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# Optimal Taxation Under Time-Inconsistent Preferences

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## *Abstract*

*We present a neoclassical model where a representative agent exhibits time-inconsistent preferences. There is a government endowed with full commitment that imposes taxes on capital and labor income to finance an exogenous stream of spending. Steady-state analysis shows that the optimal tax on capital is non-positive whereas the optimal tax on labor is zero. Off the steady state, numerically we find that the subsidy to capital income extends throughout most of the transition period. If the subsidy is larger, the planner optimally chooses to impose the burden of taxation on labor income only at the initial period, even though labor is elastically supplied.*

## *Resumen*

*En este artículo se presenta un modelo neoclásico donde un agente representativo exhibe preferencias inconsistentes en el tiempo. Hay un gobierno con pleno compromiso que fija impuestos al capital y al trabajo para financiar una secuencia exógena de gasto. El análisis en el estado estacionario muestra que el impuesto óptimo al capital es no-positivo mientras que el impuesto óptimo al trabajo es cero. Fuera del estado estacionario, numéricamente se encuentra que el subsidio al capital se extiende para la mayor parte de la transición. Si el subsidio es mayor, el planificador decide, de manera óptima, imponer la carga fiscal sobre el impuesto al trabajo sólo por un breve periodo inicial, a pesar de que el trabajo se ofrece de forma elástica.*

# 1. Introduction

In a neoclassical context, the relevance of capital accumulation to explain increases in output and consumption per capita over time is very well known. In a world with distorting taxes, a capital income tax usually affects the allocation of resources through its effects on the rate of return, yielding a lower level for the stock of capital (and thus for consumption) in the long run. Under such scenario, choosing the appropriate level of taxation that maximizes the well being of society becomes an issue of great importance.

In a seminal paper, Chamley (1986) finds that the optimal capital income tax is zero in the steady state. In a similar vein, Judd (1985) arrives to the same result under a redistributive model with capitalists and workers. Since then, there have been several papers that try to understand the conditions under which the optimality result of a zero capital income tax in the steady state does not longer hold. Examples of such papers within the representative agent framework include Kemp et al. (1993), Zhu (1995), Correia (1996), Krusell et al. (1996), Benhabib and Rustichini (1997), Judd (1997, 1999), Guo and Lansing (1999), Lansing (1999), and Chamley (2001).

The purpose of this paper is to determine the optimal path for capital and labor income taxes when the representative consumer has time-inconsistent preferences of the type first studied by Strotz (1956). Under such framework, the household typically values her utility flow differently as the planning date evolves. In particular, Laibson (1997) argues that individuals are usually impatient about consuming between today and tomorrow but are more patient about consumption choices in the distant future. In other words, this argument presumes a time-varying rate of time preference which is typically very high in the short run but much lower in the long run. Applying these insights into a neoclassical, general equilibrium model with no taxes, Barro (1999) finds that if the infinitely lived household is impatient about consuming today, is unable to commit her decisions and fully appreciates the effects of her actions on her future behavior, then the household conveniently chooses to consume a fraction of her wealth at each point in time as the solution to her inconsistency problem. It turns out that this fraction is usually higher than the standard discount rate. Thus, compared to the standard analysis with constant preferences over time, this model yields a higher consumption today and thus lower resources available for future investment.

In this paper, we extend the model of Barro (1999) by allowing for a labor-leisure choice and distorting taxes on capital and labor income necessary to finance an exogenous stream of government spending. The goal of this study is to find the sequence of optimal taxes when the household exhibits time-inconsistent preferences and the planner is endowed with full commitment in both preferences and technology. If this is the case, the planner may choose taxes optimally to provide the help necessary to a household unable to commit her decisions. For example, we find that the optimal tax on capital income is non-positive at the steady state. The intuition is simple: since the time-inconsistent consumer has a higher rate of discount in general as a result of her inconsistency problem, the stock of capital is consequently lower at the steady state. In order to restore the socially desired level of capital, the planner must provide a subsidy to capital income.

At the same time, we find that the optimal tax on labor income is zero at the steady state. Given the assumption that the government needs to finance an exogenous stream of positive spending, the optimal tax policy at the steady state implies that the sequence of taxes off the steady state deserves further study. In particular, it is of interest to know whether the burden of taxation along the transition path is imposed on capital, labor or both factors.

To analyze this issue, we follow the standard assumption in the literature whereby the government is endowed with a commitment technology in order to avoid a tax on capital income sufficiently large to cover the revenue requirements of the government. We perform numeric exercises and find that the subsidy to capital income derived from time-inconsistency arguments not only applies to the steady state but to most of the transition path. The subsidy is significantly larger as the household exhibits more impatience. In contrast, the optimal tax on labor income is positive along most of the transition. Were the household exhibits more impatience, the optimal path for labor income taxes remains essentially unchanged through time except for the initial period: the planner chooses to raise the required revenue (because of the larger subsidies to capital income) by taxing labor income heavily at the initial period, even though labor is elastically supplied.

The results of this paper heavily relies on the idea that the planner can commit her decisions at each point in time, namely that the planner both has access to a commitment technology as is usually assumed in these kind of models, and does not exhibit the time inconsistency problem of the household, even though the planner is well aware of such a problem. Although this might seem unrealistic, we believe the analysis provides a useful benchmark for tax policy and welfare.[1] On the other hand, if the planner shares the time-inconsistency problem of the household, it is possible to recover the optimality result of Chamley (1986).

[1] As emphasized by Krusell et al. (2001), normative tax policy cannot be addressed if the government's preferences are time-inconsistent.

There are three other papers in the literature closely related to ours. Laibson (1996) considers a model without capital where the technology available to consumers is linear in savings and where the government can commit to future taxes. Krusell et al. (2000) study optimal taxation in a model where the planner has the same time-inconsistency problem as the household and thus cannot commit to future taxes. In this case, the authors are particularly interested in finding whether the time-inconsistent government is able to yield an outcome with a higher welfare as compared to the household's outcome. Finally, Krusell et al. (2001) extend their previous model to consider cases in which the consumer is subject to temptation and self-control.

The remaining of the paper is as follows. Section 2 discusses the model under the alternative scenarios of full and no commitment, and solves for the optimal policy at the steady state for each case. Section 3 calibrates the model and numerically analyzes the optimal paths for capital and labor income taxes when the household is not able to commit her decisions. Section 4 concludes.

## 2. The Model

### 2.1 The Environment

We consider a standard neoclassical, deterministic exogenous growth model with infinite horizon where a representative household is endowed with perfect foresight and a single unit of time. In this framework, time may be devoted either to leisure or production activities. The household derives utility from per capita consumption $c(t)$ and raw leisure $x(t)$, where $0 \leq x(t) \leq 1$. For simplicity, the instantaneous utility function $u(c(t), x(t))$ is assumed to be continuously differentiable, strictly increasing, strictly concave and separable in its arguments.

We follow Strotz (1956) and Barro (1999) and assume that the household has a variable rate of time preference over time. In particular, the representative agent is impatient about consuming right now and fully appreciates how her actions affect her future behavior. This idea is captured by the following specification of preferences as of current date $s$:

$$U(s) = \int_{s}^{\infty} u(c(t), x(t)) \exp\left[-(\rho \cdot (t-s) + \phi(t-s)\right] dt \qquad (1)$$

where $\rho > 0$. The difference with respect to the standard model is given by the term $\phi(t-s) \geq 0$. This expression is a function of the distance in time, $v \equiv t-s \geq 0$, and captures the idea of a variable time preference. Following Laibson (1997), the function $\phi(\cdot)$ is taken to be continuous and twice differentiable with the properties $\phi'(v) \geq 0$, $\phi''(v) \leq 0$, and $\phi'(v)$ approaches zero as $v$ tends to infinity. We may normalize so that $\phi(0) = 0$. The expression $\rho + \phi'(v)$ thus denotes the instantaneous rate of time preference at the time distance $v$. From the properties on $\phi(\cdot)$, it follows that the rate of time preference is high in the near term but roughly constant at the lower value $\rho$ in the distant future.

