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Do Private Benefits Prevent the Efficient Transfer of Control in a Firm? A Mechanism Design Approach

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Abstract

In this paper we apply the mechanism design technique to study the feasibility to achieve an efficient transfer of control in a firm where the controller enjoys private benefits and the value of the firm under each potential controller is private information. Previous literature shows that, under complete information, private benefits generate inefficiencies in the transfer of control. These results suggest that, in the presence of private information, the feasibility of achieving an efficient transfer of control should be reduced by the existence of private benefits. We show that this hypothesis is not correct. Under some plausible circumstances an increase in the capacity of the stockholders to extract private benefits can help the central planner to transfer control efficiently. These circumstances depend on the status quo utilities for the players and on how the private benefits are generated.

JEL codes: C72, D82, G32, G34
Keywords: Private Benefits, Takeovers, Status Quo Utilities.

Resumen

En este artículo aplicamos la técnica de diseño de mecanismos para estudiar la factibilidad de alcanzar una transferencia eficiente de control en una firma donde el controlador disfruta de beneficios privados, y el valor de la firma bajo cada potencial controlador es información privada. Artículos previos muestran que, bajo información completa, los beneficios privados generan ineficiencias en la transferencia de control. Estos resultados sugieren que, en presencia de información incompleta, la factibilidad de alcanzar una transferencia de control eficiente debiera verse reducida por la existencia de beneficios privados. Mostraremos que esta hipótesis no es correcta. Bajo circunstancias plausibles, un incremento en la capacidad de los accionistas de extraer beneficios privados puede ayudar a un coordinador central a transferir control eficientemente. Estas circunstancias dependen de las utilidades de reserva de los jugadores y de cómo se generan los beneficios privados.

Do Private Benefits Prevent the Efficient Transfer of Control in a Firm? A Mechanism Design Approach.*

by

Roberto E. Muñoz *

Abstract

In this paper we apply the mechanism design technique to study the feasibility to achieve an efficient transfer of control in a firm where the controller enjoys private benefits and the value of the firm under each potential controller is private information. Previous literature shows that, under complete information, private benefits generate inefficiencies in the transfer of control. These results suggest that, in the presence of private information, the feasibility of achieving an efficient transfer of control should be reduced by the existence of private benefits. We show that this hypothesis is not correct. Under some plausible circumstances an increase in the capacity of the stockholders to extract private benefits can help the central planner to transfer control efficiently. These circumstances depend on the status quo utilities for the players and on how the private benefits are generated.

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1 Introduction

The allocation of control rights in a firm has been a central problem in corporate finance since Manne (1965). The main question, from a central planner's point of view, is who has to have the control rights in order to maximize the value of the firm. In reality, the firm usually already has a controller, so the question turns on the feasibility of reaching an efficient transfer of control. A particular characteristic of this problem is that the benefits for shareholders are not related to their own ability to generate value for the firm but rather to the controller's ability. From this standpoint, every shareholder is interested in assigning control to the most efficient one. However, the controller could obtain private benefits from the firm in detriment to the rest of the shareholders. This effect leads to the shareholders wanting to gain control, even if they are not the most efficient. This trade-off makes it difficult, and sometimes impossible, to reach an efficient transfer of control. In this line, some well known negative results under complete information were provided by Grossman and Hart (1988) and Harris and Raviv (1988).[1]

The sources of private benefits for the controller has been extensively described in the literature. Controllers can obtain private benefits through, for example, engaging in self-dealing, capturing corporate opportunities, obtaining excessive salaries, or simply looting the firm, among other practices. The empirical evidence is also available. Barclay and Holderness (1989) estimate that in the United States the average premium over market prices in sales of blocks exceeding 5% of equity is around 4%

[1] Many of these models, however, assume the joint existence of private benefits and a bunch of shareholders who do not have the possibility to become a controller. They can only tender or not their shares in the takeover game. The negative results depend strongly on this assumption. In this paper we study the role of private benefits when all the shareholders are able to play the takeover game and can become the controller. In this context, ex post efficiency can always be achieved in the absense of private information.

of the equity value of the firms. On the other hand, the average premium of voting shares, in relation to non voting ones, range from 13% in England (Megginson (1990)) to 82% in Italy (Zingales (1994)). Thus, the empirical importance of private benefits seems to be considerable.

The traditional financial approach to study transfers of control or *takeovers* assumes complete information and the focus is to model the inefficiencies generated in tender offers under the free rider problem (e.g. Grossman and Hart (1980)) or the pressure to tender problem (e.g. Bagnoli and Lipman (1988)). Bebchuk (1994) showed that even in the absence of these effects, there are efficiency problems involved in transactions in which an existing controller sells its control block to an acquirer. More recently, Burkart, Gromb and Panunzi (1998, 2000) studied a takeover model in the presence of private benefits for the controller. The authors point out that private parties may choose to transfer corporate control in a way that does not maximize firm value (even when the control transfer could result in an increase in firm value). That is, the very choice of the transfer procedure can be subject to agency problems. However, neither Bebchuk (1994) nor Burkart, Gromb and Panunzi (1998, 2000) have studied the role of private benefits on the efficient transfer of control in the presence of private information. In order to do that, we need to revise the mechanism design literature and its relationship to takeovers.

From the mechanism design literature, we know that only some particular environments permit the existence of an ex post efficient mechanism to trade a good. For example, Myerson and Satterthwaite (1983) show that if it is not common knowledge that gains from trade exist, then no incentive compatible, individually rational trading mechanism can be ex post efficient. This result holds in a very general setting with one seller and one buyer and where both could have private information about the valuation of the good that is being traded. Cramton, Gibbons and Klemperer (1987) show that when the players share the initial ownership of the good and each one has his own private valuation, then, under some constraints over the initial endowments, it is possible to dissolve the partnership efficiently. However, this result only considers the possibility of going from a situation where the initial ownership is shared, to a final situation where the ownership is completely concentrated in the hands of those who value the good the most. Nagarajan (1995) studies how the existence of private information in valuations affects the possibility to achieve an ex post efficient transfer of control in a firm when the initial and the final ownership structure could be shared. His most important result is that if all contenders have private information, then a *generic* efficient mechanism does not exist. In other words, it is impossible for a central planner to find a single mechanism that assigns control rights ex post efficiently for *any* initial endowment. None of these papers, however, have considered the role of private benefits. In this paper we construct over Nagarajan's model to find an answer to the question: How would the existence of private benefits affect ex post efficient transfer of control in the presence of private information about valuations?

Specifically, we want to study the existence and properties of incentive compatible (IC), individually rational (IR), ex post efficient mechanisms to transfer control of a firm in the presence of private benefits for the controller and incomplete information. We want to know under which conditions over the initial ownership structure, if any, an (IC), (IR) ex post efficient mechanism to transfer control is available for the central planner and how its existence is affected when private benefits increase.

