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# How Important is Optimal Fiscal Policy under Quasi-Geometric Discounting?

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## Abstract

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*A neoclassical model with distorting taxes on capital and labor income is presented where a representative agent is endowed with quasi-geometric discounting and derives utility from consumption and leisure. Given that full commitment is preferred to the no-commitment equilibrium, optimal fiscal policy is introduced in order to examine to what extent a benevolent planner may help the individual to overcome her lack of commitment. Numerically it is found that optimal taxation only provides between 8 and 25 percent of the payment necessary for the agent to be indifferent between the full and the no-commitment allocation.*

## Resumen

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*Este artículo presenta un modelo neoclásico con impuestos al capital y al trabajo donde un agente representativo exhibe preferencias con descuento cuasi-geométrico y obtiene utilidad por consumo y ocio. Dado que el equilibrio con compromiso pleno es preferido al equilibrio sin compromiso, se introduce una política fiscal óptima con el objeto de examinar en qué medida un planificador benevolente puede ayudar al individuo a superar su problema de falta de compromiso. De forma numérica se halla que los impuestos óptimos sólo proveen entre el 8 y 25 por ciento del pago necesario para que el individuo sea indiferente entre el equilibrio sin y con pleno compromiso.*



## Introduction

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Recently the literature has devoted particular attention to situations where individuals have quasi-geometric preferences of the type first studied by Strotz (1956) and recently re-examined by Laibson (1997), Barro (1999), Krusell et al. (2002, 2000), among others. The idea in general is that the subjective rate of time preference is non-constant over time. In particular, Laibson (1997) argues that individuals are usually impatient about consuming between today and tomorrow but are more patient about consumption choices in the distant future. In other words, as of today agents have a time-varying rate of time preference that is typically very high in the short run but much lower in the long run. As is well known, quasi-geometric discounting can create a time-consistency problem. The reason is that the individual typically values her utility flow differently as the planning date evolves. Hence, choices taken sequentially by the household will usually differ from those taken under full commitment.

Under this framework, a household with quasi-geometric discounting would definitively be better off should she had the ability to commit her choices. If this is not the case, the question is whether a benevolent planner with taxation abilities may solve the lack-of-commitment problem of the household. In a recent paper, Krusell et al. (2002) find that if the household has quasi-geometric discounting and the planner is benevolent, the closed-form (interior) solution under the decentralized allocation under no commitment yields a higher welfare as compared to the planner's allocation, even when the planner is endowed with taxation abilities and optimal taxes are time-consistent.<sup>1</sup> In other words, optimal taxation in the model cannot improve welfare and thus it is better to not have government at all. Once an elastic labor supply is taken into account and taxation abilities may be restricted, Krusell et al. (2000) find cases under which a restricted optimal tax policy may yield a higher welfare as compared to an economy with no taxes. However, in some other cases such restricted optimal tax policy yields the lowest welfare among all the tax constitutions. Moreover, multiple equilibria may be obtained when capital and labor income taxes are available. In the end, it is not possible to conclude that the restricted optimal tax policy always improves upon the decentralized allocation.

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<sup>1</sup> The explanation is that the household takes future prices as given whereas the planner may affect them at the time of making her choices. Hence for an increase in savings today, the planner perceives a lower return in the future so she saves less today compared to the household's solution. When there is short-run impatience, higher savings is desirable because the no-commitment allocation moves closer to the full commitment outcome.

Krusell and Smith (2003) further point out that the previous results in Krusell et al. (2002, 2000) should be interpreted with caution. In particular, the authors show that step-function decision rules in a recursive framework with quasi-geometric discounting may yield higher levels of lifetime utility than does the closed-form (interior) solution. In other words, an interior solution in such a case does not guarantee in general the maximum level of utility, an observation also supported by Maliar and Maliar (2003) for the case of smooth, log-linear decision rules in a recursive framework.

To circumvent these problems, this paper takes an alternative approach to Krusell et al. (2002, 2000) in order to evaluate the importance of optimal fiscal policy when the household is endowed with quasi-geometric discounting and is unable to commit. Namely, optimal tax policy is not determined recursively but in the sense of Ramsey (1927). The framework proposed is the model of Barro (1999) extended for a labor-leisure choice and distorting taxes on capital and labor income necessary to finance an exogenous stream of government spending. Given that preferences are logarithmic, it is found that if the infinitely lived household is impatient about consuming today, is unable to commit her decisions and fully appreciates the effects of her actions on her future behavior, then the household conveniently chooses to consume a constant fraction of her wealth at each point in time as the solution to her inconsistency problem. It turns out that this fraction is in fact the new subjective discount rate of the household, which is usually higher than the standard rate. Remarkably, the solution is also time-consistent: if choices are set in this manner at all future dates, then it is optimal for the household to make choices in this way at the current date.

In order to evaluate the importance of optimal taxation in this framework, three alternative economies are defined. The first of them is called the “full commitment” economy in the sense that the household with quasi-geometric preferences is unexpectedly endowed with an ability to commit from time zero on. The household thus chooses her allocation given the (constant) sequence of existing distorting taxes. The second economy (the “no commitment” economy) examines the case where the household cannot commit her choices given the existing taxes. Finally, the “Ramsey economy” is simply the solution to the Ramsey problem when the household is unable to commit. As discussed later, the full commitment equilibrium is the allocation that yields the highest utility among all. Using a welfare criterion, the paper is thus interested in evaluating the importance of optimal taxation for moving the no commitment economy closer to the full commitment economy.

The Ramsey problem in this context is solved using the primal approach in the usual fashion. First, the household with quasi-geometric discounting and

no ability to commit solves her problem in a time-consistent way. Second, taking into account the time-consistent solution by the household (including the associated higher discount rate), the benevolent planner implements an allocation consistent with a competitive equilibrium. As is standard in this type of models, it is assumed that the planner can commit to a sequence of allocations announced at time zero. In addition, the initial tax on capital income is assumed exogenous in order to avoid a capital levy that eventually replicates the first-best outcome obtained with lump-sum taxes. Under these restrictions, the Ramsey allocation thus obtained is time-consistent and improves upon the original allocation with no commitment. Although this structure might seem controversial, it provides a useful benchmark for evaluating optimal tax policy and welfare under quasi-geometric discounting in the sense that it avoids the ambiguous welfare results mentioned above.<sup>2</sup>

Using numerical methods, paths for capital, leisure and consumption are computed for each of the three economies. The advantage of this approach is that welfare may be estimated taking into account the entire transition path.<sup>3</sup> Simultaneously, it provides an additional insight about how each economy works. Paths for optimal taxes are also obtained. In particular, it is found that the optimal tax schedule features a zero tax on capital income not only at the steady-state but for most of the transition path.<sup>4</sup> This simply resembles the optimality result first posed by Chamley (1986) that capital should not be taxed in the long run. Given the sequence of government expenditures, optimal taxes on labor income are positive throughout the transition as well as in the steady state so that the budget constraint of the government is intertemporally balanced.

In this context, the optimal fiscal policy set by the planner improves household's welfare when she is not able to commit. However, when comparing this welfare gain to the gain the household could obtain should she had the ability to commit, it turns out that the planner can only provide between 8 and 25 percent of the payment necessary for the household to be indifferent between the full and the no-commitment allocation. Given that these results are valid assuming a commitment technology for the planner, a partial relaxation of this assumption would make these numbers even lower.

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<sup>2</sup> In any case, the results obtained under this framework would give an upper bound estimate for the welfare gain provided by the optimal fiscal policy. The ability to commit by a planner with quasi-geometric discounting is also assumed by Krusell et al. (2000) in the sense that the planner can commit to a tax constitution.

<sup>3</sup> As illustrated for example by Lucas (1990) and Ortigueira (1998), ignoring transitional dynamic effects may substantially overestimate welfare gains.

<sup>4</sup> This result is different to the one obtained by Krusell et al. (2000). In their model, the authors find that the time-consistent optimal policy when investment taxes are not allowed is such that labor income is taxed and capital income is subsidized when the household is impatient. This result is partially explained by their assumption of zero government expenditures: a positive tax on labor income must necessarily be accompanied by a subsidy to capital income in order to balance the budget constraint of the government.

Thus in practice the planner in this model is far from solving the no commitment problem of the household in a significant way.

The remaining of the paper is as follows. Section 2 discusses the model under the alternative scenarios of full and no commitment, and defines the Ramsey problem when the household is unable to commit. Section 3 calibrates the model and numerically finds the paths for capital, leisure and consumption for each economy, as well as the paths for optimal taxes. Next, welfare estimates including the transition path are found and optimal tax policy is evaluated. Section 4 concludes.

## 2. The Model

### 2.1 The Environment

Consider a standard neoclassical, deterministic exogenous growth model with infinite horizon where a representative household is endowed with perfect foresight and a single unit of time. In this framework, time may be devoted either to leisure or production activities. The household derives utility from per capita consumption  $c(t)$  and raw leisure  $x(t)$ , where  $0 \leq x(t) \leq 1$ . For simplicity, the instantaneous utility function  $u(c(t), x(t))$  is continuously differentiable, strictly increasing, strictly concave and separable in its arguments.

