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Moral Hazard and Firm Size: A Dynamic Analysis

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Abstract

In this paper we study how the severity of the moral hazard problem a firm faces is related to its size. We set up a dynamic agency model with capital accumulation, and show the existence of a solution to this model. To analyze the structure of the optimal contract and capital accumulation pattern under asymmetric information, we solve a numerical example. Our results show that the principal uses both present and future compensation to provide incentives to the agent at all values of the state variables. However, as capital increases, the agent's future compensation is more used by the principal for incentive provision. The capital's steady state value diminishes when the agent's lifetime expected utility increases. In the trade-off that the principal faces between compensating the agent and accumulating capital, the former prevails. Thus, asymmetric information lowers the firm's growth. In the long run, a commitment problem emerges for the principal for his expected utility consistently decreases while that of the agent increases over time.

Keywords: Asymmetric Information, Capital Accumulation, Dynamic Contract, Managerial Compensation.

Journal of Economic Literature Classification Numbers: C63, D82, G30, E22

Resumen

En este artículo se estudia cómo la severidad del problema de riesgo moral que una firma enfrenta, se relaciona con el tamaño de la misma. Se propone un modelo de agente-principal dinámico que incluye un proceso de acumulación de capital y se demuestra la existencia de un contrato óptimo en el marco de este modelo. Para analizar la estructura del contrato óptimo, se resuelve un ejemplo numérico. Los resultados del mismo indican que el principal utiliza tanto la compensación presente como la futura como instrumentos de provisión de incentivos para todos los valores de las variables de estado del modelo. Sin embargo, cuando el nivel de capital aumenta, el principal utiliza más la compensación futura para proveer incentivos al agente. El valor estacionario del capital disminuye cuando la utilidad esperada descontada del agente aumenta. Esto evidencia que el principal enfrenta un problema de decisión entre provisión de incentivos y asignación de recursos para acumular capital y que, en este problema de decisión, pareciera que el primer objetivo prevalece. Es decir, la existencia de información asimétrica dentro de la firma hace que su crecimiento, en términos de nivel de capital acumulado, se haga más lento. En el largo plazo, surge un problema de estabilidad de la relación por parte del principal pues su nivel de utilidad esperada tiene una tendencia a disminuir, mientras que lo contrario sucede con la utilidad esperada del agente.

Palabras Claves: Información asimétrica, acumulación de capital, contrato dinámico, compensación gerencial.

Clasificación de números consultados : C63, D82, G30, E22

1 Introduction

In this paper we explore how the severity of the moral hazard problem a firm faces is related to its size. For instance, Murphy (1999) says that "increased agency problems are a cost of company size that must be weighed against the benefits of expanded scale and scope". Moral hazard has been recognized as a consequence of the separation between ownership and control in modern organizations, as pointed out by Berle and Means (1932), which tends to be more evident in larger firms. However, larger firms might be better able to deal with agency problems and to achieve better risk sharing because those firms, in general, have more resources to implement long term incentive tools through promises of future expected utility to the agent or manager.

In order to provide an environment to analyze these ideas, we embed a dynamic agency model in a neoclassical growth model, which allows us to examine issues of dynamic incentive provision and firm growth simultaneously. In the first period, the principal starts with a given level of capital, and promises the agent a given level of expected discounted utility. At this point, the agent chooses a level of effort, which constitutes his private information. At the end of the period, a productivity shock is realized, conditional on the agent's effort decision, and it is observed by the principal and the agent. Then, the principal pays a salary (current compensation), and promises a level of discounted expected utility (future compensation) to the agent from the second period onwards. Finally, the principal makes the decision of how much to invest in new capital for the firm and how much to consume. In the second period, the initial capital level and the agent's promised discounted expected utility are determined by the principal's decisions in the first period, and so on.

In our model, the principal uses both future and current compensation as incentive tools for all values of the state variables. As capital increases, the principal relies more on the agent's future compensation for incentive provision, while current compensation increases and becomes more stable with respect to the different realizations of the productivity shock. That is, the principal increases the use of long term incentive tools as the firm becomes larger. This makes sense because long term incentive tools tend to be more costly for the principal to implement. On the other hand, as the agent's expected utility from his discounted lifetime income increases, the steady state values of capital under different productivity shocks tend to become lower. This result indicates that as the agent's reservation utility increases, it will become more expensive for the principal to motivate the agent even at the expense of capital accumulation. The principal, thus, faces a trade-off between providing optimal incentives to the agent and accumulating capital. These results lead to the conclusion that asymmetric information causes a lower rate of firm's growth.

In the long run, we observe that the agent's expected utility tends to increase while that of the principal has a decreasing trend which creates a commitment problem from the principal's side. This finding is consistent with those obtained in related models of dynamic agency which generate transient dynamics, as noted by Spear and Wang (2001). Also, as the firm grows and the variance of its performance increases, the sensitivity of the agent's compensation with respect to the performance of the firm decreases, which is a sign that better risk sharing is achieved in larger firms because of the agent's risk averse nature.

A branch of the literature that is related to this work includes papers that study asymmetric information issues in aggregate economies. Marcet and Marimon (1992) consider contractual elements while studying the dynamics of capital accumulation, and find that asymmetric information does not significantly affect growth. Khan and Ravikumar (2001) examine the effect of private information on growth by analyzing a model with capital accumulation where productivity shocks are privately known by the agents, who engage in long-term relationships with insurance providers. They conclude that, under private information, growth tends to be lower. In a model of a similar fashion, Khan and Ravikumar (2002) incorporate linear technologies that are susceptible to productivity shocks, which are private information. This economy shows a monotonicity property that allows them to reduce the state space, which facilitates both the analytical and the computational work. Acemoglu and Zilibotti (1999) study the capability of economies in different stages of development to create information and to deal with agency costs. Their conclusion is that as an economy accumulates more capital, more information will be generated and better risk sharing is achieved. We share with these authors the interest in analyzing capital accumulation in the presence of asymmetric information, however we focus our analysis at the firm level.

The literature on dynamic agency models is also related to this work. Wang (1997) numerically solved a dynamic agency model and, using model-generated data, obtained measures of responsiveness of CEO pay with respect to the wealth of shareholders to give a response to the puzzle presented by Jensen and Murphy (1990)[1]. One of Wang's results is that the relationship between the shareholders' wealth and the CEO future compensation is inverse, which is in contradiction with what is found in empirical studies of CEO compensation[2], and his environment does not allow him to consider the effects of firm's size. Clementi and Cooley (2001) numerically solved a dynamic agency model in which the agent's present and past effort choices affect the productivity of the firm. They let the principal borrow physical capital from an outside lender by paying a constant interest rate per period, thus allowing them to include firm size into their analysis without studying the dynamics of capital accumulation. They are able to produce model-generated data that are consistent with some empirical features of CEO compensation, which validates their hypothesis that the history of effort choices of the agent plays a role in the firm's productivity and consequently, in the agent's compensation scheme. We consider that including capital accumulation in the dynamic agency problem allows us to study the effect that the history of the realizations of a productivity shock, that is stochastically related with the agent's effort choice, has on the dynamics of the firm's growth.

This paper is organized in the following way: In the second section, we present our dynamic

[1]Jensen and Murphy (1990) empirically obtained measures of responsiveness of CEO pay that did not seem to be consistent with the predictions of agency theory. Basically, they concluded that the pay-performance sensitivities they estimated were too low and that CEOs were paid like bureaucrats.

[2]Hall and Liebman (1998) estimated measures of responsiveness using a data set that included information on CEO stock options and stock ownership. They concluded that there was a positive and strong relation between CEO pay and firm performance, and that responsiveness measures are strongly influenced by stock options and stock ownership. Furthermore, Aggarwal and Samwick (1999) provided an empirical confirmation of one of the predictions of the principal-agent model: that CEO pay's responsiveness to firm performance is decreasing with respect to the firm performance's variance. They also reported that estimates of responsiveness that do not take into account the firm performance's variance are biased toward zero.

model. In the third section, we tackle the issue of the existence of an optimal contract. In the fourth section, we solve a numerical example to study the characteristics of a solution of this model, and discuss our results. In the fifth section, we generate time series using our numerical results to study the long run pattern of behavior of the principal and the agent. An analysis of the pay-performance sensitivities obtained from our model is presented in the sixth section. Lastly, we provide some concluding remarks.

2 The Model

We embed an infinitely repeated version of the principal-agent model in a neoclassical growth model with capital accumulation. We have a principal and an agent that both maximize their respective discounted expected utilities. There is a single good, which has the role of the consumption good for both the principal and the agent, and can also be used in the capital accumulation process.

