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On the Welfare Implications of the Optimal Monetary Policy

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Abstract

We present a standard, neoclassical growth model with leisure in which a representative household must use money in order to purchase consumption goods. Taxes on money holdings, capital and labor income may be used to finance an exogenous stream of government expenditures. It is found that the optimal monetary policy is dictated by the Friedman rule if lump-sum taxes are available to the planner, but the policy becomes indeterminate if the planner has to rely on distorting taxes. Numerically, the model shows a small welfare gain from following a zero nominal interest rate rule. Once the optimal monetary policy is implemented in a second-best economy, gradual decreases in the nominal interest rate towards the Friedman rule are welfare decreasing for a series of numeric exercises.

Resumen

Se presenta un modelo neoclásico estándar de crecimiento con ocio en la función de utilidad, en el cual una familia representativa debe usar dinero para comprar bienes de consumo. En el modelo, los impuestos sobre tenencias de dinero y sobre ingreso por trabajo y capital pueden usarse para financiar una secuencia exógena de gasto de gobierno. Se encuentra que la política monetaria óptima está dada por la regla de Friedman si el planificador tiene a su disposición impuestos de suma fija. Sin embargo, dicha política se vuelve indeterminada si el planificador tiene que usar impuestos distorsionadores. Numéricamente se encuentra una pequeña ganancia en bienestar al adoptarse la regla óptima de una tasa de interés nominal igual a cero. Sin embargo, si la política monetaria óptima se implementa en un mundo second-best, el bienestar disminuye cuando la tasa de interés nominal decrece en dirección a la regla de Friedman, según se muestra en varios ejercicios numéricos.

Introduction

During the last 20 years the U.S. economy has experienced a tighter control in the rate of inflation as compared to the rates observed during the 1970s. This seems to be the result of monetary policy rules whereby the interest rates respond more aggressively today to changes in inflation and real output than in the past (cf. Taylor (1998)). Nevertheless, the interest rates still observed in practice are well above the values that are usually to be believed as theoretically optimal. The present paper is thus concerned about the welfare implications of adopting such optimal policies.

Since the seminal work of Friedman (1969), there has been a long debate about what the optimal monetary policy should be. According to Friedman, since the social production cost of money is practically zero, the government should issue money so that the opportunity cost of holding it is also zero for the public. This argument implies that the nominal interest rate should be set to zero, which corresponds to an optimal monetary policy so that the rate of inflation is negative. In contrast, Phelps (1973) argues that if the government has no access to lump sum taxes, then money holdings should be taxed as any other good. The inflation tax should thus be chosen à la Ramsey (1927), so that the marginal deadweight loss of inflation equals the marginal deadweight loss from other distorting taxes. According to Phelps, this would translate into a positive nominal interest rate and a correspondingly positive marginal deadweight loss from inflation.

More recently, Lucas and Stokey (1983), Correia and Teles (1996, 1999), and Chari et al. (1996), among others, have found that the Friedman rule is in fact optimal in the context of a variety of second-best, representative-agent general equilibrium models.[1] In contrast, Rebelo and Xie (1999) find that the Friedman rule is only one of many rules that may lead to Pareto optimality in a standard general equilibrium model with money. Mulligan and Sala-i-Martin (1997) show that it cannot be theoretically claimed that the Friedman rule is *always* optimal (or always non-optimal), but the results obtained in the literature heavily depend on the assumptions made in each model in particular. For example, the result of Correia and Teles (1999) is based on the specifications of a satiation point (in real balances) function, whereas the result by Chari et al. (1996) depends crucially on their homothetic and separability assumptions of the utility function.

Keeping in mind the present debate on the optimal monetary policy, it seems reasonable to ask about the welfare gain of moving from a moderate positive inflation rate to the inflation rate implied by the optimal monetary policy (whatever that is). In related papers, Lucas (2000) finds that the

[1] See Mulligan and Sala-i-Martin (1997) and Woodford (1990) for a detailed discussion of the literature.

welfare gain of reducing the annual inflation rate from 10 percent to zero is small (slightly less than one percent of income) and that further moving towards the optimal interest rate yields a relatively larger gain under a second-best framework with income taxes. In a first-best framework, Wolman (1997) finds that reducing inflation from a moderate level to zero brings about a relatively substantial welfare improvement, but the additional benefit of moving from zero to the optimal deflation rate is small. Overall, the total welfare gain of achieving the Friedman rule in his paper is about 0.6 percent of income.

This paper extends the work done by Lucas (2000) and Wolman (1997) in three aspects: First, we explicitly solve for the Ramsey problem in order to determine the optimal inflation rate *at the steady state*. As explained below, we show that the study of optimal policies at the steady state may yield different results from those usually found in the literature (which generally study optimality conditions off the steady state). Second, we calculate the welfare gain of such a policy including transitional dynamics. As shown by Lucas (1990) and Ortigueira (1998), the introduction of transitional dynamic effects into the analysis may bring substantial differences in results. Finally, our model is flexible enough as to allow for endogenous and exogenous growth cases.

We consider a standard, deterministic neoclassical growth model with leisure where money is introduced via a cash-in-advance constraint. After defining the household's program, we solve the corresponding Ramsey problem in terms of the dual approach and study its steady state properties.[2] In this regard, we make the distinction between first and second best frameworks. In the first case, we are able to replicate the result found by Chamley (1985) for a model with money in the utility function in which the Friedman rule is in fact optimal at the steady state. Interestingly, we find that once distorting taxes are introduced into the model the optimal monetary policy is such that the nominal interest rate is indeterminate. Therefore, the Friedman rule is only one of many interest-rate rules that may lead to Pareto optimality at the steady state (cf. Rebelo and Xie (1999)).[3] As shown later, this finding yields a continuum of optimal interest-rates (and thus optimal monetary growth rates) that may be readily evaluated in welfare terms. Most important, we are also able to show that our result does not either rely on separability assumptions in the utility function or on exogenous growth specifications. In fact, we only impose a minimum set of assumptions in order to avoid the criticisms of Mulligan and Sala-i-Martin (1997).

[2] As is well known, in the dual approach the planner solves a Ramsey problem by maximizing an indirect utility function and using the tax rates as decision variables. This method is equivalent to the primal approach, in which the planner chooses feasible allocations that are consistent with the optimizing behavior of households and firms.

[3] The result by Rebelo and Xie (1999) holds for every time *t* under a first-best framework.

Although generally of small magnitude, the welfare estimates from adopting the optimal monetary policy found in the paper show an interesting pattern: the optimal monetary policy is naturally welfare improving when lump sum taxes are available but it is not when the planner has to rely completely on distorting taxes to finance the revenue lost derived from lower taxes on money holdings. This finding thus leaves some sense of indeterminacy about the evaluation of optimal monetary policies in terms of welfare, since the result would finally depend on how the planner might end up choosing the extent of the excess burden of taxation. The welfare estimates are also found to be very sensitive to changes in the interest-elasticity of money. As discussed in the paper, this result is directly related to the argument in Phelps (1973) in which the taxation on money holdings depends heavily on the elasticity of money demand.

Our paper is naturally connected to the extensive literature on the welfare cost of inflation. With a few exceptions within the class of a representative-agent, general equilibrium analysis (cf. Cooley and Hansen (1991)), these models usually consider a first-best framework so that distortions from other types of taxation are absent. We believe a second-best specification is important since lost revenue derived from the adoption of optimal monetary policies aimed at lower interest rates may be replaced by extra distorting taxes in order to satisfy the intertemporal budget constraint of the government. This effect thus yields an extra distortion derived from changes in monetary policy. In addition, this literature usually estimates the welfare cost only at the steady state taking as a benchmark the optimal policy derived from the Friedman rule. Overall, these three aspects—the introduction of income taxation, the estimation of welfare including transitional effects and the existence of alternative optimal monetary policies—give our model some additional insights not previously considered.