Nagarajan (1995) claims that the insights from Grossman and Hart (1988) and Harris and Raviv (1988), in addition to his own, suggest that mechanisms that are not generically efficient in the incomplete information framework are even less likely to achieve generic efficiency when control benefits are introduced. In this paper we will show that this hypothesis is not correct and the role of private benefits on the efficient transfer of control is positive in some plausible environments. In other words, the presence of private benefits can facilitate the finding of an ex post efficient mechanism to transfer control, even when such a mechanism does not exist in the absence of private benefits. Two elements play a key role in order to obtain this counterintuitive result. First, each player's beliefs about how the takeover game will be solved if they stay out of the game with their shares and, second, the correlations in the amount of private benefits extracted among the challengers and the incumbent. We discuss each below.

In our model we show inductively that there exist rational beliefs such that when one player stays out of the game with his shares, he thinks that the takeover subgame played by the other agents will be solved *efficiently*. In such a context, our most important result is that the set of firms in which an ex post efficient transfer of control is achievable shrinks in the private benefits of the incumbent, but it expands in the private benefits of rivals.[2,3] In particular, we will show that an exogenous and equal increase in private benefits leads to an expansion in the set of firms where an ex post efficient mechanism to transfer control exists. The intuition behind the result is that such an increase in private benefits generates two conflicting effects. On the one hand, it increases the status quo utility for the incumbent and the informational rents of the players, which makes it more difficult to achieve ex post efficiency. On the other hand, it has the positive effect of relaxing the individual rationality constraints for players other than the incumbent, because the stock value of the firm decreases and then it is less desirable for these players to stay out of the game. In many contexts, as when the agents are equally able to extract private benefits, the second effect dominates.

[2]The relevant set of firms is defined by the set of initial ownership structures where ex post efficiency is achievable.

[3]Remember that private benefits are only enjoyed by the controller, but in the takeover process all the agents are potential controllers. As a consequence, the level of private benefits that each of them is going to extract, in case to become the controller, is relevant for the result of the takeover game.

From the previous paragraph it is clear that we can not talk about an exogenous increase in private benefits without specifying whether this increase refers to the incumbent, the rivals or all of them. The correlations in the abilities to extract private benefits among the incumbent and the rivals are important. For example, suppose that the capacity to extract private benefits is positively correlated with the bargaining power of the agents.[4] In this context, an increase in the bargaining power of the incumbent leads to an increase in the private benefits he will be able to extract when he continues in control, but it also decreases the bargaining power of the rivals, and thus the private benefits that any of them would be able to extract, if they were to become the controller, also decrease. In such a case, an increase in the bargaining power of the incumbent always decreases the set of firms where ex post efficiency is achievable.

From a methodological standpoint, this paper extends the use of mechanism design techniques introduced by Nagajaran (1995) to a richer environment and we find conditions to move away from his negative results. We also introduce and exploit a specific functional form for the *status quo* utilities that can be sustained by rational beliefs. Differently from the previous literature, these status quo utilities are endogenous in our model, because if one agent decides to stay out with his shares, he has to have beliefs about how the takeover subgame will be solved by the rest of shareholders. After all, he is going to get a percentage of the stock value of the firm generated in such a subgame.

From an applied point of view, the study of the efficient transfer of control in the presence of private benefits permits us to study a broader class of problems than those analyzed in previous literature. For example, we can study the transfer of control in non-profit organizations and sport clubs, as well as political transitions. All of these

[4] For example, if one shareholder had a better access to mass media or political authorities then he could have a higher bargaining power at the interior of the firm.

can be understood as cases where the private benefits are important in relation to the stock value of the "firm". Intuitively, these are cases where the standard notion of dividend or payoff per share does not exist due to the nature of the organization. However, the private benefits are present and play a role in the efficient transfer of control. Another interesting application is in a privatization process where, in a first stage, the government originally issued shares to the private sector but it retained control. In a second stage, however, the authority is interested in transferring control to the private sector in the most efficient way. This second stage can be discussed under our model.[5]

The rest of the paper is organized as follows. Section 2 extends Nagarajan's model by including private benefits in order to get the standard tools of mechanism design in our particular problem. In section 3 we depart from the previous literature when we establish an inductive hypothesis that endogenously defines the *status quo* utilities for the players. Section 4 presents the main results of the paper under the assumption of no deadweight loss. It shows that private benefits, in some plausible scenarios, can help the central planner to achieve ex post efficiency. Section 5 extends, when it is possible, the results of section 4 to the case in the presence of deadweight loss. In section 6 we discuss the role of the inductive hypothesis in our model and its relation to generic ex post efficiency. Section 7 presents the main conclusions and suggestions for future research. Finally, the appendix with all the proofs is contained in section 8.

[5] We are not arguing that all the privatization processes follows this two stage scheme. But those doing it could benefit with the insights from this paper.

2 Mechanism Design in the Presence of Private Benefits

The goal of this section is to extend the mechanism design approach introduced by Nagarajan (1995) in a simple way including the possibility that controller can extract private benefits. Consider a firm that is jointly owned by n stockholders. The vector $\alpha = (\alpha_1 \ldots \alpha_n)$ denotes the initial shareholdings and $\sum_{i=1}^{n} \alpha_i = 1$. In general we denote $x = (x_1 \ldots x_n)$ a vector of shareholdings resulting from the application of the mechanism with $\sum_{i=1}^{n} x_i = 1$. Control is defined as the right to operate the firm. To keep the usual notation, we assume, without loss of generality that agent 1 is the incumbent, i.e., the initial controller. All shareholders are assumed to be risk neutral, with linearly separable utility for the value of their shareholdings and money.

The potential value of the entire firm if stockholder i gains control and operates the firm is v_i. This valuation is private information at the beginning of the takeover game, but it is common knowledge that these valuations are i.i.d. with cumulative distribution F and continuous density f over the support $[\underline{v}, \overline{v}]$. It is important to emphasize that v_i is just a potential value that may or may not be reached, because when the controller extracts private benefits some deadweight loss could appear. For example, the controller could switch contracts from some suppliers to others related to him but less efficient. This operation generates private benefits for the controller but also a deadweight loss for the firm in relation to the potential value v_i.

All the agents know that each stockholder i, if he becomes the controller of the firm, will extract private benefits according to an exogenously given parameter $0 \leq \phi_i < 1$ and a function $d_i(\phi_i)$ satisfying $d_i(\phi_i) \leq \phi_i$ and $1 \geq d_i'(\phi_i) > 0$ in $[0, 1)$. By now, the generating process of ϕ_i and $d_i(\phi_i)$ has not been specified,[6] so we are free to study

[6]The idyosincratic parameter ϕ_i can be understood as an upper bound for the fraction of potential value that agent i is able to extract without causing a legal conflict with the other shareholders. We are going to make extra assumptions, if needed, to satisfy that if agent i could choose this fraction

different backgrounds that lead to the same kind of reduced form.[7] The following description illustrates the process of extraction of private benefits. If agent i gains control then $(1-\phi_i)v_i$ will be available to be shared by all the stockholders, including the controller, in proportion to their shareholdings. Note that $(1-\phi_i)v_i$ represents the *stock value* of the firm under controller i, because it is the resulting value from adding the value of all the shares when i is in control. In addition, $d_i(\phi_i)v_i$ will be kept by the controller as private benefits. Note that the private benefits are enjoyed exclusively by the controller, in addition to the return of his shares, and that $(\phi_i - d_i(\phi_i))v_i \geq 0$ represents deadweight loss.[8]

It is important to note that the presence of a deadweight loss can only be justified, in the absence of other constraints, when there is no agent i who can be the sole owner of the firm, because in such a case there is no reason to incur a deadweight loss and it is more convenient for the central planner to avoid the extraction of private benefits assigning all the shares to the agent with maximal valuation. Note, however, that in the absence of deadweight loss (i.e. when $\phi_i = d_i(\phi_i)$) the final ownership structure does not affect the total value v_i reachable under controller i, because it is just divided as a stock value of $(1-\phi_i)v_i$ plus private benefits of $\phi_i v_i$. In what follows we develop

then he would choose to extract ϕ_i instead of any other $\phi_i' < \phi_i$.