The rest of the exposition is standard. Namely, we require the household to satisfy the following present-value budget constraint:

$$\int_s^{\infty} \left[c(t) - \tilde{w}(t)(1-x(t)) - T(t)\right] \exp\left[\int_s^t \left(\tilde{r}(\upsilon) - \delta\right) d\upsilon\right] dt \leq k(s) \qquad (2)$$

for a given level of capital $k(s)$. In expression (2), $\tilde{r}(t) \equiv (1-\tau_k(t))r(t)$ and $\tilde{w}(t) \equiv (1-\tau_n(t))w(t)$ are the real rate of return on physical capital and the real wage, respectively, both expressed net of their corresponding taxes $\tau_k(t)$ and $\tau_n(t)$, and $T(t)$ denotes transfer payments. The stock of capital $k(t)$ depreciates at the constant rate $\delta$.

Firms are perfectly competitive. The technology is represented by a production function $F[k(t), 1-x(t)]$ with constant returns to scale in the stock of capital and the time devoted to working activities $1-x(t)$. We also assume that $F(\cdot)$ is continuously differentiable, increasingly monotone, concave, and satisfies well-known Inada conditions. If we let $F_i(t)$ denote the marginal product of factor of production $i = (k, n)$, profit maximization implies that both factors of production are paid their marginal products, i.e.,

$$w(t) = F_n[k(t), 1-x(t)] \qquad (3a)$$

$$r(t) = F_k[k(t), 1-x(t)] \qquad (3b)$$

The single good in this economy may be devoted either to private consumption, investment or government purchases of goods and services $G(t)$, which we assume as exogenously given. The market clearing condition is thus described by:

$$c(t)+\dot{k}(t)+\delta k(t)+G(t)=F[k(t),1-x(t)] \tag{4}$$

**DEFINITION 1.** Given $k(s) = k_s$, a competitive equilibrium is defined as the set of infinite sequences for allocations $\{c(t),\ k(t),\ x(t)\}$, factor prices $\{r(t),\ w(t)\}$, and government policy $\{G(t),\ T(t),\ \tau_k(t),\ \tau_n(t)\}$ such that:

(i) Given factor prices and government policy, the allocation $\{c(t),\ k(t),\ x(t)\}$ maximizes (1) subject to (2); and

(ii) Equation (3) and the market clearing condition (4) are satisfied.

Note that expressions (2)-(4) together imply that the present value of the government budget constraint is satisfied.

It may be readily shown that among the conditions to be fulfilled is that the marginal rate of substitution between consumption at dates $s$ and $t > s$ must equal the relative prices of these two goods:

$$\frac{u_c(t)\exp[-(\rho\cdot(t-s)+\phi(t-s))]}{u_c(s)}=\exp\left[-\int_s^t(\widetilde{r}(\upsilon)-\delta)d\upsilon\right] \tag{5}$$

where $u_i(t)$ denotes the derivative with respect to the $i$th argument, $i = (c,\ x)$. In addition, the marginal rate of substitution between leisure and consumption must be equal to the after-tax real wage:

$$\frac{u_x(t)}{u_c(t)}=\widetilde{w}(t) \tag{6}$$

Given our separability assumption between consumption and leisure, equation (5) implies that consumption evolves through time according to:

$$-\frac{d\ln u_c(t)}{dt}=\widetilde{r}(t)-\rho-\delta-\phi'(t-s) \tag{7}$$

The above expression allows us to figure out the time-consistency problem of the household more clearly: the utility-maximizing path for $c(t)$ implied by (7) holds for any arbitrary date $s$. In particular, suppose that the household initially chooses her consumption plan at time $s$. However, if the household decides to revise her plan at a later date (say, $s' > s$) then the initial plan (chosen at time $s$) does not longer maximize utility viewed as of time $s'$. Hence the representative consumer is faced with a time-consistency problem (cf. Strotz (1956)). Notice that this particular problem does not arise under a standard model in which $\phi(t-s)=0$ for all $t \geq s$.

As pointed out by Barro (1999), the solution to the time-inconsistency problem of the household depends on whether she is able to fully commit her decisions on consumption and leisure at the present time *s*. We now proceed to discuss the consequences of such behavior in more detail.

### 2.1.1 Results under Commitment

In order to gain some insight from the model, we start with the simplest case in which the household is aware of her inconsistency problem but is able to commit her decisions on current and future allocations for every $t \geq s$. The no-commitment case will be described in the following section.

Since the household is able to commit, the sequence of allocations originally chosen today are not changed over time. So when the future arrives and the household decides to re-evaluate her original consumption-leisure plan, it simply abides by the original plan. In other words, there is no internal conflict between today's self and tomorrow's self. She is able to do so either irrevocably or by imposing a penalty for her future self should she misbehave. Examples of commitment include voluntary openings of Christmas Clubs accounts or the adoption of retirement plans that impose a penalty on early withdrawals.

In order to figure out the implications of commitment in our model, consider an arbitrary date *s*. The household is assumed to commit her plans for every $t \geq s$. From the properties of $\phi(\cdot)$, the steady-state rate of time preference $\rho+\phi'(t-s)$ would be just $\rho$. If we restrict the utility function $u(c(t),x(t))$ to belong to the isoelastic family, this model would coincide with the standard model with taxes at the steady state, as seen from equation (7). However, as pointed out by Barro (1999), it is hard to imagine that the ability to commit would suddenly arise at the arbitrary date *s*. Rather it is possible to argue that if perpetual commitments on consumption and leisure are feasible, then these commitments are likely to exist in the past, even in the infinite past. Hence, current and future allocations would have been chosen earlier and so *s* would be in fact equal to minus infinity. Therefore, $\phi'(t-s)$ would be zero for all $t \geq 0$ and the rate of time preference would equal $\rho$ for all $t \geq 0$.[2] Thus the standard result would apply throughout, not only at the steady state.

### 2.1.2 Results under No Commitment

Under more realistic grounds, full commitment is not always feasible. For this reason, in this section we depart from the previous assumption that the household is able to commit her decisions on consumption and leisure chosen at time *s*. Under a no-commitment scenario, it is possible for the future household to adopt a path different from the path originally chosen. This is not because consumer's preferences have changed in any unexpected way or because the information available is now different, but simply because the representative consumer is aware that she will be a different person in the future with a new discount function. In this case, changing the original plan in the future has implications for the whole sequence of allocations from that date on. Therefore, we need to figure out how the decision on *c*(*s*) at time *s* will affect the stock of assets and how this change in assets will alter the choices of consumption in the future.[3]

Given our separability assumption between consumption and leisure, for simplicity we adopt the following logarithmic utility function throughout this section:

$$u(c(t), x(t)) = \psi \ln c(t) + (1-\psi) \ln x(t) \tag{8}$$

where $0 < \psi \leq 1$ is a parameter that measures the share of consumption in total utility.

In a model with consumption only, Barro (1999) finds that the solution to the problem without commitment is such that the household must consume a fraction of her wealth at each date. As shown in the appendix, in our model this fraction (which turns out to be constant given our specification on preferences) is expressed by the value of $\lambda$ that satisfies:

[2] Strotz (1956) in fact shows that a necessary and sufficient condition for commitment is that the instantaneous rate of time preference must be constant over time.

