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Sunspot and Sunspot-like Equilibria in an Overlapping Generations Economy with Strategic Interactions

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Abstract

In this paper we analyze the effect of market uncertainty in an overlapping generations economy with strategic interactions among agents. We demonstrate the existence of sunspot and sunspont-like equilibria in this economy. Sunspot-like equilibria are analyzed by introducing trembles in the offers of the agents, a component of the agents´ strategy set in the market game. Those trembles in offers generate strong fluctuations in equilibrium bids and prices, which allows us to conclude that trembles increase the level of indeterminacy in this economy rather than diminishing it.

Journal of Economic Literature Classifications Numbers: C72, C73, D51, L16.

Resumen

En este trabajo analizamos el efecto de la incertidumbre de mercado en una economía de generaciones solapadas con interacción estratégica entre los agentes de la misma. Demostramos la existencia de equilibrios sunspot y sunspot-like en este tipo de economía. Los equilibrios sunspot-like se analizan introduciendo perturbaciones en las ofertas de los agentes, siendo éstas un componente del conjunto de estrategias de dichos agentes en el juego de mercado. Estas perturbaciones generan fuertes fluctuaciones en las posturas de equilibrio de los agentes y, en consecuencia, en los precios de equilibrio. Esto nos permite concluir que tales perturbaciones aumentan, en vez de disminuir, el nivel de indeterminación en esta economía.

1 Introduction

The existence of randomness in competitive equilibrium outcomes in models of rational expectations with no fundamental uncertainty has been largely recognized and analyzed in the literature, see, for instance, Azariadis (1981) and, Cass and Shell (1983). As Peck and Shell (1991) aknowledge, the uncertainty that an economic actor experiences about the behavior of the others in the economy and the general environment, can lead to situations in which it is rational that this economic actor behaves differently according to the realizations of a sunspot variable, or extrinsic uncertainty. Moreover, even small degrees of market imperfections might lead to coordination problems among the agents in an economy and, consequently, to indeterminate equilibria, as pointed out by Benhabib (1998). The analysis of extrinsic uncertainty's effect on static models with imperfect competition, in which strategic interactions among market participants are explicitly modeled, is adderessed by Peck and Shell (1991). They find that sunspot and correlated equilibria exist in this kind of economies, and provide a link between these two types of equilibria. With the goal of examining whether strategic interactions among agents in models of imperfect competition contribute to the generation of business cycles, Goenka *et al.* (1998) analyze a pure exchange, dynamic model of imperfect competition. In this model, there is an overlapping generations economy in which the agents trade one commodity using the Shapley-Shubik market game. Their main conclusion is that market thickness, or the impact of an individual agent on the process of determining the equilibrium price, plays a major role in generating complicated dynamics and, consequently, business cycles. Particularly, they find conditions under which an cyclic equilibria of order two exists and, hence, they conclude that sunspot equilibria exist in this model. Finally, Manuelli and Peck (1992) study a sequence of economies with amounts of intrinsic uncertainty approaching to zero and a limiting economy, which is a pure exchange-overlapping generations economy, with no intrinsic uncertainty at all. They conclude that equilibria in the neighborhood of the limit are sunspot-like, that is, equilibria in which negligible amounts of fundamental uncertainty lead to considerable variations in prices.

In this paper, we explicitly study the effect of market uncertainty in the environment of Goenka *et al.* (1998). First, we show that sunspot equilibria exist in this type of economy by invoking the well-known result that a necessary and sufficient condition for the existence of a cyclic equilibrium of order 2, is the existence of a sunspot equilibrium with two states. This is not a surprising result, given the previously mentioned finding of Goenka *et al.* (1998). However, this analysis provides the ground for our next result, which is that small stochastic variations in the offers of the agents lead to large variations in the equilibrium bids, and, consequently, in prices. This analysis is in the spirit of Manuelli and Peck (1992). In the original version of the Shapley and Shubik's market game, agents' offers and bids constitute the strategy set of the agents. We need to fix offers in order to analyze our model because of indeterminacy. When we introduce trembles in offers, we find that they generate sunspot-like movements in equilibrium bids and prices. Hence, we may conclude that, in this case, trembles generate more indeterminacy rather than less.

The paper is organized as follows: In Section 2, we develop the overlapping generations version of the Shapley-Shubik market game with extrinsic uncertainty. In Section 3, we study the existence of stochastic steady states in our economy. We show the existence of sunspot equilibria in Section 4. In Section 5, we demonstrate the existence of sunspot-like equilibria in this model. Finally, we offer some concluding remarks.

2 The Model

In this section we develop a pure exchange market game of the type proposed by Shapley and Shubik (1977) in an overlapping generations context with extrinsic uncertainty. Time is discrete and indexed by t, where $t = 0, 1, \ldots$. In each period $n > 0$ agents are born, each of whom lives for two periods. We will not consider the case of the initial old generation because the analysis will focus on steady-state equilibria. Hence, we will assume that agents are characterized by stationary preferences and endowments.

In every period, there is a single perishable physical commodity and inside money which is storable. Uncertainty is extrinsic, that is, it does not affect the fundamentals of the model. The stochastic structure of this model can be characterized by two states of nature, $\rho \in \{\alpha, \beta\}$. These states unfold as a first order Markov process with stationary transition probabilities given by $\pi = \begin{bmatrix} \pi^{\alpha\alpha} & \pi^{\alpha\beta} \\ \pi^{\beta\alpha} & \pi^{\beta\beta} \end{bmatrix}$ where: $\pi^{\rho\alpha} + \pi^{\rho\beta} = 1$ for $\rho = \alpha, \beta$. Agents receive endowments in each period of life. We assume that endowments are stationary. So, $\omega_t = [\omega_t^t, \omega_t^{t+1}] = [\omega_1, \omega_2]$. We also assume that $0 < \omega_1 < \omega_2$, because our focus is on monetary equilibria.

The consumption of the representative young agent, indexed by $i = 1, \ldots, n$, born at period t will be denoted $[x_1^\rho, x_2^{\rho,\rho'}]_i$, where ρ' is the period $t+1$ value of the random variable ρ. Notice that $(\rho, \rho') \in \{\alpha, \beta\}^2$. For example, $x_2^{\alpha,\beta}$ is the consumption of agent i when old in the case that the value of the random variable is α when he is young, and β when he is old.

Lifetime preferences are described by a utility function u_i which maps from $\Re_+^2$ to $\Re$, and is C^r where $r \geq 4$, strictly increasing, strictly concave, and meets the Inada conditions. Specifically, the preferences of the $2n$ identical agents take the following form:

$$u_i(x_1^\rho, x_2^{\rho\rho'}) = U(x_{t,t}^\rho) + \pi^{\rho\alpha} V(x_{t,t+1}^{\rho\alpha}) + \pi^{\rho\beta} V(x_{t,t+1}^{\rho\beta})$$

where $u_i(\cdot, \cdot)$ is the von Neumann-Morgenstern expected utility function, and U and V are both smooth, strictly increasing, strictly concave and satisfy the Inada conditions. Also, $x_{t,t}^\rho$ is the consumption of the agent when young, given the realization of the random variable ρ, $x_{t,t+1}^{\rho\alpha}$ is the consumption of the agent when old, given that the random variable is ρ when he is young and α when he is old, and $x_{t,t+1}^{\rho\beta}$ is the consumption of the agent when old, given that the random variable is is ρ when he is young and β when he is old.

We take the trading mechanism of Shapley-Shubik market game (1977), where the consumption commodity is traded in a single trading post being exhanged for money. Agent i makes a non-negative offer of commodity $q_{t,t}$ when young and $q_{t,t+1}$ when old. Consumer i also bids a non-negative quantity of money $b_{t,t}^\rho$ when young and $b_{t,t+1}^{\rho\rho'}$ when old. That is,

$$q_t = [q_{t,t}, q_{t,t+1}]$$

and

$$b_t^{\rho\rho'} = [b_{t,t}^{\rho}, b_{t,t+1}^{\rho\rho'}]$$

Offers are constrained by the endowments because, offers are made in terms of the consumption good. Hence, $q_t \leq \omega_1$ and, $q_{t,t+1} \leq \omega_2$.