Following Strotz (1956), Laibson (1997) and Barro (1999), it is assumed that the household has a variable rate of time preference over time. In particular, the representative agent is impatient about consuming today but is more patient about choices in the distant future. This idea is captured by the following specification of preferences as of current date  $s$ :

$$U(s) = \int_s^{\infty} u(c(t), x(t)) \exp[-(\rho \cdot (t-s) + \phi(t-s))] dt \quad (1)$$

where  $\rho > 0$ . The difference with respect to the standard model is given by the term  $\phi(t-s) \geq 0$ . This expression is a function of the distance in time  $v \equiv t-s \geq 0$ , and captures the idea of a variable time preference. Following Laibson (1997), the function  $\phi(\cdot)$  is taken to be continuous and twice differentiable with the properties  $\phi'(v) \geq 0$ ,  $\phi''(v) \leq 0$ , and  $\phi'(v)$  approaches zero as  $v$  tends to infinity. For convenience, this function is normalized so



time preference at the time distance  $v$ . From the properties of  $\phi(\cdot)$ , it follows that the rate of time preference is high in the near term but roughly constant at the lower value  $\rho$  in the distant future. Finally, the model assumes that the household fully appreciates how her actions today affect her future behavior. In other words, the household is fully rational in this context.

The rest of the exposition is standard. Namely, it is required that the household satisfies the following present-value budget constraint:

$$\int_s^{\infty} [c(t) - \tilde{w}(t)(1-x(t)) - T(t)] \exp \left[ \int_s^t (\tilde{r}(v) - \delta) dv \right] dt \leq k(s) \quad (2)$$

for a given level of capital  $k(s)$ . In expression (2),  $\tilde{r}(t) \equiv (1 - \tau_k(t))r(t)$  and  $\tilde{w}(t) \equiv (1 - \tau_n(t))w(t)$  are the real rate of return on physical capital and the real wage, respectively, both expressed net of their corresponding taxes  $\tau_k(t)$  and  $\tau_n(t)$ , and  $T(t)$  denotes transfer payments. The stock of capital  $k(t)$  depreciates at the constant rate  $\delta$ .

Firms are perfectly competitive. Technology is represented by a production function  $F[k(t), 1-x(t)]$  with constant returns to scale in the stock of capital and the time devoted to working activities  $1-x(t)$ . The function  $F(\cdot)$  is continuously differentiable, increasingly monotone, concave, and satisfies well-known Inada conditions. Profit maximization implies that both factors of production are paid their marginal products. If  $F_i(t)$  denote the marginal product of factor of production  $i = (k, n)$ , then it must be the case that

$$w(t) = F_n[k(t), 1-x(t)] \quad (3a)$$

$$r(t) = F_k[k(t), 1-x(t)] \quad (3b)$$

The single good in this economy may be devoted either to private consumption, investment or government purchases of goods and services  $G(t)$ , which is exogenously given. The market clearing condition is thus described by:

$$c(t) + \dot{k}(t) + \delta k(t) + G(t) = F[k(t), 1-x(t)] \quad (4)$$

Thus a competitive equilibrium for this economy may be defined in the usual fashion. Note that expressions (2)-(4) together imply that the

intertemporal government budget constraint is satisfied.

It may be readily shown that among the conditions to be fulfilled is that the marginal rate of substitution between consumption at dates  $s$  and  $t > s$  must equal the relative prices of these two goods:

$$\frac{u_c(t) \exp[-(\rho \cdot (t-s) + \phi(t-s))]}{u_c(s)} = \exp\left[-\int_s^t (\tilde{r}(v) - \delta) dv\right] \quad (5)$$

where  $u_i(t)$  denotes the derivative with respect to the  $i$ th argument,  $i = (c, x)$ . In addition, the marginal rate of substitution between leisure and consumption must be equal to the after-tax real wage:

$$\frac{u_x(t)}{u_c(t)} = \tilde{w}(t) \quad (6)$$

Given the separability assumption between consumption and leisure, equation (5) implies that consumption evolves through time according to:

$$-\frac{d \ln u_c(t)}{dt} = \tilde{r}(t) - \rho - \delta - \phi'(t-s) \quad (7)$$

The above expression allows to figure out the time-consistency problem of the household more clearly: the utility-maximizing path for  $c(t)$  implied by (7) holds for any arbitrary date  $s$ . In particular, suppose that the household initially chooses her consumption plan at time  $s$ . However, if the household decides to revise her plan at a later date (say,  $s' > s$ ) then the initial plan (chosen at time  $s$ ) does not longer maximize utility viewed as of time  $s'$ . Hence the representative consumer is faced with a time-consistency problem (cf. Strotz (1956)). Notice that this particular problem does not arise under a standard model in which  $\phi(t-s) = 0$  for all  $t \geq s$ .

As pointed out by Barro (1999), the solution to the time-inconsistency problem of the household depends on whether she is able to fully commit her decisions on consumption and leisure at the present time  $s$ . Therefore, the next sections discuss the implications for each behavior in detail.

## Results under Commitment

Consider for a moment the simplest case in which the household with quasi-geometric discounting is able to commit her decisions on current and future allocations for every  $t \geq s$ . This implies that the sequence of allocations originally chosen today is not changed over time. So when the future arrives and the household decides to re-evaluate her original consumption-leisure plan, it simply abides by the original plan. In other words, there is no internal conflict between today's self and tomorrow's self. The household is able to do so either irrevocably or by imposing a penalty for her future self should she misbehave. Examples of commitment include voluntary openings of Christmas Clubs accounts or the adoption of retirement plans that impose a penalty on early withdrawals.

As explained in more detail in Barro (1999), if perpetual commitments are feasible, then these commitments are likely to exist in the past, even in the infinite past. Hence, current and future allocations would have been chosen at an arbitrary date  $s$ , where  $s$  would be in fact equal to minus infinity. Therefore,  $\phi'(t-s)$  in equation (7) would be just zero for all  $t \geq 0$  and the rate of time preference would equal  $\rho$  for all  $t \geq 0$ .<sup>5</sup> Thus the model simply reduces to the standard case, including the steady state. Of course, another possibility is that the household suddenly and unexpectedly obtains the ability at a *finite* date  $s$  to commit her choices of consumption and leisure for all future dates. This possibility will be discussed in more detail later in the paper.

## Results under No Commitment

Under more realistic grounds, full commitment is not always feasible. For this reason, this section departs from the previous assumption that the household is able to fully commit her decisions on consumption and leisure. Under a no-commitment scenario, it is possible for the future household to adopt a path different from the path originally chosen. This is not because consumer's preferences have changed in any unexpected way or because the information available is now different, but simply because the representative consumer is aware that she will be a different person in the future endowed with a new discount function. In this case, changing the original plan in the future has implications for the whole sequence of allocations from that date on. Therefore, it is important to figure out how the decision on  $c(s)$  at time  $s$

<sup>5</sup> Strotz (1956) actually shows that a necessary and sufficient condition for commitment is that the instantaneous rate of time preference must be constant over time.

affects the stock of assets and how this change in assets alters the choices of consumption in the future.<sup>6</sup>

Given the separability assumption between consumption and leisure, the following logarithmic utility function is adopted for simplicity:

$$u(c(t), x(t)) = \psi \ln c(t) + (1 - \psi) \ln x(t) \quad (8)$$

where  $0 < \psi \leq 1$  is a parameter that measures the share of consumption in total utility. In a model with consumption only and without distorting taxes, Barro (1999) finds that the solution to the problem with no commitment is such that the household must consume a constant fraction of her wealth at each date. As shown in the appendix, in the present model this fraction (which turns out to be constant given the specification on preferences) is expressed by the value of  $\lambda$  that satisfies:

$$\lambda = \frac{1}{\int_0^{\infty} \exp[-\rho v + \phi(v)] dv} \quad (9)$$

Equation (9) may alternatively be expressed as:

$$\lambda = \frac{\int_0^{\infty} [\rho + \phi'(v)] \exp[-(\rho v + \phi(v))] dv}{\int_0^{\infty} \exp[-(\rho v + \phi(v))] dv} \quad (10)$$

since the numerator in (10) is equal to unity. Notice that expression (9) reduces to  $\lambda = \rho$  for the standard case in which  $\phi(v) = 0$  for all  $v$ . From (10),  $\lambda$  may be interpreted as a time-invariant weighted average of the instantaneous rates of time preference  $\rho + \phi'(v)$ . From the properties of the  $\phi(\cdot)$  function, it follows that  $\rho \leq \lambda \leq \rho + \phi'(0)$ . In other words, the fraction  $\lambda$  of wealth chosen by the household has a value between the long-run rate of time preference  $\rho$  and the short-run, instantaneous rate  $\rho + \phi'(0)$ . Alternatively, since  $\lambda \geq \rho$  the time-inconsistent household cannot be more patient than her full committed self.