[3] From the first-order conditions of the household, it is possible to define leisure explicitly in terms of consumption. Hence the no commitment problem may be reduced to the analysis of consumption decisions only.

$$\lambda = \frac{1}{\int_0^\infty \exp[-\rho v + \phi(v)]dv} \tag{9}$$

Equation (9) may alternatively be expressed as:

$$\lambda = \frac{\int_0^\infty [\rho + \phi'(v)]\exp[-(\rho v + \phi(v)]dv}{\int_0^\infty \exp[-(\rho v + \phi(v)]dv} \tag{10}$$

since the numerator in (10) is equal to unity. Notice that expression (9) reduces to $\lambda = \rho$ for the standard case in which $\phi(v)$ = 0 for all $v$. From (10), $\lambda$ may be interpreted as a time-invariant weighted average of the instantaneous rates of time preference $\rho + \phi'(v)$. From the properties of the $\phi(\cdot)$ function, it follows that $\rho \leq \lambda \leq \rho + \phi'(0)$. In other words, the fraction $\lambda$ of wealth chosen by the household has a value between the long-run rate of time preference $\rho$ and the short-run, instantaneous rate $\rho + \phi'(0)$. Alternatively, since $\lambda \geq \rho$ we may conclude that the time-inconsistent household cannot be more patient than her full committed self.

The solution to the no-commitment problem provided by (9) implies that consumption must now evolve through time according to:

$$\frac{\dot{c}(t)}{c(t)} = \tilde{r}(t) - \delta - \lambda \tag{11}$$

From previous discussion, it is readily noticed that if expression (11) holds, it yields a solution that is time-consistent from the point of view of the representative consumer. In other words, if $c(t)$ is chosen as the fraction $\lambda$ of wealth at all future dates, then the household will also choose present consumption in the same way in order to maximize her utility. In this sense, the marginal rate of substitution between consumption at dates $s$ and $t > s$ is now given by:

$$\frac{u_c(t)\exp[-\lambda \cdot (t-s)]}{u_c(s)} = \exp\left[-\int_s^t (\tilde{r}(\upsilon) - \delta)d\upsilon\right] \tag{12}$$

Finally, it may be easily verified that the marginal rate of substitution between consumption and leisure is still given by expression (6).

## 2.2 The Second-Best Problem

The purpose of this section is to characterize a solution to the optimal taxation problem in terms of Ramsey (1927). The method chosen is the primal approach whereby the planner announces a feasible allocation (subject to the relevant constraints) that is consistent with the optimizing behavior of private agents. This method may be roughly implemented through the following steps: First, the household and firms solve their maximization problem taking factor prices and government policy as given. Prices and taxes are then solved in terms of the corresponding allocation so that the intertemporal constraint of the household may be expressed in terms of quantities only. Finally, the planner solves for the Ramsey allocation by maximizing utility subject to the implementability constraint (to be defined later) and the feasibility constraint. Prices and taxes consistent with a competitive equilibrium may then be recovered from the previous step.

Before proceeding, there are two important comments to make. As mentioned in the introduction, we are interested in those cases under which the planner can provide help to the consumer through optimal fiscal policy. Therefore, we assume that the planner is endowed with full commitment. Second, it turns out that the solution to the second-best problem depends heavily on whether the household is able to fully commit her decisions, as it will become clear below.

### 2.2.1 Optimal Taxation with Full Commitment

As discussed earlier, under full commitment it is possible to recover the properties of the standard Ramsey-Cass-Koopmans model so that the household in fact discounts utility at the rate $\rho$. In such a situation, it may be easily shown that the solution to the second-best problem yields a zero capital income tax in the long run (cf. Chamley (1986)). In this particular case, the private discount rate is equal to the social discount rate so the capital stock in the long run is at the optimally social level: no subsidies to capital income are required.

### 2.2.2 Optimal Taxation with No Commitment

The results on optimal taxation discussed above are substantially modified once we assume the household is not able to commit her consumption-leisure decisions from today on. In order to define the second-best problem of the planner, first we need to get rid of prices and taxes from the household's program so that the planner's problem may be expressed in terms of quantities only. Plugging (6) and (12) into the household's budget constraint (2) yields:

$$\int_s^{\infty} \exp[-\lambda \cdot (t-s)][u_c(t)c(t) - u_x(t)(1-x(t)) - u_c(t)T(t)]dt = u_c(s)k(s) \qquad (13)$$

Equation (13) is usually known as the implementability constraint, expressed in terms of quantities only. Now that we have all the expressions required to solve for the second-best problem, as of time *s* it is possible to describe the Ramsey problem as the program:

$$\max_{c(t),x(t)} \int_s^{\infty} u(c(t),x(t)) \exp[-\rho \cdot (t-s)]dt \qquad (P)$$

subject to the implementability constraint (13) and the feasibility constraint (4). In expression (P), notice that the planner has the same utility function $u(c(t),x(t))$ as the representative consumer but discounts utility at the standard rate $\rho$.[4] To make this exercise interesting, we follow the standard assumption of taking $\tau_k(s)$ as given in order to avoid a capital levy at time *s*.[5] In addition, we simplify the problem by assuming that the planner has a commitment technology that allows her to bind herself to a particular sequence of allocations announced at time *s*.

For convenience, let us define the function

$$W(c(t),x(t),q(t,s)) = u(c(t),x(t)) + q(t,s)[u_c(t)c(t) - u_x(t)(1-x(t)) - u_c(t)T(t)] \qquad (14)$$

[4] An earlier example of divergence between private and social rates of discount is provided in Ramsey (1928). He argues that the discounting of utility for future generations is "ethically indefensible". Therefore, he sets $\rho = 0$ regardless of the value of the discount rate of private households. For alternative examples, see Fisher (1980) and Calvo and Obstfeld (1988).

[5] An additional restriction is that the tax rate on capital at time *s* must be bounded by above. Otherwise, investment at time *s* may be zero. See Jones et al. (1993) for a further discussion.

where $q(t,s) \equiv \gamma_1 \exp(\rho - \lambda)(t-s)$ is a non-negative variable and $\gamma_1 \geq 0$ is the time-invariant, Lagrange multiplier associated with the implementability constraint (13). Notice that $\gamma_1$ may be interpreted as the marginal excess burden of taxation: it is strictly positive if it is necessary for the planner to use any distorting taxes and zero otherwise.[6] Given the fact that $\rho \leq \lambda$, the expression $q(t, s)$ converges to zero as time $t$ goes to infinity. The Ramsey problem is thus reduced to solving the following Hamiltonian $H$ as of time $s$:

$$H = e^{-\rho(t-s)}\{W(c(t),x(t),q(t,s))$$

$$+\gamma_2(t)[F(k(t),1-x(t))-c(t)-\delta k(t)-G(t)]\}-\Lambda \qquad (15)$$

where $\Lambda \equiv \gamma_1 u_c(s)k(s)$ and $k(s) = k_s$ is given. Here, $\gamma_2(t) > 0$ denotes the marginal social value of goods. It is important to remark that, since the constraint (13) faced by the household is already taken into account in the Ramsey problem, the allocation announced by the planner (the Ramsey equilibrium) will be consistent with the allocation that would be chosen by utility-maximizing agents. This idea is described more formally in the following definition:

**DEFINITION 2.** Let $\Pi$ denote the set of tax policies for which a competitive equilibrium exists. A Ramsey equilibrium is thus an infinite sequence of allocation rules $\{c(t), x(t), k(t)\}$, tax policy $\tau(t) = \{\tau_k(t), \tau_n(t) | (\tau_k(t), \tau_n(t)) \in \Pi\}$ and prices $\{r(t), w(t)\}$ such that:

(i) The policy $\tau$ solves program (P) subject to (13); and
(ii) For every policy $\tau'$, the allocation $\{c(\tau'), x(\tau'), k(\tau')\}$ and the price system $\{r(\tau'), w(\tau')\}$, together with the policy $\tau'$, constitute a competitive equilibrium.