There exists the possibility for agent i born in period t to save, and her savings are denoted by m_t^{ρ}. Therefore, her strategy set is the following:

$$S_i = \{[(b_{t,t}^{\rho}, b_{t,t+1}^{\rho\rho'}), (q_{t,t}, q_{t,t+1}), m_t^{\rho}] \in \Re^{10}/(q_{t,t}, q_{t,t+1}) \gg 0\}$$

Denote a strategy profile for all the consumers as:

$$\sigma_i = (S_i)_{i\in G_t, t\geq 0} = [(b_{t,t}^{\rho}, b_{t,t+1}^{\rho\rho'})_i, (q_{t,t}, q_{t,t+1})_i]_{i\in G_t, t\geq 0}$$

and σ_{-i} denotes the strategies of all consumers other than agent i.

In the trading process, every consumer is assigned a proportion of the aggregate offer of the consumption commodity in relation to the weight that her bid has on the aggregate bid. And, each consumer is assigned a proportion of the aggregate inside money bid in relation to the weight that her offer has in the aggregate offer of the consumption good. We now introduce the following notation:

$$\begin{aligned} B_{t,t}^{\rho} &= nb_{t,t}^{\rho} \\ B_{t,t+1}^{\rho\rho'} &= nb_{t,t+1}^{\rho\rho'} \end{aligned}$$

and,

$$\begin{aligned} Q_{t,t} &= nq_{t,t} \\ Q_{t,t+1} &= nq_{t,t+1} \end{aligned}$$

to denote the aggregate bids and offers of the consumers born in t and $t+1$ respectively.

Let:

$$B_t^{\rho} = B_{t-1,t}^{\rho} + B_{t,t}^{\rho}$$

denote the aggregate inside money bid in period t, and

$$Q_t = Q_{t-1,t} + Q_{t,t}$$

denote the aggregate offer of the consumption good at time t.

Similarly,

$$B_{t+1,t+1}^{\rho\rho'} = nb_{t+1,t+1}^{\rho\rho'}$$

and

$$Q_{t+1,t+1} = nq_{t+1,t+1}$$

Therefore, we have

$$B_{t+1}^{\rho\rho'} = B_{t,t+1}^{\rho\rho'} + B_{t+1,t+1}^{\rho\rho'}$$

and,

$$Q_{t+1} = Q_{t,t+1} + Q_{t+1,t+1}$$

Then, a young agent born at time t will obtain the following allocations of the consumption good under the trading process of the Shapley-Shubik market game:

$$x_{t,t}^{\rho} = \omega_1 - q_{t,t} + \frac{Q_t}{B_t^{\rho}} b_{t,t}^{\rho} \tag{1}$$

$$x_{t,t+1}^{\rho\rho'} = \omega_2 - q_{t,t+1} + \frac{Q_{t,t+1}}{B_{t,t+1}^{\rho\rho'}} b_{t,t+1}^{\rho\rho'} \tag{2}$$

The ratio B/Q can be interpreted as the price of consumption in terms of money.

The lifetime budget constraints of a typical young consumer, given her share of the aggregate bids, are the following:

$$b_{t,t}^{\rho} + m_t^{\rho} = \frac{B_t^{\rho}}{Q_t} q_{t,t} \tag{3}$$

$$b_{t,t+1}^{\rho\rho'} - m_t^{\rho} = \frac{B_{t+1}^{\rho\rho'}}{Q_{t+1}} q_{t,t+1} \tag{4}$$

Eliminating the savings, we can reduce these two budget constraints into a single one, which is:

$$b_{t,t}^{\rho} + b_{t,t+1}^{\rho\rho'} = \frac{B_t^{\rho}}{Q_t} q_{t,t} + \frac{B_{t+1}^{\rho\rho'}}{Q_{t+1}} q_{t,t+1} \tag{5}$$

Since in this situation all the agents are affected by the sunspot activity symmetrically, we wish to consider the correlated equilibria of this game, assuming that the information partitions of all the players are identical. Since the set of pure strategy Nash equilibria is a subset of the set of correlated equilibria, being a correlated equilibrium any probability distribution over the pure strategy Nash equilibria, we consider first the Nash equilibria in pure strategies of this game. Hence, we need to express the price ratio in terms of the bids and offers of the other agents:

$$b_{t,t}^{\rho} Q_t = B_t^{\rho} q_{t,t} - Q_t m_t^{\rho}$$

$$b_{t,t}^{\rho} Q_t = [b_{t,t}^{\rho} + \hat{B}_t^{\rho}] q_{t,t} - Q_t m_t^{\rho}$$

where $\hat{B}_t^{\rho} = B_t^{\rho} - b_{t,t}^{\rho}$. Also, $\hat{Q}_t = Q_t - q_{t,t}$. So,

$$b_{t,t}^{\rho}[\hat{Q}_t + q_{t,t}] = b_{t,t}^{\rho} q_{t,t} + \hat{B}_t^{\rho} q_{t,t} - Q_t m_t^{\rho}$$

Simplifying, we get the following result:

$$b_{t,t}^{\rho} = \frac{\hat{B}_t^{\rho} q_{t,t} - Q_t m_t^{\rho}}{\hat{Q}_t} \tag{6}$$

and hence,

$$b_{t,t}^{\rho} + m_t^{\rho} = \left[\frac{\hat{B}_t^{\rho} - m_t^{\rho}}{\hat{Q}_t}\right] q_{t,t}$$

Define:

$$\tilde{B}_{t+1}^{\rho\rho'} = B_{t+1}^{\rho\rho'} - b_{t,t+1}^{\rho\rho'}$$

and,

$$\tilde{Q}_{t+1} = Q_{t+1} - q_{t,t+1}$$

Similarly, we obtain this expression:

$$b_{t,t+1}^{\rho\rho'} - m_t^{\rho} = \left[\frac{\tilde{B}_{t+1}^{\rho\rho'} + m_t^{\rho}}{\tilde{Q}_{t+1}}\right] q_{t,t+1} \tag{7}$$

Since $B_t^{\rho} = \hat{B}_t^{\rho} + b_{t,t}^{\rho}$, subtituting into (6) we obtain:

$$B_t^{\rho} = \hat{B}_t^{\rho} + \left[\frac{\hat{B}_t^{\rho} - m_t^{\rho}}{\hat{Q}_t}\right] q_{t,t} - m_t^{\rho}$$

$$B_t^{\rho} = \hat{B}_t^{\rho} + \left[\frac{\hat{B}_t^{\rho} - m_t^{\rho}}{\hat{Q}_t}\right] [Q_t - \hat{Q}_t] - m_t^{\rho}$$

$$B_t^{\rho} = \left[\frac{\hat{B}_t^{\rho} - m_t^{\rho}}{\hat{Q}_t}\right] Q_t$$

Hence:

$$\frac{Q_t}{B_t^{\rho}} = \frac{\hat{Q}_t}{\hat{B}_t^{\rho} - m_t^{\rho}}$$

Substituting this expression and (3) into (1):

$$x_{t,t}^{\rho} = \omega_1 - q_{t,t} + \left[\frac{\hat{Q}_t}{\hat{B}_t^{\rho} - m_t^{\rho}}\right]\left[\frac{\hat{B}_t^{\rho} - m_t^{\rho}}{\hat{Q}_t} q_{t,t} - m_t^{\rho}\right]$$

$$x_{t,t}^{\rho} = \omega_1 - \frac{\hat{Q}_t}{\hat{B}_t^{\rho} - m_t^{\rho}} m_t^{\rho}$$

Similarly,

$$x_{t,t+1}^{\rho\rho'} = \omega_2 + \frac{\tilde{Q}_{t+1}}{\tilde{B}_{t+1}^{\rho\rho'} + m_t^\rho} m_t^\rho$$