The paper is divided as follows. The next section presents the basic model as well as some basic extensions. A characterization of the optimal monetary policy is then provided and defined along the balanced growth path. The third section discusses the calibration of the model for particular utility and production functions and shows the results on the welfare gain of moving towards the optimal monetary rule. Section four concludes by including some ways in which the model may be extended.

2. The Optimal Monetary Policy at Steady State

2.1 The Model

We study a standard, deterministic growth model with infinite horizon where a representative agent is endowed with perfect foresight and a single unit of time that may be devoted to leisure, working or "schooling" activities. The household derives utility from consumption $c(t)$ and raw leisure $x(t)$ where $0 \le x(t) \le 1$. The instantaneous utility function $U(c(t), x(t))$ is bounded, continuously differentiable, strictly increasing, strictly concave and non-separable in its arguments. Hence the household maximizes the discounted sum of utilities over time at the rate $\rho > 0$ according to:

$$\int_0^\infty e^{-\rho t} U[c(t), x(t)]dt \tag{2.1}$$

The household keeps assets in the form of money holdings $m(t)$ and ownership claims on physical capital $k(t)$ and public debt $b(t)$. Let us denote $a(t) = k(t) + b(t)$ as the level of private assets (not including money). Since there is no uncertainty, public debt is perfectly substitutable with capital and thus must yield the same rate of return. The representative agent is also endowed with a stock of human capital $h(t)$. Accordingly, her flow budget constraint is given by:

$$c(t) + \dot{a}(t) + q(t) = \tilde{r}(t)a(t) + \tilde{w}(t)u(t)h(t) \tag{2.2}$$

where $q(t) \equiv \dot{m}(t) + \pi(t)m(t)$ represents gross investment in real money balances, $\pi(t)$ is the inflation rate, and $\tilde{r}(t) = [1 - \tau_k(t)]r(t)$ and $\tilde{w}(t) = [1 - \tau_n(t)]w(t)$ are the real rate of return on physical capital and the real wage respectively, both expressed net of taxes. Furthermore, $\tau_k(t)$ and $\tau_n(t)$ denote taxes on capital income and labor income, respectively, and $u(t)$ is the fraction of time that the household devotes to the production of the single good with $0 \le u(t) \le 1$. For simplicity, we assume no depreciation of physical capital, so gross investment $i(t)$ equals net investment:

$$\dot{k}(t) = i(t) \tag{2.3}$$

Money is valued in this economy since it is required to purchase consumption goods. We generalize an otherwise standard cash-in-advance constraint by introducing the degree of liquidity constraint faced by the household, namely the fraction of consumption goods that must be purchased with money. Such exogenous fraction is denoted as ϕ, where $0 < \phi \le 1$. Therefore,

$$\phi c(t) \le m(t) \tag{2.4}$$

In order to allow for endogenous growth, we assume that human capital accumulation may be affected by the way households allocate their time. Accordingly, if we denote the fraction of time devoted to schooling activities by $v(t)$, the law of motion for human capital is given by:

$$\dot{h}(t) = h(t)H[v(t)] \tag{2.5}$$

where $H(\cdot)$ is a continuously differentiable, increasing and concave function, and $0 \leq v(t) \leq 1$.

Finally, we must have:

$$u(t) + v(t) + x(t) = 1 \tag{2.6}$$

We assume perfect competition in the firms sector. The technology is represented by a production function $F[k(t), u(t)h(t)]$ with constant returns to scale in the stock of physical capital $k(t)$ and the effective amount of labor $u(t)h(t)$. We also assume that $F[k(t), u(t)h(t)]$ is continuously differentiable, concave, increasingly monotone and satisfies well-known Inada conditions.

Profit maximization implies that both factors of production are paid their marginal products, i.e.,

$$w(t) = F_n[k(t), u(t)h(t)] \tag{2.7a}$$
$$r(t) = F_k[k(t), u(t)h(t)] \tag{2.7b}$$

where $F_i(t)$ denotes the marginal product of factor of production *i* = *k*, *n*. Since the production function is constant returns to scale, the following properties must hold:

$$F_{nk}(t)[u(t)h(t)] = -F_{kk}(t)k(t) \tag{2.8a}$$
$$F_{kn}(t)k(t) = -F_{nn}(t)[u(t)h(t)] \tag{2.8b}$$

The single good produced in this economy may be devoted either to consumption, investment $i(t)$ or government purchases of goods and services $G(t)$. In this sense, the role of the government is to provide currency and to impose taxes on capital income, labor income and money holdings in order to finance the exogenous stream of government expenditures $G(t)$. Any difference between revenue taxation and expenditures is traded through bonds by the government. Money is issued at the rate $\mu(t) \equiv \dot{M}(t)/M(t)$, where $M(t)$ is the (nominal) money supply. Equilibrium in the money market is thus reached when the nominal price level $P(t)$ adjusts so that real money demand equals real money supply, $m(t) = M(t)/P(t)$.[4] Thus,

$$\dot{m}(t)/m(t) = \mu(t) - \pi(t) \tag{2.9}$$

Accordingly, the amount of revenue raised by the government through money creation at time t is just $\dot{M}(t)/P(t) = \mu(t)m(t)$. Therefore, at each point in time the government's budget constraint must satisfy:

$$G(t) + \tilde{r}(t)b(t) = \dot{b}(t) + \mu(t)m(t) + \tau_k(t)r(t)k(t) + \tau_n(t)w(t)u(t)h(t) \tag{2.10}$$

DEFINITION 1. Given $k(0) = k_0$, $h(0) = h_0$, $b(0) = b_0$ and $M(0) = M_0$, a *competitive equilibrium* is defined as the set of infinite sequences for allocations $\{c(t), i(t), k(t), h(t), m(t), u(t), x(t), v(t)\}$, factor prices $\{r(t), w(t)\}$, and government policy $\{\tau_k(t), \tau_n(t), \mu(t), b(t), G(t)\}$ such that:

(i) Given factor prices and government policy, the allocations $\{c(t), i(t), k(t), h(t), m(t), u(t), x(t), v(t)\}$ maximizes (2.1) subject to (2.2) – (2.6);

(ii) The sequence $\{k(t), h(t), m(t), u(t), r(t), w(t), \tau_k(t), \tau_n(t), \mu(t), b(t), G(t)\}$ satisfies equations (2.7) and (2.10); and

(iii) The goods market clears:[5]

$$c(t) + i(t) + G(t) = F[k(t), u(t)h(t)]$$

2.2 The Second-Best Problem

In this section we provide a characterization of the solution to an optimal taxation problem à la Ramsey (1927) using a dual approach. As is well known, the Ramsey problem in such case is basically solved as follows. First, the household maximizes the utility function (2.1) subject to the constraints (2.2) – (2.6), taking factor prices and government policy as given. The corresponding competitive allocations are then expressed in terms of prices and taxes so that an indirect utility function may be obtained. This allows us to eliminate explicitly the allocations from the program so that the planner can be thought of as directly choosing prices and taxes subject to constraints that ensure the existence of competitive allocations. The prices and taxes

[4] We assume that the price level $P(t)$ is continuous. As discussed by Chamley (1985), this condition is necessary and sufficient so that the solution to the Ramsey problem is time-consistent.