It may be readily verified that the Ramsey equilibrium must satisfy, among other restrictions, the following first-order conditions:

$$W_c(c(t),x(t),q(t,s)) = \gamma_2(t), \qquad t > s \qquad (16a)$$

$$W_x(c(t),x(t),q(t,s)) = \gamma_2(t)F_n(t), \qquad t \geq s \qquad (16b)$$

$$W_c(c(s),x(s)) = \gamma_2(s) + \gamma_1 u_{cc}(s)k(s), \qquad t = s \qquad (16c)$$

[6] To see this more clearly, we may think of a first-best model in which the planner maximizes the utility of the household subject only to the feasibility constraint. This model is thus equivalent to setting $\gamma_1 = 0$ in expression (15) below.

$$\dot{\gamma}_2(t) = \gamma_2(t)[\rho + \delta - F_k(t)], \qquad t \geq s \qquad (16d)$$

plus the standard transversality condition for the stock of capital. Here, $W_i(\cdot)$ denotes the derivative with respect to the *i*th argument, $i = (c, x, q)$.

The computation of the Ramsey equilibrium proceeds as follows: Suppose for a moment that the value for $\gamma_1$ is known. Then the feasibility constraint plus the system (16) pin down the whole sequence for $c(t)$, $x(t)$, $k(t)$ and $\gamma_2(t)$ for $t \geq s$. Labor and capital income taxes may be then recovered from (6) and (12), respectively, whereas factor prices are given by (3). Finally, we need to check that the implementability constraint (13) is satisfied for given $\gamma_1$ and initial value $k(s) = k_s$.

We restrict now our analysis to the steady state. Contrary to what we find in the full commitment case, it is possible to show that the optimal tax on capital income under no commitment is non-positive and depends heavily on the share $\lambda$ of wealth. This result is provided in the following proposition:

**PROPOSITION.** Suppose the Ramsey equilibrium converges to a steady state. If the time-inconsistent household is not able to commit her decisions and the planner is endowed with full commitment, the following must hold at the steady state:

(i) The optimal tax on capital income is given by the expression[7]

$$\tau_k^* = \frac{\rho - \lambda}{\rho + \delta} \leq 0 \qquad (17)$$

(ii) The optimal tax on labor income is zero.

*Proof.* To prove part (i), simple manipulations of expressions (11), (16a) and (16d) evaluated at the steady state yield the desired result. To prove part (ii), it is important to recall that $q^* = 0$. Steady state versions of expressions (16a), (16b) and (6) complete the proof. ■

The intuition of expression (17) is straightforward: Since the time-inconsistent consumer with no commitment has a higher rate of discount in general, the steady state level of capital is consequently lower. The planner must thus provide a subsidy to capital in order to restore it to its socially

[7] Steady-state values are denoted with an asterisk.

desired level. Notice that for the standard case in which $\phi(v)=0$ for all $v$, we know that $\lambda = \rho$ and so capital income is not taxed at the steady state (cf. Chamley (1986)). Finally, it is important to remark that $\tau_k^*$ does not depend on the level of the marginal excess burden of taxation $\gamma_1$ (cf. Judd (1997), Guo and Lansing (1999)).

The next issued is to figure out what the model implies for optimal taxation off the steady state. Given the assumption that the government has an exogenous sequence of government consumption, the results in the above proposition imply that the dynamics of taxes along the entire transition deserve further study. In particular, we need to check whether the subsidy to capital income holds for some or most of the transition path and if the tax on labor income asymptotically decreases over time or follow a different pattern. For that purpose, we rely on numerical methods as discussed in section 3.

## 2.3 A Note on Time-Inconsistent Plans

So far the results obtained in the previous section have made the assumption that the planner has a commitment technology that allows her to honor the plan originally announced at time *s*. In this section we discuss briefly the relevance of such an assumption.

It is well known that if the government has no access to a commitment technology, the solution to the Ramsey problem is time inconsistent from the point of view of the planner: since the capital stock is fixed at the initial period *s*, the planner (acting in the best interest of the household) has an incentive to deviate from her original plan to take advantage of a capital levy. In this regard, Xie (1997) provides a counterexample under the neoclassical framework in which the solution to the second-best problem is time consistent. For particular logarithmic preferences on consumption and leisure, the author shows that the current allocation depends on the current (after tax) rate of return on capital only. Therefore, future after-tax rates of return do not have an impact on current decisions on consumption and leisure, and so the policy originally announced by the planner is time-consistent.

The example provided by Xie (1997) is of particular interest since it points out a special case previously ignored in the literature. Nevertheless, we need to point out that his result depends heavily on the specification of preferences. For a Cobb-Douglas utility function like the one used in this paper, it is possible to show that current consumption depends on the present value of net wages (see expression A2 in the appendix), which is discounted by the sequence for the after-tax rate of return on capital. In other words, future taxes on capital income may have an effect on current consumption in our model and so the time-inconsistency problem is still present, despite the fact that preferences on consumption and leisure are logarithmic.

## 3. A Numeric Characterization of Optimal Paths

### 3.1 Preliminaries

The purpose of this entire section is to characterize the solution path for optimal capital and labor income taxes when the household is not able to commit her decisions. The problem is thus basically to solve the dynamic system described in (16) (plus the feasibility constraint and well known transversality conditions) for a given value of $\gamma_1$. As discussed earlier, the solution to such a system yields the whole sequence for $c(t)$, $x(t)$, $k(t)$ and $\gamma_2(t)$ for $t \geq s$. The entire sequence for optimal taxes may then be recovered from expressions (6) and (12).

We rely on numeric methods to solve for the Ramsey equilibrium. When solving numerically the system of differential equations, we are faced in principle with at least two choices. One of them is the familiar shooting method: given $k(s)$, guess some initial values for $c(s)$ and $x(s)$ so that the system converges sufficiently close to their steady-state values. As is well known, using shooting methods for solving an infinite-horizon model is difficult and time-consuming, especially in those cases in which there are at least two control variables. A better alternative in such a case is to use a reverse shooting method (cf. Judd (1998), Mulligan and Sala-i-Martin (1991))[8]. The basic idea is to transform a boundary value problem (like the one implied in the shooting method) to an initial value problem in which the "initial" conditions of the system are given by their steady-state conditions. Under this procedure, the stable manifold is easier to estimate numerically and less time-consuming. This is basically the method we use throughout this paper.

The algorithm for the numeric computation of system (16) is as follows. We make an initial guess for the value of the marginal excess burden of taxation $\gamma_1$ and solve the problem numerically. Once the entire sequence of allocations $\{c(t),\ x(t),\ k(t),\ \gamma_2(t)\}$ is obtained, we check whether the

[8] Mulligan and Sala-i-Martin (1991) alternatively refer to this method as the time-elimination method because time plays no role when solving numerically the dynamic system of differential equations.

implementability constraint (13) is satisfied. If it is not (as it might be expected), we continue adjusting the value of $\gamma_1$ until such condition is met. The resulting sequence of allocations is then used to compute the sequence of optimal taxes. Finally, we check if the government budget constraint is balanced (in a present-value sense). Since the tax on capital income is assumed to be exogenous at time $t = s$ in order to avoid a capital levy, we adjust the initial tax on labor income $\tau_n(s)$ so that the government budget constraint holds.