Time is discrete and is indexed as $t = 1, 2, ...$ The principal is the owner of the production technology, and the agent operates it by exerting some effort. We assume that the output of the production process is public information, while the effort choice of the agent constitutes his private information. In this environment, the inability of the principal to observe the manager's effort choice might create a dilemma to the principal in terms of allocation of limited resources both to provide optimal incentives to the agent and to accumulate capital. We denote the capital stock at the beginning of period t by k_t, and the manager's choice of effort per unit of capital available at period t by a_t. We assume that $k_t \in [\underline{k}, \overline{k}] \in \Re_+$. We also make the assumption and that $a_t \in A$, where $A = [\underline{a}, \overline{a}] \in \Re_+$ and a_t is continuous in this interval.

The production technology is characterized by the following expression:

$$y_t = \theta_t f(k_t),$$

where θ_t represents a productivity shock that behaves according to the time invariant distribution function $G(\theta_t|a_t)$. We assume that G has a density denoted by g, and that g is twice continuously differentiable with respect to a. We also assume that for a fixed a, the distribution is *i.i.d.* from one period to the next, and that the support of the productivity shock distribution is compact.

At time t, the principal pays the agent a compensation of c_t, which is supposed to be non-negative. We assume that the principal does not have access to any type of credit, and consequently, the principal has to compensate the agent, accumulate capital for the firm and consume himself with the resources produced by the firm. Therefore, the following resource constraint needs to be satisfied in this environment:

$$c_t + i_t \leq \theta_t f(k_t),$$

where, i_t represents the amount of investment resources in period t, and $i_t \geq 0$. Also, in every period the next period capital must satisfy:

$$k_{t+1} = (1 - \delta)k_t + i_t,$$

where, $\delta \in (0,1)$ denotes the depreciation rate, and i_t represents the amount of investment resources in period t.

Thus, the reduced version of the resource constraint is:

$$c_t \leq \theta_t f(k_t) + (1-\delta)k_t - k_{t+1}.$$

We assume that the principal is risk-neutral, and that the agent is risk-averse. The agent's preferences are given by the utility function $u(c_t, m(a_t)l(k_t))$, which is assumed to be bounded, strictly increasing and strictly concave in c_t, and strictly decreasing in $m(a_t)l(k_t)$. The motivation for having $m(a_t)l(k_t)$ as an argument of the agent's utility function is to incorporate the idea that as the amount of capital stock increases, the agent's managerial effort will become more complex. We assume that $m(a_t)$ is an increasing and convex mapping with respect to a_t, as is commonly assumed in standard agency models. The only restriction we impose on the function $l(k_t)$ is that it should be increasing with respect to k_t. We also assume that $u(c_t, m(a_t)l(k_t))$ is additively separable in the arguments c_t and $m(a_t)l(k_t)$.

To introduce dynamic elements in this environment, we allow the principal and the agent to employ history-dependent pure strategies, as in Wang (1997). The principal's problem is to construct a sequence of effort recommendations $\left\{a_t(h^{t-1})\right\}_{t=1}^{\infty}$, and a sequence of compensation schemes for the agent $\left\{c_t(h^t)\right\}_{t=1}^{\infty}$, where $h^t = \{y_1, y_2, ..., y_t\}$, in order to maximize the principal's lifetime discounted expected utility subject to the incentive compatibility constraint and the participation constraint, which promises an expected discounted utility of w_0 to the agent. Let σ denote a contract, where $\sigma = \{a_t(h^{t-1}), c_t(h^t)\}_{t=1}^{\infty}$. Also, the principal has to make a decision on the sequence of future capital levels $\left\{k_{t+1}(h^t)\right\}_{t=1}^{\infty}$. Notice that the lifetime discounted expected wealth of the principal is affected by the process of capital accumulation, that depends on the level of activity given by the realization of the random variable θ_t, which, in turn, is conditional on the sequence of effort decisions made by the agent. Thus, the principal's strategy consists of the sequence $\{c_t(h^t), k_{t+1}(h^t)\}_{t=1}^{\infty}$, and the agent's strategy consists of the sequence $\{a_t(h^{t-1})\}_{t=1}^{\infty}$.

The continuation profile of a contract σ from date $t+1$ on, given h_t, is denoted as $\sigma|h_t$. Conditional on the agent following the action recommendation given by $\sigma|h_t$, then the continuation value for the expected discounted utility of the agent is denoted by $w(\sigma|h_t)$, and that of the principal is denoted by $v(\sigma|h_t)$.

A contract $\sigma = \{a_t(h^{t-1}), c_t(h^t)\}_{t=1}^{\infty}$is *feasible* if the effort choice of the agent belongs to A and the reduced resource constraint is satisfied in every period, given the history of outputs:

$$a_t(h^{t-1}) \in A, \forall t \geq 1, \forall h^{t-1}, \tag{1}$$

$$0 \leq c_t(h^t) \leq \theta_t f(k_t) + (1-\delta)k_t - k_{t+1}, \forall t \geq 1, \forall h^t. \tag{2}$$

A contract $\sigma = \{a_t(h^{t-1}), c_t(h^t)\}_{t=1}^{\infty}$is *incentive compatible* if:

$$a_t(h^{t-1}) \in \arg\max_a \int_\theta \left\{u(c_t(h^t), m(a)l(k_t)) + \beta w(\sigma|h^t)\right\} g(\theta_t|a)d\theta, \forall t \geq 1, \forall h^t, \tag{3}$$

where, $\beta \in (0,1)$ denotes the discount factor of both the principal and the agent. Since in this environment, $a_t(h^{t-1})$ is a continuous variable, we use the first-order approach to incentive compatibility, which is not universally valid. To ensure the validity of this approach, we assume that the Monotone Likelihood Ratio Property and the Convexity of the Conditional Distribution Condition hold, following Rogerson (1985) and Spear and Srivastava (1987). This constraint ensures that the agent will not deviate from the principal's effort recommendation plan in any future date, from period $t+1$ on.

Let Ω be the set of capital levels and expected discounted utilities of the agent that can be generated by a feasible, and incentive compatible contract:

$$\Omega \equiv \left\{(k,w) \in \Delta \mid \exists \sigma \ s.t. 1,\ 2,\ 3, \text{ and, } w(\sigma|h^0) = w\right\},$$

where $\Delta \in \Re^2$ is the space in which (k,w) is allowed to take values. Assume Δ is nonempty and compact, and that it is endowed with a structure such that Ω is nonempty as well. Note that the agent is promised a level of expected discounted utility of w. The promise-keeping constraint is expressed as an equality, which is a valid representation, given the assumption of the separability of the agent's utility function in c and $h(a)l(k)$ (see Grossman and Hart (1983).)

For every (k,w), the principal's problem is:

$$\max_{\sigma} v(\sigma|h^0) \ s.t. 1,\ 2,\ 3, \text{ and, } w(\sigma|h^0) = w.$$

The solution of the above problem would be the optimal contract that ensures a lifetime discounted expected utility of w. We assume that both parties are committed to the contract. For every $(k,w) \in \Omega$, we define the following set:

$$\Phi(k,w) = \left\{v(\sigma|h^0) \mid 1,\ 2,\ 3, \text{ and, } w(\sigma|h^0) = w\right\},$$

where, Φ is the set of feasible and incentive compatible expected discounted utilities of the principal given (k,w).

3 Existence of an Optimal Contract

To prove that such a contract σ exists, we need to show that the set of the principal's expected discounted utilities that are feasible and incentive compatible, $\Phi(k,w)$, is compact.

Proposition 1. $\Phi(k,w)$ is compact, $\forall (k,w) \in \Omega$.

We follow the same strategy as in Wang (1997) of constructing an optimal contract σ_∞ with the desirable property of having a finite collection of bounded sequences of effort recommendations, agent's current compensation and, in our case, future capital decisions, such that in every period there exist converging subsequences of each of those sequences.