<sup>6</sup> From the first-order conditions of the household, it is possible to define leisure explicitly in terms of consumption. Hence the no commitment problem may be reduced to the analysis of consumption decisions only.

The solution to the no-commitment problem provided by (9) implies that consumption must now evolve through time according to:

$$\frac{\dot{c}(t)}{c(t)} = \tilde{r}(t) - \delta - \lambda \quad (11)$$

From previous discussion, it is readily noticed that if expression (11) holds, it yields a solution that is time-consistent from the point of view of the representative consumer. In other words, if  $c(t)$  is chosen as the fraction  $\lambda$  of wealth at all future dates, then the household will also choose present consumption in the same way in order to maximize her utility. In such a case, the marginal rate of substitution between consumption at dates  $s$  and  $t > s$  is now given by:

$$\frac{u_c(t) \exp[-\lambda \cdot (t-s)]}{u_c(s)} = \exp\left[-\int_s^t (\tilde{r}(v) - \delta) dv\right] \quad (12)$$

Finally, it may be easily verified that the marginal rate of substitution between consumption and leisure is still given by expression (6).

### Comparing Full versus No-Commitment Solutions in the Model with Taxes and Leisure

As it may be readily inferred from above, as long as  $\lambda > \rho$  the full commitment solution when the ability to commit is acquired in the infinite past yields a higher path of consumption and capital over time as compared to the solution with no commitment. A more interesting picture emerges if it is rather assumed that the household with quasi-geometric preferences suddenly and unexpectedly obtains the ability at some finite date  $s$  to commit her allocations for all future dates. Thus equation (7) is satisfied for  $t > s$ . If this is the case, the new full commitment framework yields an asymptotic constant rate of time preference  $\rho$ . In other words, the new full commitment regime still features a higher steady state for consumption and capital as compared to the no-commitment solution with  $\lambda > \rho$ .

The difference between these two solutions arises in the short-run. Once the household is unexpectedly able to commit her decisions forever on starting at time  $s$ , her rate of time preference at date  $s$  in fact rises from  $\lambda$  to  $\rho + \phi'(0)$ . In other words, the unexpected commitment ability initially makes

households less patient. From that date on, the instantaneous rate of time preference declines steadily and asymptotically approaches  $\rho < \lambda$ .

As described by Barro (1999), the introduction of full commitment at a finite date  $s$  has the effect of raising  $c(s)$  (and thus leisure at time  $s$ ). This behavior is possible by forcing some future selves to save and work more than under no commitment. Since the household is able to commit future saving and hours worked,  $k(t)$  eventually rises above the no commitment level despite the initial increase in both consumption and leisure at date  $s$ . This behavior will be later confirmed in the numeric section below.

Since the (unexpected) full commitment allocation is preferred to the allocation with no commitment in a model with constant distorting taxes like the one presented here, the issue of interest now is to analyze whether optimal fiscal policy may alleviate the household's lack of commitment in a significant way. In other words, the question is whether optimal fiscal policy may move the economy under no commitment sufficiently close to the solution under (unexpected) full commitment.<sup>7</sup> Before trying to tackle this problem, it is useful to describe first the optimal fiscal policy of the model.

### 2.1.1 *The Second-Best Problem*

The purpose of this section is to characterize a solution to the optimal taxation problem in terms of Ramsey (1927). The method chosen is the primal approach whereby the benevolent planner announces a feasible allocation (subject to relevant constraints) that is consistent with the optimizing behavior of private agents. This method may be roughly implemented through the following steps. First, the household and firms solve their maximization problem taking factor prices and government policy as given. Prices and taxes are then solved in terms of the corresponding allocation so that the intertemporal constraint of the household may be expressed in terms of quantities only (the so called "implementability constraint"). Finally, the planner solves for the Ramsey allocation by maximizing utility subject to the implementability constraint and the feasibility constraint. Prices and taxes consistent with a competitive equilibrium may then be recovered from the previous step.

As mentioned in the previous section, the analysis for the optimal tax policy is restricted to situations where the household is not able to commit

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<sup>7</sup> As discussed later, the answer to this question is not trivial. For example, the steady state level of capital under the optimal tax policy may be or may be not larger than the corresponding level under the full commitment economy due to taxation and impatience effects moving in opposite directions.

her decisions. As discussed above, in such a case the household solves her problem in a time consistent way by consuming a constant fraction  $\lambda$  of her wealth. This is equivalent to say that the instantaneous rate of time preference of the household is now given by  $\lambda > \rho$ . Since the planner is benevolent and subject to the same time-inconsistency problem, she discounts instantaneous utility at the same rate  $\lambda$ .<sup>8</sup>

In order to define the second-best problem of the planner, prices and taxes need to be eliminated from the household's program so that the planner's problem may be expressed in terms of quantities only. Plugging (6) and (12) into the household's budget constraint (2) yields the familiar implementability constraint, expressed in terms of quantities only:

$$\int_s^{\infty} \exp[-\lambda \cdot (t-s)] [u_c(t)c(t) - u_x(t)(1-x(t)) - u_e(t)T(t)] dt = u_c(s)k(s) \quad (13)$$

According to the discussion above, as of time  $s$  the *Ramsey problem* is thus described by

$$\max_{c(t), x(t)} \int_s^{\infty} u(c(t), x(t)) \exp[-\lambda \cdot (t-s)] dt \quad (P)$$

subject to the implementability constraint (13) and the feasibility constraint (4). As is standard in the literature, the government is assumed to have a commitment technology that binds a particular sequence of allocations announced at time  $s$ . To make this exercise interesting, the initial tax rate on capital income  $\tau_k(s)$  is taken as given in order to avoid a capital levy at time  $s$ .<sup>9</sup> Overall, these assumptions are imposed to guarantee that the solution announced by the planner is time-consistent.

For convenience, define the function

$$W(c(t), x(t), \gamma_1) = u(c(t), x(t)) + \gamma_1 [u_c(t)c(t) - u_x(t)(1-x(t)) - u_e(t)T(t)] \quad (14)$$

where  $\gamma_1 \geq 0$  is the time-invariant, Lagrange multiplier associated with the implementability constraint (13). Notice that  $\gamma_1$  may be interpreted as the

<sup>8</sup> Recall that the problem is solved backwards. So the assumption here is that the household first solves her problem in a time-consistent manner. Then the planner solves her own problem by taking into account the new discount factor  $\lambda$  of the household as implied by the time-consistent solution of the previous step.

<sup>9</sup> An additional restriction is that the tax rate on capital at time  $s$  must be bounded by above. Otherwise, investment at time  $s$  may be zero. See Jones et al. (1993) for a further discussion.

marginal excess burden of taxation: it is strictly positive should the planner have to rely on distorting taxes and zero otherwise.<sup>10</sup> The Ramsey problem is thus conveniently reduced to solving the following Hamiltonian  $H$  as of time  $s$ :

$$H = e^{-\lambda(t-s)} \{W(c(t), x(t), \gamma_1) + \gamma_2(t)[F(k(t), 1 - x(t)) - c(t) - \delta k(t) - G(t)]\} - \Lambda \quad (15)$$

where  $\Lambda \equiv \gamma_1 u_c(s)k(s)$  and  $k(s) = k_s$  is given. Here,  $\gamma_2(t) > 0$  denotes the marginal social value of goods. It is important to remark that, since the constraint (13) faced by the household is already taken into account in the Ramsey problem, the allocation announced by the planner (the Ramsey equilibrium) will be consistent with the allocation that would be chosen by utility-maximizing agents.

It may be readily verified that the Ramsey equilibrium must satisfy the following first-order conditions:

$$W_c(c(t), x(t), \gamma_1) = \gamma_2(t), \quad t > s \quad (16a)$$

$$W_x(c(t), x(t), \gamma_1) = \gamma_2(t)F_n(t), \quad t \geq s \quad (16b)$$

$$W_c(c(s), x(s), \gamma_1) = \gamma_2(s) + \gamma_1 u_{cc}(s)k(s), \quad t = s \quad (16c)$$

$$\dot{\gamma}_2(t) = \gamma_2(t)[\lambda + \delta - F_k(t)], \quad t \geq s \quad (16d)$$

plus the standard transversality condition for the stock of capital. Here,  $W_i(\cdot)$  denotes the derivative with respect to the  $i$ th argument,  $i = (c, x)$ .

The computation of the Ramsey equilibrium proceeds as follows: Suppose for a moment that the value for  $\gamma_1$  is known. Then the feasibility constraint plus the system (16) pin down the whole sequence for  $c(t)$ ,  $x(t)$ ,  $k(t)$  and  $\gamma_2(t)$  for  $t \geq s$ . Labor and capital income taxes may then be recovered from (6) and (12), respectively, whereas factor prices are given by (3). Finally,  $\gamma_1$  and the equilibrium allocation must be such that the implementability constraint (13) is satisfied for a given initial value  $k(s) = k_s$ .