Before estimating the model numerically, we first need to define a functional form for $\phi(v)$. As noted earlier, the instantaneous rate of time preference $\rho + \phi'(v)$ reflects short-term impatience if it is high when $v$ is small, and declines gradually to $\rho$ as $v$ becomes large. Following Barro (1999), a functional form that captures this idea is given by:

$$\phi'(v) = be^{-\zeta v} \tag{18}$$

where $b = \phi'(0) \geq 0$ and $\zeta > 0$ denotes the constant rate at which $\phi'(v)$ declines from $\phi'(0)$ to zero. Integration of (18) along with the boundary condition $\phi(0) = 0$ allows us to get an expression for $\phi(v)$:

$$\phi(v) = (b/\zeta)(1 - e^{-\zeta v}) \tag{19}$$

This expression may be substituted into (9) in order to get a numeric value for $\lambda$. From (19), it may be shown that either a higher $b$ or a lower $\zeta$ yield a higher value for $\lambda$ (i.e., more impatience).

Finally, we also need to define a production function. For simplicity, we abide by the standard assumption that the function $F(\cdot)$ is of the Cobb-Douglas form $F[k(t), 1 - x(t)] = Ak(t)^{\alpha}(1 - x(t))^{1-\alpha}$ with $A > 0$ and $0 < \alpha \leq 1$.

The calibration of the model is made so that parameter values are consistent with the observations for the U.S. economy. We choose $A = 1$, $\alpha = 0.33$ and $\delta = 0.10$. The last two values are roughly standard in the literature. Given such parameter values, $\rho = 0.03$ is fixed to yield a capital-output ratio of about 2.5 whereas $\psi = 0.28$ is chosen so that the household allocates about one-third of her endowed time to working activities.

The parameters that define the time-inconsistency function are as follows. The rate at which $\phi'(v)$ declines from $\phi'(0)$ to zero is given by $\zeta = 0.75$ whereas the value for $b = \phi'(0)$ is given by $b = 0.23$. These parameters (along with the discount rate $\rho$) imply a value for $\lambda$ of about 0.04 according to expression (9), which we believe it is reasonable.[9] Overall, the values for $\zeta$, $\rho$ and $b$ are such that $\phi'(v)$ gets close to zero a few years in the future, an observation made by Laibson (1997) and considered also by Barro (1999).[10]

It remains to describe the parameters of fiscal policy. Following Jones et al. (1993), we choose government expenditures $G(t)$ and transfer payments $T(t)$ to be a fixed proportion of total output at each period. From time-series observations, we set the share of $G(t)$ with respect to total output at 0.21. In principle, since the model may involve large lump sum taxes in order to finance the sequence of subsidies to capital income, we start the numeric simulations assuming $T(t)$ = 0 for all $t$. Later on we also allow for alternative values of $T(t)$. Finally, the (exogenous) initial tax on capital income is fixed at its historical average value. This value turns out to be $\tau_k(s)$ = 0.43 according to the estimations provided in Mendoza et al. (1994). As noted earlier, the initial tax on labor income is conveniently fixed to balance the present value of the government's budget constraint.

## 3.2 Results

As mentioned above, it is of interest to examine numerically the entire sequence of capital and labor income taxes since the government has a positive spending to fulfill. In addition, we need to figure out the effects of the household's time inconsistency on such paths. Figure 1 gives a numeric answer to these two issues. The first graph shows the sequence of the optimal tax on capital income for alternative values of $b$ and assuming that $T(t)$ = 0 for all $t$.[11] For convenience, only the relevant period of analysis is shown. A value of $b = 0.01$ yields a value for $\lambda$ near $\rho = 0.03$, so that the corresponding sequence for $\tau_k(t)$ nearly reflects the optimal path under standard preferences (cf. Chamley (1986)). In such a case, the figure illustrates how, after taking the exogenous initial tax $\tau_k(s)$ as given, the economy drastically drops to its zero long-run value in just a few periods. Alternative values for $b$ and their corresponding paths for $\tau_k(t)$ are also reported. In particular, $b$ = 0.23 and $b = 0.50$ correspond to $\lambda$ = 0.040 and $\lambda = 0.057$, respectively, so that the latter case reflects more impatience. Again, we find that after an initial period of exogenous taxation the tax rate immediately drops and then

[9] The steady state version of equation (11) implies that $\lambda$ must be equal to the (after tax) steady state rate of return (net of depreciation). As a comparison, the steady state rate of return (before taxes) is usually estimated to be about six percent.

[10] As a comparison, Barro (1999) chooses $b = 0.50$, $\zeta = 0.50$ and $\rho = 0.02$ at the benchmark. These numbers yield $\lambda = 0.052$ which, according to our previous discussion, might seem a relatively large value in a model with taxes like ours.

[11] The number of periods for the numeric computations is fixed at $t = 100$.

gradually moves to its long run value.[12] Overall, it is readily available how more impatient economies face substantially larger subsidies to capital income.

[12] Judd (1999) shows that, except for an initial period, the *average* tax on capital income is zero for standard infinite-horizon models.