[7]For example, it is possible that the value of ϕ_i is the result of a common knowledge technology to extract private benefits. Alternatively, it could be the result of some kind of bargaining power of individual i in front to the others. Depending on the case, we could assume that these parameters increase or decrease together or, they move in entirely different directions.

[8]In the complete information model of Burkart, Gromb and Panunzi (2000), ϕ_i depends on the shares that the final controller collects, and the selected ϕ_i is the result of an optimization problem for the controller. We have avoided that approach in this paper because in such a case an endogenous problem arise since the shares themselves (and thus ϕ_i) are an object to be designed in the mechanism as a function of the vector of valuations v. Nevertheless, we keep the notation as close as possible to that paper to refer private benefits and, in a companion paper, we discuss the share-dependant case.

a general setup in order to study, in the following sections, the cases in the absence and in the presence of deadweight loss. The main results of the paper do not depend on this factor.

We assume that ϕ_i and $d_i(\phi_i)$ are common knowledge so they are known and incorporated by the agents in the static takeover game. Moreover, the revelation principle allows us to use only equivalent direct revelation games where the information variable is just the vector of valuations. These are takeover games where each stockholder reports his valuation to a disinterested coordinator, in our case the central planner, who then determines the outcome of the takeover game according to a direct revelation takeover mechanism. Such mechanism must satisfy two standard conditions: individual rationality (IR) and incentive compatibility (IC). Individual rationality means that each stockholder must be willing to participate in the game. Incentive compatibility means that each stockholder has the incentive to reveal his type (in this case his valuation) truthfully.

Formally, $\forall v = (v_1 \ldots v_n) \in [\underline{v}, \overline{v}]^n$ and $\alpha = (\alpha_1 \ldots \alpha_n) \in \Delta^{n-1}$, a direct revelation takeover mechanism $\tau(v|\alpha) = \langle x(v|\alpha), y(v|\alpha), p(v|\alpha), 1_{\{y|\alpha\}} \rangle$ consist of:

i.- A share allocation correspondence $x(v|\alpha) = \{x_1(v|\alpha) \ldots x_n(v|\alpha)\}$ where $x_i(v|\alpha)$ is the fraction of shares allocated to stockholder i subject to $\sum_{i=1}^n x_i(v|\alpha) = 1$.

ii.- A vote allocation correspondence $y(v|\alpha) = \{y_1(v|\alpha) \ldots y_n(v|\alpha)\}$ where $y_i(v|\alpha)$ is the fraction of votes allocated to stockholder i subject to $\sum_{i=1}^n y_i(v|\alpha) = 1$.

iii.- A payment correspondence $p(v|\alpha) = \{p_1(v|\alpha) \ldots p_n(v|\alpha)\}$ where $p_i(v|\alpha)$ is the net monetary payment made to stockholder i for any transaction, subject to budget balance, i.e., $\sum_{i=1}^n p_i(v|\alpha) = 0$.

iv.- A voting game rule for awarding control $1_{\{y|\alpha\}} = \{1^1_{\{y|\alpha\}} \ldots 1^n_{\{y|\alpha\}}\}$ where $1^i_{\{y|\alpha\}} = 1$ if stockholder i wins control in the voting game and zero otherwise, subject to $\sum_{i=1}^n 1^i_{\{y|\alpha\}} = 1$.

It is important to note that the direct mechanism depends not only on the private reports of the stockholders, but also on the initial ownership structure. Moreover, following Nagarajan (1995), given that the stockholders do not receive any direct utility from votes, we can simplify the mechanism by assuming that the mechanism designer directly assigns control as a function of reports rather than using a voting game. Henceforth, the simplified takeover mechanism is given by $\tau(v|\alpha) = \langle x(v|\alpha), p(v|\alpha), 1_{\{T\}}(v|\alpha)\rangle$, where $1_{\{T\}}(v|\alpha) = \{1_{\{T_1\}}(v|\alpha) \ldots 1_{\{T_n\}}(v|\alpha)\}$ with $1_{\{T_i\}}(v|\alpha) = 1$ if i gains control and zero otherwise.

2.1 The Interim Utilities

The potential value of the firm under the mechanism and in the absence of deadweight loss is given by: $V^p(v) = \sum_{i=1}^n v_i 1_{\{T_i\}}(v)$. However, due to the presence of private benefits, it is convenient to define the *stock value of the firm* under the mechanism as: $V(v) = \sum_{i=1}^n (1-\phi_i) v_i 1_{\{T_i\}}(v)$. The payoff or utility to i under the mechanism is then given by:

$$U_i(v|\tau) = x_i(v) \sum_{k=1}^n (1-\phi_k) v_k 1_{\{T_k\}}(v) + d_i(\phi_i) v_i 1_{\{T_i\}}(v) + p_i(v)$$

The first term corresponds to the benefits for shareholder i when the firm is controlled by some shareholder k. The second term is the private benefit for shareholder i when he is in control. The third term is the transfer received by i in the mechanism.

To keep the standard notation, we define:

$N = \{1 \ldots n\}$, $D = [\underline{v}, \overline{v}]^n$, $D_{-i} = [\underline{v}, \overline{v}]^{n-1}$, $dF_n(v) = dF(v_1) \ldots dF(v_n)$, $dF_{-i}(v) = dF(v_1) \ldots dF(v_{i-1}) dF(v_{i+1}) \ldots dF(v_n)$.

Using this notation, the expected utility for i when his valuation is v_i, conditional on the use of a mechanism τ, is given by:

$$
\begin{aligned}
U_i[v_i|\tau] &= (1-\phi_i)v_i \int_{D_{-i}} x_i(v)1_{\{T_i\}}(v)dF_{-i}(v_{-i}) \\
&\quad + \sum_{k\neq i} \int_{D_{-i}} (1-\phi_k)v_k x_i(v)1_{\{T_k\}}(v)dF_{-i}(v_{-i}) \\
&\quad + d_i(\phi_i)v_i \int_{D_{-i}} 1_{\{T_i\}}(v)dF_{-i}(v_{-i}) \\
&\quad + \int_{D_{-i}} p_i(v)dF_{-i}(v_{-i})
\end{aligned}
$$

To simplify the notation we can define $\forall i,k \in N$:

$X[v_i|\tau] = \int_{D_{-i}} x_i(v)1_{\{T_i\}}(v)dF_{-i}(v_{-i})$

$Z_k[v_i|\tau] = \int_{D_{-i}}(1-\phi_k)v_k x_i(v)1_{\{T_k\}}(v)dF_{-i}(v_{-i})$

$I[v_i|\tau] = \int_{D_{-i}} 1_{\{T_i\}}(v)dF_{-i}(v_{-i})$

$P_i[v_i|\tau] = \int_{D_{-i}} p_i(v)dF_{-i}(v_{-i})$

Using this notation we have:

$$U_i[v_i|\tau] = (1-\phi_i)v_i X[v_i|\tau] + \sum_{k\neq i} Z_k[v_i|\tau] + d_i(\phi_i)v_i I[v_i|\tau] + P_i[v_i|\tau]$$

$U_i[v_i|\tau]$ are known as the *interim utilities* for the agents and they are the cornerstone for the study of the optimal mechanism design in any problem.