<sup>10</sup> To see this more clearly, one may think of a first-best model in which the planner maximizes the utility of the household subject only to the feasibility constraint. This model is thus equivalent to setting  $\gamma_1 = 0$  in expression (15) below.



income.<sup>11</sup> This is just the standard optimality result first studied by Chamley (1986). On the other hand, it may be shown that the optimal tax on labor income is positive in general and depends on the value of the Lagrange multiplier  $\gamma$ . As it is found later numerically, the planner chooses to tax labor income heavily not only at the steady state but throughout the entire transition path in order to finance her consumption requirements while leaving capital income mostly untaxed.

### 3. A Numeric Characterization of Equilibrium Paths

#### 3.1 Preliminaries

So far the paper has described three alternative economies: the full commitment economy, the no-commitment economy and the optimal taxation economy under no commitment (the "Ramsey economy"). The next step is thus to numerically characterize the allocation paths for each of these economies. As described later, this approach will provide to be useful at the time of comparing welfare among the different allocations.

Before estimating the model numerically, functional forms for the subjective discount factor  $\phi(v)$  and technology need to be chosen. As noted earlier, the instantaneous rate of time preference  $\rho + \phi'(v)$  reflects short-term impatience if it is high when  $v$  is small, and declines gradually to  $\rho$  as  $v$  becomes large. Following Barro (1999), a functional form that captures this idea is given by:

$$\phi'(v) = be^{-\zeta v} \quad (18)$$

where  $b = \phi'(0) \geq 0$  and  $\zeta > 0$  denotes the constant rate at which  $\phi'(v)$  declines from  $\phi'(0)$  to zero. Integration of (18) along with the boundary condition  $\phi(0) = 0$  yield an expression for  $\phi(v)$ :

$$\phi(v) = (b/\zeta)(1 - e^{-\zeta v}) \quad (19)$$

This equation may be substituted into (9) in order to get a numeric value for  $\lambda$ . From (19), it may be shown that either a higher  $b$  or a lower  $\zeta$  yield a

<sup>11</sup> To see this, simple manipulations of expressions (11), (16a) and (16d) evaluated at the steady state yield this result.

higher value for  $\lambda$  (i.e., more impatience). For simplicity, technology is described by a standard Cobb-Douglas production function of the form  $F[k(t), 1 - x(t)] = Ak(t)^\alpha (1 - x(t))^{1-\alpha}$  with  $A > 0$  and  $0 < \alpha \leq 1$ . Preferences are still defined by (8) for each of the three economies.

Parameters related to quasi-hyperbolic preferences are determined as follows. According to Laibson's (1997) observations, the parameter  $\zeta$  must be at least 0.50 per year whereas  $b = \phi'(0)$  must be around 0.50 per year. Overall these values imply that  $\phi'(v)$  gets close to zero a few years in the future. Since reliable point estimates for  $b$  and  $\zeta$  are unknown, the numeric analysis below allows for alternative values of  $b$  and  $\zeta$ . In the particular case  $\rho = 0.03$ ,  $b = 0.50$  and  $\zeta = 0.75$ , the value for  $\lambda$  is about 0.057 according to expression (9).

The remaining calibration of the model is made so that parameter values are consistent with the long-run observations for the U.S. economy. First, the values  $A = 1$ ,  $\alpha = 0.33$  and  $\delta = 0.05$  are conveniently chosen:  $A$  is simply normalized and the last two of them are roughly standard in the literature. In the model, government expenditures  $G(t)$  and transfer payments  $T(t)$  are assumed to be a fixed proportion of total output at each period, as in Jones et al. (1993). From time-series observations, the share of  $G(t)$  with respect to total output is set at 0.21. The share of transfers in total output is determined following the observations made by Jones et al. (1993), so this number is set at 0.07. In addition, the tax on capital income is fixed at its historical average value. This value turns out to be  $\tau_k = 0.43$  according to the estimations provided by Mendoza et al. (1994).

It still remains to set values for  $\rho$ ,  $\psi$  and  $\tau_n$ , which are determined simultaneously. In particular,  $\rho = 0.03$  is fixed to yield a steady-state capital-output ratio under the full commitment economy of about 2.4.<sup>12</sup> For the particular case  $\rho = 0.03$ ,  $b = 0.50$  and  $\zeta = 0.75$ , the share of consumption  $\psi$  in the instantaneous utility function is set at 0.38 so that the household with no commitment allocates about one-third of her endowed time to working activities at the steady state. Finally, the tax on labor income is fixed at 0.19 in order to balance the government budget constraint of the economy with no commitment at the steady-state.

The algorithm for the numeric computation of the Ramsey system described by equation (16) deserves to be described in some detail. First, an

<sup>12</sup> Recall that neither the no-commitment nor the Ramsey economy are determined by  $\rho$  at the steady state. On the other hand, the steady state of the full commitment economy does not depend on the time-variant discount factor  $\phi(v)$ .

initial guess for the value of the marginal excess burden of taxation  $\gamma_1$  is made so that the system may be solved numerically. Once the entire sequence  $\{c(t), x(t), k(t), \gamma_2(t)\}$  is obtained, it is checked whether the implementability constraint (13) is satisfied. If it is not (as it might be expected), a new guess for the value of  $\gamma_1$  is made. This new value defines a new sequence, and the implementability constraint is evaluated once again and so on. Hence this algorithm continues until condition (13) is met. The resulting allocation is then used to recover the sequence of optimal taxes, according to equations (6) and (11).

## Results

The goal now is to describe the properties for each of the three models (the full commitment, the no commitment and the Ramsey economies) both at the steady state as well as along the transition path. To fix ideas, a full commitment economy is understood here as the sudden and unexpected ability acquired by the household to commit her allocations for all future dates, starting from some finite date  $s$ . For all the exercises below, such a finite date  $s$  is equivalent to the initial period of analysis which is conveniently normalized to zero. With respect to notation,  $k_{FC}^*$  for example denotes the steady state capital stock under full commitment, and  $k_{NC}^*$  and  $k_{OP}^*$  the corresponding steady state values under no commitment and the Ramsey policy, respectively, and so on. In terms of exposition, it is convenient for a moment to restrict the numeric analysis to the full and no commitment economies. The Ramsey economy is described later in detail.

Table 1 presents the steady state estimates of capital, leisure and consumption for each of these economies under alternative values of  $b$  and  $\zeta$ . The third column yields the corresponding value for the discount rate  $\lambda$  that applies under no commitment. As discussed before, the allocation under full commitment is not affected by either  $b$  or  $\zeta$ . This is reflected in the columns three to six which show the steady state values for capital, leisure and consumption under full commitment. Since  $\lambda > \rho$  for all the parameter values considered, the steady state estimates for capital and consumption (leisure) under full commitment are larger (smaller) than their corresponding values under no commitment. As for the economy with no commitment in isolation, higher values of  $\lambda$  (higher impatience) imply lower values for the capital stock and consumption in the long-run.

**Table 1**  
**Steady State Estimates**  
**For the Full Commitment, No Commitment and Ramsey Economies**

$b$	$\zeta$	$\lambda$	$k_{FC}^*$	$x_{FC}^*$	$c_{FC}^*$	$k_{NC}^*$	$x_{NC}^*$	$c_{NC}^*$	$k_{OP}^*$	$x_{OP}^*$	$c_{OP}^*$
0.10	0.50	0.036	1.173	0.673	0.340	1.040	0.675	0.329	2.160	0.709	0.343
0.25	0.50	0.048	1.173	0.673	0.340	0.849	0.679	0.311	1.743	0.716	0.326
0.50	0.50	0.076	1.173	0.673	0.340	0.572	0.686	0.278	1.152	0.727	0.294
0.10	0.75	0.034	1.173	0.673	0.340	1.082	0.674	0.333	2.251	0.707	0.346
0.25	0.75	0.041	1.173	0.673	0.340	0.949	0.677	0.321	1.959	0.712	0.336
0.50	0.75	0.057	1.173	0.673	0.340	0.742	0.682	0.299	1.511	0.720	0.315

Transitional dynamics for capital, leisure and consumption under full and no commitment are shown by the solid and dashed lines in figures 1 to 3 for the particular case  $b = 0.50$  and  $\zeta = 0.75$ .<sup>13</sup> An important thing to notice is the sudden increase in full committed leisure and consumption at time zero compared to the no commitment path, followed by a temporary smooth fall. However, this fall is short-lived (about 1.3 and 0.6 years, respectively) and the variables smoothly increase thereafter towards their corresponding steady-state values. Not surprisingly, capital stock under full commitment is temporarily below its corresponding path under no commitment. These observations are just consistent with the discussion above.