Then, a typical young consumer's utility maximization problem becomes:

$$\max_{m_t^\rho} U\left(\omega_1 - \frac{\hat{Q}_t}{\hat{B}_t^\rho - m_t^\rho} m_t^\rho\right) + \pi^{\rho\alpha} V\left(\omega_2 + \frac{\tilde{Q}_{t+1}}{\tilde{B}_{t+1}^{\rho\alpha} + m_t^\rho} m_t^\rho\right)$$
$$+\pi^{\rho\beta} V\left(\omega_2 + \frac{\tilde{Q}_{t+1}}{\tilde{B}_{t+1}^{\rho\beta} + m_t^\rho} m_t^\rho\right) \qquad (8)$$

The first order conditions associated with this optimization problem are:

$$\left[-U'\left(\omega_1 - \frac{\hat{Q}_t}{\hat{B}_t^\rho - m_t^\rho} m_t^\rho\right)\left(\frac{\hat{Q}_t}{\hat{B}_t^\rho - m_t^\rho} + \frac{\hat{Q}_t}{(\hat{B}_t^\rho - m_t^\rho)^2} m_t^\rho\right)\right]$$
$$+\pi^{\rho\alpha}\left[V'\left(\omega_2 + \frac{\tilde{Q}_{t+1}}{\tilde{B}_{t+1}^{\rho\alpha} + m_t^\rho} m_t^\rho\right)\left(\frac{\tilde{Q}_{t+1}}{\tilde{B}_{t+1}^{\rho\alpha} + m_t^\rho} - \frac{\tilde{Q}_{t+1}}{(\tilde{B}_{t+1}^{\rho\alpha} + m_t^\rho)^2} m_t^\rho\right)\right]$$
$$+\pi^{\rho\beta}\left[V'\left(\omega_2 + \frac{\tilde{Q}_{t+1}}{\tilde{B}_{t+1}^{\rho\beta} + m_t^\rho} m_t^\rho\right)\left(\frac{\tilde{Q}_{t+1}}{\tilde{B}_{t+1}^{\rho\beta} + m_t^\rho} - \frac{\tilde{Q}_{t+1}}{(\tilde{B}_{t+1}^{\rho\beta} + m_t^\rho)^2} m_t^\rho\right)\right] = 0 \qquad (9)$$

Definition 1. The stochastic extension of the market game with symmetric information is the strategic game characterized by:

1. $2n$ players.
2. A finite set $\Omega = \{\alpha, \beta\}$ of states of nature.

and, for each player:

1. u_i which is the von Neumann-Morgenstern utility function.
2. The strategy set S_t.
3. Transition probabilities that follow a stationary first order Markovian process characterized by: $\pi = \begin{bmatrix} \pi^{\alpha\alpha} & \pi^{\alpha\beta} \\ \pi^{\beta\alpha} & \pi^{\beta\beta} \end{bmatrix}$ where: $\pi^{\rho\alpha} + \pi^{\rho\beta} = 1$, $\pi^{\rho\rho'} \geq 0$ for $\rho, \rho' = \alpha, \beta$.

Definition 2. The Nash equilibrium in pure strategies for the stochastic extension of the market game is a sequence of bids $\{b_{t,t}^\rho, b_{t,t+1}^{\rho\rho'}\}_{t=1,2,\dots}$ such that:

1. Offers that are exogenously given by $\{q_{t,t}, q_{t,t+1}\}_{t=1,2,\dots}$ The reason for fixing offers is to deal with the indeterminacy of this model.
2. Every agent's strategy $(b_{t,t}^\rho, b_{t,t+1}^{\rho\rho'})_i$ and m_t^ρ is a best response to the bids of other agents, $(b_{t,t}^\rho, b_{t,t+1}^{\rho\rho'})_{-i}$, which are taken as given.
3. $\forall t$, $m_t^\rho = \bar{m}$, where $\bar{m}$ is the exogenously given stock of fiat money.

3 Stochastic Steady States

We now address the question of the existence of stochastic steady state equilibria in the context of this model. Following Spear (1985), a stationary sequence of equilibrium prices can be defined as one in which the components of that sequence depend only on the state realized at a given point in time. We are only interested in analyzing the equilibrium when the market for the consumption commodity is open; that is, bids and offers are strictly positive. The existence of a stationary equilibrium with zero bids and offers is a trivial question, because this equilibrium is self-enforcing.

Assumption 1. In the stationary environment, bids depend only on the state realized at time t, that is, $b^{\rho}_{t,t} = b^{\rho}_1$ and $b^{\rho\rho'}_{t,t+1} = b^{\rho'}_2$.

Assumption 2. In the stationary environment, offers are not time-dependent, that is, $q_{t,t} = q_1$ and $q_{t,t+1} = q_2$.

By definition, in equilibrium $m^{\rho}_t = \bar{m}$, which can be explained by the role of "the beginning" in the stationary environment. Suppose that the initial old generation has $\bar{m} > 0$ as an endowment of fiat money. Then, the first young generation will receive a transfer of $\bar{m}$, no matter what state of nature has been realized at period 1. This transfer will be the same for the following generation, and so on.

We will consider an economy in which there is only one consumer belonging to each living generation at a given point in time. We maintain the assumption that consumers are identical except for their date of birth.

Proposition 1. In an economy with a single consumer in each generation and a single consumption commodity at each date, a stationary equilibrium exists with non-zero net trade.

Proof. The first order conditions that characterize a stochastic stationary equilibrium are the following:

$$\left[-U'\left(\omega_1 - \frac{q_2\bar{m}}{b^{\rho}_2 - \bar{m}}\right)\left(\frac{q_2}{b^{\rho}_2 - \bar{m}} + \frac{q_2\bar{m}}{(b^{\rho}_2 - \bar{m})^2}\right)\right]$$
$$+\pi^{\rho\alpha}\left[V'\left(\omega_2 + \frac{q_1\bar{m}}{b^{\alpha}_1 + \bar{m}}\right)\left(\frac{q_1}{b^{\alpha}_1 + \bar{m}} - \frac{q_1\bar{m}}{(b^{\alpha}_1 + \bar{m})^2}\right)\right]$$
$$+\pi^{\rho\alpha}\left[V'\left(\omega_2 + \frac{q_1\bar{m}}{b^{\beta}_1 + \bar{m}}\right)\left(\frac{q_1}{b^{\beta}_1 + \bar{m}} - \frac{q_1\bar{m}}{(b^{\beta}_1 + \bar{m})^2}\right)\right] = 0$$

or,

$$\left[-U'\left(\omega_1 - \frac{q_2\bar{m}}{b^{\rho}_2 - \bar{m}}\right)\left(\frac{q_2 b^{\rho}_2}{(b^{\rho}_2 - \bar{m})^2}\right)\right]$$
$$+\pi^{\rho\alpha}\left[V'\left(\omega_2 + \frac{q_1\bar{m}}{b^{\alpha}_1 + \bar{m}}\right)\left(\frac{q_1 b^{\alpha}_1}{(b^{\alpha}_1 + \bar{m})^2}\right)\right]$$
$$+\pi^{\rho\alpha}\left[V'\left(\omega_2 + \frac{q_1\bar{m}}{b^{\beta}_1 + \bar{m}}\right)\left(\frac{q_1 b^{\beta}_1}{(b^{\beta}_1 + \bar{m})^2}\right)\right] = 0$$

and, the budget constraints can be written as follows:

$$b^{\rho}_1 + \bar{m} = \frac{b^{\rho}_2 + b^{\rho}_1}{q_2 + q_1} q_1$$

$$b_2^{\rho'} - \bar{m} = \frac{b_2^{\rho'} + b_1^{\rho'}}{q_2 + q_1} q_2$$

We need to show that non-zero solutions exist to equation (10). To do so, we consider two cases:

(i)*Sell-all equilibria:* Let $q_1 = \omega_1$ and $q_2 = \omega_2$. Then (10) becomes:

$$\left[-U'\left(\omega_1 - \frac{\omega_2 \bar{m}}{b_2^\rho - \bar{m}}\right)\left(\frac{\omega_2 b_2^\rho}{(b_2^\rho - \bar{m})^2}\right)\right]$$

$$+\pi^{\rho\alpha}\left[V'\left(\omega_2 + \frac{\omega_1 \bar{m}}{b_1^\alpha + \bar{m}}\right)\left(\frac{\omega_1 b_1^\alpha}{(b_1^\alpha + \bar{m})^2}\right)\right]$$

$$+\pi^{\rho\alpha}\left[V'\left(\omega_2 + \frac{\omega_1 \bar{m}}{b_1^\beta + \bar{m}}\right)\left(\frac{\omega_1 b_1^\beta}{(b_1^\beta + \bar{m})^2}\right)\right] = 0$$

From the budget constraints, the following equation results:

$$b_2^\rho = \bar{m} + \frac{\omega_2}{\omega_1}(b_1^\rho + \bar{m})$$

By manipulating (12), we can obtain the following result:

$$\omega_1 - \frac{\omega_2 \bar{m}}{b_2^\rho - \bar{m}} = \frac{\omega_1 b_1^\rho}{b_1^\rho + \bar{m}}$$

If wesubstitute (12) into the expression $\frac{\omega_2 b_2^\rho}{(b_2^\rho - \bar{m})^2}$, as showed below:

$$\frac{\omega_2 b_2^\rho}{(b_2^\rho - \bar{m})^2} = \omega_2\left[\bar{m} + \frac{(b_1^\rho + \bar{m})\omega_2)}{\omega_1}\right]\left[\frac{\omega_1}{(b_1^\rho + \bar{m})\omega_2)}\right]^2$$

we obtain our next expression:

$$\frac{\omega_2 b_2^\rho}{(b_2^\rho - \bar{m})^2} = \frac{\omega_1[\bar{m}(\omega_1 + \omega_2) + b_1^\rho \omega_2]}{(b_1^\rho + \bar{m})^2 \omega_2}$$

Now, we define the following function:

$$h^\rho(b_1^\alpha, b_1^\beta) = -U'\left(\frac{\omega_1 b_1^\rho}{b_1^\rho + \bar{m}}\right)\left(\frac{\omega_1[\bar{m}(\omega_1 + \omega_2) + b_1^\rho \omega_2]}{(b_1^\rho + \bar{m})^2 \omega_2}\right)$$

$$+\pi^{\rho\alpha}\left[V'\left(\omega_2 + \frac{\omega_1 \bar{m}}{b_1^\alpha + \bar{m}}\right)\left(\frac{\omega_1 b_1^\alpha}{(b_1^\alpha + \bar{m})^2}\right)\right]$$

$$+\pi^{\rho\alpha}\left[V'\left(\omega_2 + \frac{\omega_1 \bar{m}}{b_1^\beta + \bar{m}}\right)\left(\frac{\omega_1 b_1^\beta}{(b_1^\beta + \bar{m})^2}\right)\right]$$

The equilibrium situation requires that: $h(b_1^\alpha, b_1^\beta) = \begin{bmatrix} h^\alpha(b_1^\alpha, b_1^\beta) \\ h^\beta(b_1^\alpha, b_1^\beta) \end{bmatrix} = 0$

Now, we define the following function:

$$H(b_1^\alpha, b_1^\beta) = -U'\left(\frac{\omega_1 b_1^\alpha}{b_1^\alpha + \bar{m}}\right)\left(\frac{\omega_1[\bar{m}(\omega_1+\omega_2) + b_1^\alpha \omega_2]}{(b_1^\alpha + \bar{m})^2\omega_2}\right)$$

$$+U'\left(\frac{\omega_1 b_1^\beta}{b_1^\beta + \bar{m}}\right)\left(\frac{\omega_1[\bar{m}(\omega_1+\omega_2) + b_1^\beta \omega_2]}{(b_1^\beta + \bar{m})^2\omega_2}\right)$$

At equilibrium, it must be true that $H(b_1^\alpha, b_1^\beta) = 0$. For any $b_1^\beta > 0$, if $b_1^\alpha \to 0$ then:

$$\lim_{b_1^\alpha \to 0} -U'(0)\left(\frac{\omega_1 \bar{m}(\omega_1+\omega_2)}{\omega_2 \bar{m}^2}\right)$$

$$+U'\left(\frac{\omega_1 b_1^\beta}{b_1^\beta + \bar{m}}\right)\left(\frac{\omega_1[\bar{m}(\omega_1+\omega_2) + b_1^\beta \omega_2]}{(b_1^\beta + \bar{m})^2\omega_2}\right)$$

Hence, by the Inada conditions, $lim_{b_1^\alpha \to 0} H(b_1^\alpha, b_1^\beta) = -\infty$.
If $b_1^\alpha \to \infty$, then $\lim_{b_1^\alpha \to \infty} H(b_1^\alpha, b_1^\beta)$ is indeterminate.
Applying L'Hopital Rule we obtain:

$$\lim_{b_1^\alpha \to \infty} H(b_1^\alpha, b_1^\beta) = \lim_{b_1^\alpha \to \infty} U'(\omega_1)\left(\frac{\omega_1}{2(b_1^\alpha + \bar{m})}\right)$$

$$+U'\left(\frac{\omega_1 b_1^\beta}{b_1^\beta + \bar{m}}\right)\left(\frac{\omega_1[\bar{m}(\omega_1+\omega_2) + b_1^\beta \omega_2]}{(b_1^\beta + \bar{m})^2\omega_2}\right)$$

Then, $H(b_1^\alpha, b_1^\beta)$ tends to some positive number as $b_1^\alpha \to \infty$. Hence, there exists $b_1^\alpha > 0$ such that $H(b_1^\alpha, b_1^\beta) = 0$.

Now, let

$$g_\alpha(b_1^\alpha) = \frac{\omega_1[\bar{m}(\omega_1+\omega_2) + b_1^\alpha \omega_2]}{(b_1^\alpha + \bar{m})^2\omega_2}$$

Hence,

$$\frac{dH}{db_1^\alpha} = -U'' g_\alpha - U' g_\alpha^{'}$$

and,

$$g_\alpha^{'} = -\frac{\omega_1[\bar{m}(2\omega_1+\omega_2) + \omega_2 b_1^\alpha]}{\omega_2(b_1^\alpha + \bar{m})^3} < 0$$

Since $g_\alpha > 0$ and U is strictly increasing and concave, then $\frac{dH}{db_1^\alpha} > 0$.

From the Implicit Function Theorem (IFT), it follows that there exists a unique C^1 function f such that $b_1^\alpha = f(b_1^\beta)$ and $H(b_1^\alpha, b_1^\beta) = 0$.

To complete the proof of the existence of an stochastic steady state equilibrium, we need to show that there exists b_1^β such that $h^\alpha(f(b), b) = 0$.

Claim 1. f is monotonically increasing.

Proof. By the IFT, we have that $f'(b) = -\frac{H_\beta}{H_\alpha}$, where $H_\rho = \frac{dH}{db_1^\rho}$. From previous calculations we know that $H_\alpha > 0$. To know the sign of H_β we proceed similarly as we did with H_α. That is, we define:

$$g_\beta(b_1^\beta) = \frac{\omega_1[\bar{m}(\omega_1 + \omega_2) + \omega_2 b_1^\beta]}{\omega_2(b_1^\beta + \bar{m})^2}$$

Then, $\frac{dH}{db_1^\beta} = U'' g_\beta + U' g_\beta'$. We also have that:

$$g_\beta' = -\frac{\omega_1[\bar{m}(2\omega_1 + \omega_2) + \omega_2 b_1^\beta]}{\omega_2(b_1^\beta + \bar{m})^3} < 0$$

It follows that $\frac{dH}{db_1^\beta} < 0$. Hence $f'(b) > 0$.