We will now formulate the Bellman equation to solve this infinite-horizon optimization problem. For all $(k,w) \in \Omega$, we have that the best level of expected discounted utility that can be achieved by the principal through the characterized contract $\sigma|h^0$ is given by:

$$v^*(k,w) \equiv \max\{v(\sigma|h^0) \in \Phi(k,w)\}$$

Now, we define the operator T that maps from the space of bounded and continuous functions $v : \Omega \to \Re$ into itself, with the sup norm, by:

$$T(v)(k,w) = \max \int_\theta \left\{ \theta f(k) - c - k' + (1-\delta)k + \beta v(k',w') \right\} g(\theta|a) d\theta$$

$$s.t. \int_\theta \left\{ u(c, m(a)l(k)) + \beta w' \right\} g(\theta|a) d\theta = w \tag{4}$$

$$a \in \arg\max_a \int_\theta \left\{ u(c, m(a)l(k)) + \beta w' \right\} g(\theta|a) d\theta \tag{5}$$

$$0 \leqslant c \leqslant \theta f(k) - k' + (1-\delta)k \tag{6}$$

$$a \in A \tag{7}$$

$$(k', w') \in \Omega \tag{8}$$

where the decision variables in the optimization process are the following: $a = a(k,w)$, $c = c(\theta,k,w)$, $k' = k(\theta,k,w)$, and $w' = w(\theta,k,w)$. The solution of this problem is Markovian stationary, and perfect in the sense that no deviation from the agent is expected in any period. Given that the just mentioned decision variables are expressed in stationary terms, then the history of the realizations of the output distribution is being summarized by the state variables (k,w). At this point, we need to demonstrate that $v^*(k,w)$ is a fixed point of T.

Proposition 2. $v^*(k,w) = T(v^*)(k,w)$.

As in Wang (1997), we construct a feasible and incentive compatible contract σ by letting $a(k,w)$, $c(\theta,k,w)$, $k(\theta,k,w)$, and $w(\theta,k,w)$ denote the solution of the dynamic optimization problem associated with the definition of the operator $T(v^*)(k,w)$.

Provided that we have a Bellman equation to solve for an optimal contract, we can operate recursively, as in Spear and Srivastava (1987). That is, in period 1, given the values of the state variables k_1 and w_1, the agent chooses an effort level of $a_1 = a(k_1,w_1)$. At the end of period 1, the productivity shock θ_1 is realized, conditional on a_1, and observed by both the principal and the agent. A level of compensation of $c_1 = c(\theta_1,k_1,w_1)$ is paid to the agent by the principal. Also, a level of discounted expected utility of $w_2 = w(\theta_1,k_1,w_1)$ is promised to the agent by the principal. At this point the principal makes the decision on how much capital to accumulate in the next period, $k_2 = k(\theta_1,k_1,w_1)$, and finally the principal gets a level of consumption in period 1 of $(\theta_1 f(k_1) - c_1 - k_2 + (1-\delta)k_1)$, and a level of expected discounted utility of $v_1 = v^*(k_2,w_2)$. Now, the state variables are given by the pair (k_2,w_2). And the story is repeated for every period.

Given that Ω is a convex subset of $\Re^2$, that Φ is non-empty, compact-valued and continuous, that the return function is bounded and continuous, and that $\beta \in (0,1)$, then we have that the operator T has a fixed point with the standard properties. This means that the principal's problem defined in the last section has a solution, that can be obtained by a value function iteration process.

To perform the value function iteration process, we need first to find the set Ω. with this purpose we use the approach proposed by Abreu, Pierce and Stacchetti (1990). We have to demonstrate that Ω is self-generating, and this will allow us to device an algorithm to compute Ω.

We assume that k is restricted to take values on a closed and bounded subset of $\Re_+$, denoted X_k. Let

$$\Omega_0^k = \{k : k \in X_k = [k_{\min}, k_{\max}]\}$$

The state variable w is allowed to take values on a closed and bounded subset of $\Re_+$, denoted $X_{w(k)}$. Let

$$\Omega_0^w(k) = \left\{w(k) : k \in \Omega_0^k \text{ and } w \in X_{w(k)} = [w_{\min}(k), w_{\max}(k)]\right\}$$

We set $w_{\min}$ arbitrarily to a very small positive number.

To obtain $w_{\max}(k)$, we solve the following dynamic optimization problem:

$$W(k) = \max_{(a,c,k')} \int_\theta \{u(c, m(a)l(k)) + \beta W(k')\}\, g(\theta|a)d\theta$$

$$s.t.\ \ 0 \leqslant c \leqslant \theta f(k) - k' + (1-\delta)k$$

The solution of this problem exists given that u is bounded, strictly increasing and strictly concave in c, and strictly decreasing in $h(a)l(k)$. Also, the constraint space is convex with respect to the control variables of this problem. By solving this problem, we obtain the set of maximal and feasible values that the agent's future discounted expected utility can take for each level of capital that belongs to Ω_0^k.

Let

$$\Omega_0 = \{(k, w) \mid k \in \Omega_0^k \text{ and } w \in \Omega_0^w(k)\}$$

We will now use the concept of self-generation of Abreu, Pierce and Stacchetti (1990). Let us define an operator B such that for any arbitrary $\Sigma \in \Re^2$:

$$B(\Sigma) = \{(k, w) \mid \exists \{a, c, k', w'\}\ s.t. 4,\ 5,\ 6,\ 7, \text{ and } (k', w') \in \Sigma\}$$

The operator B is monotone in the following sense: $\Sigma_1 \subseteq \Sigma_2$, implies that $B(\Sigma_1) \subseteq B(\Sigma_2)$. We say that Σ is self-generating if $\Sigma \subseteq B(\Sigma)$.

Proposition 3. (a) Ω is self-generating. (b) If Σ is self-generating, then $B(\Sigma) \subseteq \Omega$.

We follow the same strategy of Wang (1997) of constructing the desired space of the feasible and incentive compatible values of the agent's expected utility with the difference that we include capital through the agent's utility function and the feasibility constraint.

Proposition 4. (a) $\Omega = B(\Omega)$. (b) Let $X_0 = \Delta$, and let $X_{n+1} = B(X_n)$, for $n = 0, 1, 2, \ldots$ Then, $\lim_{n \to \infty} X_n = \Omega$.

This proposition is equivalent to Proposition 2 in Wang (1997), where a proof can be found. It ensures us that if we start with a set Δ, and iterate on it using the operator B, we will converge to the set Ω. Moreover, this set Ω, by part (a) of Proposition 3, is a fixed point of the operator B.

In this section we have proved that our contracting problem admits a Bellman equation representation which can be solved by using the contraction mapping theorem. We also established the validity of the self-generation concept in our environment which provides an algorithm to compute the space of the feasible and incentive compatible values of the agent's expected utility.

4 A Numerical Example

To study the characteristics of a solution of the previous model and to perform a comparative static analysis, we solve a numerical example. First, we specialize the model. The preferences of the agent are assumed to be represented by the utility function $u(c, h(a)l(k)) = \sqrt{c} - ak$. Given our assumption of a continuum of effort levels, and that $A = [0, \overline{a}]$, where $\overline{a} \in \Re_+$; we need to set a numerical value for a high enough such that it will not perturb the numerical solution We set $\overline{a} = 20.0$, after performing some initial numerical exercises. We assume that the technology shock can take two values, $\{\theta_1, \theta_2\} = \{0.5\ ,\ 2.0\}$, with probabilities $\exp(-a)$ and $1 - \exp(-a)$ respectively. The production function is $f(k) = k^{\varepsilon}$, where $\varepsilon \in (0, 1)$. For this particular example, we assume that $\varepsilon = 0.36$ and that $\beta = 0.9633$. We also assume that $\delta = 0.1$. We must clarify that this is just a numerical experiment and that we do not intend to calibrate this model.

We construct a grid with $N1$ equidistant points over the continuous and compact interval $[k_{\min}, k_{\max}]$, in which the state variable k can take values. We also build a grid with $N2$ equidistant points over the continuous and compact interval $[w_{\min}, w_{\max}(k = k_{\max})]$, in which the state variable w is allowed to take values. We set $N1 = 10$, and $N2 = 100$.