The following step is to quantify how large is the difference in welfare between these two economies because this number would give an idea about the potential for optimal tax policy to improve welfare. In order to give a reasonable answer to this issue, the method suggested by Lucas (1987) is followed. For the model under study, a welfare gain is estimated as the constant consumption supplement  $\mu$  under no commitment so that the household is indifferent between the full and the no commitment allocation, namely

$$\int_0^{\infty} u[(1 + \mu)c_{NC}(t), x_{NC}(t)] \exp(-\lambda t) dt = \int_0^{\infty} u[c_{FC}(t), x_{FC}(t)] \exp[-\rho - \phi(t)] dt \quad (20)$$

<sup>13</sup> All the simulations for the three economies throughout the paper assume an initial value for  $k(0)$  to be 25 percent of the steady-state value  $k_{NC}^*$ .

where the normalization  $s = 0$  is used and the function  $\phi(t)$  is given by (19).

Results for the welfare gain  $\mu$  are provided in the third column of table 2 for alternative values of  $b$  and  $\zeta$ . The results show that there are substantial welfare gains from adopting a full commitment policy. Naturally, these potential welfare gains are larger as the household with no commitment becomes more impatient. Thus the conclusion derived from this analysis is that optimal fiscal policy might in principle play an important role in helping to alleviate the household's lack of commitment: rather than following the no commitment path, the planner may help the household by announcing an optimal allocation (in the second-best sense) as of time zero that moves the no commitment economy to a new allocation path closer to the full commitment equilibrium. This optimal path naturally improves upon the path under no commitment in the sense that household's welfare is increased.

**Table 2**  
**Welfare estimates**  
**For alternative values of  $\lambda$**

$b$	$\zeta$	$\mu$	$\hat{\mu}$	$\hat{\mu}/\mu$
0.10	0.50	0.026	0.005	0.213
0.25	0.50	0.071	0.006	0.082
0.50	0.50	0.165	0.016	0.096
0.10	0.75	0.018	0.004	0.245
0.25	0.75	0.047	0.005	0.098
0.50	0.75	0.105	0.015	0.139

Consider now the economy with no commitment. Rather than following such a path, as of time zero the benevolent planner unexpectedly announces the Ramsey allocation which is time-consistent according to earlier discussion. In order to figure out the Ramsey economy more clearly, the sequence of optimal taxes for capital and labor income are depicted in figures 4 and 5 for  $\zeta = 0.75$  and alternative values of  $b$ .

For each value of  $b$ , the capital income tax immediately drops at time zero from its starting value of 0.43 to a value of 0.046. Thereafter the tax

decreases smoothly towards zero. On the other hand, the tax on labor income immediately jumps at time zero from its initial value of 0.19 to a value of 0.422 for the particular case  $b = 0.50$ . Subsequently the labor income tax decreases smoothly towards a long-run value slightly lower than 0.39. In both cases, optimal taxes are lower along the transition path as  $b$  increases: as the household becomes more impatient, the planner chooses to tax capital and labor income less heavily in order to compensate for the long-run negative effects of higher impatience. Finally, for illustrative purposes the corresponding sequences for government expenditure and revenue associated with the Ramsey economy are shown in figure 6. This figure is qualitatively similar to the results found by Jones et al. (1993): there is an initial period (of about 6 years for the parameter specification) in which the government builds a budget surplus in order to finance future deficits.

Steady state values for capital, leisure and consumption under the optimal Ramsey policy are shown in the last three columns of table 1. Compared to the full commitment allocation at the steady state, there is not a clear relationship among the variables of interest. For example, if  $\lambda$  is relatively small and close to  $\rho$ ,  $k_{OP}^*$  is substantially higher than  $k_{FC}^*$ . The reason is that a zero capital income tax in the long run creates a positive effect on long-run capital that overcomes the negative “impatience” effect derived from a higher discount rate ( $\lambda > \rho$ ). As  $\lambda$  becomes larger, the impatience effect dominates the zero tax effect so that  $k_{OP}^*$  may be in fact below  $k_{FC}^*$ . This whole result is reflected in long-run consumption:  $c_{OP}^*$  is higher than  $c_{FC}^*$  only for low values of  $\lambda$ . Finally, since long-run optimal taxes on labor income are higher than its corresponding initial value of 0.19, steady-state leisure under the Ramsey economy is larger than its full commitment value.

Transitional dynamics for capital, leisure and consumption in the Ramsey economy are illustrated by the dash-dot curves in figures 1 to 3 for the particular case  $b = 0.50$  and  $\zeta = 0.75$ . Here, the capital stock for the Ramsey economy is the highest in every period because the impatience effect is relatively small. With respect to leisure, it is well known that there are opposite effects occurring at the time the Ramsey policy is announced. First, non-human wealth at time zero is below its long-run value by construction. Second, the after-tax marginal product of capital is now higher at the time the optimal tax on capital income is implemented. Third, the after-tax marginal product of labor at time zero is now lower since the planner implements a higher tax on labor income as compared to the original value for  $\tau_n$ . The first two effects induce the household to work more hours; the third effect works in the opposite direction. Given the particular preferences and technology under study, the first two effects dominate so the household

effect works in the opposite direction. Given the particular preferences and technology under study, the first two effects dominate so the household responds to the Ramsey policy by supplying a higher working time at time zero. Thereafter, working time decreases smoothly over time to its long-run value.<sup>14</sup> Finally, the effects of the Ramsey policy on consumption are such that  $c_{OP}(t)$  is below  $c_{NC}(t)$  for about eleven years but the rewards from previous higher saving and work are felt thereafter.

In order to evaluate the importance of optimal fiscal policy in welfare terms, a similar procedure as the one described by equation (20) is adopted. Here the constant  $\hat{\mu}$  denotes the compensating consumption supplement so that the allocation with no commitment yields the same welfare as the Ramsey allocation. Estimates for  $\hat{\mu}$  are shown in the fourth column of table 2. Not surprisingly, the Ramsey allocation improves upon the no commitment allocation, but such an improvement is relatively small in general. The explanation may be partially found in figures 2 and 3: once the optimal tax policy is implemented, both consumption and leisure under the Ramsey policy significantly fall below their corresponding values under the no commitment economy during the first years after the announcement. Only after some years consumption and leisure under the Ramsey economy are above their no commitment values.

Finally, the last column of table 2 evaluates the importance for the Ramsey allocation to alleviate the no commitment problem. For example, when  $b = 0.10$  and  $\alpha = 0.50$ , optimal fiscal policy only provides about 23 percent of the total consumption supplement necessary for the household to be indifferent between the full and the no commitment allocation. Thus for the parameter values considered, the Ramsey policy only covers between 8 and 25 percent of the total consumption supplement. Therefore, according to the model a benevolent planner with taxation abilities may only play a minor role in helping the household with quasi-hyperbolic discounting to solve her lack of commitment.

#### 4. Final Remarks

The goal of this paper has been to quantify the importance of the optimal fiscal policy as a tool to alleviate the no-commitment problem of a household with quasi-geometric discounting. For this purpose, three alternative

<sup>14</sup> An alternative explanation is the following: since savings are higher as a result of the optimal fiscal policy at time zero, current consumption is lower. Since consumption and leisure are normal goods in this model, less leisure is consumed at time zero as well.

from that date on. In the long-run, this economy behaves exactly as the standard case with a constant discount rate. When the household is unable to commit her choices, the problem is solved in a time-consistent manner by discounting utility at a new constant discount factor which is simply a weighted average of the instantaneous rates of time preference. Thus the household with no commitment exhibits more impatience than her full commitment counterpart in general. Finally, the Ramsey economy is simply the solution to the Ramsey problem under no commitment. Here the benevolent planner is endowed with a commitment technology in order to avoid the conflicting welfare results about optimal fiscal policy found in Krusell et al. (2000).

As the numerical analysis illustrates, there exist large welfare gains should the household with no commitment had an unexpected ability to commit. Therefore, the intervention of a benevolent planner setting taxes on capital and labor income optimally in order to partially alleviate this lack of commitment might be justified in principle. However, it is found that the optimal fiscal policy in such a case may provide only between 8 and 25 percent of the total payment necessary for the time-inconsistent household to be just indifferent between the full and the no commitment allocation. It is important to remark that this result holds under the strong assumption that the planner is endowed with a commitment technology. Given that the planner is benevolent and shares the quasi-geometric discounting of the household, one would argue that a more reasonable scenario would be to assume a partial commitment technology for the planner. If that were the case, such a planner would be even less helpful to the household and thus the numbers provided above would be even lower. Therefore, according to the model it seems that optimal fiscal policy is far from solving the no-commitment problem in a satisfactory way.

There are several factors (both internal and external to the household) that influence a household's ability to commit her consumption choices. Laibson (1997) and Barro (1999) provide several examples of public policies as well as institutional and market mechanisms related to this issue. The existence of legal constraints on credit markets that inhibit excessive consumer spending through borrowing, or the penalties imposed on retirement benefits for withdrawals made before reaching the full retirement age, are only a few real-world examples of mechanisms that may be interpreted as commitment devices. Of course, the degree of commitment is also related to the self-discipline of the household, a situation in which cultural factors might play an important role.