Note that $f(0) = 0$. If this were not true, the first term of H would remain bounded while the second term would tend to $= +\infty$. So, consider:

$$h^\alpha(f(b), b) = -U'\left[\frac{\omega_1 f}{f + \bar{m}}\right]\left[\frac{\omega_1[\bar{m}(\omega_1 + \omega_2) + \omega_2 f]}{\omega_2(f + \bar{m})^2}\right]$$

$$+\pi^{\rho\alpha} V'\left[\omega_2 + \frac{\omega_1 \bar{m}}{f + \bar{m}}\right]\left[\frac{f\omega_1}{(f - \bar{m})^2}\right]$$

$$+\pi^{\rho\beta} V'\left[\omega_2 + \frac{\omega_1 \bar{m}}{b + \bar{m}}\right]\left[\frac{b\omega_1}{(b - \bar{m})^2}\right]$$

As $b \to 0$,then:

$$\lim_{b\to 0} h^\alpha(f(b), b) = \lim_{b\to 0} -U'(0)\left[\frac{\omega_1(\omega_1 + \omega_2)\bar{m}}{\omega_2 \bar{m}^2}\right]$$

By the Inada conditions, we can affirm that $h^\alpha(f(b), b) \to -\infty$ as $b \to 0$.

Now, we need to show that as $b \to \infty$, $h^\alpha(f(b), b)$ tends to some positive number. Consider:

$$h^\alpha(f(b), b)\left[\frac{\omega_1[\bar{m}(\omega_1 + \omega_2) + \omega_2 f]}{\omega_2(f + \bar{m})^2}\right] = -U'\left[\frac{\omega_1 f}{f + \bar{m}}\right]$$

$$+\pi^{\rho\alpha} V'\left[\omega_2 + \frac{\omega_1 \bar{m}}{f + \bar{m}}\right]\left[\frac{f\omega_1}{(f - \bar{m})^2}\right]\left[\frac{\omega_1[\bar{m}(\omega_1 + \omega_2) + \omega_2 f]}{\omega_2(f + \bar{m})^2}\right]$$

$$+\pi^{\rho\beta} V'\left[\omega_2 + \frac{\omega_1 \bar{m}}{b + \bar{m}}\right]\left[\frac{b\omega_1}{(b - \bar{m})^2}\right]\left[\frac{\omega_1[\bar{m}(\omega_1 + \omega_2) + \omega_2 f]}{\omega_2(f + \bar{m})^2}\right]$$

If $h^\alpha < 0$ for large b, the lilmit on the right hand side (RHS) must be negative.

$$lim_{b\to\infty} RHS = -U'(\omega_1) + \pi^{\rho\alpha} V'(\omega_2)$$

$$+\pi^{\rho\beta}V'(\omega_2)\lim_{b\to\infty}\frac{\omega_2\omega_1 b(f+\bar{m})^2}{\omega_1(b-\bar{m})^2(\bar{m}(\omega_1+\omega_2)+f\omega_2)}$$

The last limit can be obtained from the function H by noting that:

$$\frac{U'\left[\frac{\omega_1 f}{f+\bar{m}}\right]}{U'\left[\frac{\omega_1 b}{b+\bar{m}}\right]}=\frac{\omega_1(f+\bar{m})^2(\bar{m}(\omega_1+\omega_2)+b\omega_2)}{\omega_1(b+\bar{m})^2(\bar{m}(\omega_1+\omega_2)+f\omega_2)}$$

The right hand side of this expression has the same limit as $b\to\infty$ as the expression above. This limit is therefore $\frac{U'(\omega_1)}{U'(\omega_1)}=1$. Thus,

$$\lim_{b\to\infty}RHS=-U'(\omega_1)+\pi^{\rho\alpha}V'(\omega_2)+\pi^{\rho\beta}V'(\omega_2)=-U'(\omega_1)+V'(\omega_2)$$

Given ω_1, this expression can be made as large as possible by taking $\omega_2\approx 0$. Thus, as b gets large, $h^\alpha>0$. By the Intermediate Value Theorem, there exists a b^* such that $h^\alpha(f(b^*),b^*)=0$.

(ii) *General offers*: We consider $(q_1,q_2)\gg 0$. The proof proceeds as above by fixing $(\bar{q}_1,\bar{q}_2)\gg 0$. Now, the proof is complete.

In this section, we demonstrated that a stationary equilibrium exists in this economy with trades that are non trivial.

4 Cycles and Sunspot Equilibria

A very well known result says that a necessary and sufficient condition for the existence of a cyclic equilibrium of order 2, is the existence of a sunspot equilibrium with two states [1]. Moreover, a sunspot equilibrium with two states can degenerate into a 2-cycle in which the matrix of transition probabilities has zeros in the diagonal, i.e., $\pi=\begin{bmatrix}0&1\\1&0\end{bmatrix}$.

Goenka *et al.* (1998) provide a condition under which a cyclic equilibrium of order 2 exists for the deterministic model of overlapping generations with market games, which is:

$$R_2(\omega_2+\hat{\theta})>\frac{(\omega_2+\hat{\theta})(\tilde{Q}-2\hat{\theta})}{\hat{\theta}(\tilde{Q}-\hat{\theta})}+\frac{\omega_2+\hat{\theta}}{\hat{\theta}}\left[\frac{2\hat{\theta}+\hat{Q}}{\hat{Q}+\hat{\theta}}+R_1(\omega_1-\hat{\theta})\frac{\hat{\theta}}{\omega_1-\hat{\theta}}\right]$$

where:

- $\theta_t=\frac{\hat{Q}m}{\hat{B}_t-m}$ and $\theta_t=\phi(\theta_{t+1})$.
- $\phi'(\hat{\theta})<-1$.
- $R_2(\cdot)$: Relative risk aversion of an old agent.
- $R_1(\cdot)$: Relative risk aversion of an young agent.

[1] See for example Azariadis and Guesnerie(1986)

Assuming that $\frac{U'(\omega_1)}{V'(\omega_2)} < \frac{1}{3}$ and that $R_2(0) > 0$, Goenka *et al.* (1998) prove that this condition for the existence of a two-cycle will hold at $\tilde{Q} = 0$ and, by continuity, for small positive values of $\tilde{Q}$.

A two-cycle equilibrium is one in which either:

$$-U'\left(\omega_1 - \frac{q_2\bar{m}}{b_2^\alpha - \bar{m}}\right)\left(\frac{q_2 b_2^\alpha}{(b_2^\alpha - \bar{m})^2}\right) + \pi^{\alpha\beta}V'\left(\omega_2 + \frac{q_1\bar{m}}{b_1^\beta - \bar{m}}\right)\left(\frac{q_1 b_1^\beta}{(b_1^\beta - \bar{m})^2}\right) = 0 \tag{10}$$

or,

$$-U'\left(\omega_1 - \frac{q_2\bar{m}}{b_2^\beta - \bar{m}}\right)\left(\frac{q_2 b_2^\beta}{(b_2^\beta - \bar{m})^2}\right) + \pi^{\beta\alpha}V'\left(\omega_2 + \frac{q_1\bar{m}}{b_1^\alpha - \bar{m}}\right)\left(\frac{q_1 b_1^\alpha}{(b_1^\alpha - \bar{m})^2}\right) = 0 \tag{11}$$

assuming that $\frac{U'(\omega_1)}{V'(\omega_2)} < \frac{1}{3}$ and that $R_2(0) > 0$.