Our set of constraints becomes:

$$a \in A \tag{9}$$

$$0 \le c_i \le \theta_i k^{\varepsilon} - k_i' + (1 - \delta)k,\ i = 1, 2 \tag{10}$$

$$\arg\max_a\ [\sqrt{c_1} - ak + \beta w_1'] \exp(-a) + [\sqrt{c_2} - ak + \beta w_2'](1 - \exp(-a)) \tag{11}$$

$$[\sqrt{c_1} - ak + \beta w_1'] \exp(-a) + [\sqrt{c_2} - ak + \beta w_2'](1 - \exp(-a)) = w. \tag{12}$$

So, the algorithm proposed by Abreu, Pierce and Stacchetti (1990) to compute the set of admissible points, described in the first section, can be rewritten in the following way. Let

$$\Omega \equiv \left\{ \begin{array}{c} (k, w) \mid \exists \{c_i, a, w_i', k_i'\} \text{ s.t. } 9,\ 10,\ 11,\ 12\ , \\ \text{and } (k_i', w_i') \in \Omega,\ i = 1, 2 \end{array} \right\}.$$

Now, we have to construct the initial Ω_0. The details of its construction are given in Appendix B.2. This set is defined as follows:

$$\Omega_0 = \left\{ (k, w) \mid k \in \Omega_0^k \text{ and } w \in \Omega_0^w(k) \right\}.$$

Finally, we iterate to find the optimal state space by finding a fixed point of the operator B:

$$\Omega_{t+1} = B(\Omega_t) = \left\{ \begin{array}{c} (k, w) \mid \exists \{k_i', w_i', a, c_i\} \text{ s.t. } 9,\ 10,\ 11,\ 12, \\ \text{and } (k_i', w_i') \in \Omega_t,\ i = 1, 2 \end{array} \right\}.$$

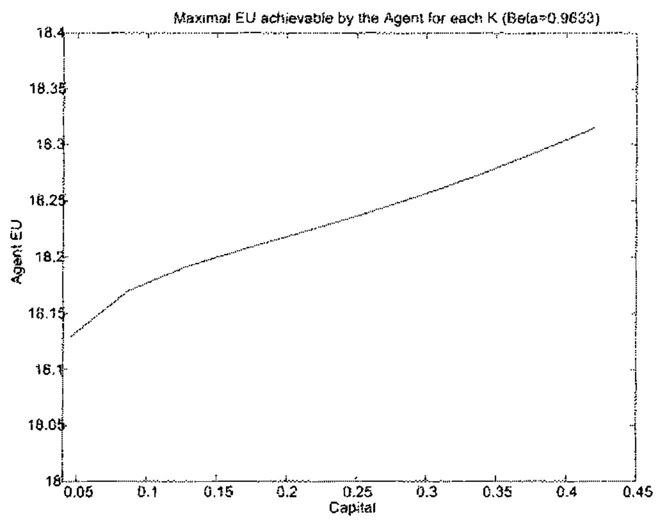

Figure 1: Space of Feasible Levels of the Agent's Expected Utility ($\beta = 0.9633$)

4.1 Results

We used the parametric approach to value function iteration to obtain the solution of the above specialization of our model. Our computational strategy is described in the appendix. We will now show our results and discuss our findings. In Figure 1 we depict the upper frontier of the agent's lifetime expected utility that resulted from finding the maximal values of the agent's expected utility subject to the resource constraints. As today's capital increases, it will be feasible for the agent to achieve higher levels of lifetime expected utility, as a result of higher levels of the consumption good that can be produced through the technology.

The value function that we obtained is a smooth surface which depends on both the current level of capital and the lifetime expected utility of the agent. Since the informativeness of the three dimensional graph of this value function is limited[3], we are going to present two-dimensional graphs. In Figure 2, we depict the value function depending only on the lifetime expected utility of the agent and keeping capital constant at several levels, specified in the graph, using a continuous line. In the same figure, we also show the value function that resulted from the solution of the standard dynamic agency model without capital accumulation (benchmark model), using a dashed line. We have selected this model as the benchmark model since we aim to emphasize the novel aspects that capital accumulation introduces in this context. To compute the benchmark model we used a fixed level of capital and the realizations of the productivity shock marked the difference between the high and low production level. We repeated the procedure with several capital levels in order to make comparisons with the results of the model with capital accumulation.

The value function is decreasing and concave with respect to the lifetime expected utility of the

[3]This graph can be seen in Di Giannatale (2001)

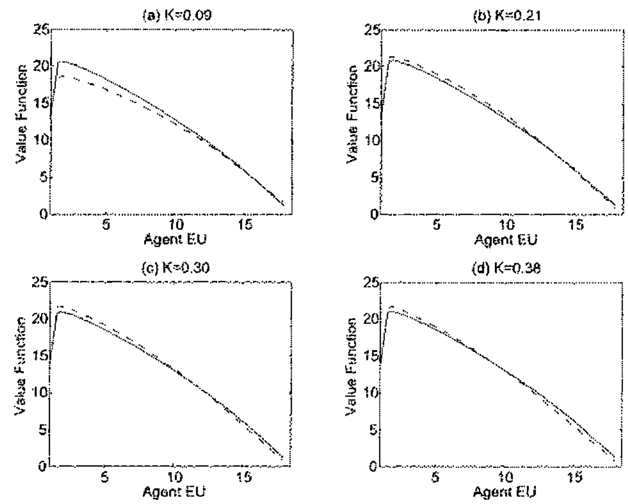

Figure 2: 2-D Value Function: With Capital [-] and Without Capital [- -]

agent in both cases (except for the very low values of the lifetime expected utility of the agent where it is increasing), which is consistent with the predictions of the standard dynamic principal-agent model. In part (a) of this figure, where the fixed capital level is $K = 0.09$, we observe that the value function of our model dominates the value function of the benchmark model. In parts (b), (c), and (d), where the capital is fixed at higher values, we have that the value function of the benchmark model dominates the value function of our model up to a certain level of the lifetime expected utility of the agent, which decreases with the level of capital, after which there is a flip in the dominance pattern.

In Figure 3, we plot the value function of our model depending only on the current level of capital, keeping constant the lifetime expected utility of the agent. We consider several levels of the lifetime expected utility of the agent, specified in the graph. We observe that the value function is increasing and concave with respect to the today's level of capital, a result which is typical in the neoclassical growth model with a decreasing returns to scale technology. We confirm that the value function decreases with respect to the level of the agent's lifetime expected utility.

We will now discuss how the incentive tools work in this model. In Figure 4(a) we plot the policy rules of the agent's promised discounted expected utility, keeping the level of current capital constant ($k = 0.09$). As expected, the agent will achieve a higher level of promised discounted expected utility in the event of the high productivity shock. Abstracting from the lowest values of the agent's current lifetime utility, observe that as the current lifetime expected utility of the agent increases, the separation of those policies rules decreases. This means that as the current lifetime expected utility of the agent increases, this incentive tool loses effectiveness. This is in line with the concavity of the value function with respect to the lifetime expected utility of the agent; which implies that as the latter increases, it becomes more costly to the principal, in terms of

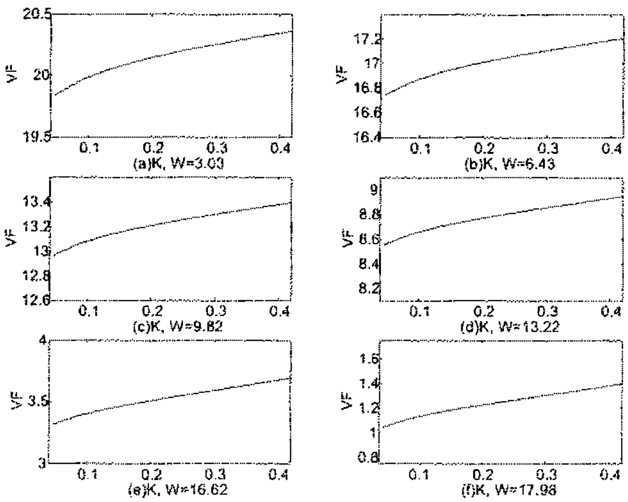

Figure 3: Another 2-D Value Function

expected utility, to compensate the agent using future discounted expected utility. In Figure 4(b), we depict the same laws of motion corresponding to the benchmark model. Note that for this level of capital, ($k = 0.09$), the spread is higher in the benchmark model. In parts (c) and (d) of the same figure, we plot again the mentioned law of motions for each model but for a different capital level ($k = 0.21$). Furthermore, we present the last observation in a summarized way in Figure 5, which depicts the behavior of the spread of high and low shock policy rules of the agent's promised discounted expected utility for several capital levels ($k = 0.09$, $k = .21$, $k = 0.30$, and $k = 0.38$). The higher the curve, the higher the associated capital level is. Thus, from the graphs, we could say that the principal relies more on this incentive tool for incentive provision as the firm's physical capital grows.