Commitment mechanisms as well as institutional and cultural factors vary greatly across societies. As discussed above, economies that feature a better capacity to commit future consumption should exhibit a lower effective rate of time preference and thus higher levels of capital and consumption in the long run. For those economies in which commitment mechanisms are weak or non-enforceable, it might seem that a natural instrument to partially alleviate this problem is fiscal policy. However, as shown in this paper, the help provided by taxes set in an optimal manner is far from satisfactory. Accordingly, it seems that the solution to this lack-of-commitment problem should be rather found in institutional and cultural factors which are usually beyond the control of the planner.

## Appendix

The objective in this section is to derive an expression for the share of wealth that the household must consume when she is not able to commit her future choices. For that purpose, the analysis in section III of Barro (1999) is extended for a model with taxes and endogenous labor supply as provided by (8).

In order to solve for such an expression, consider the household choosing consumption at time  $s$  as the constant flow  $c(s)$  over the short discrete interval  $[s, s+\varepsilon]$ . The value for  $\varepsilon$  will eventually approach zero and thereby generate results for continuous time. Hence as of time  $s$  the utility in expression (1) may alternatively be written as

$$\begin{aligned}
 U(s) &= \int_s^{s+\varepsilon} [\psi \ln c(t) + (1-\psi) \ln x(t)] \exp[-(\rho \cdot (t-s) + \phi(t-s))] dt \\
 &\quad + \int_{s+\varepsilon}^{\infty} [\psi \ln c(t) + (1-\psi) \ln x(t)] \exp[-(\rho \cdot (t-s) + \phi(t-s))] dt \\
 &\approx \varepsilon [\psi \ln c(s) + (1-\psi) \ln x(s)] + \int_{s+\varepsilon}^{\infty} [\psi \ln c(t) + (1-\psi) \ln x(t)] e^{[-(\rho \cdot (t-s) + \phi(t-s))]} dt
 \end{aligned}$$

where the approximation arises from taking  $e^{[-(\rho \cdot (t-s) + \phi(t-s))]}$  as equal to unity over the interval  $[s, s+\varepsilon]$ . The above result is thus given in terms of consumption and leisure. However, it is more convenient to work out an expression in terms of consumption only. To this purpose, condition (6) and the utility function (8) in the text are used in order to express leisure as a function of consumption. Replacing this result into the above approximation yields:

$$U(s) \approx \varepsilon \ln c(s) + \int_{s+\varepsilon}^{\infty} \ln c(t) e^{[-(\rho \cdot (t-s) + \phi(t-s))]} dt + \Gamma \tag{A1}$$

where the expression

$$\Gamma \equiv \varepsilon(1-\psi) \ln \left[ \frac{1-\psi}{\psi \tilde{w}(s)} \right] + (1-\psi) \int_{s+\varepsilon}^{\infty} \ln \left[ \frac{1-\psi}{\psi \tilde{w}(t)} \right] e^{[-(\rho \cdot (t-s) + \phi(t-s))]} dt$$

is independent of the  $c(t)$  path.

When the household picks  $c(s)$  at time  $s$ , the consumption path  $c(t)$  for  $t \geq s + \varepsilon$  is affected through the stock of assets  $k(t + \varepsilon)$  available at time  $s + \varepsilon$ . In order to determine the welfare-maximizing choice of  $c(s)$ , the representative consumer needs to know both the relationship between  $c(s)$  and  $k(s + \varepsilon)$  as well as the relationship between  $k(s + \varepsilon)$  and the choices of  $c(t)$  for  $t \geq s + \varepsilon$ .

The solution to the first part of this problem may be directly determined by taking a linear approximation to the household's budget constraint over the interval  $(s, s + \varepsilon)$ . This procedure yields the expression  $d[k(s + \varepsilon)]/d[c(s)] \approx -\varepsilon$  (see Barro (1999) for details). To obtain a result for the second part, it is conjectured that the income and substitution effects associated with future interest rates would cancel under logarithmic utility, even though the rate of time preference is variable and the household cannot commit her decisions. This implies that there must be a constant fraction  $\lambda$  of wealth so that

$$c(t) = \lambda[k(t) + \bar{w}(s)] \quad (\text{A2})$$

where  $\bar{w}(s)$  denotes the present value of wage income (net of taxes) as of time  $s$ . It is important to remark that the conjectured fraction  $\lambda$  need not equal the constant fraction  $\rho$  of wealth that would be obtained under a standard model. Given the conjecture, consumption should grow over time at the rate  $\tilde{r}(t) - \delta - \lambda$  for  $t \geq s + \varepsilon$ . Therefore, for any  $t \geq s + \varepsilon$  it must be the case that

$$\ln c(t) = \ln c(s + \varepsilon) + \Psi(t, s + \varepsilon) \quad (\text{A3})$$

where  $\Psi(t, s + \varepsilon) \equiv \int_{s + \varepsilon}^t [\tilde{r}(v) - \delta - \lambda] dv$  is also a term independent of the  $c(t)$  path. Plugging equation (A3) into (A1) leads to:

$$U(s) \approx \varepsilon \ln c(s) + \ln c(s + \varepsilon) \int_{s + \varepsilon}^{\infty} e^{-[\rho \cdot (t-s) + \phi(t-s)]} dt + \int_{s + \varepsilon}^{\infty} \Psi(t, s + \varepsilon) e^{-[\rho \cdot (t-s) + \phi(t-s)]} dt + \Gamma \quad (\text{A4})$$

Finally, define the integral

$$\Omega \equiv \int_0^{\infty} e^{-[\rho v + \phi(v)]} dv \quad (\text{A5})$$

which corresponds to the first integral in (A4) as  $\varepsilon$  goes to zero.

Now it is possible to estimate the marginal effect of  $c(s)$  on the instantaneous utility  $U(s)$ . Such effect is given by:

$$\frac{d[U(s)]}{d[c(s)]} \approx \frac{\varepsilon}{c(s)} + \frac{\Omega}{c(s+\varepsilon)} \cdot \frac{d[c(s+\varepsilon)]}{d[k(s+\varepsilon)]} \cdot \frac{d[k(s+\varepsilon)]}{d[c(s)]}$$

From the discussion above, setting the previous derivative to zero implies that:

$$c(s) = c(s+\varepsilon)/\Omega\lambda$$

If the conjecture on  $\lambda$  is correct, then  $c(s+\varepsilon)$  must approach  $c(s)$  as  $\varepsilon$  goes to zero. Hence, it must be the case that:

$$\lambda = \frac{1}{\int_0^{\infty} e^{-[\rho v + \phi(v)]} dv} \quad (\text{A6})$$

This is just equation (9) in the text. It is also equal to the expression obtained in Barro (1999) for a model with no taxes and consumption in the utility function only.

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Figure 1  
Capital Stock  
Full, No Commitment and Ramsey Economies  
Solid line: Full commitment; dashed line: No commitment; dash-dot line: Ramsey