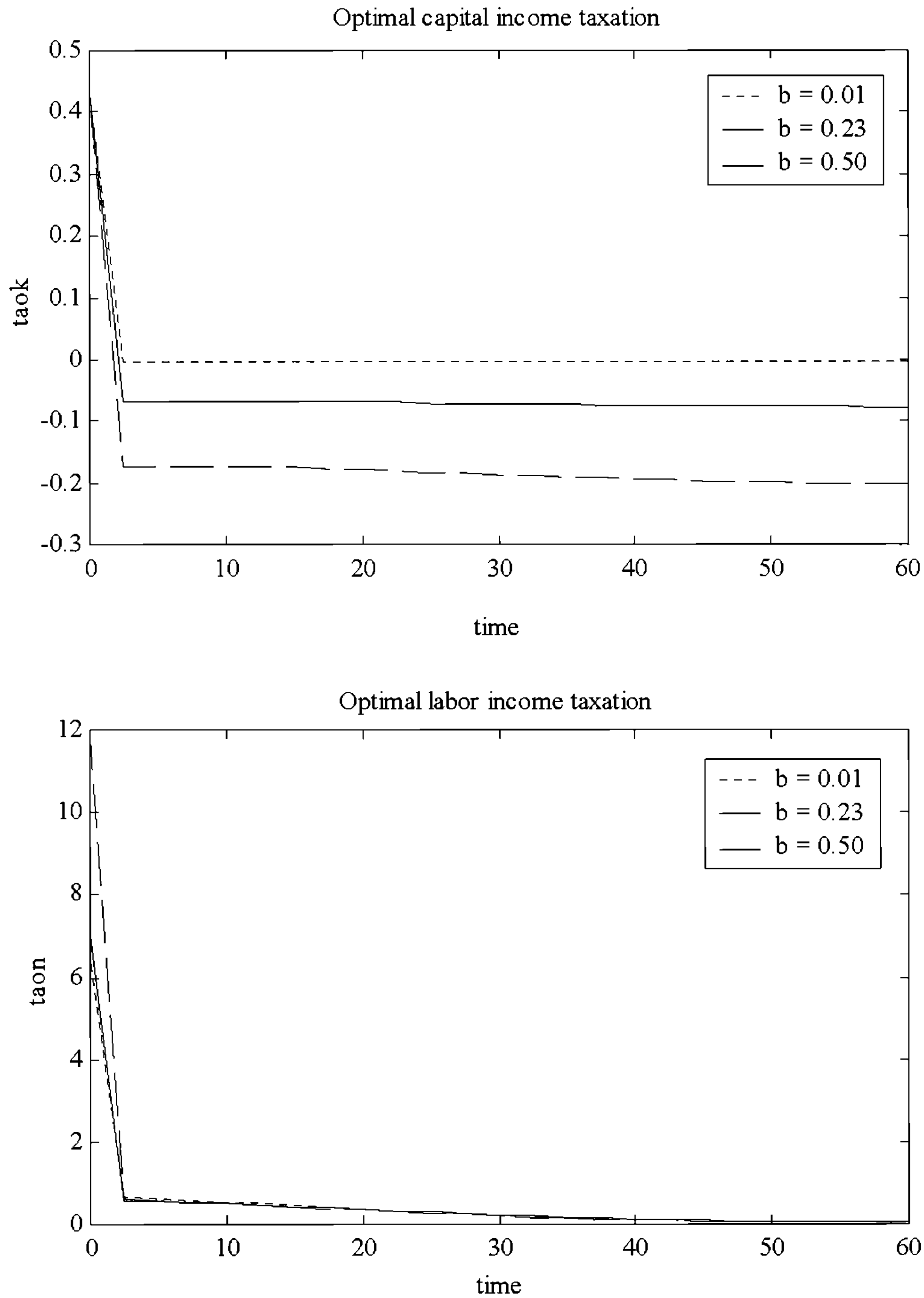


**Figure 1. Effects of Impatience on Optimal Taxation (Transfer Share = 0)**

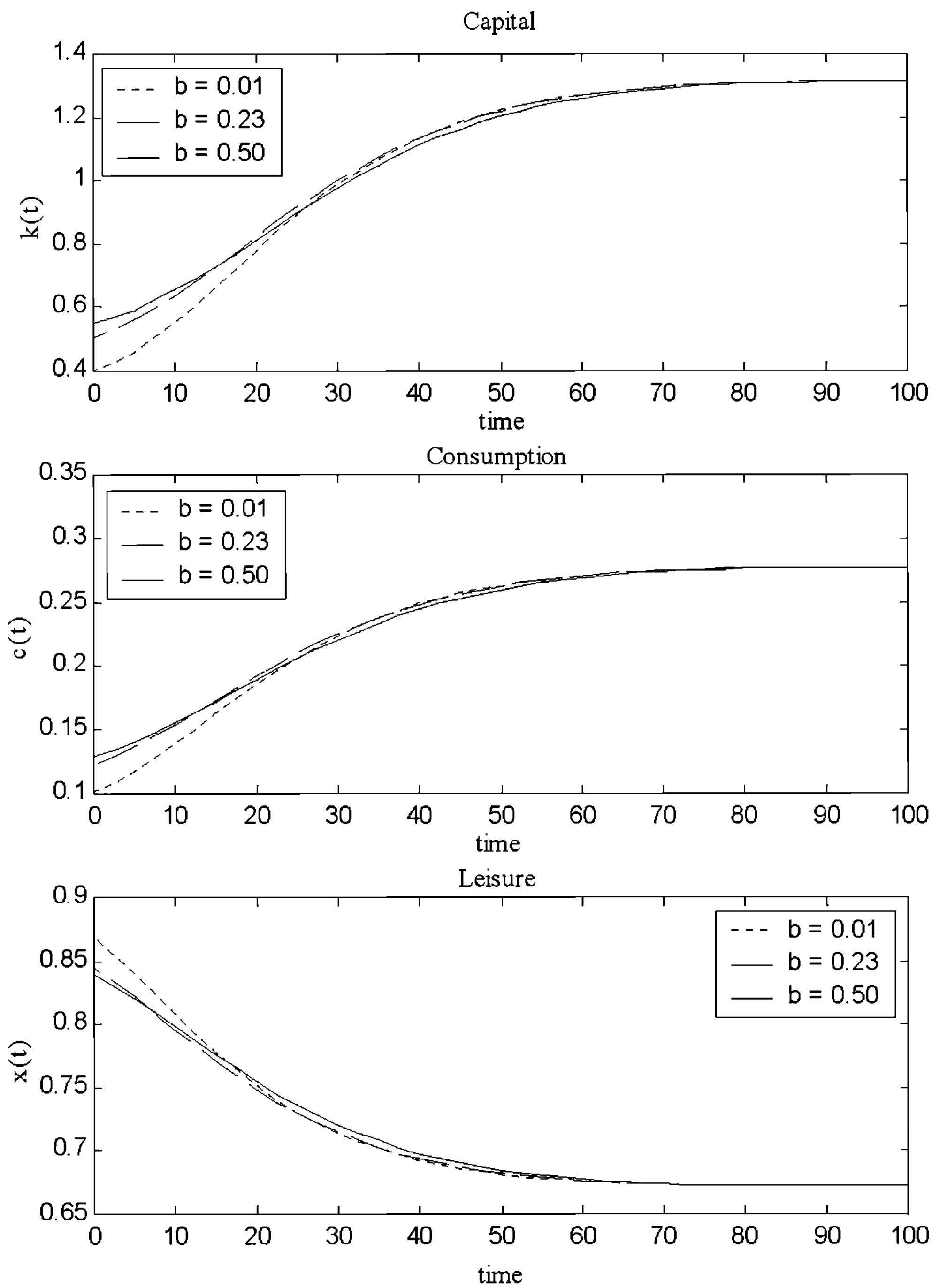


**Figure 2. Effects of Impatience on Real Variables (Transfer Share = 0)**

The second part of figure 1 shows the path for optimal labor taxes over the relevant transition period, also under alternative values of *b*. As noticed earlier, more impatience implies larger subsidies to capital along most of the transition path. Accordingly, higher taxes on labor income are needed to finance a constant stream of government spending. As the bottom part of figure 1 shows, the planner optimally chooses to tax labor heavily at the initial period, even though labor is elastically supplied. After a few years, the path for $\tau_n(t)$ is nearly the same regardless of the degree of impatience shown by the consumer.

Figure 2 denotes the optimal paths for capital, consumption and leisure under alternative values of *b*. As it may be noticed for all cases, the initial value of each variable depends on the value for *b*. The reason is simply because different degrees of impatience imply different paths for capital and labor income taxes and thus different values on the marginal excess burden of taxation $\gamma_1$. As it turns out, the relation between *b* and $\gamma_1$ is not necessarily monotonic since there are two effects moving in opposite directions: a higher *b* implies more subsidies to capital that help decrease the burden of taxation; on the other hand, higher impatience brings about a higher initial tax on labor income, which increases the value of $\gamma_1$.[13]

The next series of exercises assume alternative values for the ratio of lump-sum transfers to output, keeping *b* fixed at its benchmark value. In particular, we consider either a 5 percent lump sum tax out of income (which implies a negative transfer share value) or a 5 percent lump sum transfer (also out of income). The first part of figure 3 shows how the sequence of $\tau_k(t)$ is unaffected by alternative transfer share values. As it may be noticed, the burden of taxation relies again on labor income taxation, not only at the beginning of time but along the transition path as well, as depicted in the bottom part of figure 3.

Figure 4 illustrates the effects of different transfer shares out of output on capital, consumption and leisure. From the previous analysis, we know that a positive transfer implies larger taxes on labor income for every t in order to satisfy the government budget constraint, while keeping the tax on capital income practically unchanged. Given such a lump sum transfer, we might expect lower values for the stock of capital, consumption and hours worked. Figure 4 shows how the effects of alternative transfer shares are felt

[13] In particular, we find that $\gamma_1 = \{0.193, 0.019, 0.023\}$ corresponds to $b = \{0.01, 0.23, 0.50\}$ under the benchmark economy, respectively.