The existence of the above cyclic stationary equilibria above described might be proved by showing that non-zero solutions to (14) and (15) exist. Let $q_1 = \epsilon_1$ and $q_2 = \epsilon_2$, where $\epsilon_1 \gg 0$ and $\epsilon_2 \gg 0$ but small enough so that we have small values of $\tilde{Q}$. Then (14) becomes:

$$-U'\left(\omega_1 - \frac{\epsilon_2\bar{m}}{b_2^\alpha - \bar{m}}\right)\left(\frac{\epsilon_2 b_2^\alpha}{(b_2^\alpha - \bar{m})^2}\right) + \pi^{\alpha\beta}V'\left(\omega_2 + \frac{\epsilon_1\bar{m}}{b_1^\beta - \bar{m}}\right)\left(\frac{\epsilon_1 b_1^\beta}{(b_1^\beta - \bar{m})^2}\right) = 0$$

The rest of the proof proceeds like in the proof for the existence of an stochastic stationary equilibrium. Additionally, we need to prove that $b_1^\alpha \neq b_1^\beta$, to ensure that this equilibrium is a sunspot equilibrium. This can be done by showing that the stochastic Nash equilibrium associated with the Markovian transition probability structure depends continuously on the probabilities.

By assumption, in the stationary environment we have that the following hold:

$$\begin{aligned} b_{t,t}^\rho &= b_1^\rho \\ b_{t,t+1}^{\rho\rho'} &= b_2^{\rho'} \\ q_{t,t} &= q_1 \\ q_{t,t+1} &= q_2 \end{aligned}$$

From the budget constraints we obtained the following results:

$$\begin{aligned} \frac{Q_t}{B_t^\rho} &= \frac{\hat{Q}_t}{\hat{B}_t^\rho - m_t} \\ \frac{Q_{t,t+1}}{B_{t,t+1}^{\rho\rho'}} &= \frac{\tilde{Q}_{t+1}}{\tilde{B}_{t+1}^{\rho\rho'} - m_t} \end{aligned}$$

which in the stationary environment become:

$$\begin{aligned} \frac{Q}{B^\rho} &= \frac{\hat{Q}}{\hat{B}^\rho - \bar{m}} \\ \frac{Q}{B^{\rho'}} &= \frac{\tilde{Q}}{\tilde{B}^{\rho'} - \bar{m}} \end{aligned}$$

meaning that:

$$\frac{\hat{Q}}{\hat{B}^{\rho}-\bar{m}}=\frac{\tilde{Q}}{\tilde{B}^{\rho}-\bar{m}}$$

It follows that:

$$\tilde{B}^{\rho}=\frac{\tilde{Q}(\hat{B}^{\rho}-\bar{m})}{\hat{Q}}-\bar{m}$$

In an economy with a single consumer in each generation, the above expression becomes:

$$b_1^{\rho}=\frac{q_1(b_2^{\beta}-\bar{m})}{q_2}-\bar{m}$$

Substituting the above result into the first order conditions that characterize a stochastic stationary equilibrium, we obtain:

$$h^{\alpha}(b_1^{\alpha},b_1^{\beta},b_2^{\alpha})=-U'\left(\omega_1-\frac{q_2\bar{m}}{b_2^{\alpha}-\bar{m}}\right)\left(\frac{q_2}{b_2^{\alpha}-\bar{m}}+\frac{q_2\bar{m}}{(b_2^{\alpha}-\bar{m})^2}\right)$$
$$+\pi^{\alpha\alpha}V'\left(\omega_2+\frac{q_2\bar{m}}{b_2^{\alpha}-\bar{m}}\right)\left(\frac{q_2}{b_2^{\alpha}-\bar{m}}-\frac{(q_2)^2\bar{m}}{q_1(b_2^{\alpha}-\bar{m})^2}\right)$$
$$+\pi^{\alpha\beta}V'\left(\omega_2+\frac{q_2\bar{m}}{b_2^{\beta}-\bar{m}}\right)\left(\frac{q_2}{b_2^{\beta}-\bar{m}}-\frac{(q_2)^2\bar{m}}{q_1(b_2^{\beta}-\bar{m})^2}\right)=0$$

and,

$$h^{\beta}(b_1^{\alpha},b_1^{\beta},b_2^{\beta})=-U'\left(\omega_1-\frac{q_2\bar{m}}{b_2^{\beta}-\bar{m}}\right)\left(\frac{q_2}{b_2^{\beta}-\bar{m}}+\frac{q_2\bar{m}}{(b_2^{\beta}-\bar{m})^2}\right)$$
$$+\pi^{\beta\alpha}V'\left(\omega_2+\frac{q_2\bar{m}}{b_2^{\alpha}-\bar{m}}\right)\left(\frac{q_2}{b_2^{\alpha}-\bar{m}}-\frac{(q_2)^2\bar{m}}{q_1(b_2^{\alpha}-\bar{m})^2}\right)$$
$$+\pi^{\beta\beta}V'\left(\omega_2+\frac{q_2\bar{m}}{b_2^{\beta}-\bar{m}}\right)\left(\frac{q_2}{b_2^{\beta}-\bar{m}}-\frac{(q_2)^2\bar{m}}{q_1(b_2^{\beta}-\bar{m})^2}\right)=0$$

Let $\theta^{\rho}=\frac{q_2\bar{m}}{b_2^{\rho}-\bar{m}}$ and $\bar{m}=1$.
Then,

$$h^{\alpha}=-U'(\omega_1-\theta^{\alpha})\left(\theta^{\alpha}+\frac{\theta^{\alpha}}{b_2^{\alpha}-\bar{m}}\right)+\pi^{\alpha\alpha}V'(\omega_2+\theta^{\alpha})\left(\theta^{\alpha}-\frac{\theta^{\alpha}q_2}{q_1(b_2^{\alpha}-\bar{m})}\right)$$

$$+\pi^{\alpha\beta}V'(\omega_2+\theta^{\beta})\left(\theta^{\beta}-\frac{\theta^{\beta}q_2}{q_1(b_2^{\beta}-\bar{m})}\right)=0$$

Knowing that $b_2^{\rho}-\bar{m}=\frac{q_2\bar{m}}{\theta^{\rho}}$ and that $\pi^{\alpha\alpha}=1-\pi^{\alpha\beta}$, we have that:

$$h^{\alpha}=-U'(\omega_1-\theta^{\alpha})\theta^{\alpha}\left(\frac{q_2+\theta^{\alpha}}{q_2}\right)+(1-\pi^{\alpha\beta})V'(\omega_2+\theta^{\alpha})\theta^{\alpha}\left(\frac{q_1-\theta^{\alpha}}{q_1}\right)$$

$$+\pi^{\alpha\beta}V'(\omega_2+\theta^\beta)\theta^\beta\left(\frac{q_1-\theta^\beta}{q_1}\right)=0$$

Similarly,

$$h^\beta=-U'(\omega_1-\theta^\beta)\theta^\beta\left(\frac{q_2+\theta^\beta}{q_2}\right)+\pi^{\beta\alpha}V'(\omega_2+\theta^\alpha)\theta^\alpha\left(\frac{q_1-\theta^\alpha}{q_1}\right)$$

$$+(1-\pi^{\beta\alpha})V'(\omega_2+\theta^\beta)\theta^\beta\left(\frac{q_1-\theta^\beta}{q_1}\right)=0$$

Now we need to show that the following Jacobian determinant is different from zero:

$$|D_\pi h|=\begin{vmatrix}\frac{\partial h^\alpha}{\partial\pi^{\alpha\beta}} & \frac{\partial h^\alpha}{\partial\pi^{\beta\alpha}}\\ \frac{\partial h^\beta}{\partial\pi^{\alpha\beta}} & \frac{\partial h^\beta}{\partial\pi^{\beta\alpha}}\end{vmatrix}$$

where:

- $\frac{\partial h^\alpha}{\partial\pi^{\alpha\beta}}=-V'(\omega_2+\theta^\alpha)\theta^\alpha\left(\frac{q_1-\theta^\alpha}{q_1}\right)+V'(\omega_2+\theta^\beta)\theta^\beta\left(\frac{q_1-\theta^\beta}{q_1}\right)$
- $\frac{\partial h^\alpha}{\partial\pi^{\beta\alpha}}=0$
- $\frac{\partial h^\beta}{\partial\pi^{\alpha\beta}}=0$
- $\frac{\partial h^\beta}{\partial\pi^{\beta\alpha}}=V'(\omega_2+\theta^\alpha)\theta^\alpha\left(\frac{q_1-\theta^\alpha}{q_1}\right)-V'(\omega_2+\theta^\beta)\theta^\beta\left(\frac{q_1-\theta^\beta}{q_1}\right)$