To continue with the description of the incentive tools of this model, in Figures 6(a) and 7(a) we show the current compensation of our model's agent for the high and low productivity shock respectively, keeping constant the capital level ($k = 0.09$ and $k = 0.21$, respectively). Current compensation is non-decreasing with respect to the current level of the agent's lifetime expected utility. Note that, as expected, the current compensation of the agent is higher when, relative to when the low shock is observed, the high productivity shock is realized. Also, the separation between those two schedules becomes larger as the level of the lifetime expected utility of the agent increases. This is compatible with the result observed for the laws of motion of the promised discounted future utility of the agent. That is, as the current level of the lifetime expected utility of the agent becomes larger, the incentive tool that becomes more effective (and less costly to the principal) is the current compensation. However, it must be said that both incentive tools are operating at all levels of the current lifetime expected utility of the agent. In Figures 6(b) and 7(b), we show the optimal current compensation schedules of the agent that result from numerically

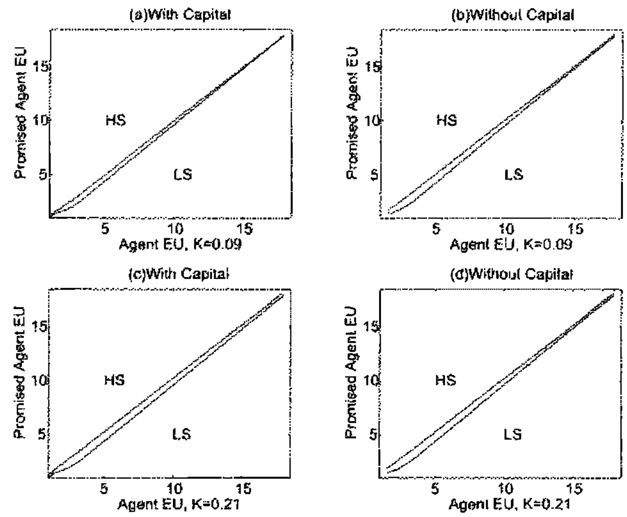

Figure 4: 2-D View of the Laws of Motion of the Agent's Discounted Expected Utility

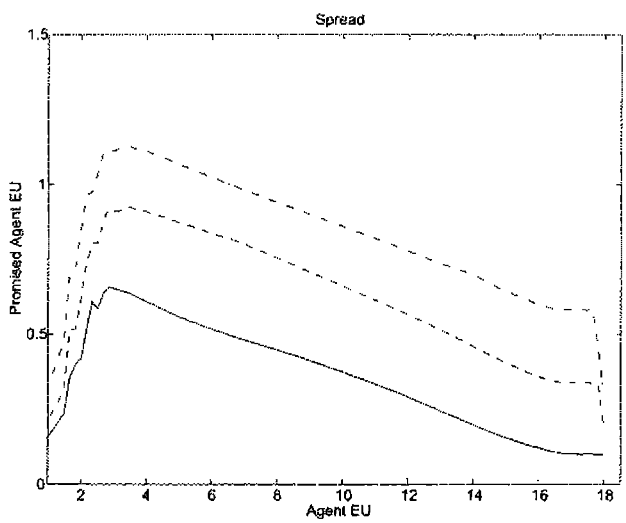

Figure 5: Spread of the Agent's Expected Utility

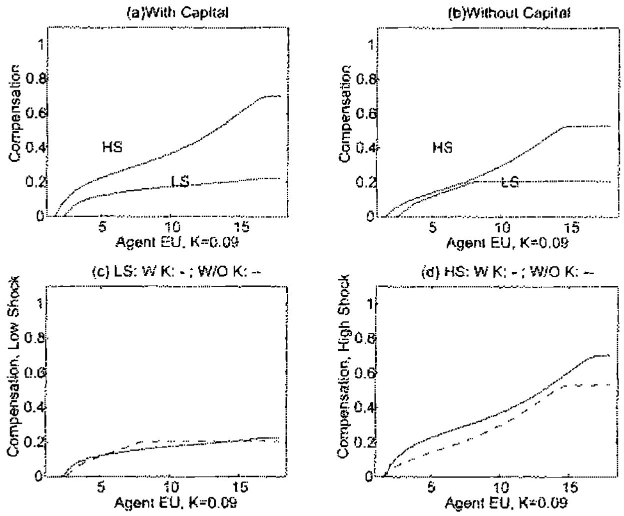

Figure 6: 2-D View of the Optimal Compensation of the Agent

solving the benchmark model. We observe that the pattern of behavior is similar to what we can see in our model, however the separation between the high and low shock optimal current compensation schedules is lower in the standard dynamic agency model for the lower capital level. This can be confirmed by looking at Figure 6(c) and Figure 6(d), which show the agent's optimal current compensation paths of our model (in continuous line) and the benchmark model (in dashed line) for the high and low shocks respectively. On the other hand, we can note that as the capital level increases, the difference between current compensation for the low and high shocks diminishes for the case of our model. That is, our model passes from having a bigger difference between the current compensation schedules for the low and high shocks for the lowest capital level considered, to having the lower difference between those schedules for the highest capital level considered. Therefore, we can say that as capital increases, the principal tends to rely more on the promised discounted expected utility of the agent as an incentive tool.

It is noticeable that for very high values of the state variable w as the capital level increases, the pattern of the compensation of the agent in the case of the realization of the high productivity shock, shows non-monotonicities for the higher capital values plotted. These results might be due to problems of the computational program in dealing with the upper boundaries of the agent's expected utility policy rules. We performed additional numerical exercises to see whether we could improve these results. First, we used a denser grid for the higher values of the state variable w, and we obtained similar results to those showed above. We also performed another experiment in which we doubled the number of grid points for the state variable w with respect to the original number of grid points we considered for this variable. That is, originally we considered 100 grid points for w, and for this experiment we considered 200 of evenly spaced grid points for w, with the result that the non-monotonicities in the schedule of agent's compensation for the high shock

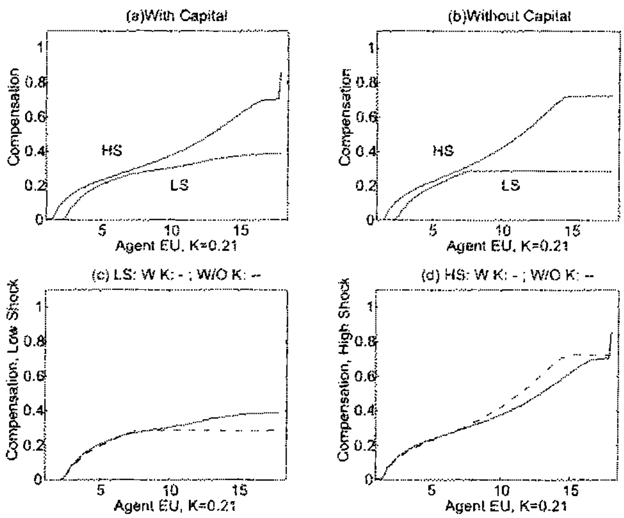

Figure 7: 2-D View of Optimal Compensation of the Agent (Continuation)

realization for the highest values of w could still be observed[4].

In Figures 8 and 9, we plot the policy rules of capital by confronting the levels of current capital with the levels of future capital for each productivity shock, and keeping the value of the lifetime expected utility of the agent fixed at several levels specified in the graphs. In the graphs, we depicted the 45° line, the policy rule of capital for the high shock as a continuous line, and the one corresponding to the low shock as a dashed line. We notice that the steady state of capital decreases when the current lifetime expected utility of the agent increases. Also, the steady state of capital corresponding to the low productivity shock is higher than that in the case of the high shock, up to a critical level of the lifetime expected utility of the agent at which there is a flip in this pattern of behavior. That is, for values of the lifetime expected utility of the agent higher than this critical level, the steady state of capital given by the high shock law of motion of future capital is higher than that given by the low shock law of motion of future capital. Looking at Figures 8 and 9, we can also notice that the law of motion of future capital in the high productivity event is decreasing for most of the values of current capital and the fixed level of lifetime expected utility of the agent. Moreover, this law of motion is almost always close to the steady state value of capital given by the intersection of this law of motion and the 45° line. The law of motion of future capital in the event of the low productivity shock is non-decreasing with respect to current capital, and non-increasing with respect to the different fixed levels of the lifetime expected utility of the agent. It must be pointed out that the laws of motion of future capital for each realization of the productivity shock, for the different fixed levels of the current lifetime expected utility of the agent, converge to the corresponding steady state, in the sense that once they intersect the 45° line

[4]For more details of these exercises, see Di Giannatale (2001).

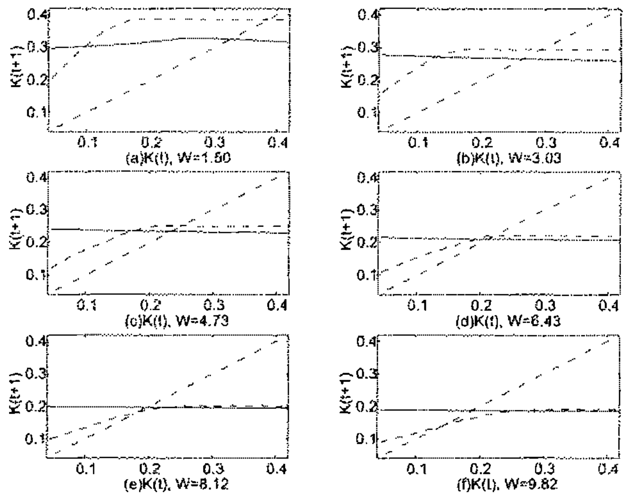

Figure 8: 2-D Laws of Motion of Capital

they will lie below it. Thus, as the agent's current lifetime expected utility increases the steady state of capital will be higher in the case of the high productivity shock, however the steady state of capital 's trend is to decrease both for the high and low productivity shocks.