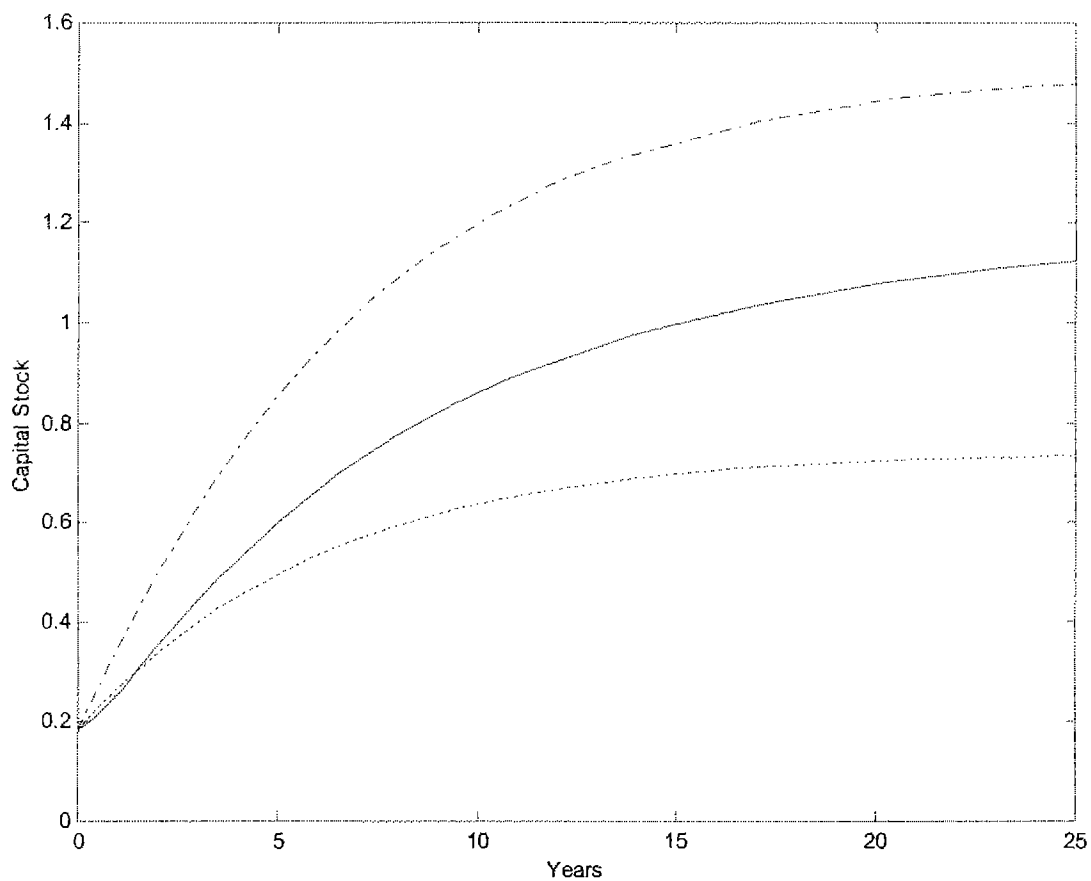


Figure 2  
Leisure  
Full, No Commitment and Ramsey Economies  
Solid line: Full commitment; dashed line: No commitment; dash-dot line: Ramsey

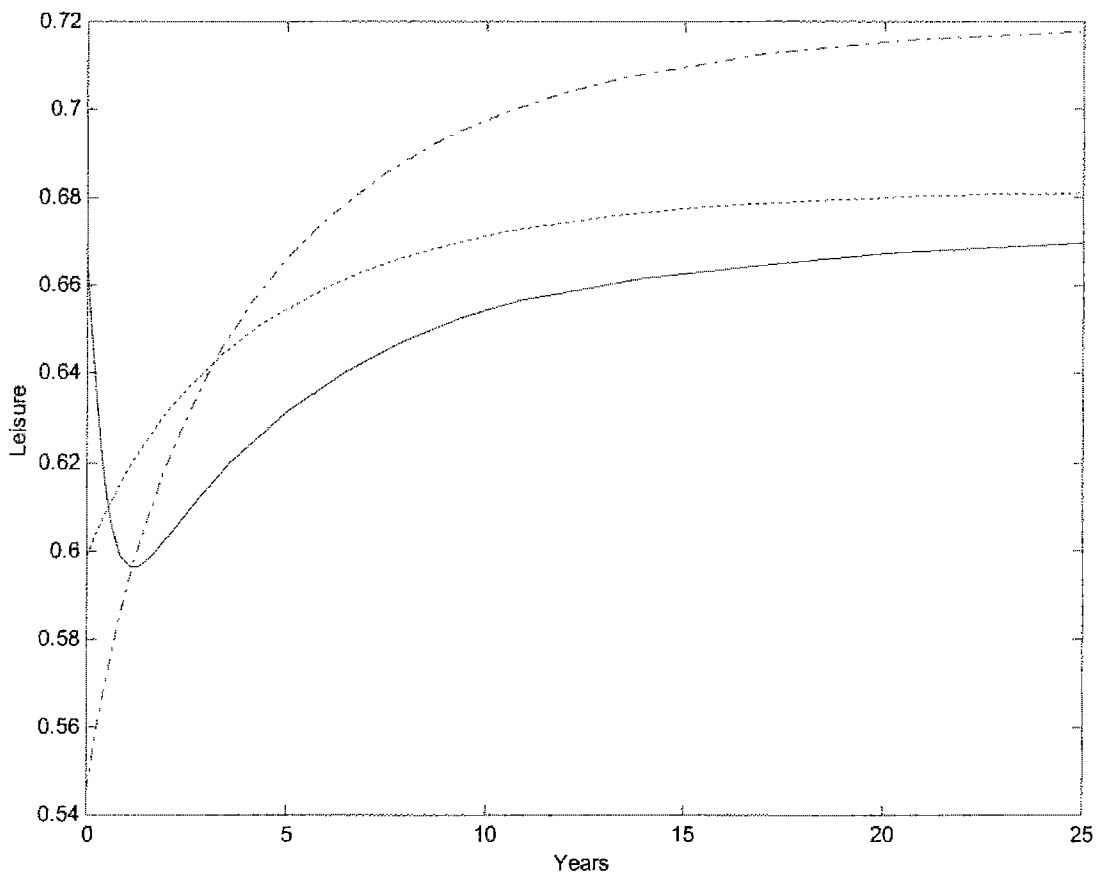


Figure 3  
Consumption  
Full, No Commitment and Ramsey Economies  
Solid line: Full commitment; dashed line: No commitment; dash-dot line: Ramsey