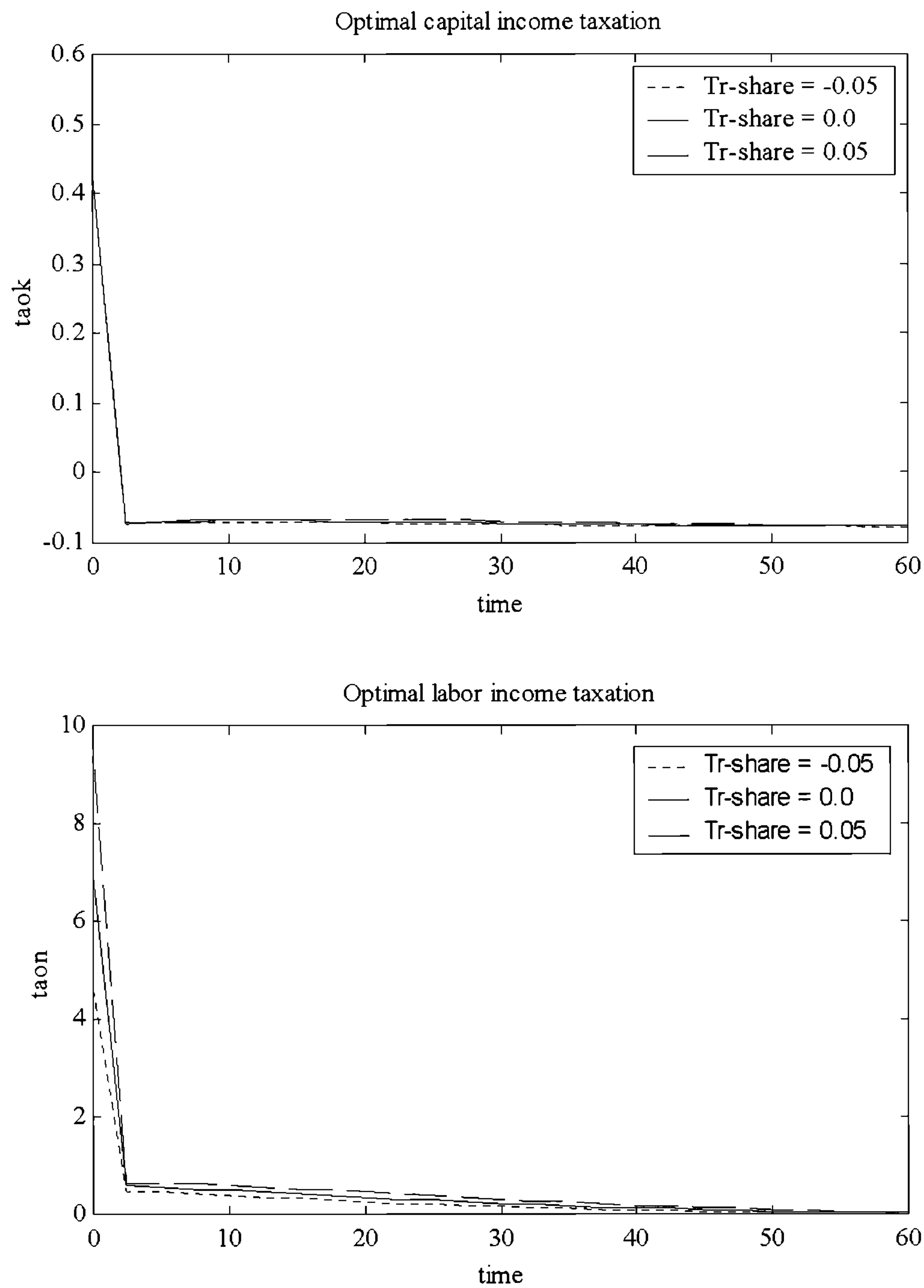


**Figure 3. Optimal Taxation under Alternative Transfer Shares (*b* = 0.23)**

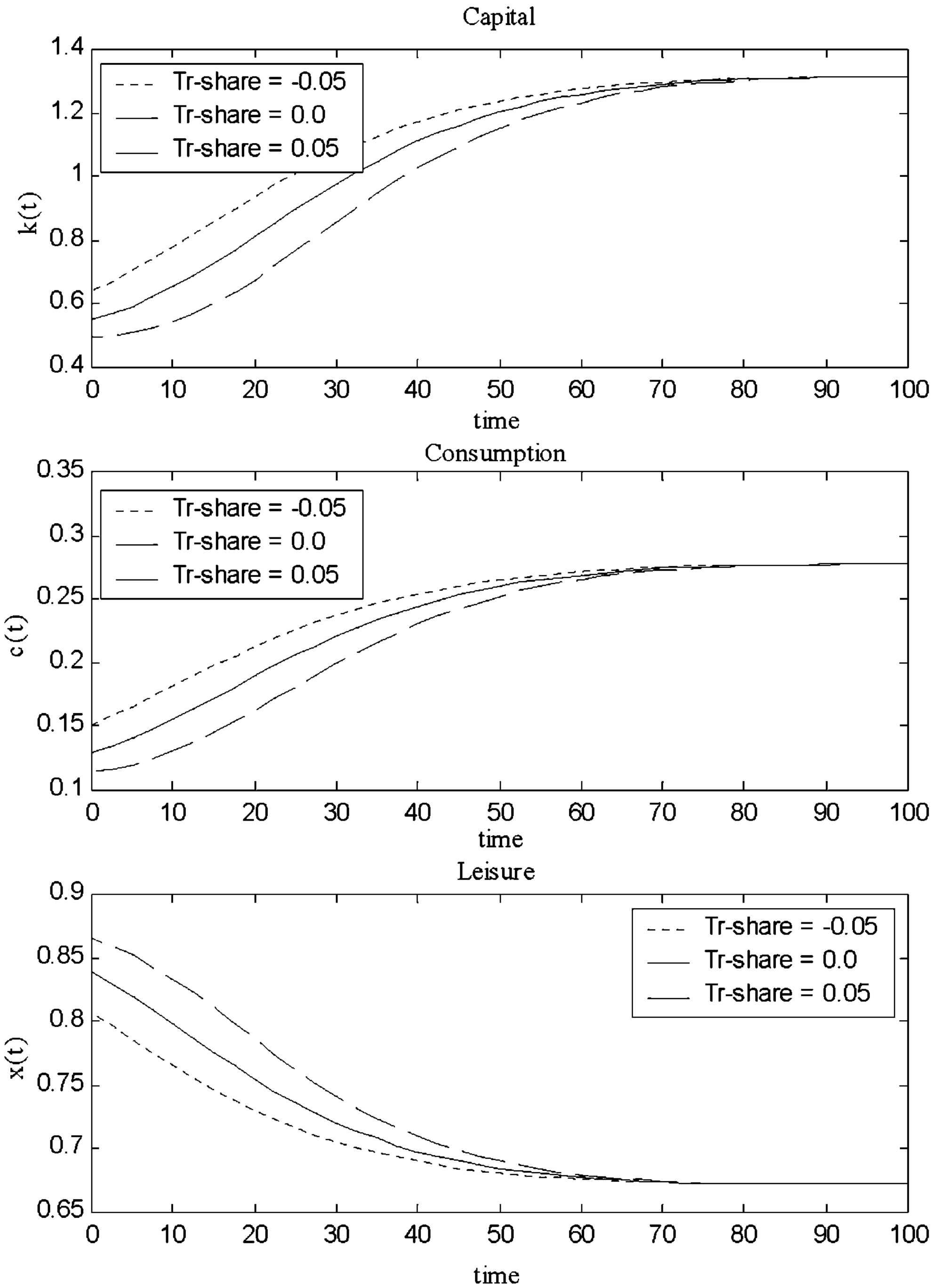


**Figure 4. Effect of Alternative Transfer Shares on Real Variables (*b* = 0.23)**

throughout the entire transition path and how these effects are relatively important. For example, a decrease in the transfer share from 5 percent to minus 5 percent out of income (and thus a corresponding decrease in distorting taxation) brings about an increase of 29 percent in the initial level of capital.