[5] Walras law guarantees that equilibrium in the money market, $m(t) = M(t)/P(t)$, is satisfied if conditions (i), (ii) and (iii) are satisfied.

announced by the planner are thus consistent with the optimization behavior of households and firms. Throughout this analysis, we assume that the planner has access to a commitment technology so that she abides by the taxes originally announced.

It may be readily verified that the solution to the representative agent's program is defined by the following first-order conditions:

$$U_c(t) = \lambda_1(t) + \phi\lambda_2(t) \tag{2.11a}$$

$$U_x(t) = \lambda_3(t) \tag{2.11b}$$

$$\lambda_1(t)\widetilde{w}(t)h(t) = \lambda_3(t) \tag{2.11c}$$

$$h(t)H'[v(t)]\lambda_4(t) = \lambda_3(t) \tag{2.11d}$$

$$\lambda_1(t) = \lambda_5(t) \tag{2.11e}$$

$$\dot{\lambda}_1(t) = \lambda_1(t)[\rho - \widetilde{r}(t)] \tag{2.11f}$$

$$\dot{\lambda}_4(t) = \lambda_4(t)\{\rho - H[v(t)]\} - \lambda_1(t)\widetilde{w}(t)u(t) \tag{2.11g}$$

$$\dot{\lambda}_5(t) = \rho\lambda_5(t) + \lambda_1(t)\pi(t) - \lambda_2(t) \tag{2.11h}$$

$$\lambda_2(t)[m(t) - \phi c_1(t)] = 0\,,\ \ \lambda_2(t) \geq 0 \tag{2.11i}$$

plus some well-known transversality conditions. In the above conditions, $U_i(t)$ denotes the derivative with respect to the *i*th argument, *i* = (*c*, *x*), and $\lambda_1(t)$, $\lambda_2(t)$, $\lambda_3(t)$, $\lambda_4(t)$ and $\lambda_5(t)$ represent the shadow prices of physical capital, the cash-in-advance constraint, the time-allocation constraint, human capital and money holdings, respectively.

Manipulation of (2.11e), (2.11f) and (2.11h) leads to the following expression:

$$\lambda_2(t) = \lambda_1(t)R(t) \tag{2.12}$$

where $R(t) \equiv \pi(t) + \widetilde{r}(t)$ denotes the nominal interest rate (net of taxes). Notice that $\pi(t)$ and $\widetilde{r}(t)$ are one-to-one, linear arguments of the nominal interest rate. From (2.12), note that if $R(t) > 0$ then $\lambda_2(t) > 0$ since the shadow price $\lambda_1(t)$ is strictly positive. Therefore, (2.11i) implies $m(t) = \phi c(t)$ so that the cash-in-advance constraint is strictly binding. On the other hand, if $R(t) = 0$ then $\lambda_2(t) = 0$ and the cash-in-advance constraint is just binding. In other words, we always have $m(t) = \phi c(t)$.

Since the utility function is non-separable in its arguments, substitution of (2.12) into (2.11a) allows us to implicitly define consumption and leisure in terms of the rate of inflation, prices and the stock of human capital:

$$c(t) = \varphi\left[R\left(\pi(t), \tilde{r}(t)\right), \tilde{w}(t), \lambda_1(t), h(t)\right] \tag{2.13a}$$

$$x(t) = \chi\left[R\left(\pi(t), \tilde{r}(t)\right), \tilde{w}(t), \lambda_1(t), h(t)\right] \tag{2.13b}$$

We will denote φ_i and χ_i as the corresponding derivative with respect to $i = (R, \tilde{w}, \lambda_1, h)$.

Using (2.13), it is convenient to define the indirect utility function $V(\cdot)$ as

$$V\left[R\left(\pi(t), \tilde{r}(t)\right), \tilde{w}(t), \lambda_1(t), h(t)\right] \equiv U(c(t), x(t)) \tag{2.14}$$

In addition, differentiation of (2.13a) with respect to time leads to:

$$\dot{c}(t) = \varphi_R(t)\dot{\pi}(t) + \varphi_R(t)\dot{\tilde{r}}(t) + \varphi_w \dot{\tilde{w}}(t) + \varphi_\lambda(t)\dot{\lambda}_1(t) + \varphi_h \dot{h}(t) \tag{2.15}$$

Once the allocation is expressed in terms of the rate of inflation, prices and the stock of human capital, the planner needs to choose the relevant instruments so that the utility of the representative household is maximized.[6] Obviously, the planner's program is constrained so that the market clearing condition in the goods market plus the government's budget constraint (2.10) are satisfied.

From (2.9), (2.10) and (2.15), we find convenient to solve the planner's problem in terms of variables $\dot{\pi}(t)$, $\dot{\tilde{r}}(t)$ and $\dot{\tilde{w}}(t)$. As it turns out, these are the only control variables in the second-best program to be defined below.[7] To make the problem interesting, we need to impose some conditions on the tax of capital income. In particular, since physical capital is given at time zero, we follow the standard procedure of fixing $\tau_k(0)$ exogenously in order to

[6] From (2.9) in the text, the planner may indistinctly choose either $\mu(t)$ or $\pi(t)$ as the relevant monetary policy instrument.

[7] The state variables are given by $k(t)$, $b(t)$, $h(t)$, $\lambda_1(t)$, $\lambda_4(t)$, $\pi(t)$, $\tilde{w}(t)$ and $\tilde{r}(t)$.

avoid a capital levy.[8] In order to simplify the exposition, we momentarily assume exogenous growth so that $v(t) = 0$. This allows us to get rid of equations (2.11d) and (2.11g).

Using (2.14), the *Ramsey problem* is thus to solve

$$\max \int_0^\infty e^{-\rho t} V\left[R\left(\pi(t), \widetilde{r}(t)\right), \widetilde{w}(t), \lambda_1(t), h(t)\right] dt \qquad \text{(P)}$$

subject to

$$\varphi(t) + i(t) + G(t) = F\left[k(t), u(t)h(t)\right] \qquad (2.16)$$

$$G(t) + \widetilde{r}(t)b(t) = \dot{b}(t) + \dot{c}(t) + \pi(t)\varphi(t) + \left[r(t) - \widetilde{r}(t)\right]k(t) + \left[w(t) - \widetilde{w}(t)\right]u(t)h(t) \qquad (2.17)$$

$$u(t) = 1 - \chi(t) \qquad (2.18)$$

and (2.11f), where $\dot{c}(t)$ is given by (2.15). The first restriction in (P) is simply the feasibility constraint expressed in terms of prices and the stocks of human and physical capital whereas the second restriction is the budget constraint for the government after substituting (2.7), (2.9) and the definitions for $\widetilde{r}(t)$ and $\widetilde{w}(t)$ into (2.10). Notice that the amount of working time $u(t)$ in program (P) is also well defined in terms of inflation, prices and the stock of human capital, as given by (2.13b) and (2.18). Finally, note that the first order conditions of the private sector are embedded in the planner's problem through the functional forms (2.13) and equation (2.11f).

DEFINITION 2. A *Ramsey equilibrium* is an infinite sequence of policies $\tau(t) = \left\{\dot{\widetilde{r}}(t), \dot{\widetilde{w}}(t), \dot{\pi}(t)\right\}$, allocation rules $\{c(\tau), k(\tau), b(\tau), h(\tau), m(\tau), u(\tau), x(\tau)\}$ and prices $\left\{r(\tau), w(\tau), R(\tau), \lambda_1(\tau)\right\}$ such that:

(i) The policy τ solves program (P) subject to (2.17); and
(ii) For every policy τ', the allocations $\{c(\tau'), k(\tau'), b(\tau'), h(\tau'), m(\tau'), u(\tau'), x(\tau')\}$ and the price system $\left\{r(\tau'), w(\tau'), R(\tau'), \lambda_1(\tau')\right\}$, together with the policy τ', constitute a competitive equilibrium.