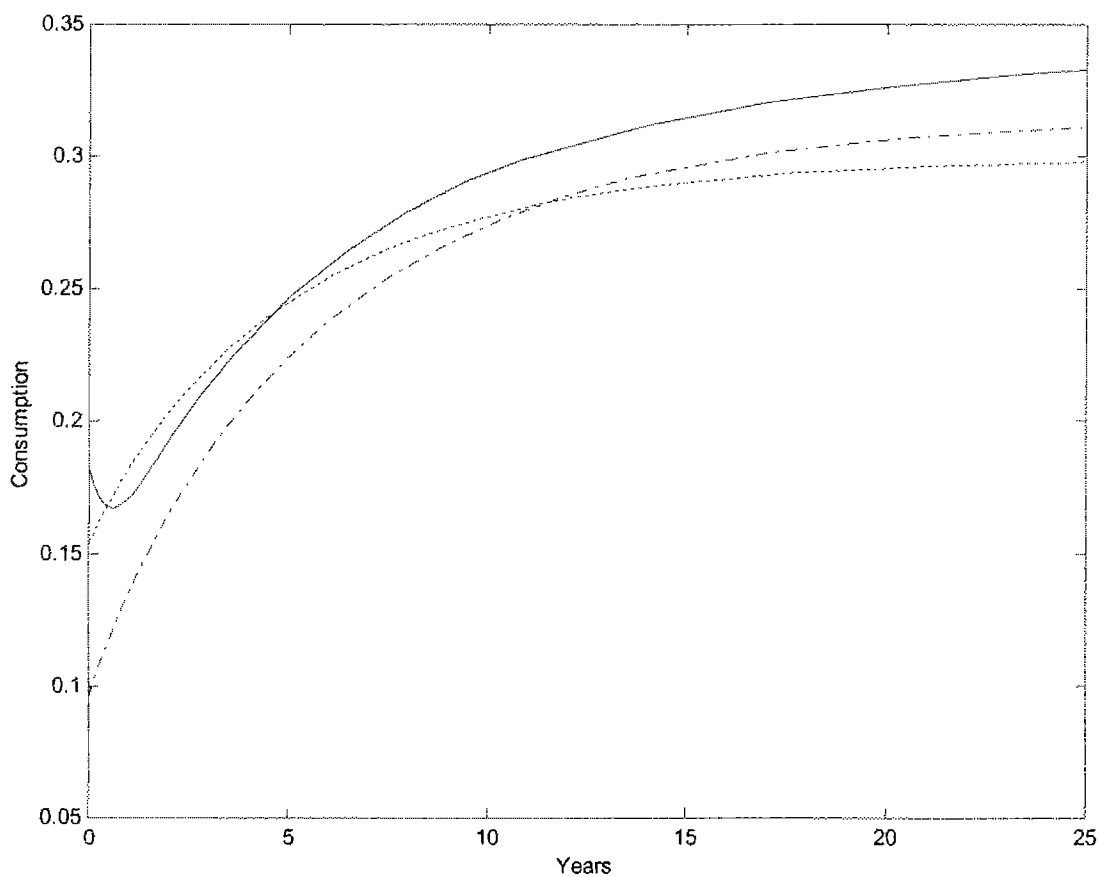




Figure 4  
Optimal Capital Income Taxation  
Under Alternative Values for  $b$

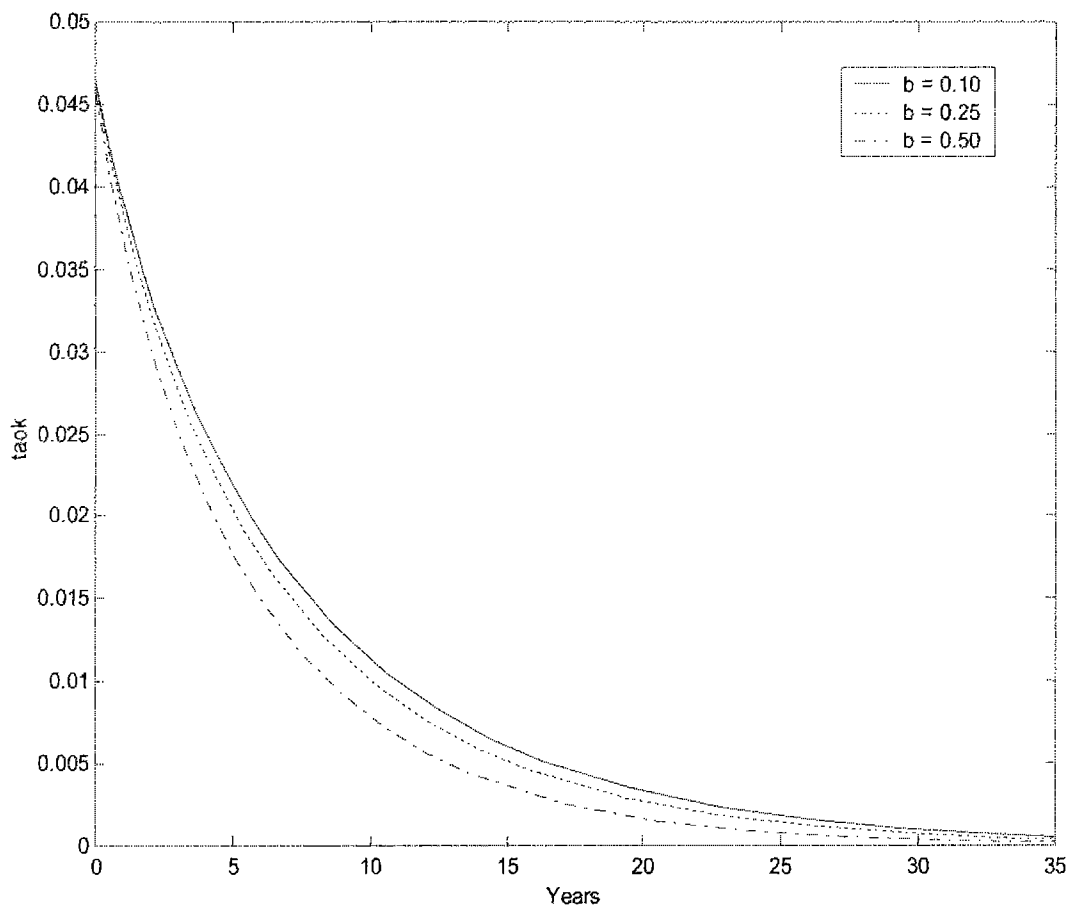


Figure 5  
Optimal Labor Income Taxation  
Under Alternative Values for  $b$

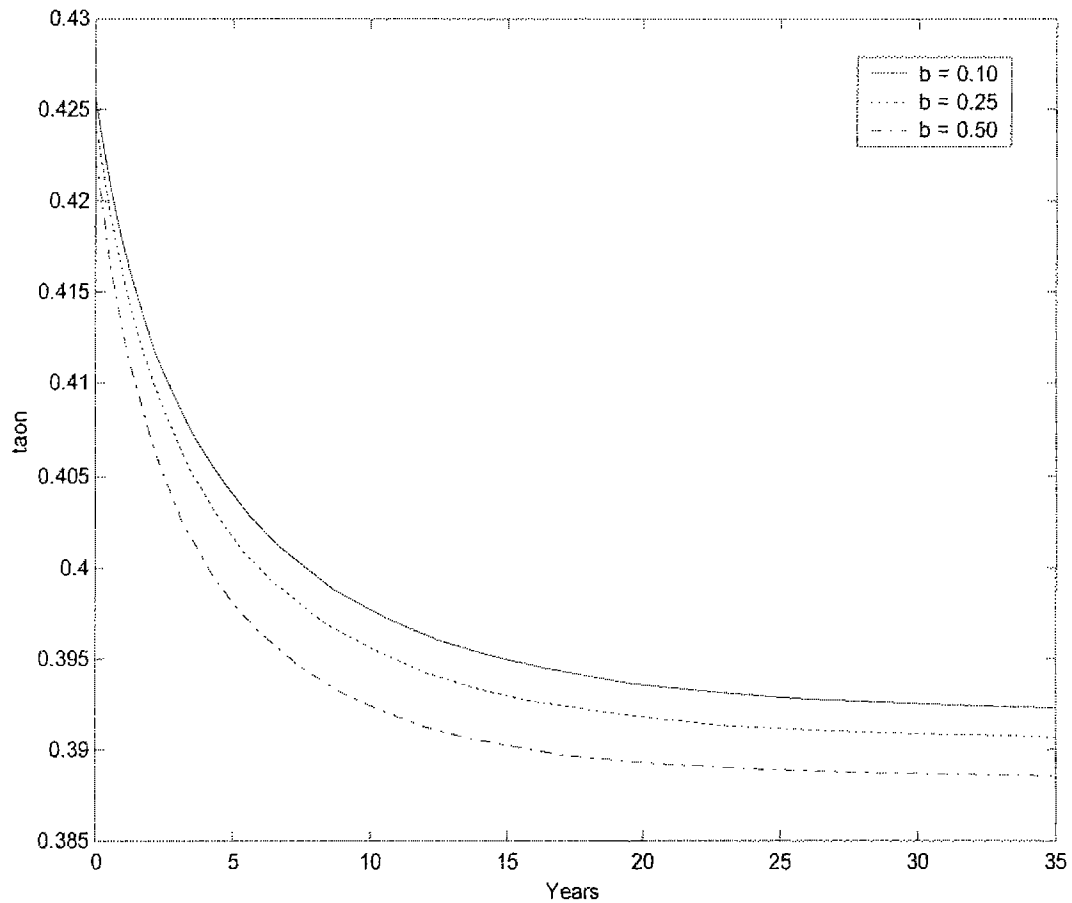
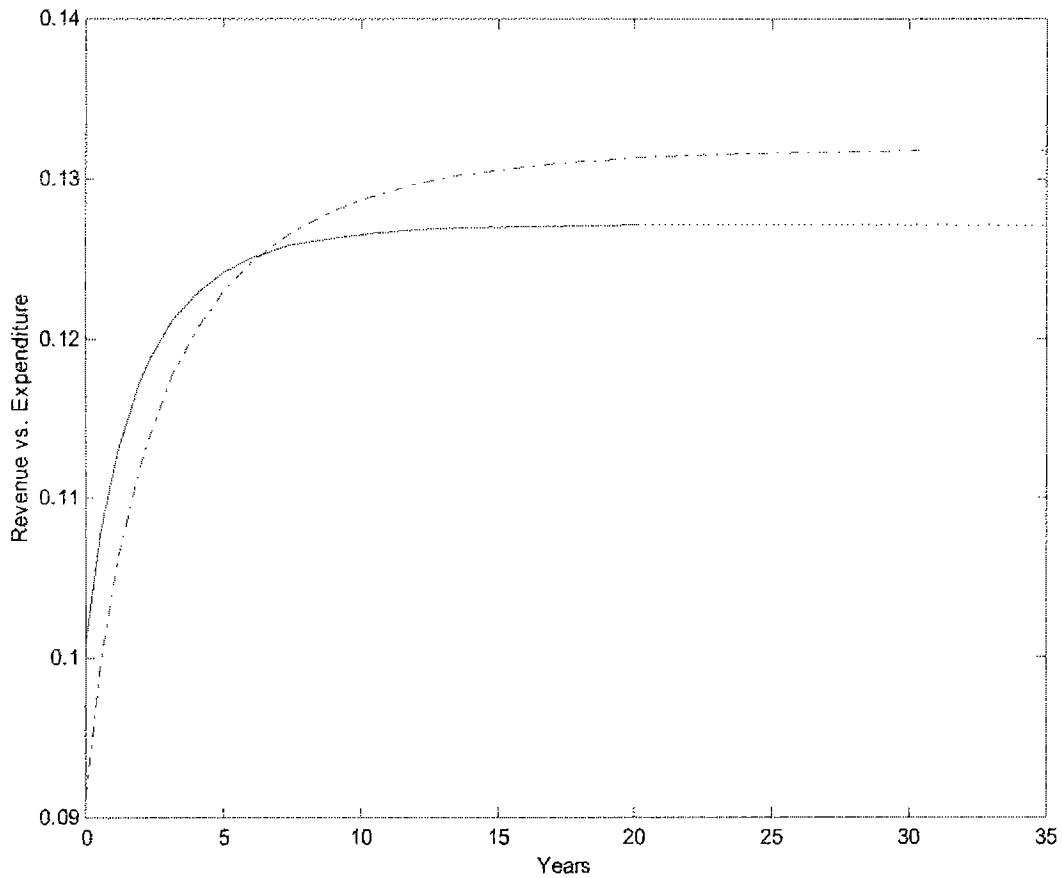


Figure 6  
Government Revenue and Expenditure under Optimal Fiscal Policy  
 $b = 0.50, \zeta = 0.75$   
Solid line: Revenue; dash-dot line: Expenditure





## Novedades

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