Summarizing our analysis, we say, first, that when the level of capital increases, the principal uses the promised discounted expected utility of the agent as the dominant incentive tool. Also, the principal pays higher and closer salaries to the agent when both the high and low shocks are observed. That is, as capital increases, future compensation becomes the dominant tool for achieving risk sharing. However, future compensation becomes more costly to the principal as the lifetime expected utility of the agent increases. As the lifetime current expected utility of the agent increases, the steady state value of capital decreases for both the high and low shocks. Therefore, the principal in our model faces a conflict between accumulating capital and minimizing agency costs. In the principal's conflict in allocating resources, incentive provision seems to be favoured over capital accumulation.

5 Simulation of Time Series

We now generate time series from our numerical results in order to draw some conclusions about the long-term behavior of the principal and the agent. We performed a simulation for 200 periods, considering combinations of ten (10) equidistant levels of initial capital (from the range of possible values of capital), and ten (10) equidistant levels of initial lifetime expected utility of the agent (from the range of possible values of the lifetime expected utility of the agent). The following plots only show a few of those combinations for the purpose of understandability.

In Figure 10, we plot the simulated value function for combinations of four levels of initial

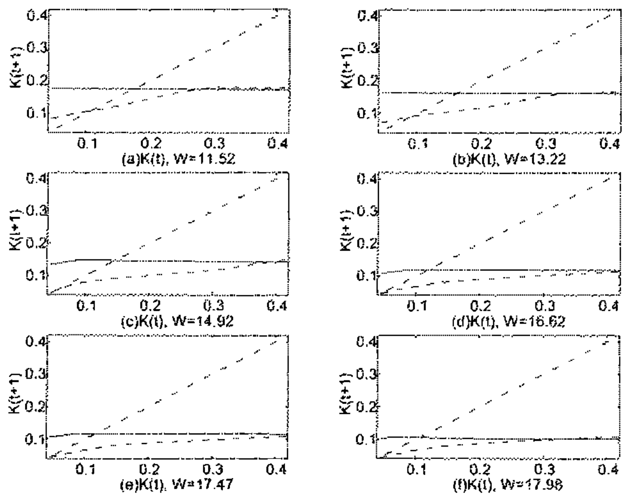

Figure 9: 2-D Laws of Motion of Capital (Continuation)

capital, specified in the graph, and two levels of initial lifetime expected utility of the agent, which are $w1 = 3.27$ and $w2 = 7.69$. In each of the boxes, the path that corresponds to the specified initial capital level and an initial lifetime expected utility of the agent of $w1$ is represented with the continuous line, and the one that corresponds to $w2$ is represented with the dashed line. We can observe that the expected wealth of the principal fluctuates over time, but tends to decrease. Also, that the level of the expected wealth of the agent is lower as the initial level of the lifetime expected utility of the agent increases.

On the other hand, the expected utility of the agent has an increasing trend, even though it also shows fluctuations with the passing of time, as we can see in Figure 11. In each of the boxes, the path that corresponds to the specified initial capital level and an initial lifetime expected utility of the agent of $w1$ is represented with the continuous line, and the one that corresponds to $w2$ is represented with the dashed line. Note that the level of the expected utility of the agent is higher as the initial level of the lifetime expected utility of the agent increases.

The time series results suggest that in the long run, the principal is likely to face a commitment problem in this environment. That is, the principal tends to use more and more resources in incentive provision, limiting the firm's growth and his own consumption. We then conclude that asymmetry of information about the agent's effort lowers the firm's growth. Probably, the fact that the principal has no access to any form of credit plays an important role in this result and considering some market for credit in this environment might prove useful. However, access to credit might not offer a complete answer to this story because still incentive provision might dominate over other uses of the available resources. Therefore, this analysis might also be enriched with the consideration of distinct agent's disciplinary measures, such as takeovers and compensation limits.

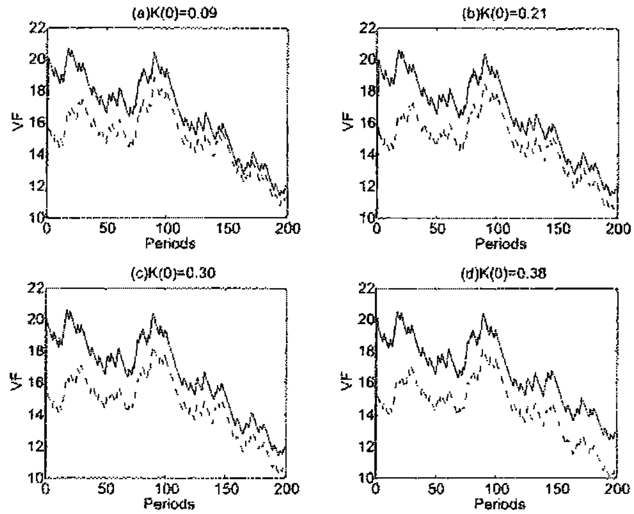

Figure 10: Simulation Results: Value Function With Capital Accumulation

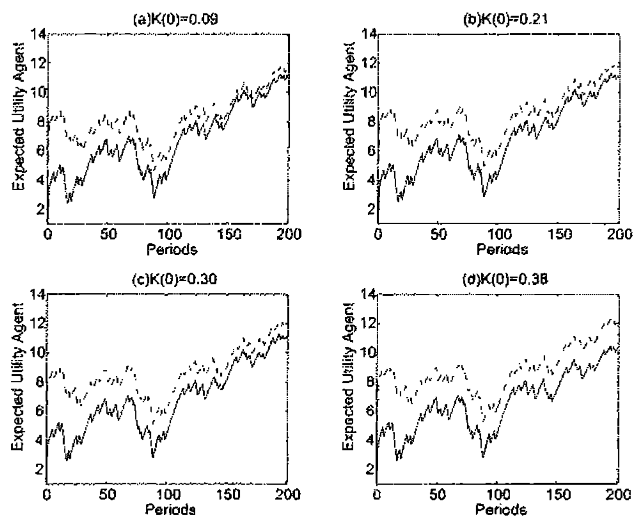

Figure 11: Simulation Results: Discounted Expected Utility of the Agent

6 A comment on pay-performance sensitivities

In this section, we analyze the behavior of pay-performance sensitivities as the firm grows to understand better our risk sharing results. We generated data from our numerical results to compute those sensitivities, considering a situation with 400 agents or CEOs and 15 periods[5]. We selected combinations of ten (10) equidistant levels of initial capital (from the range of possible values of capital), and ten (10) equidistant levels of initial lifetime expected utility of the agent (from the range of possible values of the lifetime expected utility of the agent). For each of those 100 pairs of current capital and current lifetime expected utility of the agent, we produced time series of 15 periods of length for 400 CEOs.

We first estimated the sensitivity of future compensation of the agent to the performance of the firm, using the following equation:

$$\Delta w_t = \alpha^w + \beta_1^w \Delta V_t + \beta_2^w \Delta V_{t-1}$$

where, Δw_t is the change in the agent's promised discounted expected utility during the current period, ΔV_t is the change in the expected wealth of the agent during the current period, and ΔV_{t-1} is the change in the expected wealth of the agent during the immediate previous period. Our measure of future compensation is given in utility terms. We selected two lags in the variation of the performance of the firm, measured as changes in the value function, to capture some features of our model that will be explained later.

We used the level of the firm's stock of physical capital as our definition of firm size. That is, we computed the pay-performance sensitivities by grouping firms with the same initial capital level. Given that we performed our simulation by considering ten initial levels of capital, we also estimated the regression equations for the same ten initial capital levels.

From our results, presented in Table 1, we conclude that the immediately previous lag is not significant and of much lower impact explaining the sensitivity of future compensation with respect to the firm performance. As the level of initial capital increases, the significance and impact of this sensitivity decreases, except for $k_5 = 0.215$. The significance and impact of the second lag is higher. We can also state that from $k_5 = 0.215$ on, the magnitude and significance of this sensitivity is weakly decreasing, and this can be related to the fact that as the level of initial capital increases, the variance of the value function increases too.