[8] The tax rate on capital at time zero must also be bounded by above. Otherwise, the investment at time 0 may be zero. See Jones et al. (1993) for a further discussion.

The implementation of the second-best policy involves the solution of problem (P) by the planner. After this solution is computed, the planner announces the values of the capital income tax, the wage tax and the inflation rate (alternatively, the money growth rate). The household and firms then chooses the same program as that obtained in the solution to (P) since their optimizing behavior is already taken into account in the second-best problem (P).

Without any proof, from now on we assume that there exists a unique, saddle-path-stable manifold that satisfies the first order conditions for problem (P), and that such path converges to a steady state.[9] Obviously, the optimizing path also satisfies well-known transversality conditions for problem (P). Let $\gamma_1(t)$, $\gamma_2(t)$, $\gamma_3(t)$, $\gamma_4(t)$, $\gamma_5(t)$ and $\gamma_6(t)$ denote the corresponding co-state variables associated with expressions (2.16), (2.17), the laws of motion for $\pi(t)$, $\tilde{r}(t)$ and $\tilde{w}(t)$, and equation (2.11f) respectively. From the maximum principle, after some simplifications, the use of (2.8) and the properties of expression (2.14), the system of equations that characterizes an equilibrium for the planner's problem may be reduced to:[10]

$$\dot{\gamma}_1 = (\rho - r)\gamma_1 + \gamma_2(\tilde{r} - r) \tag{2.19a}$$

$$\dot{\gamma}_2 = (\rho - \tilde{r})\gamma_2 \tag{2.19b}$$

$$\dot{\gamma}_3 = (\rho + \pi)\gamma_3 - (U_c - \gamma_1)\varphi_R - \Gamma\chi_R - \gamma_2\varphi \tag{2.19c}$$

$$\dot{\gamma}_4 = (\rho + \pi)\gamma_4 - (U_c - \gamma_1)\varphi_R - \Gamma\chi_R + \gamma_2(b + k + \varphi_\lambda\lambda_1) + \gamma_6\lambda_1 \tag{2.19d}$$

$$\dot{\gamma}_5 = (\rho + \pi)\gamma_5 - (U_c - \gamma_1)\varphi_w - \Gamma\chi_w + \gamma_2(1 - \chi)h \tag{2.19e}$$

$$\dot{\gamma}_6 = \rho\gamma_6 - (U_c - \gamma_1)\varphi_\lambda - \Gamma\chi_\lambda - \gamma_2\pi\varphi_\lambda - (\gamma_2\varphi_\lambda + \gamma_6)(\rho - \tilde{r}) \tag{2.19f}$$

where $\Gamma = U_x - \gamma_1 wh - \gamma_2(w - \tilde{w})h$. Note that $\gamma_1 > 0$ represents the *social* marginal value of the single good in the economy. In general, γ_1 is not necessarily equal to the *private* marginal value of the good, λ_1, due to the second-best nature of the program. Similarly, $\gamma_2 \geq 0$ denotes the social marginal value of the public debt. Following Chamley (1985), it may also be interpreted as the marginal excess burden of taxation. Notice that $\gamma_2 = 0$

[9] The numeric analysis in next section finds that the saddle-path stable manifold is in fact locally unique for the relevant parameter space.
[10] In the remaining of this section we drop the time script unless otherwise noticed.

corresponds to a first-best economy for every *t*.[11] Similarly, a positive value for γ_2 implies that the planner has to rely on distorting taxes in order to balance its budget.

At this time, it is useful to remark that the focus of this paper is to solve for the optimal monetary policy derived from the solution to the Ramsey problem. As noticed by (2.10), the adoption of such policy implies an imbalance in the government's budget constraint. If the optimal monetary policy is dictated by lower taxes on money holdings, we assume that the planner has two alternative ways of raising the lost revenue implied by such policy: either through lump sum or distorting taxation. Given the restrictions imposed on the taxation of physical capital at time zero, the only way in which the planner may increase revenue through lump-sum taxes is by allowing the initial level of debt $b(0)$ to be endogenous. In this event, if the planner is able to manipulate $b(0)$, lump sum transfers of private assets $a(0)$ between the government and the household would be possible (cf. Chamley (1985)).[12]

As it turns out, the way in which the government is able to raise extra revenue is crucial to determine the optimal monetary policy at the steady state. This result is shown by the proposition below which summarizes our main finding:

PROPOSITION. Suppose there is a unique path that solves program (P) and that such path converges to a steady state, provided it exists. Let R^* denote the steady-state nominal interest rate.[13] Hence, the following must hold in the long run:

(i) The unique optimal monetary policy corresponds to the Friedman rule $R^* = 0$ if only lump-sum taxes are available to balance the present value of the government's budget constraint (i.e., γ_2 = 0).

(ii) The optimal monetary policy is such that R^* is indeterminate if distorting taxes are available to balance the present value of the government's budget constraint (i.e., γ_2 > 0).

Proof: See the appendix. ■

The proposition above is in some way an extension of the result in Rebelo and Xie (1999) for a first-best, general equilibrium model with money. In their model, the authors show that it is optimal to set a constant, non-negative

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[11] As discussed below, the case for a first-best economy may be obtained, for example, by allowing $b(0)$ to be endogenous.

[12] A negative transfer from the government to the household is equivalent to a lump sum tax.

[13] Steady-state values are represented with an asterisk.

nominal interest rate for every *t* since such policy makes the effective price of consumption in the monetary, competitive equilibrium model equal to the price of consumption in the real, competitive equilibrium case. Although restricted to the steady state, our result in part (ii) implies that the Friedman rule $R = 0$ is just one of many rules that may lead to optimality even when other distorting taxes are available. If only lump sum taxes are at the disposal of the planner, the proposition resembles the result found in Chamley (1985) for a model with money in the utility function and labor taxes only.

The proposition also yields a strong and transparent result since it is based on a standard neoclassical model with a minimum of assumptions. For example, we neither require consumption and leisure to be separable as in Chari et al. (1996) nor to impose additional restrictions on the taxes of physical capital and labor income at the steady state. Furthermore, the model may be easily extended to include endogenous growth. This implies that (2.11d) and (2.11g) need now to be added to the planner's problem (P). Following a similar procedure as in the appendix, it is possible to show that the proposition above still holds.

2.3 Describing an Optimal Monetary Policy

We would like to end this section by briefly describing the optimal monetary policy along the balanced growth path. A previous step in this goal is to define particular utility and technology functions. In the first case, we propose a utility function that exhibits a constant elasticity of intertemporal substitution in consumption so a balanced growth path exists. Namely,

$$U\left[c(t), x(t)\right] = \frac{\left[c(t)^{\psi} x(t)^{1-\psi}\right]^{1-\sigma} - 1}{1-\sigma} \tag{2.20}$$

for $\sigma > 0$, $\sigma \neq 1$ and $0 < \psi \leq 1$, and $U\left[c(t), x(t)\right] = \psi \log c(t) + (1-\psi) \log x(t)$ if $\sigma = 1$. The production function $F(t)$ is given by a standard Cobb-Douglas specification $F(t) = Ak(t)^{\alpha} (u(t)h(t))^{1-\alpha}$ where $0 < \alpha < 1$ and $A > 0$. Following Lucas (1990), human capital technology $H(\cdot)$ is expressed as $H\left[v(t)\right] = Bv(t)^{\eta}$ where $0 \leq \eta \leq 1$ represents the constant elasticity of the learning function with respect to time devoted to human capital accumulation, and $B > 0$ is the constant marginal productivity of human capital.