Then:

$$|D_\pi h|=\left[V'(\omega_2+\theta^\alpha)\theta^\alpha\left(\frac{q_1-\theta^\alpha}{q_1}\right)-V'(\omega_2+\theta^\beta)\theta^\beta\left(\frac{q_1-\theta^\beta}{q_1}\right)\right]^2$$

To show that $|D_\pi h|\neq 0$, we need to show that $q_1\neq\theta^\alpha$ and $q_1\neq\theta^\beta$, and that

$$V'(\omega_2+\theta^\alpha)\theta^\alpha\left(\frac{q_1-\theta^\alpha}{q_1}\right)\neq V'(\omega_2+\theta^\beta)\theta^\beta\left(\frac{q_1-\theta^\beta}{q_1}\right)$$

Since $\theta^\alpha=\frac{q_2}{b_2^\alpha-1}$ and $\theta^\beta=\frac{q_2}{b_2^\beta-1}$, and by assumption $\omega_1>\omega_2>0$, $q_1\leq\omega_1$, and $q_2\leq\omega_2$; then it follows that:

$$q_1\neq\theta^\alpha$$
$$q_1\neq\theta^\beta$$

Then, it remains to show the following claim.

Claim 2. In order for a sunspot equilibrium to exist the following condition must hold:

$$V'(\omega_2+\theta^\alpha)\theta^\alpha\left(\frac{q_1-\theta^\alpha}{q_1}\right)\neq V'(\omega_2+\theta^\beta)\theta^\beta\left(\frac{q_1-\theta^\beta}{q_1}\right)$$

Proof. Suppose not. Then:

$$V'(\omega_2+\theta^\alpha)\theta^\alpha\left(\frac{q_1-\theta^\alpha}{q_1}\right)=V'(\omega_2+\theta^\beta)\theta^\beta\left(\frac{q_1-\theta^\beta}{q_1}\right) \tag{12}$$

Assume that $\theta^\alpha \neq \theta^\beta$ since the cyclic equilibrium is non-degenerate.

The first order condition for an agent born in state α who faces state β in the following period is:

$$-U'(\omega_1-\theta^\alpha)\theta^\alpha\left(\frac{q_2+\theta^\alpha}{q_2}\right)+\pi^{\alpha\beta}V'(\omega_2+\theta^\beta)\theta^\beta\left(\frac{q_1-\theta^\beta}{q_1}\right)=0$$

Similarly, the first order condition for an agent born in state β who faces state α in the following period is:

$$-U'(\omega_1-\theta^\beta)\theta^\beta\left(\frac{q_2+\theta^\beta}{q_2}\right)+\pi^{\beta\alpha}V'(\omega_2+\theta^\alpha)\theta^\alpha\left(\frac{q_1-\theta^\alpha}{q_1}\right)=0$$

In a cyclic equilibrium it must be true that $\pi^{\alpha\beta}=\pi^{\beta\alpha}=1$, then:

$$U'(\omega_1-\theta^\alpha)\theta^\alpha\left(\frac{q_2+\theta^\alpha}{q_2}\right)=V'(\omega_2+\theta^\beta)\theta^\beta\left(\frac{q_1-\theta^\beta}{q_1}\right)$$

and,

$$U'(\omega_1-\theta^\beta)\theta^\beta\left(\frac{q_2+\theta^\beta}{q_2}\right)=V'(\omega_2+\theta^\alpha)\theta^\alpha\left(\frac{q_1-\theta^\alpha}{q_1}\right)$$

Then, by (16), we can say that the two first order conditions degenerate into a single condition:

$$U'(\omega_1-\hat\theta)\hat\theta\left(\frac{q_2+\hat\theta}{q_2}\right)=V'(\omega_2+\hat\theta)\hat\theta\left(\frac{q_1-\hat\theta}{q_1}\right)$$

where $\hat\theta=\theta^\alpha=\theta^\beta$, which contradicts the assumption that θ^α and θ^β are cyclic values.

Therefore, for a sunspot equilibrium to exist it must be true that:

$$V'(\omega_2+\theta^\alpha)\theta^\alpha\left(\frac{q_1-\theta^\alpha}{q_1}\right)\neq V'(\omega_2+\theta^\beta)\theta^\beta\left(\frac{q_1-\theta^\beta}{q_1}\right)$$

In this section, we have shown that sunspot equilibria exist in this model by using a condition provided by Goenka *et al.* (1998), in which strategic interactions and not risk-aversion generate the result.

5 Sunspot-like Equilibria

In this section we show that sunspot equilibria depend continuously on the offers. By showing this, we will be able to demosntrate that small stochastic variations in offers (perfectly correlated with the sunspots) can generate relatively large variations in the Nash equilibrium bids. This is along the lines of a result of Manuelli-Peck (1992), which establishes that if sunspot equilibria exist in a deterministic model, then it is possible to verify the existence of sunspot-like equilibria in the model perturbed by small amounts of fundamental uncertainty.

We now have that the offers of agent i are $q^{\rho}_{t,t}$ when young and $q^{\rho\rho'}_{t,t+1}$ when old, and that $q^{\rho\rho'}_{t} = [q^{\rho}_{t,t}, q^{\rho\rho'}_{t,t+1}]$. Offers are still constrained by the endowments in the following way: $q^{\rho}_{t,t} \leq \omega_1$ and $q^{\rho\rho'}_{t,t+1} \leq \omega_2$.

In aggregate terms, we have that $Q^{\rho}_{t,t} = nq^{\rho}_{t,t}$, $Q^{\rho\rho'}_{t,t+1} = nq^{\rho\rho'}_{t,t+1}$, and $Q^{\rho\rho'}_{t+1,t+1} = nq^{\rho\rho'}_{t+1,t+1}$. Consequently, $Q^{\rho}_{t} = Q^{\rho}_{t-1,t} + Q^{\rho}_{t,t}$, and $Q^{\rho\rho'}_{t+1} = Q^{\rho\rho'}_{t,t+1} + Q^{\rho\rho'}_{t+1,t+1}$. Similarly, $\hat{Q}^{\rho}_{t} = Q^{\rho}_{t} - q^{\rho}_{t,t}$, and $\tilde{Q}^{\rho\rho'}_{t+1} = Q^{\rho\rho'}_{t+1} - q^{\rho\rho'}_{t,t+1}$.

In the stationary environment we have that the offers depend only on the state realized at time t, that is, $q^{\rho}_{t,t} = q^{\rho}_{1}$ and $q^{\rho\rho'}_{t,t+1} = q^{\rho'}_{2}$.The first order conditions that characterize the stationary equilibrium are:

$$h^{\alpha} = -U'(\omega_1 - \theta^{\alpha})\theta^{\alpha}\left(\frac{q^{\alpha}_2 + \theta^{\alpha}}{q^{\alpha}_2}\right) + (1-\pi^{\alpha\beta})V'(\omega_2 + \theta^{\alpha})\theta^{\alpha}\left(\frac{q^{\alpha}_1 - \theta^{\alpha}}{q^{\alpha}_1}\right)$$

$$+\pi^{\alpha\beta}V'(\omega_2 + \theta^{\beta})\theta^{\beta}\left(\frac{q^{\beta}_1 - \theta^{\beta}}{q^{\beta}_1}\right) = 0$$

and,

$$h^{\beta} = -U'(\omega_1 - \theta^{\beta})\theta^{\beta}\left(\frac{q^{\beta}_2 + \theta^{\beta}}{q^{\beta}_2}\right) + \pi^{\beta\alpha}V'(\omega_2 + \theta^{\alpha})\theta^{\alpha}\left(\frac{q^{\alpha}_1 - \theta^{\alpha}}{q^{\alpha}_1}\right)$$

$$+(1-\pi^{\beta\alpha})V'(\omega_2 + \theta^{\beta})\theta^{\beta}\left(\frac{q^{\beta}_1 - \theta^{\beta}}{q^{\beta}_1}\right) = 0$$