2.2 The Central Planner's Problem

Making use of the revelation principle, we assume that each agent reports to the central planner, truthfully or not, his valuation v_i. To distinguish the true valuation from the reported one, we denote $\widehat{v}_i$ the reported valuation. In what follows we look for a mathematically convenient way to express the (IC) and (IR) constraints.

Define the takeover mechanism $\tau = \langle x, p, 1_{\{T\}} \rangle$ as *incentive feasible* if it is (*interim Bayesian*) *incentive compatible* as well as *interim individually rational.* The takeover mechanism $\tau = \langle x, p, 1_{\{T\}} \rangle$ is incentive compatible if and only if honest reporting of each player's valuation forms a Bayesian Nash equilibrium. That is, if and only if for each stockholder i,

$$U_i[v_i|\tau] \geq (1-\phi_i)v_i X[\widehat{v}_i|\tau] + \sum_{k \neq i} Z_k[\widehat{v}_i|\tau] + d_i(\phi_i)v_i I[\widehat{v}_i|\tau] + P_i[\widehat{v}_i|\tau]$$

for all $v_i, \widehat{v}_i \in [\underline{v}, \overline{v}]$. The following lemma characterizes the conditions for incentive compatibility.

Lemma 1: *A direct revelation takeover mechanism $\tau = \langle x, p, 1_{\{T\}} \rangle$ is Bayesian interim incentive compatible if and only if $U_i[\cdot|\tau]$ is convex with*:

$U_i'[v_i|\tau] = (1-\phi_i)X[v_i|\tau] + d_i(\phi_i)I[v_i|\tau] > 0$ *a.e.*

where $(1-\phi_i)X[\cdot|\tau] + d_i(\phi_i)I[\cdot|\tau]$ *is non decreasing* $\forall i \in N$.

The lemma shows that incentive compatibility can be characterized, given ϕ_i, in terms of the expected probability of becoming the controller and the expected share that agent i would receive in case of becoming the controller.

On the other hand, the interim individual rationality condition requires that each stockholder must be willing to participate in the game, given his own valuation. In other words, the takeover mechanism must guarantee that the interim utility participating in the game is at least as much as the *status quo* utility for each player.[9] The next assumption is useful to simplify the incumbent's individual rationality constraint.

Assumption 1 (*Veto Power (VP)*). *The incumbent is the only agent with veto power against the mechanism.*

This is a standard assumption in the literature, despite that in reality some hostile takeovers do occur. One way to justify the assumption is to assume that the

[9] The *status quo* utility for agent i is the utility that he would receive if he did not play the takeover game, staying out with his shares α_i.

transfer rule establishes that the control is assigned to the agent who collects 50% or more of the shares (simple majority rule) and that initially the incumbent satisfies this requirement. An alternative way to justify Assumption 1 is to assume that the incumbent is the sole owner of some asset which is basic for the operation of the firm, as for example, the right to sign on behalf of the firm. Finally, in a second stage of a privatization process the government naturally has veto power over the mechanism, but in such a case we need to distinguish between the government as a central planner who designs the mechanism pursuing to maximize social welfare and the government as the incumbent of the firm.[10]

Under Assumption 1, if the incumbent stayed out of the game, the game would not be played and there would not be transfer of control, therefore his status quo utility is $\alpha_1(1-\phi_1)v_1 + d_1(\phi_1)v_1$ which represent the benefits he expects to get in case of keeping control. However, the challengers have no veto power so the status quo utility for each of them depends on what happens with the game played by the other $n-1$ players if the non player decides to stay out of the game retaining his shares. The definition of the status quo utilities will depend naturally on the solution of a takeover game with $n-1$ players, but this problem requires its own status quo utilities so we need to solve the problem for $n-2$ players and so on. In Nagarajan's paper, to avoid this nested problem, the author uses the fact that the valuations are assumed to be independent and therefore, at the interim stage, the expected status quo value of the firm for each challenger is a constant, bounded by the expected value of the higher order statistic among $n-1$ valuations. Instead, in our paper we explicitly consider the role of all the subgames involved. In order to simplify this problem, we are going to impose a condition so that the subgame played by l players ($l < n$) be independent

[10]The distinction is important, otherwise the central planner would maximize the benefits for the incumbent. The distinction is also plausible, because the government usually incorporates welfare considerations in its decisions.

of who stays out. So far this condition is not guarantee because ϕ_i and $d_i(\phi_i)$ depend on i, and thus, for example, the subgame played by $n-1$ players changes depending on who is out. One way to solve this problem is the following assumption:

Assumption 2 (*Symmetry of Challengers (SOC)*). *$\phi_i = \phi$ and $d_i(\cdot) = d(\cdot)$, $\forall i \in \{2 \dots n\}$.*

Under symmetry of challengers, the game played by the $n-1$ other players does not depend on which challenger remains out of the game.[11] As a consequence, the interim individual rationality constraints are:

$$\begin{aligned} \hat{U}_1(v_1|\tau) &= U_1(v_1|\tau) - [\alpha_1(1-\phi_1) + d_1(\phi_1)]\, v_1 \geq 0 \qquad (1) \\ \hat{U}_i(v_i|\tau) &= U_i(v_i|\tau) - \alpha_i(1-\phi_0)v_0 \geq 0, \quad \forall i = 2 \dots n \text{ and } v \in D \end{aligned}$$

Consider agent $i \neq 1$. Given that he does not have veto power, if he keeps his initial shares and stays out of the takeover game, he does not prevent the game being played by the other $n-1$ players. Consequently, he needs to establish some "beliefs" about how this subgame is played. Specifically, he needs to figure out the expected stock value of the firm if he stays out and the game is played by the other shareholders. Under the SOC assumption, this expected stock value is independent of i and therefore we denote it $(1-\phi_0)v_0$. In order to induce agent i to participate in the game, the central planner has to compensate him by at least $\alpha_i(1-\phi_0)v_0$.

In what follows the whole expression $(1-\phi_0)v_0$ will be used as a notation to refer to the expected stock value of the firm from the perspective of any agent i, other

[11] In the second stage of a privatization process this assumption means that the fraction of private benefits extracted under private and public administration of the firm is different, but it does not differ accross private agents.

than the controller, who decides to stay out.[12] The specific value for $(1-\phi_0)v_0$ will depend on the players' beliefs about how the subgames are solved, and it will play a central role in the paper.

