if } n = t^a \right\} \\
&\left. +n\pi^P((C_n^e + C^F), C^N))\right]
\end{aligned}$$

or

$$\begin{aligned}
\lim_{\beta^P \to 1} \Pi^P(F,N) - \Pi^P(a,N) &= \frac{1}{9}\left[\begin{array}{c} (\pi^P((C_0^e + C_0^P + C^F), C^N) - \pi^P(C_0^e + C_{t^a}^p + C^F, C^N) + \\ n(\pi^P((C_n^e + C^F), C^N) - (\pi^P(C^N, C^N))) + \end{array} \right.
\end{aligned}$$

$$\left\{ \left\{ \begin{array}{c} (t^a - n)(\pi^P((C_n^e + C^F), C^N) - \pi^P(C^N, C^N)) \\ \text{if } n \leq t^a - 1 \end{array} \right\} \begin{array}{c} \\ \\ 0 \text{ if } n = t^a \end{array} \right\} \Bigg]$$

Consequently, $\lim_{\beta^P \to 1} \Pi^P(F, N) - \Pi^P(a, N) > 0$, since $\pi^P((C_n^e + C^F), C^N) - \pi^P(C^N, C^N) > 0$ (A3), and $\pi^P((C_0^e + C_0^P + C^F), C^N) - \pi^P(C_0^e + C_{t^a}^p + C^F, C^N) > 0$, provided that $C_{t^a}^P \geq C_0^P$ for all $t^a \leq t^g - 1$ (by assumption). Then, we announce the following result:

$$\left\{ \begin{array}{c} \text{There is a } \beta_2^P \in (0,1) \text{ such that} \\ \text{for every } \beta^P > \beta_2^P \text{ we have that} \\ \text{for any } \{C_t(a)\} \in A_N \text{ with} \\ t^a \leq t^g - 1, t^g \geq t^a + n, \\ \text{and } t^a \geq n, \text{ whe have} \\ \Pi^P(F, N) > \Pi^P(a, N) \end{array} \right\} \quad (27)$$

2.1.P.1.2 Suppose now that $t^g < t^a + n$. Observe that, provided the result in (24), it suffices to show that $\lim_{\beta^P \to 1} \Pi^P(a, N) < \infty$. Indeed, since $C_{t^g}(a) = (C_0^e + C_{t^g - t^a}^p + C^F)$ with $t^g - t^a < n$ and the foreign firm enters the market, $\pi^P(C_{t^g}(a), C_{t^g}(N)) = \pi^P(C_{t^g}(a), C^N) = 0$, due to A2 (the foreign technology is not adopted with sufficient precedence in order of time). Therefore, we have the following result:

$$\left\{ \begin{array}{c} \text{There is a } \beta_3^P \in (0,1) \text{ such that} \\ \text{for every } \beta^P > \beta_3^P \text{ we have that} \\ \text{for any } \{C_t(a)\} \in A_N \text{ with} \\ t^a \leq t^g - 1, \text{ and } t^g < t^a + n, \text{ whe have} \\ \Pi^P(F, N) > \Pi^P(a, N) \end{array} \right\} \quad (28)$$

2.1.P.2 It rests now to analyze the case $t^a > t^g - 1$.

Now, once again, provided the result in (24), it suffices to show that $\lim_{\beta^P \to 1} \Pi^P(a, N) < \infty$. Applying the same argument as in (2.1.P.1.2), we have that $\pi^P(C_{t^g}(a), C_{t^g}(N)) = \pi^P(C_{t^g}(a), C^N) = 0$, and therefore $\lim_{\beta^P \to 1} \Pi^P(a, N) < \infty$. Hence, the following result holds:

$$\left\{ \begin{array}{c} \text{There is a } \beta_4^P \in (0,1) \text{ such that} \\ \text{for every } \beta^P > \beta_3^P \text{ we have that} \\ \text{for any } \{C_t(a)\} \in A_N \text{ with} \\ t^a > t^g - 1, \text{ we have} \\ \Pi^P(F, N) > \Pi^P(a, N). \end{array} \right\} \quad (29)$$

Consequently, we can announce the following

Proposition 10 *There is a* $\hat{\beta}^P \in (0,1)$ *such that if* $\beta^P > \hat{\beta}^P$, *if* $\{C_t(a)\} \in S$, *then* $\Pi^P(F, N) > \Pi^P(a, N)$.

Proof: Take $\hat{\beta}^P = \max\left\{\beta_i^P \mid \text{with } \beta_i^P \text{ given in (25),(26), (27), (28) and (29) }\right\}$ and apply the remark 4.

Now, applying the prepositions 3 and 4, the demonstration of the part (1) of theorem 2 is done.

2.2 Proof of the part (2).

The argumentation here is totally analogous to the proof in (2.1.I.), and hence omitted.

3 Proof of the theorem 3.

The argumentation here is totally analogous to the proof in (2.1.P.), and hence omitted.

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