Once the technology and utility functions are specified, we study the full model (i.e., $v(t) > 0$) and proceed to describe the system along the balanced growth path. Let *g* denote the endogenous growth rate along such path. Thus,

$$\dot{c}(t)/c(t) = \dot{m}(t)/m(t) = \dot{k}(t)/k(t) = \dot{h}(t)/h(t) = \dot{b}(t)/b(t) = g$$

and $\dot{u}(t)/u(t) = \dot{x}(t)/x(t) = \dot{v}(t)/v(t) = 0$ along the balanced growth path. It may be readily verified that the transversality condition for the consumer problem derived under this specification is given by $\psi(1 - \sigma)B < \rho$.

As is familiar, it is convenient to redefine variables in terms of the stock of human capital so that $z(t) \equiv k(t)/h(t)$ and $c(t)/h(t)$. Hence, it may be shown that a balanced path is described by the values of z^*, $(c/h)^*$, u^*, x^*, v^* and g that satisfy

$$g = \left(\frac{1}{\phi}\right)\left[1 + \phi(\mu^* + \widetilde{r}^*) - \frac{\psi\widetilde{w}^* x^*}{(1-\psi)(c/h)^*}\right] \tag{2.21a}$$

$$(c/h)^* + (G/h)^* = Az^{*\alpha} u^{*1-\alpha} - gz^* \tag{2.21b}$$

$$g[\psi(1-\sigma) - 1] = \rho - \widetilde{r}^* \tag{2.21c}$$

$$g = B(1 - u^* - x^*)^\eta \tag{2.21d}$$

$$g\left[\psi(1-\sigma) + \frac{\eta u^*}{1 - u^* - x^*}\right] = \rho \tag{2.21e}$$

together with the time constraint (2.6). Note that equations (2.21d) and (2.21e) are dropped from the analysis if the model exhibits exogenous growth (i.e., if $g = 0$). If endogenous growth is assumed instead, we allow $G(t)$ to grow at the endogenous rate g in order to avoid that the ratio G/h goes to zero in the limit.

Now we are finally able to determine the optimal monetary policy $\hat{\mu}$.[14] We may combine the optimal version of the Fisher equation $R(t) \equiv \pi(t) + \widetilde{r}(t)$ and expressions (2.9) and (2.21c) to obtain:

$$\hat{\mu} = \hat{R} - \rho + \psi(1-\sigma)g \tag{2.22}$$

Therefore, $\hat{\mu}$ is a function of the nominal interest rate $\hat{R}$. Note that expression (2.22) simply reduces to $\hat{\mu} = \hat{R} - \rho$ if the model belongs to the exogenous growth case. Under this situation, we may easily recover the

[14] We denote optimal steady-state values for tax instruments $\{\tau_k, \tau_n, \mu\}$ with a hat " ^ ".

standard optimality result that the growth rate of money must be equal to the negative of time preference as $\hat{R} = 0$.

The description of the system along the balanced growth path provides a good intuition for the indeterminacy result obtained earlier. For simplicity, consider the exogenous growth case in the system (2.21a) - (2.21e). For a given value of $\hat{\tau}_k$ and $(c/h)^*$, equations (2.21b), (2.21c) plus the time constraint (2.6) determine the steady-state values for z^*, u^* and x^*. In addition, any combination of policies $\hat{\tau}_n$ and $\hat{\mu}$ that satisfies equation (2.21a) yields a solution to the steady-state allocation $(c/h)^*$. From the Ramsey problem, such combination is constrained to satisfy the present value of the government's budget constraint (2.10). Therefore, the growth rate of money at the steady state is indeterminate and so is the nominal interest rate. If distorting taxes were absent from the analysis, then $\hat{\tau}_n = 0$ and so (2.21a) simply reduces to:

$$(1+\phi\hat{R})(1-\psi)(c/h)^* = \psi w^* x^*$$

Since the corresponding steady state, competitive allocation in a first-best model with no money is given by $(1 - \psi)(c/h)^* = \psi w^* x^*$, optimality in this case requires $\hat{R} = 0$. Finally, it may be easily verified that a similar argument applies for the endogenous growth version of the model.

3. Welfare Estimates of Adopting an Optimal Monetary Policy

3.1 Preliminaries

In this section we provide some welfare estimates of adopting an optimal monetary policy $\hat{\mu}$. It is important to remark that our welfare exercises do not involve the adoption of optimal fiscal and monetary policies simultaneously. As emphasized in the introduction, we are only concerned about the welfare gain of moving from an existing monetary policy to some optimal policy defined by a predetermined nominal interest rate $\hat{R}$. Alternatively, we could solve for optimal fiscal policies at the steady state and leave the optimal monetary policy to be determined residually to satisfy the budget constraint of the government. However, in such a case the welfare effect from an optimal monetary policy alone would be harder to quantify given the series of distorting effects occurring simultaneously.

In order to provide a numeric answer, we proceed as follows. We assume that the economy is initially along a balanced growth path. Let $(c(\tau), x(\tau))$ denote the corresponding path associated with the existing tax policy $\tau \equiv \tau(\tau_k, \tau_n, \mu)$ and $k^*(0)$, $h^*(0)$, and $b^*(0)$ as the initial endowments of physical and human capital as well as debt, respectively. To simplify the analysis, we let the planner to choose an interest rate a priori when lump sum taxes are not available. Hence, at time zero the planner unexpectedly announces a nominal interest rate $\hat{R}$ that defines an optimal policy $\hat{\mu}$ according to expression (2.22). The rate $\hat{R}$ announced is assumed to be strictly lower than the existing nominal interest rate as of time zero. Accordingly, the economy moves out of its initial allocation in order to converge to its new balanced growth path (provided it exists), which is denoted as $(c(\hat{\mu}), x(\hat{\mu}))$. We assume that the household has perfect foresight once the policy is announced.

Here it is important to remark that the allocation implied by the optimal monetary policy $\hat{\mu}$ is not necessarily the second-best Pareto allocation. In this regard, when computing our welfare estimates we are not interested in considering the optimal allocation itself but in the process whereby such an allocation may be attained. Putting differently, we perform what is called in the public finance literature as a *partial welfare improvement* exercise (cf. Atkinson and Stiglitz (1980), chapter 12).

Following Lucas (1987), we may define ζ as the compensating consumption supplement necessary for the household to be indifferent between the existing policy τ and the policy implied by the nominal interest rate $\hat{R}$. In other words, the welfare gain ζ of adopting the optimal monetary policy is implicitly given by:

$$\int_0^\infty e^{-\rho t} U\left[(1+\zeta)c(\tau), x(\tau)\right] dt = \int_0^\infty e^{-\rho t} U\left[c(\hat{\mu}), x(\hat{\mu})\right] dt \qquad (3.1)$$

From (2.10), the adoption of an optimal policy $\hat{\mu}$ brings about an imbalance in the government budget constraint. In order to solve for this issue, we divide our welfare exercises in two parts. First, we assume that the lost revenue derived from the adoption of policy $\hat{\mu}$ is replaced by extra lump-sum taxes. In the second set of exercises we let the lost revenue to be replaced by higher labor income taxes.[15] In both cases, the tax changes are of such magnitude so that the intertemporal budget constraint of the government—i.e., the *present value* of equation (2.10)—holds. In any event, we find convenient to follow Chari et al. (1994) and think of lump-sum transfers $T(t)$ as obligations by the government to pay a fixed amount in present-value terms. Under this interpretation, transfers are just equivalent

[15] It may be shown that the solution to the Ramsey problem above involves no taxation of capital income at the steady state (cf. Chamley (1986)). In a similar vein, Chari et al. (1994) numerically solve a stochastic standard neoclassical model and find that optimal labor income taxes are nearly constant across time. Taking these two facts into account, we prefer to raise revenue through τ_n since presumably such policy would resemble the solution to the Ramsey problem more closely.

to government debt.[16] This allows us to redefine the present value of equation (2.10) in terms of lump-sum transfers rather than bonds. This alternative specification is used when solving numerically the intertemporal budget constraint problem of the government.