## 4. Final Remarks

The purpose of this paper has been to investigate the sequence of optimal taxes when a representative consumer has time-inconsistent preferences. When the household is able to fully commit her choices at each time, the model simply reduces to the standard Ramsey-Cass-Koopmans model and thus the standard results of Chamley (1986) apply throughout. Once the household is unable to commit her decisions, she solves her inconsistency problem by consuming a constant fraction of her wealth at every point in time. It turns out that this fraction is simply a weighted average of the instantaneous rates of time preference, so the household in fact exhibits more impatience than under the standard model. This impatience is reflected in a lower steady state value for capital that requires a subsidy to capital income at the steady state in order to eliminate such a distortion. As the numerical analysis illustrates, the subsidy extends not only at the steady state but also to most of the transition path. On the other hand, this result implies that the burden of taxation is imposed on labor income. Nonetheless, labor income taxes are only increased heavily during the first periods of transition.

There are several factors (both internal and external to the household) that influence a household's ability to commit her consumption choices. Laibson (1997) and Barro (1999) provide several examples of public policies as well as institutional and market mechanisms related to this issue. The existence of legal constraints on credit markets that inhibit excessive consumer spending through borrowing, or the penalties imposed on retirement benefits for withdrawals made before reaching the full retirement age, are only a few real-world examples of mechanisms that may be interpreted as commitment devices. Of course, the degree of commitment is also related to the self-discipline of the household, a situation in which cultural factors might play an important role.

We should expect that such types of mechanisms, policies and cultural schemes vary greatly across societies. As we have seen, economies that feature a better capacity to commit future consumption should exhibit a lower effective rate of time preference and thus higher levels of capital in the long run. If the planner is fully able to commit her announcements, these economies should display higher taxes on capital income relative to those with weaker institutions responsible for enforcing commitment. Unfortunately, these differences among societies are in principle hard to quantify since the instantaneous rate of time preference cannot be observed directly from the data. Nonetheless, the exploration of such conjecture may well deserve further study in the future.

## Appendix

The objective in this section is to derive an expression for the share of wealth that the household must consume when she is not able to commit her future choices. For that purpose, we extend the analysis in section III of Barro (1999) for a model with taxes and the logarithmic utility function provided in (8).

In order to solve for such an expression, we imagine the household choosing $c(t)$ at time $s$ as the constant flow $c(s)$ over the short discrete interval $[s, s+\varepsilon]$. The value for $\varepsilon$ will eventually approach zero and thereby generate results for continuous time. Accordingly, as of time $s$ the utility in expression (1) may alternatively be written as

$$
\begin{aligned}
U(s) = & \int_{s}^{s+\varepsilon} [\psi \ln c(t) + (1-\psi)\ln x(t)] \exp[-(\rho\cdot(t-s)+\phi(t-s))]dt \\
& + \int_{s+\varepsilon}^{\infty} [\psi \ln c(t) + (1-\psi)\ln x(t)] \exp[-(\rho\cdot(t-s)+\phi(t-s))]dt \\
& \approx \varepsilon[\psi \ln c(s) + (1-\psi)\ln x(s)] + \int_{s+\varepsilon}^{\infty} [\psi \ln c(t) + (1-\psi)\ln x(t)] e^{[-(\rho(t-s)+\phi(t-s))]}dt
\end{aligned}
$$

where the approximation arises from taking $e^{[-(\rho(t-s)+\phi(t-s)]}$ as equal to unity over the interval $[s, s+\varepsilon]$. The above result is thus given in terms of consumption and leisure. However, it is more convenient to work out an expression in terms of consumption only. To this purpose, we may use condition (6) in the text and the utility function (8) in order to express leisure as a function of consumption. Replacing this result into the above approximation yields:

$$U(s) \approx \varepsilon \ln c(s) + \int_{s+\varepsilon}^{\infty} \ln c(t) e^{-[\rho(t-s)+\phi(t-s)]} dt + \Gamma \qquad \text{(A1)}$$

where the expression

$$\Gamma \equiv \varepsilon(1-\psi)\ln\left[\frac{1-\psi}{\psi\widetilde{w}(s)}\right]+(1-\psi)\int_{s+\varepsilon}^{\infty}\ln\left[\frac{1-\psi}{\psi\widetilde{w}(t)}\right]e^{-[\rho\cdot(t-s)+\phi(t-s)]}dt$$

is independent of the $c(t)$ path.

When the household picks $c(s)$ at time $s$, the consumption path $c(t)$ for $t \geq s+\varepsilon$ is affected through the stock of assets $k(t+\varepsilon)$ available at time $s+\varepsilon$. In order to determine the welfare-maximizing choice of $c(s)$, the representative consumer needs to know both the relationship between $c(s)$ and $k(s+\varepsilon)$ and the relationship between $k(s+\varepsilon)$ and the choices of $c(t)$ for $t \geq s+\varepsilon$.

The solution to the first part of this problem may be directly determined by taking a linear approximation to the household's budget constraint over the interval $(s, s+\varepsilon)$. This procedure yields the expression $d[k(s+\varepsilon)]/d[c(s)] \approx -\varepsilon$ (see Barro (1999) for details). To obtain a result for the second part, we may conjecture that the income and substitution effects associated with future interest rates would cancel under logarithmic utility, even though the rate of time preference is variable and the household cannot commit her decisions. This implies that there must exist a constant fraction $\lambda$ of wealth so that

$$c(t) = \lambda[k(t)+\overline{w}(s)] \qquad \text{(A2)}$$

where $\overline{w}(s)$ denotes the present value of wage income (net of taxes) as of time $s$. It is important to remark that the conjectured fraction $\lambda$ need not equal the constant fraction $\rho$ of wealth that would be obtained under a standard model. Given the conjecture, consumption must grow over time at the rate $\widetilde{r}(t)-\delta-\lambda$ for $t \geq s+\varepsilon$. Therefore, for any $t \geq s+\varepsilon$ it must be the case that

$$\ln c(t) = \ln c(s+\varepsilon) + \Psi(t, s+\varepsilon) \qquad \text{(A3)}$$

where $\Psi(t, s+\varepsilon) \equiv \int_{s+\varepsilon}^{t}[\widetilde{r}(\upsilon)-\delta-\lambda]d\upsilon$ is also a term independent of the $c(t)$ path. Plugging equation (A3) into (A1) leads to:

$$U(s) \approx \varepsilon \ln c(s) + \ln c(s+\varepsilon)\int_{s+\varepsilon}^{\infty} e^{-[\rho\cdot(t-s)+\phi(t-s)]}dt$$

$$+\int_{s+\varepsilon}^{\infty}\Psi(t,s+\varepsilon)e^{-[\rho(t-s)+\phi(t-s)]}dt+\Gamma \qquad \text{(A4)}$$

Finally, we define the integral

$$\Omega\equiv\int_{0}^{\infty}e^{-[\rho v+\phi(v)]}dv \qquad \text{(A5)}$$

which corresponds to the first integral in (A4) as $\varepsilon$ goes to zero.

Now we are able to estimate the marginal effect of $c(s)$ on the instantaneous utility $U(s)$. Such effect is given by:

$$\frac{d[U(s)]}{d[c(s)]}\approx\frac{\varepsilon}{c(s)}+\frac{\Omega}{c(s+\varepsilon)}\cdot\frac{d[c(s+\varepsilon)]}{d[k(s+\varepsilon)]}\cdot\frac{d[k(s+\varepsilon)]}{d[c(s)]}$$

From the discussion above, setting the previous derivative to zero implies that:

$$c(s)=c(s+\varepsilon)/\Omega\lambda$$

If the conjecture on $\lambda$ is correct, then $c(s+\varepsilon)$ must approach $c(s)$ as $\varepsilon$ goes to zero. Hence, it must be the case that:

$$\lambda=\frac{1}{\int_{0}^{\infty}e^{-[\rho v+\phi(v)]}dv} \qquad \text{(A6)}$$

This is just equation (9) in the text. It is also equal to the expression obtained in Barro (1999) for a model with no taxes and consumption in the utility function only.

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