Going back to equation (1), we can simplify the individual rationality constraints noting that $\hat{U}_1(v_1|\tau)$ is convex over $[\underline{v},\overline{v}]$ and therefore, it has a minimum at some $v_m \in [\underline{v},\overline{v}]$. On the other hand, for $i=2\ldots n$ we know that $\hat{U}_i(v_i|\tau)$ has a minimum at $\underline{v}$. The individual rationality constraints for all types of individuals are satisfied if and only if they are satisfied for the types with lowest utilities. In other words, the set of constraints in (1) can be reduced to:

$$\begin{aligned} \hat{U}_1(v_m|\tau) &= U_1(v_m|\tau) - [\alpha_1(1-\phi_1) + d_1(\phi_1)]\, v_m \geq 0 \\ \hat{U}_i(\underline{v}|\tau) &= U_i(\underline{v}|\tau) - \alpha_i(1-\phi_0)v_0 \geq 0, \quad \forall i = 2\ldots n \end{aligned} \tag{2}$$

Following the standard strategy in the mechanism design literature, we must provide a characterization of the set of incentive feasible direct revelation takeover mechanisms. The following theorem provides such a characterization:

[12] The idea of this notation is to make explicit that agent i's expected stock value of the firm is obtained from the stock value of the firm for the agents playing the game. These stock values have the form $(1-\phi_k)v_k$ with $k \neq i$.

Theorem 1: *For any incentive compatible takeover mechanism $\tau = \langle x, p, 1_{\{T\}} \rangle$ we have:*

$$
\begin{aligned}
& U_1[v_m|\tau] + \sum_{i=2}^{n} U_i[\underline{v}|\tau] = \\
& \sum_{i=1}^{n}(1-\phi_i) \int_D \left[v_i - \frac{(1-F(v_i))}{f(v_i)} x_i(v) \right] 1_{T_i}(v) dF_n(v) \\
& + \sum_{i=1}^{n} d_i(\phi_i) \int_D \left[v_i - \frac{(1-F(v_i))}{f(v_i)} \right] 1_{T_i}(v) dF_n(v) \\
& + \int_{D_{-1}} \int_{\underline{v}}^{v_m} \left[\frac{(1-\phi_1)x_1(v) + d_1(\phi_1)}{f(v_1)} \right] 1_{T_1}(v) dF_n(v)
\end{aligned} \tag{3}
$$

Moreover, under the (VP) and (SOC) assumptions, for any share correspondence $x(v)$ and control rule $1_{\{T\}}(v)$, there exists a payment correspondence $p(v)$ such that $\tau = \langle x, p, 1_{\{T\}} \rangle$ is incentive feasible iff:

a.- The function $(1-\phi_i)X[\cdot|\tau] + d_i(\phi_i)I[\cdot|\tau]$ is non decreasing $\forall i \in \{1 \ldots n\}$, and

b.- The function:

$$
\begin{aligned}
& W[\alpha, F, n, \phi_1, \phi|\tau] \equiv \\
& \sum_{i=1}^{n}(1-\phi_i) \int_D \left[v_i - \frac{(1-F(v_i))}{f(v_i)} x_i(v) \right] 1_{T_i}(v) dF_n(v) \\
& + \sum_{i=1}^{n} d_i(\phi_i) \int_D \left[v_i - \frac{(1-F(v_i))}{f(v_i)} \right] 1_{T_i}(v) dF_n(v) \\
& + \int_{D_{-1}} \int_{\underline{v}}^{v_m} \left[\frac{(1-\phi_1)x_1(v) + d_1(\phi_1)}{f(v_1)} \right] 1_{T_1}(v) dF_n(v) \\
& -(1-\alpha_1)(1-\phi_0)v_0 - [\alpha_1(1-\phi_1) + d_1(\phi_1)]v_m
\end{aligned} \tag{4}
$$

satisfies $W[\alpha, F, n, \phi_1, \phi|\tau] \geq 0$.

The intuition behind this theorem is as follows. In equation (3) we establish that, if the mechanism is incentive compatible, the sum of utilities for the lowest type's stockholders is equal to the expected achievable value of the firm, net of the informational rents required to induce them to reveal their types truthfully. The intuition for the second part of the theorem is easily understood in the only if part. When the mechanism is incentive feasible it is both Bayesian interim incentive compatible and interim individually rational. The incentive compatibility was characterized in Lemma 1 by the condition (a) in Theorem 1 and moreover it implies that the first part of Theorem 1 holds. On the other hand, equation (4) is a direct consequence of adding all the individual rationality constraints; the sum of utilities for the lowest type's stockholders must be greater than or equal to the sum of the status quo utilities.

We define an *ex post efficient* takeover mechanism as a mechanism that transfers the control to the agent able to generate the maximum *achievable* value of the firm.[13] Formally, the takeover mechanism $\tau = \langle x, p, 1_{\{T\}} \rangle$ is *ex post efficient* if and only if:

$$1_{\{T_i\}}(v) = \begin{cases} 1 & \text{if} \quad [(1-\phi_i) + d_i(\phi_i)]v_i \geq [(1-\phi_j) + d_j(\phi_j)]v_j \quad \forall\ i, j = 1 \ldots n \\ 0 & \text{otherwise} \end{cases}$$

In the context of this paper, ex post efficiency does not mean maximizing either the potential value of the firm, $V^p(v) = \sum_{i=1}^n v_i 1_{\{T_i\}}(v)$, or the stock value of the firm, $V(v) = \sum_{i=1}^n (1-\phi_i) v_i 1_{\{T_i\}}(v)$. The extraction of private benefits is not contractible so we know that whoever becomes the controller, he will extract private benefits possibly at the cost of some deadweight loss and consequently, we can not maximize the potential value of the firm because it is not reachable. On the other hand, the stock value of the firm does not incorporate the private benefits for the controller

[13] An alternative way to define ex post efficiency is to assume that the central planner wants to maximize only the stock value of the firm to protect minority shareholders. This consideration could be relevant in repeated games if we consider incentives to invest. However, this is beyond the scope of this paper.

which also have a social value. We could also try to maximize a weighted sum of stock value and private benefits, but in the static context of this paper we do not have a good argument to weight differently both kinds of benefits. As a result, we maximize the *achievable* value of the firm defined as the simple sum of the stock value and the private benefits.