3.2 Calibration

For clarity purposes, we make the distinction between those parameters borrowed from the literature and those specially calibrated to match some observed variables for the U.S. economy. We also proceed as Lucas (1990) and normalize initial output and the initial stock of human capital to unity.

Unless otherwise noticed, the following values for parameters are standard in the literature (cf. Kydland and Prescott (1982)):

(i) $\alpha = 0.36$;

(ii) $\eta = 0.8$. The value is taken from Lucas (1990). Since there may be no general agreement on this parameter, we also present alternative values later on;

(iii) $\sigma = 1.5$;

(iv) $\psi = 0.43$. We take this value from Krusell and Rios-Rull (1999). In the endogenous growth version of the model, we use $\psi = 0.45$ to get a more accurate value for worked hours along the balanced growth path;

(v) $\mu = 0.067$. This is the number for the average growth rate of the monetary base in the U.S. for the period 1960 - 1997;

(vi) $\tau_k = 0.43$ and $\tau_n = 0.25$. These are the averages for the effective tax rates in the U.S. on physical capital and labor income, respectively as reported in Mendoza et al. (1994);

(vii) $G/h = 0.21$. This value is taken from Lucas (1990). Given our normalization in both output and human capital, this number means that 21 percent of output is devoted to government consumption of goods and services.

Parameters that are specially calibrated include the following:

(i) $A = 1.1155$. This value yields an output equal to 1 under the exogenous growth version of the model. The corresponding value under endogenous growth is $A = 1.3885$;

(ii) $B = 0.1093$. The marginal productivity of human capital is fixed so that the endogenous growth rate along the balanced path

[16] It may be shown that the proposition in section 2 is unaffected if bonds are replaced by lump-sum transfers.

equals 2.16 percent. From (2.9), this value is just consistent with the 4.54 average percent of inflation observed during the period 1960 – 1997 and the value for μ provided above;

(iii) ρ = 0.035. This parameter is calibrated so that the steady-state real rate of return on capital (before taxes) is 6.1 percent under the exogenous growth case;

(iv) ϕ = 0.15. The share of consumption goods that must be paid with money is fixed so that the seigniorage-output ratio is 0.54 percent, which compares well according to the values reported by Cooley and Hansen (1991).

Overall, the parameterization above yields the following values for the exogenous growth case:

$(k/h)^* = 5.863$ $\quad (c/h)^* = 0.790$ $\quad u^* = 0.312$ $\quad x^* = 0.688$

and the following values for the endogenous growth framework:

$(k/h)^* = 3.339$ $\quad (c/h)^* = 0.718$ $\quad u^* = 0.304$ $\quad x^* = 0.564$

$v^* = 0.132$ $\quad g = 0.0216$

The values given above yield steady-state transfers in the amount of $(T/h)^*$ = 0.110 and $(T/h)^*$ = 0.112 in the exogenous and endogenous growth case, respectively so that the government's budget constraint is balanced at the steady state. These numbers are closely related to the value of 12 percent reported in Chari et al. (1994). Finally, the value for the nominal interest rate under the benchmark economy is either 0.081 or 0.107, depending on whether the economy exhibits exogenous or endogenous growth, respectively.

3.3 Results

Before presenting the results, there are some important remarks to be made. As mentioned earlier, the adoption of the optimal policy $\hat{\mu}$ brings about an imbalance in the present value of the government budget constraint. When only distorting taxes are available, we need to find the value for τ_n so that the present-value version of expression (2.10) is satisfied. The steps involved in this computation are as follows: First, we start with a policy $\hat{\mu}$ and a guessed number for τ_n in order to calculate the corresponding present value of government revenue along the transition path defined by policy $\hat{\mu}$. Next, since the guessed value for τ_n usually defines a new lump-sum transfer payment T/h along the balanced growth path as implied by (2.10), we compute the corresponding present value for total government spending along the transition path defined also by policy $\hat{\mu}$.[17] Finally, we check whether the present value of government revenue is equal to the present value of government spending. If it is not (as it is mostly the case), we continue adjusting the free tax policy instrument until the present-value version of equation (2.10) is satisfied.

For the numeric exercises shown below, we find a unique negative eigenvalue in both exogenous and endogenous growth cases. This means that, at least for the parameter space under study, there is a locally unique, saddle path stable manifold. We also find a unique steady state for all the exercises considered. This last result is not trivial given the specification (2.1) in which utility is a function of non-qualified leisure.[18]

Table 1 presents the welfare gain of adopting alternative optimal monetary policies under the exogenous growth case. The first column simply lists the alternative nominal interest rates announced by the planner whereas the second column is the implied optimal growth rate of money according to the exogenous growth version of equation (2.22). The welfare gain (in consumption terms) of adopting such a policy is presented in the last column. Part A assumes that the lost revenue derived from the adoption of policy $\hat{\mu}$ is replaced by lump sum taxation. In such case, the first row simply states that the welfare gain of adopting the Friedman rule with respect to the existing policy is about 0.23 percent in consumption terms.

[17] As in the case of government expenditures, we let lump sum transfers T grow at the endogenous rate g so that the ratio T/h moves away from zero in the limit.

Table 1
Welfare gain of adopting an optimal monetary policy
Benchmark economy, exogenous growth
(Numbers in percentage terms)

A. Lump-sum taxation

$\hat{R}$	$\hat{\mu}$	ζ
0.0	-3.5	0.233

B. Distorting taxation

$\hat{R}$	$\hat{\mu}$	τ_n	ζ
8.0	4.5	25.0	-0.001
6.0	2.5	25.4	-0.058
4.0	0.5	25.9	-0.115
3.5	0.0	26.0	-0.130
2.0	-1.5	26.3	-0.174
0.0	-3.5	26.8	-0.234

Now we consider the case when revenue is fully raised through distorting taxes, specifically labor income taxes. In particular, the third column in part B of table 1 presents the value of τ_n necessary so that the present value of the government budget constraint is satisfied. Obviously, the value of τ_n increases as the economy moves toward the Friedman rule according to the alternative values of $\hat{R}$ announced by the planner. In contrast to the previous result, the adoption of an optimal monetary policy with lower nominal interest rates is not welfare improving, although the loss is relatively small (about 0.23 percent when adopting the Friedman rule). In fact, it is clear from part B that the Friedman rule yields the maximum welfare *loss* possible: any positive level of $\hat{R}$ gives a higher welfare to the household. Relative to the Friedman rule, table 1 also shows that about 56 percent of the total welfare loss is derived from adopting a nominal interest rate that yields a zero growth rate of money (and thus a zero inflation rate).[19]

[18] Ladron-de-Guevara et al. (1999) find multiplicity of steady states under a relatively broad parameter space in a model very similar to ours with no money and non-distorting taxes. As shown by the authors, the existence of equilibria is explained by the introduction of non-qualified leisure in the utility function.

19 Relative to the Friedman rule, we find that around 57 percent of the total welfare gain is obtained from a non-optimal policy of zero inflation in the lump-sum taxation case. This number decreases to 43 percent in the endogenous growth case.