We also estimated the sensitivity of present compensation of the agent to the performance of the firm using this equation:

$$\Delta c_t = \alpha^c + \beta_1^c \Delta V_t + \beta_2^c \Delta V_{t-1}$$

where, Δc_t is the change in the agent's present compensation during the current period, and ΔV_t and ΔV_{t-1} are defined as before. The agent's present compensation is also given in utility

[5]The selections of number of agents and time periods were made to resemble the structure of the data sets used by the authors of some empirical CEO's compensation papers, for instance Hall *et al.* (1998) and Aggarwal *et al.* (1999).

Table 1: Pay-Performance Sensitivities for Future Pay

Capital	α^w	t	β_1^w	t	β_2^w	t	F	$\overline{R}^2$	Mean VF	Var VF
$k_1 = 0.079$	0.0560	35.12	0.0032	0.69	0.0336	7.21	26.30	0.0011	16.57	6.84
$k_2 = 0.113$	0.0555	34.94	0.0029	0.62	0.0340	7.31	26.94	0.0011	16.54	6.89
$k_3 = 0.147$	0.0552	34.81	0.0026	0.55	0.0341	7.34	27.14	0.0011	16.51	6.93
$k_4 = 0.181$	0.0548	34.62	0.0025	0.53	0.0337	7.25	26.45	0.0011	16.50	6.96
$k_5 = 0.215$	0.0546	34.55	0.0028	0.59	0.0341	7.35	27.24	0.0011	16.48	6.99
$k_6 = 0.250$	0.0543	34.95	0.0020	0.43	0.0336	7.26	26.52	0.0011	16.46	7.04
$k_7 = 0.284$	0.0540	34.32	0.0019	0.40	0.0335	7.26	26.49	0.0011	16.44	7.05
$k_8 = 0.318$	0.0538	34.21	0.0015	0.33	0.0333	7.20	26.02	0.0011	16.43	7.07
$k_9 = 0.352$	0.0535	34.03	0.0010	0.23	0.0325	7.05	24.90	0.0010	16.42	7.07
$k_{10} = 0.386$	0.0532	33.91	0.0008	0.18	0.0325	7.06	24.94	0.0010	16.40	7.07

Table 2: Pay-Performance Sensitivities for Current Pay

Capital	α^w	t	β_1^w	t	β_2^w	t	F	$\overline{R}^2$	Mean VF	Var VF
$k_1 = 0.079$	0.0037	22.02	0.0273	55.92	0.0050	10.17	1621.74	0.063	16.57	6.84
$k_2 = 0.113$	0.0037	21.80	0.0276	56.34	0.0051	10.27	1647.16	0.064	16.54	6.89
$k_3 = 0.147$	0.0037	21.83	0.0281	57.35	0.0050	10.26	1704.75	0.066	16.51	6.93
$k_4 = 0.181$	0.0036	21.60	0.0283	57.78	0.0049	10.00	1726.25	0.067	16.50	6.96
$k_5 = 0.215$	0.0036	21.55	0.0286	58.30	0.0050	9.92	1755.13	0.068	16.48	6.99
$k_6 = 0.250$	0.0036	21.57	0.0289	58.78	0.0049	9.68	1781.56	0.069	16.46	7.04
$k_7 = 0.284$	0.0036	21.58	0.0291	59.76	0.0047	9.69	1840.05	0.071	16.44	7.05
$k_8 = 0.318$	0.0036	21.53	0.0293	60.05	0.0048	9.76	1858.14	0.072	16.43	7.07
$k_9 = 0.352$	0.0036	21.48	0.0293	60.30	0.0048	9.76	1873.76	0.072	16.42	7.07
$k_{10} = 0.386$	0.0035	21.27	0.0294	60.36	0.0048	9.74	1876.83	0.073	16.40	7.07

terms to make comparisons with the sensitivities of future compensation. Our results, presented in Table 2, allow us to conclude that the significance and magnitude of the sensitivity associated with the first lag increase as the level of initial capital increases. However, the magnitude and significance of the sensitivity associated with the second lag is weakly decreasing with respect to the initial level of capital, except for $k_5 = 0.215$.

From the results of Table 1 and Table 2 we see that future compensation is affected by more distant lags and present compensation by nearer lags. That is, history explains better the movements in the agent's future compensation while current events explain better the happenings of current compensation of the agent. Secondly, we observe that as firms grow, the sensitivity of future compensation with respect to firm performance decreases while the sensitivity of current compensation with respect to firm performance is weakly increasing. Thus, we can conclude that as the firm grows and its performance becomes more variable, the principal relies more on future compensation to provide incentives, but the link between the agent's wealth and firm's performance becomes weaker given the risk-averse nature of the agent.

7 Concluding Remarks

Some of our results are in line with several findings reported in the empirical literature about CEO compensation. For example, the pay-performance sensitivities that we obtained using our model-generated data reflect that there is an inverse relationship between between pay-performance sensitivities and firm size. This is one of the stylized facts regarding CEO compensation that Murphy (1999) reports, as well as the observations that larger firms pay more to their CEOs and that the compensation component that establishes a stronger link between CEO pay and firm's performance is future compensation. Moreover, Clementi and Cooley (2000) inferred from the empirical literature on CEO compensation that the contemporaneous effect of firm performance on CEO compensation is lower than the cumulative effect, that includes lagged information. With our model-generated data, we obtained the result that the history of the firm's performance has a stronger effect on future compensation and the current firm's performance has a stronger effect on the present compensation of the CEO. These facts underscore the importance of future compensation as a component of CEO pay and that future compensation tends to be a larger component of CEO's compensation in larger firms. In our model, we obtained as a result that the principal tends to rely more on future compensation for incentive provision as the firm's capital is higher. Then, we conclude that, in fact, as firms grow, their moral hazard problem becomes more severe in that the shareholders need to implement a compensation scheme that ensures a stronger relationship between the performance of the firm (or the interests of the shareholders) and the CEO's compensation. However, larger firms have more resources to deal with agency problems such as long-term incentive tools, and, thus, better risk-sharing can be achieved in those firms.

Finally, we would like to bring into the discussion that our model predicts a problem of commitment for the principal in the long-run, given our time series results. The cause of this problem may lie in our assumption that the principal does not have access to the credit markets. Therefore, it might be interesting to explore the possibility of open credit markets in this environment. On the other hand, this might suggest that the principal needs to implement other types of incentive tools in order to align the interests of the agent with the principal's interests. In this line of thinking, the effects of takeover threats and reputational issues may constitute a productive line of future research.

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1 Appendix

In this appendix we describe our computational strategy. The first step is to numerically find the set of maximal and feasible values of the agent's future discounted expected utility, in order to construct the initial state space. The computation of those values is done by using a parametric approach to value function iteration, and selecting cubic splines as the approximating scheme. Denote the value function as $\widehat{W}(k;u)$ where $u \in \Re^{N1}$is the associated set of parameters. The corresponding recursive algorithm, presented in the same fashion as Judd (1998), is:

Algorithm 1:

- *Initialization*: Make initial guess $\widehat{W}(k;u_0)$ where $u_0 \in \Re^{N1}$ is the initial set of parameters, and choose stopping criterion $\pi > 0$.
- *Step 1*: For each point in the one-dimensional approximation grid, $\Omega_0^k = \{k(T) : T = 1, ..., N1\}$; solve for $(c_1, c_2, k_1', k_2', a)$ by finding the maximal value of the current expected utility of the agent with respect to those decision variables, subject to the feasibility or resource constraints.
- *Step 2*: Compute:

$$\begin{aligned} w_{n(T)} &= [\sqrt{c_1^*} + \beta\widehat{W}(k_1';u_n)]\exp(-a^*) \\ &\quad +[\sqrt{c_2^*} + \beta\widehat{W}(k_2';u_n)](1-\exp(-a^*)) - a^*k,\ \forall T \end{aligned}$$

where n denotes the current iteration of the algorithm.

- *Step 3:* Update the set of parameters $u_{n+1} \in \Re^{N1}$ such that $\widehat{W}(k;u_{n+1})$ approximates the (w_n, k) data.
- *Step 4:* If $\left|\widehat{W}(k;u_{n+1}) - \widehat{W}(k;u_n)\right| < \pi$, then stop; else go to Step 1.