Define $\mathcal{F}_n[\phi_1, \phi, F|\tau]$ as the set of firms where the takeover mechanism $\tau = \langle x, p, 1_{\{T\}} \rangle$ is incentive feasible. By Theorem 1 we can write:

$\mathcal{F}_n[\phi_1, \phi, F|\tau] = \{\alpha \mid \ \alpha \in \Delta^{n-1}, W[\alpha, F, n, \phi_1, \phi|\tau] \geq 0\}$

From Theorem 1, equation (4), it is also clear that $\mathcal{F}_n$ is defined just by α_1. We are interested in studying how $\mathcal{F}_n$ is affected when private benefits increase once we have imposed ex post efficiency. For example, if $\mathcal{F}_n$ expands when private benefits increase, then the set of firms where the transfer of control can be achieved ex post efficiently also expands. In our model the private benefits are given by $d_i(\phi_i)v_i$ where $d_i^{'}(\phi_i) > 0$ in $[0, 1)$ so the problem becomes the analysis of what happens to $\mathcal{F}_n$ when ϕ or ϕ_1 increases.[14] More precisely, we want to study the effect of an increase in private benefits over $\mathcal{F}_n$ for some predetermined and commonly used *transfer rules* in the mechanism. A *transfer rule* usually takes the form:

$$1_{\{T_i\}}(v) = \begin{cases} 1 & \text{if} \ \ x_i(v) \geq k \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

where k is a constant that defines which proportion of the shares (votes) is required to get control of the firm. The most popular ones are the *simple majority rule*, where

[14]Note that the condition $d_i^{'}(\phi_i) > 0$ in $[0, 1)$ implies that an increase in private benefits necessarily decreases the stock value of the firm under $i's$ control, $(1 - \phi_i)v_i$. A related question, which is not considered in this paper, is what happen if, given ϕ_i, agent i becomes more efficient in the extraction of private benefits. In this case the function $d_i(\cdot)$ itself would change, permitting a higher level of private benefits keeping constant the stock value of the firm.

$k = 0.5 + \epsilon$ and the supermajority rule where $k >> 0.5$, for example, $k = 0.75$.

A natural hypothesis to test is that if we are not able to find an ex post efficient and incentive feasible mechanism in the absence of private benefits, then in the presence of private benefits the situation should be worse. That is:

$\mathcal{F}_n[\phi_1, \phi, F|\tau] \subseteq \mathcal{F}_n[0, 0, F|\tau] \qquad \forall \ \ 0 \leq \phi, \phi_1 < 1$

As we will see, our model shows that this hypothesis is not correct in general, but it holds for some particular cases studied in the literature. In order to provide a proof, we are going to work with a simplified model that satisfies Theorem 1.

2.3 Specializing the Model

The expressions that appear in Theorem 1 are complicated, and we can achieve simplicity by making the following assumption:

Assumption 3 *(Public Incumbent's Valuation(PIV)). The incumbent valuation* v_1 *is not stochastic and it is common knowledge.*

Assumption 3 can be justified intuitively as follows. The value of the firm that each agent is able to generate could be a function of an agent's exogenously given ability to create value. When a particular agent is in control, his ability to create value, and therefore the potential value of the firm itself, is revealed. At the beginning of the takeover game, only the incumbent's ability has been revealed.

Now we can establish the following result:

Proposition 1. *Under the (VP), (SOC) and (PIV) assumptions, Theorem 1 holds with:*

$$
\begin{aligned}
W[\alpha, F, n, \phi_1, \phi|\tau] \;\; = \;\; & \sum_{i=2}^{n} \int_{D_{-1}} \Big\{ (1-\phi+d(\phi))v_i - (1-\phi_1+d_1(\phi_1))v_1 \\
& -\frac{(1-F(v_i))}{f(v_i)} \left[(1-\phi)x_i(v)+d(\phi)\right] \Big\} 1_{T_i}(v) dF_{-1}(v) \\
& -(1-\alpha_1)\left[(1-\phi_0)v_0-(1-\phi_1)v_1\right] \qquad (6)
\end{aligned}
$$

The inequality $W \geq 0$ is easier to interpret using equation (6) than equation (4). The expected achievable value of the firm net of the achievable value under the incumbent's control and net of informational rents, must be greater than or equal to the sum of the net increase in the expected stock value of the firm for the players other than the incumbent. Moreover, given that the incumbent has no private information, he does not receive informational rents.

Two important properties can be observed from Proposition 1. First, the informational rents for each rival are increasing in the shares assigned to him when he becomes the controller. As a consequence, the central planner will assign the minimum number of shares, consistent with the transfer rule, to the agent who becomes the controller i.e. $x_i(v) = k$. Second, as long as $d'(\phi) > k$,[15] the informational rents are increasing in ϕ. It is then apparent that an increase in ϕ could prevent the efficient transfer of control, for a given value of α_1, because the mechanism has to pay higher informational rents. This argument, however, ignores the effect of an increase of ϕ on the status quo utilities of the rivals or, more precisely, on $(1-\phi_0)v_0$. We will show that, under some reasonable assumptions, the net effect could generates an increase in W, which expands the set of values for α_1 where an efficient transfer of control is achievable.

[15] This is the case, for example, in the absence of deadweight loss because $d(\phi) = \phi$.

3 The Model with Endogenous Status Quo Utilities

In this section we introduce one of the main technological innovations of the paper which is to endogeneize the status quo utilities for the players. We are going to establish an induction hypothesis: in each stage player i set his status quo utilities under the belief that in all the subgames of the stage the takeover game will be solved ex post efficiently if he stays out. This is an induction hypothesis because what we want to find are precisely the conditions so that an ex post efficient mechanism exists. We will show later that these beliefs are indeed rational.

Induction Hypothesis *(Efficient Solution in Smaller Games (ESSG)). Consider any l players game with $2 < l \leq n$. The challengers expect that an ex post efficient takeover mechanism will be applied in the $l-1$ stockholder game if they stay out of the game. In other words*:

$$(1-\phi_0)v_0 = E\left\{\frac{(1-\phi_0)}{(1-\phi_0+d_0(\phi_0))}\left[Max\left\{(1-\phi_1+d_1(\phi_1))v_1,(1-\phi+d(\phi))\hat{v}_{l-2}\right\}\right]\right\}$$

where $\hat{v}_{l-2}$ is the higher order statistic among $l-2$ valuations and $d_0(\cdot)$ is the private benefit associated with the agent who becomes the controller in the $(l-1)$ takeover game and where the maximum is reached.

Note first that for two players an ex post efficient assignment of control is always feasible because, by Assumption 3, the incumbent does not have private information. On the other hand, the intuition behind the expression for $(1-\phi_0)v_0$ is that the challengers assume that, if they stay out of the game, the central planner will apply an ex post efficient mechanism to the active players in order to assign control. As a consequence, the new controller will be the active player who maximizes the achievable value of the firm, that is, the stock value of the firm plus private benefits.

However, the player who stays out only receives benefits from the stock value of the firm associated to the agent who becomes the controller, and this stock value is known only in expectation at the moment when the decision is made if it is convenient or not to play the game.

The central role of the induction hypothesis can be easily understood if, instead of it, the challengers expect that the incumbent will never lose control. In that case $v_0 = v_1$ and $\phi_0 = \phi_1$ and consequently $(1-\phi_0)v_0$ does not depend on ϕ and then, as long as $d'(\phi) > k$, an increase in ϕ necessarily diminishes $W[\alpha, F, n, \phi_1, \phi|\tau]$, making it more difficult to transfer control efficiently. This alternative assumption, however, is not rational because there always exists a probability greater than zero that the incumbent loses control in the takeover game.