Table 2
Sensitivity of welfare gain to changes in benchmark parameter values under the Friedman rule ($\hat{R}$ = 0)
Exogenous growth

A. Lump-sum taxation

Parameter	Value	ζ (%)
σ	1.5	0.233
σ	1.01	0.240
σ	2.0	0.226
σ	3.0	0.216
σ	5.0	0.203
ψ	0.30	0.272
ψ	0.40	0.242
ψ	0.50	0.211
ψ	0.60	0.177
ϕ	0.25	0.390
ϕ	0.50	0.794
ϕ	0.75	1.210
ϕ	1.00	1.637

B. Distorting taxation

Parameter	Value	τ_n (%)	ζ (%)
σ	1.5	26.8	-0.234
σ	1.01	26.8	-0.246
σ	2.0	26.8	-0.225
σ	3.0	26.7	-0.212
σ	5.0	26.7	-0.197
ψ	0.30	26.6	-0.207
ψ	0.40	26.7	-0.233
ψ	0.50	26.8	-0.226
ψ	0.60	26.9	-0.198
ϕ	0.25	27.9	-0.413
ϕ	0.50	31.0	-0.961
ϕ	0.75	34.3	-1.682
ϕ	1.00	37.8	-2.642

A sensitivity analysis for some of the parameters of the model under the Friedman rule policy is presented in table 2. Again, the results are differentiated between lump sum and distorting taxation. The first row in each case denotes the welfare gain or loss under the benchmark case in order to facilitate the analysis. In both cases, it may be noticed that the estimates for ζ are relatively unchanged with respect to the benchmark for alternative values of σ and ψ. However, the results are highly sensitive to alternative values for the degree of liquidity constraint ϕ: for ϕ = 1, the welfare gain (loss) is as high as 1.6 (2.6) percent in consumption terms under lump sum (distorting) taxation.

A similar picture is obtained under the endogenous growth case. Table 3 presents some welfare estimates under alternative predetermined nominal interest rates. In addition, we include estimates of the corresponding endogenous growth rates as shown in the last column. For example, the first row in part A states that the welfare gain of adopting the Friedman rule yields a welfare gain of around 0.22 percent. Interestingly, this number is very similar to the exogenous growth case specification. Such policy also implies a slightly increase in the endogenous growth rate from the benchmark value of 2.16 to 2.20 percent.

Table 3
Welfare gain of adopting an optimal monetary policy
Benchmark economy, endogenous growth
(Numbers in percentage terms)

A. Lump-sum taxation

$\hat{R}$	$\hat{\mu}$	ζ	g
0.0	-3.99	0.224	2.199

B. Distorting taxation

$\hat{R}$	$\hat{\mu}$	τ_n	ζ	g
8.0	4.01	25.5	-0.036	2.152
6.1	2.16	25.8	-0.061	2.148
6.0	2.01	25.8	-0.063	2.148
4.0	0.01	26.2	-0.091	2.143
2.0	-1.99	26.6	-0.119	2.139
0.0	-3.99	26.9	-0.148	2.134

Under distorting taxes, we find again that the Friedman rule yields the maximum welfare loss and that the relationship between $\hat{R}$ and ζ is monotonic for the exercises considered. In particular, the adoption of a zero nominal interest rate gives a welfare loss of about 0.15 percent, a number slightly smaller than under the exogenous framework. In either case, the changes in the welfare estimates are still relatively small. Finally, we observe that about 41 percent of the total welfare loss is explained by adopting a policy of zero inflation, as it may be seen from the second row of part B.

Sensitivity analysis for the endogenous growth case is carried out in table 4. For each scenario, again we find that reasonable changes in either σ, ψ or η do not significantly affect the welfare estimates under the benchmark case although the growth rate g is in fact notably modified. Nevertheless, the share of consumption goods paid with money ϕ once again yields relatively large changes in the estimates for ζ: the welfare cost of adopting the Friedman rule may be as high as 2.25 percent in consumption terms although still smaller than the corresponding estimate in the exogenous growth case.

In order to clarify the relatively large effect of parameter ϕ on welfare, it is useful to get an expression for the interest-elasticity of money $\varepsilon(t)$ implied by the model. By manipulating (2.11a) – (2.11c) and (2.12), this expression is given by:

$$\varepsilon(t) = \frac{-\phi R(t)}{1+\phi R(t)} \tag{3.2}$$

The interest-elasticity of money is thus only a function of the exogenous parameter ϕ and the nominal interest rate $R(t)$. From equation (3.2), it is evident that a higher ϕ makes money demand more interest-elastic. Following the arguments of Phelps (1973), this implies that the planner should rely relatively less on money taxation and more on other type of taxes when the household is more liquidity constrained. Therefore, we should expect a higher welfare gain when lump sum taxes are available and ϕ is higher. This is just what the first part of tables 2 and 4 reflects for alternative values of ϕ. In a similar way, a higher ϕ would optimally lead the planner to rely relatively less on money taxation and more on distorting taxes once lump sum instruments are no longer available. Hence, welfare loses would be even greater as ϕ increases, a fact that the second part of tables 2 and 4 displays.

Table 4
Sensitivity of welfare gain to changes in benchmark parameter values under the Friedman rule ($\hat{R}$ = 0)
Endogenous growth

A. Lump-sum taxation

Parameter	Value	ζ (%)	g (%)
σ	1.5	0.224	2.20
σ	1.01	0.171	3.06
σ	2.0	0.236	1.77
σ	3.0	0.241	1.33
σ	5.0	0.243	0.94
ψ	0.30	0.279	0.79
ψ	0.40	0.239	1.73
ψ	0.50	0.209	2.63
ψ	0.60	0.177	3.37
ϕ	0.25	0.381	2.20
ϕ	0.50	0.804	2.20
ϕ	0.75	1.264	2.20
ϕ	1.00	1.759	2.20
η	0.65	0.248	3.24
η	0.95	0.204	0.83

B. Distorting taxation

Parameter	Value	τ_n (%)	ζ (%)	g (%)
σ	1.5	26.9	-0.148	2.13
σ	1.01	26.7	-0.089	2.98
σ	2.0	27.1	-0.175	1.72
σ	3.0	27.2	-0.199	1.29
σ	5.0	27.4	-0.217	0.91
ψ	0.30	26.9	-0.212	0.75
ψ	0.40	26.9	-0.164	1.66
ψ	0.50	26.9	-0.135	2.57
ψ	0.60	26.9	-0.109	3.32
ϕ	0.25	28.2	-0.275	2.09
ϕ	0.50	31.4	-0.711	1.97
ϕ	0.75	34.8	-1.350	1.84
ϕ	1.00	38.3	-2.254	1.71
η	0.65	26.9	-0.143	3.19
η	0.95	27.0	-0.166	0.73

Before closing this section, it is relevant to compare our results with those found in the literature. The first issue is related to the welfare cost of inflation. As mentioned earlier, the representative-agent, general equilibrium literature usually excludes the effect of distorting taxes into the analysis by assuming a first-best model. In such a case, the welfare cost of a 5 percent rate of inflation (relative to the Pareto optimal allocation) is generally below one percent of income (cf. Lucas (2000)). In this sense, our model also provides evidence of low welfare costs in general even under a second-best framework. Once distorting taxes are taken into account, Cooley and Hansen (1991) find that moving from an existing tax policy (including taxes on money holdings) towards a zero inflation policy brings about more costs than benefits. When the effects of such policy on the intertemporal budget constraint of the government are fully incorporated into the analysis (as in our model), the results of table 1 and 3 provide supporting evidence in the same direction as Cooley and Hansen (1991).