We need to show that $|D_{\tilde{Q}_1}h| \neq 0$, where:

$$|D_{\tilde{Q}_1}h| = \begin{vmatrix} \frac{\partial h^{\alpha}}{\partial q^{\alpha}_1} & \frac{\partial h^{\alpha}}{\partial q^{\beta}_1} \\ \frac{\partial h^{\beta}}{\partial q^{\alpha}_1} & \frac{\partial h^{\beta}}{\partial q^{\beta}_1} \end{vmatrix}$$

where:

- $\frac{\partial h^{\alpha}}{\partial q^{\alpha}_1} = (1-\pi^{\alpha\beta})V'(\omega_2 + \theta^{\alpha})\left(\frac{\theta^{\alpha}}{q^{\alpha}_1}\right)^2 = 0$ since in a cyclic equilibrium $\pi^{\alpha\beta} = 1$.
- $\frac{\partial h^{\alpha}}{\partial q^{\beta}_1} = V'(\omega_2 + \theta^{\beta})\left(\frac{\theta^{\beta}}{q^{\beta}_1}\right)^2$
- $\frac{\partial h^{\beta}}{\partial q^{\alpha}_1} = V'(\omega_2 + \theta^{\alpha})\left(\frac{\theta^{\alpha}}{q^{\alpha}_1}\right)^2$

- $\frac{\partial h^\beta}{\partial q_1^\beta} = (1 - \pi^{\beta\alpha})V'(\omega_2 + \theta^\beta)\left(\frac{\theta^\beta}{q_1^\beta}\right)^2 = 0$ since in a cyclic equilibrium $\pi^{\beta\alpha} = 1$.

Hence,

$$|D_{\tilde{Q}_1} h| = -V'(\omega_2 + \theta^\alpha)V'(\omega_2 + \theta^\beta)\left(\frac{\theta^\alpha}{q_1^\alpha}\right)^2\left(\frac{\theta^\beta}{q_1^\beta}\right)^2$$

Provided that $q_1^\rho \neq 0$, $q_2^\rho \neq 0$ and $\omega_2 \neq 0$, we can ensure that $|D_{\tilde{Q}_1} h| \neq 0$.

Since $\theta^\rho = \frac{q_2^\rho}{b_2^\rho - 1}$, determining whether the first order conditions depend continuously on the $\tilde{Q}_2$'s demands a more careful analysis.

By the Implicit Function Theorem we have that:

$$|D_\theta h| \times D_{\tilde{Q}_2}\theta + |D_{\tilde{Q}_2} h| = 0$$

where:

$$|D_\theta h| = \begin{vmatrix} \frac{\partial h^\alpha}{\partial \theta^\alpha} & \frac{\partial h^\alpha}{\partial \theta^\beta} \\ \frac{\partial h^\beta}{\partial \theta^\alpha} & \frac{\partial h^\beta}{\partial \theta^\beta} \end{vmatrix}$$

where:

- $\frac{\partial h^\alpha}{\partial \theta^\alpha} = U''(\omega_1 - \theta^\alpha)\theta^\alpha\left(\frac{q_2^\alpha + \theta^\alpha}{q_2^\alpha}\right) - U'(\omega_1 - \theta^\alpha)\left(\frac{q_2^\alpha + 2\theta^\alpha}{q_2^\alpha}\right)$
- $\frac{\partial h^\alpha}{\partial \theta^\beta} = V''(\omega_2 + \theta^\beta)\theta^\beta\left(\frac{q_1^\beta - \theta^\beta}{q_1^\beta}\right) - U'(\omega_2 + \theta^\beta)\left(\frac{q_1^\beta - 2\theta^\beta}{q_1^\beta}\right)$
- $\frac{\partial h^\beta}{\partial \theta^\alpha} = V''(\omega_2 + \theta^\alpha)\theta^\alpha\left(\frac{q_1^\alpha - \theta^\alpha}{q_1^\alpha}\right) - U'(\omega_2 + \theta^\alpha)\left(\frac{q_1^\alpha - 2\theta^\alpha}{q_1^\alpha}\right)$
- $\frac{\partial h^\beta}{\partial theta^\beta} = U''(\omega_1 - \theta^\beta)\theta^\beta\left(\frac{q_2^\beta + \theta^\beta}{q_2^\beta}\right) - U'(\omega_1 - \theta^\beta)\left(\frac{q_2^\beta + 2\theta^\beta}{q_2^\beta}\right)$

We need to show now that the above Jacobian determinant is different from zero, so the Implicit Function Theorem can be applied. To avoid its computation, we take a related path. We argue that the correspondent Jacobian matrix is full-rank generically. If this matrix were less than full-rank we could restore it to a full rank by perturbing the second derivatives of the utility functions in a way that leaves unchanged the cyclic equilibrium, which only involves first derivatives.

Now, we need to show that the following Jacobian determinant is different from zero:

$$|D_{\tilde{Q}_2} h| = \begin{vmatrix} \frac{\partial h^\alpha}{\partial q_2^\alpha} & \frac{\partial h^\alpha}{\partial q_2^\beta} \\ \frac{\partial h^\beta}{\partial q_2^\alpha} & \frac{\partial h^\beta}{\partial q_2^\beta} \end{vmatrix}$$

where:

- $\frac{\partial h^\alpha}{\partial q_2^\alpha} = U'(\omega_1 - \theta^\alpha)\left[\frac{\theta^\alpha}{q_2^\alpha}\right]^2$
- $\frac{\partial h^\alpha}{\partial q_2^\beta} = 0$

- $\frac{\partial h^\beta}{\partial q_2^\alpha} = 0$

- $\frac{\partial h^\beta}{\partial q_2^\beta} = U'(\omega_1 - \theta^\beta)\left[\frac{\theta^\beta}{q_2^\beta}\right]^2$

As long as $\theta^\alpha \neq 0$, $\theta^\beta \neq 0$, $q_2^\alpha \neq 0$, $q_2^\beta \neq 0$, we can conclude that $|D_{\tilde{Q}_2} h| \neq 0$.

The result that sunspot-like equilibria exist in this model is now established. Moreover, we can now say that trembles in the offers of the agents, which are basically part of the their strategy.set, generate sunspot-like movements in equilibrium bids and prices. That is, trembles do not decrease but increase the level of indeterminacy of this economy. Now, we may ask the question of why agents would make mistakes when formulating their offers. An explanation may come from our assumption that agents are risk averse and, therefore, they might want to transfer wealth from one state to the other in the presence of extrinsic uncertainty, given that sunspot equilibria exist in this framework. In the process of transferring wealth among states, a given agent will probably consume less than what is available to him in one state, in order to consume more in another state. Hence, agents might make slight mistakes while formulating their offers. As a program for next research, it would be useful to model securities game associated with our model with extrinsic uncertainty and, try to answer the question of what might cause trembles in the agents' offers.

6 Conclusions

We have demonstrated that for the deterministic overlapping generations version of the Shapley-Shubik market game, the equilibrium bids vary accordingly to the realizations of a two-state sunspot variable. This result is not surprising since it was implied by Goenka *et al.* (1998). However, that analysis provided the ground to show that sunspot-like equilibria exist in our model. That is, when there is a small amount of intrinsic uncertainty in the offers of the agents, a element of the strategy set of the agents in the market game, the equilibrium bids and prices experience large variations. This result allows us to conclude that trembles in the offers of the agents increase the level of indeterminacy in this economy with strategic interaction rather than diminishing it.

Further research should first tackle the question of what causes the agent's trembles in formulationg offers. With this purpose, it would be a good idea to extend the game with extrinsic uncertainty to include securities that allow the agents to transfer wealth among the states of nature. Also, we will consider to introduce production in this economy to further investigate the effect of uncertainty in the offers of the agents and how this kind of uncertainty is related to the generation of business cycles.

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