Assumption 4 *(Increasing Stock Value (ISV)). Consider any l players game with $2 < l \leq n$. The expected stock value of the firm under the mechanism in the $(l-1)$ takeover game is greater than the stock value when the takeover is blocked by the incumbent. In other words, $(1-\phi_0)v_0 > (1-\phi_1)v_1$.*

Assumption 4 establishes that the takeover game creates value and we use it to show that the set of firms where an ex post efficient transfer of control is achievable in the n players game, $\mathcal{F}_n[\phi_1, \phi, F|\tau]$, not only is defined exclusively by α_1 (see Theorem 1) but it also takes the form:

$$\mathcal{F}_n[\phi_1, \phi, F|\tau] = \{\alpha_1| \quad \max\{\underline{\alpha}_1(\phi_1, \phi, F, \tau, n), 0\} \leq \alpha_1 \leq 1\} \tag{7}$$

where $\underline{\alpha}_1$ is defined by the condition $W[(\underline{\alpha}_1, \alpha_{-1}), F, n, \phi_1, \phi|\tau] = 0$.[16]

The reason why the shape of the set $\mathcal{F}_n[\phi_1, \phi, F|\tau]$ is simplified is because in this case $W[\alpha, F, n, \phi_1, \phi|\tau]$ is a non-decreasing function of α_1 (see equation (6)).

[16] The value of $\underline{\alpha}_1$ consistent with the condition $W = 0$ can be negative. In such a case, $\mathcal{F}_n$ becomes $[0, 1]$.

Consequently, the constraint $W[\alpha, F, n, \phi_1, \phi|\tau] \geq 0$ in the definition of $\mathcal{F}_n$ is satisfied above a lower bound $\underline{\alpha}_1$.[17] Given Assumption 4 it is clear that the analysis of how $\mathcal{F}_n$ is affected when ϕ or ϕ_1 increases is reduced to the analysis of how $\underline{\alpha}_1(\phi_1, \phi, F, \tau, n)$ is affected.

In what follows we work with the model in two different environments, with and without deadweight loss.

4 Solving in the Absense of Deadweight Loss

In this section our focus will be to find conditions over α_1 such that an ex post efficient mechanism exists in the n players game. By now, in addition to the assumptions, we work under the *ESSG* hypothesis. In section 6 we discuss the role of the induction hypothesis in the subgames and how the existence result is afected when we impose that beliefs have to be self consistent. Consider the following additional assumption:

Assumption 5 *(No Deadweight Loss (NDL)). Independently of who becomes the controller, we assume that there is no deadweight loss involved, that is:* $d(\phi) = \phi$ *and* $d_1(\phi_1) = \phi_1$.

Assumption 5 simplifies considerably the calculations and makes evident that the main results of the paper are not based on the existence and behavior of the deadweight loss. In the next section we will discuss how to extend the results to the case where a deadweight loss exists.

Under Assumptions 1 to 5 and *ESSG*, the expression for W is given by (8).

[17] This is not true, for example, when v_1 is stochastic, because in such a case a high enough α_1 can prevent an ex post efficient transfer of control due to the informational rents for the incumbent.

$$
\begin{aligned}
&W[\alpha, F, n, \phi_1, \phi|\tau] = \\
&\sum_{i=2}^{n} \int_{D_{-1}} \left\{ v_i - v_1 - \frac{(1-F(v_i))}{f(v_i)} \left[(1-\phi)x_i(v) + \phi\right] \right\} 1_{T_i}(v) dF_{-1}(v) \\
&-(1-\alpha_1)\left[(1-\phi_0)v_0 - (1-\phi_1)v_1\right] \qquad (8)
\end{aligned}
$$

$$
\begin{aligned}
\text{where} \qquad (1-\phi_0)v_0 &= E\left\{(1-\phi_0)\left[Max\left\{v_1, \hat{v}_{n-2}\right\}\right]\right\} \\
&= (1-\phi)\int_{v_1}^{\overline{v}} v \; dF^{n-2}(v) + (1-\phi_1)v_1 F^{n-2}(v_1)
\end{aligned}
$$

Based on (8) we can get some intuition about the role of Assumption 4. Suppose that, in the previous expression for $(1-\phi_0)v_0$, the maximum is reached for $i \neq 1$. If $\phi > \phi_1$ it could be the case that $(1-\phi)\hat{v}_{n-2} \leq (1-\phi_1)v_1$. Therefore, if v_1 is sufficiently close to $\underline{v}$ then $(1-\phi_0)v_0 \approx (1-\phi)\hat{v}_{n-2} \leq (1-\phi_1)v_1$, which is the opposite of the assumption. It is then clear that the assumption imposes a constraint on the relation between ϕ and ϕ_1. A simple sufficient condition that guarantee Assumption 4 is to assume that if $v_i > v_j$ then $(1-\phi_i)v_i > (1-\phi_j)v_j$. This condition is similar to Assumption 1 in Harris and Raviv (1988) and it says that a ranking based on potential value and stock value must be the same.[18] In our model, given that the parameters ϕ and ϕ_1 are exogenously given and that all the $\{v_i\}_{i=2}^{n}$ are coming from the same distribution F, the sufficient condition becomes $\phi \leq \phi_1$.

We are going to analyze two cases, the totally symmetric and the non-symmetric case under a generic transfer rule given by (5). We are going to discuss the implications of requiring ex post efficiency, given the transfer rule, in the set of *incentive feasible* mechanisms.

[18] More generally, in order to consider also the case in the presence of deadweight loss, we should assume that a ranking based on achievable value and stock value must be the same.

4.1 The Totally Symmetric Case

In this case $\phi = \phi_1$ and $d(\cdot) = d_1(\cdot)$ so, after imposing ex post efficiency, (8) becomes:

$$
\begin{aligned}
&W[\alpha, F, n, \phi, \phi|\tau] = \int_{v_1}^{\overline{v}} (v_i - v_1) dF^{n-1}(v_i) \\
&- [\phi + (1-\phi)k] \int_{v_1}^{\overline{v}} \frac{1-F(v_i)}{f(v_i)} dF^{n-1}(v_i) \\
&-(1-\alpha_1)(1-\phi) \left[\int_{v_1}^{\overline{v}} v_i \; dF^{n-2}(v_i) - v_1 \left(1 - F^{n-2}(v_1)\right) \right]
\end{aligned} \tag{9}
$$

The effect of an increase in private benefits on $\underline{\alpha}_1$ is given by the following proposition.

Proposition 2. *Under the (VP), (SOC), (PIV) and (NDL) assumptions and the induction hypothesis (ESSG), if* $\phi = \phi_1$ and $d(\cdot) = d_1(\cdot)$ *then:*

$$
\frac{\partial \underline{\alpha}_1(\phi, \phi, F, \tau, n)}{\partial \phi} \leq 0 \qquad\qquad \forall n > 2
$$

where $\underline{\alpha}_1(\phi, \phi, F, \tau, n)$ *is defined by the condition* $W[(\underline{\alpha}_1, \alpha_{-1}), F, n, \phi, \phi|\tau] = 0$.

Note first that Proposition 2 does not require Assumption 4 (ISV). The reason is that, under symmetry, (ISV) is automatically satisfied because:

$(1-\phi_0)v_0 = E\left\{(1-\phi)\left[Max\left\{v_1, \hat{v}_{n-2}\right\}\right]\right\} = (1-\phi)E\left\{\left[Max\left\{v_1, \hat{v}_{n-2}\right\}\right]\right\}$.