The second step is to find the numerical solution to the principal's dynamic optimization problem. This is a complex process because of the dimension of the state space, as well as the number of decision variables. Then, it is important to provide the computational algorithm that solves this problem with the best initial guess possible. We obtain the initial guess by solving a simpler dynamic problem, which is the complete information version of our model. In this case, we assume that $w = w_1' = w_2'$, and, thus, we have only one state variable. The problem is:

$$V(k) = \max_{(c,k_1',k_2',a)} \begin{array}{l} [\theta_1 f(k) - k_1' + \beta V(k_1')]\exp(-a) \\ +[\theta_2 f(k) - k_2' + \beta V(k_2')](1-\exp(-a)) - c + (1-\delta)k \end{array}$$

$$s.t.\ \sqrt{c} - ak = (1-\beta)w$$

$$0 \leqslant c \leqslant \theta_i f(k) - k_i' + (1-\delta)k, \quad i = 1,2$$

The computation of the optimal contract sequence related to the above problem is done by using a parametric approach to value function iteration, and using cubic splines as the approximating scheme of the value function, $\widehat{V^1}(k; e^1)$, where $e^1 \in \Re^{N1}$ is the set of parameters that defines the spline. The corresponding recursive algorithm is:

Algorithm 2:

- *Initialization:* Make initial guess $\widehat{V^1}(k; e_0^1)$, and choose stopping criterion $\pi > 0$. Create a one-dimensional grid with $N2$ equidistant points between $w_{\min}$ and $w_{\max}(k = k_{\max})$. Notice that $w_{\max}(k = k_{\max})$ is the value corresponding to the solution of the problem solved with Algorithm 1 when $k = k_{\max}$.

- *Step 1:* For each value in the above grid, create a one-dimensional approximation grid $\Omega_0^k = \{k(T) : T = 1, ..., N1\}$.

- *Step 2:* For each of those points, solve for the optimal values of (a, k_1', k_2') by finding the optimum of the current expected utility of the principal with respect to those decision variables, subject to the resource constraints.

- *Step 3:* Compute the optimal compensation for the risk-neutral agent, which is equal for each realization of θ, using the optimal values of (k_1', k_2', a), and the following version of the participation constraint:
$$\sqrt{c} = (1-\beta)w + a^* k_T$$

- *Step 4:* Compute:

$$\begin{aligned} v_{(T)}^1(w) &= [\theta_1 k_T^{\varepsilon} - k_1'^* + \beta\widehat{V^1}(k_1'^*; e_n^1)]\exp(-a^*) \\ &\quad + [\theta_2 k_T^{\varepsilon} - k_2'^* + \beta\widehat{V^1}(k_2'^*; e_n^1)](1 - \exp(-a^*)) - c^* \ + (1-\delta)k_T \ , \ \forall T \end{aligned}$$

where n denotes the current iteration of the algorithm corresponding to a given value of w.

- *Step 5:* Update the set of parameters $e_{n+1}^1 \in \Re^{N1}$such that $\widehat{V^1}(k; e_{n+1}^1)$ approximates the $(v_n^1(w), k)$ data.

- *Step 6:* If $\left|\widehat{V^1}(k; e_{n+1}^1) - \widehat{V^1}(k; e_n^1)\right| < \pi$, then go to Step 7; else go to Step 2.

- *Step 7:* Check whether all the points of the w-grid have been exhausted. If so, stop; else go to Step 1.

Now we proceed to find the numerical solution of the principal's problem with incomplete information. We use again a parametric approach to value function iteration, and cubic splines as the approximating scheme of the value function $\widehat{V^2}(k, w; e^2)$, where $e^2 \in \Re^{N1\times N2}$ are the parameters that characterize the spline. The following algorithm summarizes the procedure:

Algorithm 3:

- *Initialization*: Use as the initial guess $\widehat{V^2}(k, w; e_0^2)$ the $(v_{n+1}^1(w), k)$ data for each value of w, and choose stopping criterion $\pi > 0$.

- *Step 1*: For each point in the two-dimensional approximation grid, $\Omega_0 = \{(k, w) \mid k \in \Omega_0^k$ and $w \in \Omega_0^w(k)\}$, compute the set of admissible points Ω_n using the specialization of the Abreu, Pierce, Stacchetti's algorithm previously described; where $n = 0, 1, 2, ...$, denotes the current iteration of the algorithm.

- *Step 2*: Within Ω_n, solve for the optimal values of $(a, k_1', k_2', w_1', w_2')$ by finding the optimum of the current expected utility of the principal with respect to those decision variables, subject to the resource constraints.

- *Step 3*: Compute the optimal compensation for the risk-neutral agent for each realization of θ, using the optimal values of $(k_1', k_2', w_1', w_2', a)$, and the following simplification of the incentive compatibility and participation constraints:

$$\begin{aligned}
\sqrt{c_1^*} &= w_S - \beta w_1'^* + k_T(1 + a^* - \exp(a^*)) \\
\sqrt{c_2^*} &= w_S - \beta w_2' + k_T(1 + a^*)
\end{aligned}$$

- *Step 4*: Compute:

$$\begin{aligned}
v_{n(T,S)}^2 &= [\theta_1 k_T^\epsilon - c_1^* - k_1'^* + \beta\widehat{V^2}(k_1'^*, w_1'^*; e_n^2)]\exp(-a^*) \\
&+ [\theta_2 k_T^\epsilon - c_2^* - k_2'^* + \beta\widehat{V^2}(k_2'^*, w_2'^*; e_n^2)](1 - \exp(-a^*)) + (1-\delta)k_T \quad \forall T\ , \forall S
\end{aligned}$$

- *Step 5:* Update the set of parameters $e_{n+1} \in \Re^{N1\times N2}$such that $\widehat{V^2}(k, w; e_{n+1}^2)$ approximates the (v_n^2, k, w) data.

- *Step 6*: If $\left|\widehat{V^2}(k, w; e_{n+1}^2) - \widehat{V^2}(k, w; e_n^2)\right| < \pi$, then stop; else go to Step 1.

As a benchmark case, we also compute the solution of a standard dynamic agency model with no capital accumulation, using the specifications described above. The resource constraints in this model set upper bounds to the current compensation of the agent in the following fashion:

$$0 \leqslant c \leqslant \theta_i f(\widehat{k}), \quad i = 1, 2$$

where $\widehat{k}$ is some chosen capital value, so that we will be able to keep using the production function. However, we now assume that there is neither depreciation nor accumulation of capital.

We use again a parametric approach to value function iteration, and cubic splines as the approximating scheme of the value function $\widehat{V^3}(w; e^3)$, where $e^3 \in \Re^{N2}$ are the parameters that characterize the spline. The following algorithm summarizes the procedure:

Algorithm 4:

- *Initialization*: Make initial guess $\widehat{V^3}(w; e_0^3)$, and choose stopping criterion $\pi > 0$. Create a one-dimensional approximation grid with $N2$ equidistant points computed over the range $[w_{\min}, w_{\max}(k = k_{\max})]$, where $w_{\min}, w_{\max}(k = k_{\max})$ are as defined in the above algorithm. We denote the space of w that contains those values as Ω_0^3.

- *Step 1*: For each point in the one-dimensional grid over Ω_0^3, compute the set of admissible points Ω_n using the specialization of the Abreu, Pierce, Stacchetti's algorithm previously described (and making it one-dimensional, as in Wang (1997)); where $n = 0, 1, 2, ...,$ denotes the current iteration of the algorithm.

- *Step 2*: Within Ω_n, solve for the optimal values of (w_1', w_2', a) by finding the optimum of the current expected utility of the principal with respect to those decision variables, subject to the resource constraints.

- *Step 3*: Compute the optimal compensation for the risk-neutral agent for each realization of θ, using the optimal values of (w_1', w_2', a), and the following simplification of the incentive compatibility and participation constraints:

$$
\begin{aligned}
\sqrt{c_1^*} &= w_S - \beta w_1'^* + \widehat{k}(1 + a^* - \exp(a^*)) \\
\sqrt{c_2^*} &= w_S - \beta w_2' + \widehat{k}(1 + a^*)
\end{aligned}
$$

- *Step 4*: Compute:

$$
\begin{aligned}
v_{n(S)}^3 &= [\theta_1 \widehat{k}^\epsilon - c_1^* + \beta \widehat{V^3}(w_1'^*; e_n^3)] \exp(-a^*) \\
&\quad + [\theta_2 \widehat{k}^\epsilon - c_2^* + \beta \widehat{V^3}(w_2'^*; e_n^3)](1 - \exp(-a^*)) \ \forall S
\end{aligned}
$$

- *Step 5:* Update the set of parameters $e_{n+1} \in \Re^{N2}$such that $\widehat{V^2}(w; e_{n+1}^3)$ approximates the (v_n^3, w) data.

- *Step 6*: If $\left|\widehat{V^3}(w; e_{n+1}^3) - \widehat{V^3}(w; e_n^3)\right| < \pi$, then stop; else go to Step 1.

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