The second issue has to do with the effects of inflation and distorting taxes on growth. From the last column of table 3 part A, it is noticeable that the endogenous growth rate g slightly increases when the growth rate of money is decreased, and slightly decreases once fiscal and monetary policies are combined (recall that the value for g under the benchmark case is 2.16 percent). The small negative relationship between growth and the rate of inflation are well in accord to what is found elsewhere (cf. Chari et al. (1995)) whereas the small negative impact of taxes on growth is also found to be the most plausible case (cf. Stokey and Rebelo (1995)).

The final point is related to the importance (in welfare terms) from following the Friedman rule. For example, if a central bank or planner is committed to follow a policy of zero inflation, the issue is how much extra welfare could be gained if the Friedman rule is followed instead. Lucas (2000) finds that, starting from a nominal interest rate of 8 percent, around 38 percent of the total welfare gain (with respect to the Friedman rule) is obtained under a policy of zero inflation in a first-best, exogenous growth framework. In contrast, Wolman (1997) reports that between 75 and 90 percent of the maximum welfare gain may be achieved with zero inflation when using a money demand function that nests the specification in Lucas (2000). In our model we have found earlier that such gain is about 57 percent under the first-best, exogenous growth framework. In other words, once transitional dynamics are taken into account, the relative gain from following a policy of zero inflation in our model is of a magnitude between those reported in Lucas and Wolman. Of course, if only taxes on labor income are available to the planner, our results show that a policy of zero inflation cannot be welfare improving.

Concluding Remarks

We have presented a standard neoclassical growth model with leisure where a representative household must use money in order to purchase consumption goods. The model is then used to find the optimal monetary policy at the steady state and to estimate the welfare gain of following such a policy. In the first case, it is found that the optimal policy is dictated by the Friedman rule if lump-sum taxes are available to the planner, but the optimal policy becomes indeterminate if the planner has to rely on distorting taxes to finance lost revenue. When estimating the welfare effects of such policies, we find a small welfare gain from following a zero nominal interest rate policy under a first-best framework. Otherwise, optimal monetary policies dictated by lower nominal interest rates (and thus lower rates of inflation) are not welfare improving when lost revenue is fully raised through distorting taxation. As mentioned in the text, this is perfectly consistent with the results from a partial welfare improvement exercise as ours (cf. Atkinson and Stiglitz (1980)). Nevertheless, the welfare losses are of small magnitude in general regardless of whether the model exhibits exogenous or endogenous growth.

The result that the optimal monetary policy has relatively small welfare consequences in general is of particular relevance in those cases where the planner is concerned about the effect of such a policy on the well being of the household. Nevertheless, we believe there are two issues that deserve further exploration in future work. First, although we have found that our welfare results are generally robust to alternative values, the parameter that measures the relative amount of consumption goods that must be purchased with money has a great effect on welfare. As explained in the text, the interest-elasticity of money is positively related to the value of such parameter and thus our results are in accordance to the intuition provided by Phelps (1973) in which interest-inelastic goods should be taxed more heavily. Accordingly, it may be interesting to extend the model along the line of models like those by Gillman (1993) and Lacker and Schreft (1996) in which the cash-in-advance constraint is endogeneized. In particular, this specification would allows us to check if the household is able to mitigate the large welfare losses derived from optimal policies under the second best framework. Second, Lucas (2000) reports that welfare may not be reliable estimated for low levels of the interest rate under the existing standard general equilibrium models with money. This is not a trivial issue. In fact, we believe it is a very interesting area of research that may certainly give a sharper answer to an old but surely relevant question in monetary theory.

Appendix: Proof of proposition

The goal in this section is to provide a proof for the proposition in the text. The proof outlined below applies specifically to the exogenous growth case, although it may be easily extended to the endogenous growth framework. First, we proceed to evaluate the first-order conditions from the Ramsey problem at the steady state. Combining (2.19c), (2.19d) in the text and the fact that $\gamma_3 = \gamma_4$ yields:

$$\gamma_6^* \lambda_1^* = -\gamma_2^* \left(\varphi^* + k^* + b^* + \varphi_\lambda^* \lambda_1^*\right) \tag{A.1}$$

Next, multiply the steady-state versions of (2.19c), (2.19e) and (2.19f) by 1 + R^*, $\tilde{w}^*$ and λ_1^* respectively in order to get:

$$\Gamma^* \left(1 + R^*\right)\chi_R^* = \left(1 + R^*\right)\left((\rho + \pi^*)\gamma_3^* - \gamma_2^* \varphi^*\right) - \left(1 + R^*\right)\varphi_R^* (U_c^* - \gamma_1^*) \tag{A.2}$$

$$\Gamma^* \tilde{w}^* \chi_w^* = \tilde{w}^* \left[\left(\rho + \pi^*\right)\gamma_5^* + \gamma_2^* (1 - \chi^*) h^*\right] - \tilde{w}^* \varphi_w^* \left(U_c^* - \gamma_1^*\right) \tag{A.3}$$

$$\Gamma^* \lambda_1^* \chi_\lambda^* = \lambda_1^* \left(\rho \gamma_6^* - \gamma_2^* \pi^* \varphi_\lambda^*\right) - \lambda_1^* \varphi_\lambda^* \left(U_c^* - \gamma_1^*\right) \tag{A.4}$$

where $\Gamma^* = U_x^* - \gamma_1^* w^* h^* - \gamma_2^* (w^* - \tilde{w}^*) h^*$. Finally, (2.13a), (2.13b) and the use of the implicit function theorem give the following result that holds for every *t*:[20]

$$(1 + R)\varphi_R + \tilde{w}\varphi_w - \lambda_1 \varphi_\lambda = 0 \tag{A.5}$$

$$(1 + R)\chi_R + \tilde{w}\chi_w - \lambda_1 \chi_\lambda = 0 \tag{A.6}$$

First we assume that only lump sum taxes are available at the time of implementing the optimal monetary policy, i.e. $\gamma_2 = 0$ for every t. From (A.1), this implies $\gamma_6^* = 0$. Substitute (A.2) – (A.4) into (A.6) evaluated at steady state to get:

$$-\left(U_c^* - \gamma_1^*\right)\left((1 + R^*)\varphi_R^* + \tilde{w}^* \varphi_w^* - \lambda_1^* \varphi_\lambda^*\right) + \left(\rho + \pi^*\right)\left(\gamma_3^* (1 + R^*) + \gamma_5^* \tilde{w}^*\right) = 0$$

[20] The implicit function theorem is well defined given the properties of the *U* function.

Using the steady-state version of (A.5) and the fact from the maximum principle that $-\varphi_R = \gamma_3/\gamma_2$ and $-\varphi_w = \gamma_5/\gamma_2$ for every t, the above expression is reduced to

$$R^* \lambda_1^* \varphi_\lambda^* = 0 \tag{A.7}$$

which implies R^* = 0. Therefore, the Friedman rule is the unique optimal monetary policy. This proves part (i).

The next part of the proof allows $\gamma_2(t)$ to be strictly positive. Note that (2.16) and (2.17) evaluated at steady state yield:

$$\tilde{w}^*(1-\chi^*)h^* = (1+\pi^*)\varphi^* - \rho(k^* + b^*) \tag{A.8}$$

Now substitute expressions (A.1) – (A.4) and (A.8) into the steady state version of equation (A.6). After some simple manipulations, we obtain the following:

$$\left((1+R^*)\varphi_R^* + \tilde{w}^*\varphi_w^* - \lambda_1^*\varphi_\lambda^*\right)\left(\gamma_2^* + \lambda_1^*\right)R^* = 0 \tag{A.9}$$

that necessarily holds by the steady state version of (A.5). Therefore, any value of R^* satisfies equation (A.9) and so the nominal interest rate is indeterminate. This proves part (ii).

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