The result of Proposition 2 is important because it says that when the private benefits increase, the set of firms $\mathcal{F}_n[\phi, \phi, F|\tau]$, where there exists an ex post efficient and incentive feasible takeover mechanism, is expanded. That is, using (7), we have:

$\mathcal{F}_n[\phi, \phi, F|\tau] \subseteq \mathcal{F}_n[\phi + d\phi, \phi + d\phi, F|\tau]$

This result conflicts with the intuition provided in the literature that in the presence of private benefits the ex post efficiency should be more difficult to reach than in the absence of them. In fact, what Proposition 2 says is precisely the opposite; when private benefits increase it is easier to reach ex post efficiency. The intuition for this result is behind equation (9). An increase in ϕ generates two conflicting effects. On the one hand, the informational rents increase, making it more difficult to transfer control efficiently. On the other hand, it reduces the net increase in the expected stock value of the firm, relaxing the individual rationality constraint, which facilitates an efficient transfer of control. Proposition 2 says that the second effect dominates.

4.2 The Non-Symmetric Case

This is the original case represented by equation (8). From the analysis of the symmetric case we know that $\mathcal{F}_n[\phi, \phi, F|\tau]$ is increasing in the amount of private benefits. However, the symmetry imposed over the private benefits among the controller and the challengers could be hiding a more complex effect. Our goal here is to study whether the private benefits of controller and challengers have different effects on $\mathcal{F}_n$.

The following proposition establishes the main result for this case.

Proposition 3. *Under Assumptions 1 to 5 and the induction hypothesis (ESSG), the following inequalities hold for all* $n > 2$*:*

$$\frac{\partial \underline{\alpha}_1(\phi_1, \phi, F, \tau, n)}{\partial \phi} \leq 0$$

$$\frac{\partial \underline{\alpha}_1(\phi_1, \phi, F, \tau, n)}{\partial \phi_1} \geq 0$$

where $\underline{\alpha}_1(\phi_1, \phi, F, \tau, n)$ *is defined by the condition* $W[(\underline{\alpha}_1, \alpha_{-1}), F, n, \phi_1, \phi|\tau] = 0$.

The interpretation of these inequalities is that the role of private benefits is asymmetric between the incumbent and the challengers. The set $\mathcal{F}_n[\phi_1, \phi, F|\tau]$ expands in the private benefits of challengers but decreases in the private benefits of the incumbent for $n > 2$. However, under symmetry, the first effect dominates the second. It is also important to note that Propositions 2 and 3 hold for *any* distribution F on $[\underline{v}, \overline{v}]$ and for *any* transfer rule of the form given by (5).

The intuition behind Proposition 3 is easily understood analyzing the formula (8) and comparing the implications of different beliefs in a benchmark case and in our model.

<u>The Beliefs in the Benchmark Case</u>

Suppose first, as a benchmark case, that player i's beliefs about what happens with the stock value of the firm when he stays out of the game are such that $(1-\phi_0)v_0$ is constant with respect to ϕ and ϕ_1. In this case, from (8) we would have:

$\frac{\partial W}{\partial \phi} \leq 0$ when ϕ increases, because the informational rents required by the challengers increase.

$\frac{\partial W}{\partial \phi_1} \leq 0$ when ϕ_1 increases, because it is more difficult to compensate the initial controller when he loses control.

Assumption 4 implies that $\frac{\partial W}{\partial \alpha_1} \geq 0$, therefore the above inequalities and the definition of $\underline{\alpha}_1$ would lead to $\frac{\partial \underline{\alpha}_1}{\partial \phi} \geq 0$ and $\frac{\partial \underline{\alpha}_1}{\partial \phi_1} \geq 0$. These inequalities imply that the set $\mathcal{F}_n$ of firms where an ex post efficient transfer of control is feasible shrinks in both ϕ and ϕ_1. In other words, to compensate the negative effect of an increase in private benefits over W, we would have to consider higher values of α_1 in order to satisfy $W \geq 0$, so $\mathcal{F}_n$ would always shrink. Consequently, it is clear that the beliefs about which will be the stock value of the firm if player i stays out of the game, $(1-\phi_0)v_0$, plays a central role in explaining the signs we got in Propositions 2 and 3 (mainly $\frac{\partial \underline{\alpha}_1}{\partial \phi} \leq 0$).

<u>The Beliefs in our Model</u>

In opposition to the benchmark case, our induction hypothesis establishes that the beliefs are such that, at the interim stage, each player thinks that if he stays out of the game, the central planner will apply an ex post efficient mechanism in the takeover subgame. This beliefs leads to the expression for $(1-\phi_0)v_0$ in (8). The following intuition is also based on equation (8).

Consider the game with n players, $n > 2$. Under the $(ESSG)$ hypothesis, when ϕ increases we have two effects on W. First, the informational rents increase which has a negative effect on W. On the other hand, it is easier to satisfy the individual rationality constraints for the challengers, because the expected stock value in the $(n-1)$ takeover game, $(1-\phi_0)v_0$, decreases. This second effect has a positive impact on W. It is clear, from Proposition 3, that the second effect dominates, making $\underline{\alpha}_1$ decreasing in ϕ.

The comparative static for ϕ_1 is similar. In response to an increase in ϕ_1 there is a direct negative effect on W in (8), because it is more difficult to satisfy the individual rationality constraint for the incumbent. However, it is easier to satisfy the individual rationality constraints for the challengers. From Proposition 3, the first effect dominates so that $\underline{\alpha}_1$ is increasing in ϕ_1 making more difficult to transfer control efficiently.

In the symmetric case, where $\phi = \phi_1$, the positive effect of ϕ on W overrules the negative effect of ϕ_1.

The opposite effects of ϕ and ϕ_1 over $\underline{\alpha}_1$ lead us to the discussion of how plausible it is that ϕ and ϕ_1 move together, in opposite directions or independently. For example, if there exists a publicly known technology to extract private benefits, then it is reasonable to assume that ϕ and ϕ_1 move together. On the contrary, if the capacity to extract private benefits depends on the relative bargaining power of the players, when one player becomes stronger the others become weaker, leading to an inverse relationship between ϕ and ϕ_1. An in depth study of the properties of private benefits in particular applications is needed.[19]

We also need to check that the beliefs are indeed rational or self consistent. In

[19] For example, in the second stage of a privatization process the bargaining game seems more plausible.

other words, is it rational to believe that in every subgame the central planner can apply an ex post efficient mechanism to transfer control? The inductive hypothesis is formally analyzed in section 6, but the intuition is the following. When we have just two firms, an ex post efficient transfer of control is always possible, because the incumbent does not have private information in our model. Consider the game for $n > 2$ players. For any given stage involving j active players with $2 \leq j \leq n$, ex post efficiency is achievable over a subset of initial ownership structures such that $\alpha_1 \geq \underline{\alpha}_1^j$ (according to Proposition 2 or 3) with initial condition $\underline{\alpha}_1^2 = 0$. Taking the intersection of all these subsets we can guarantee that the beliefs are self consistent.

4.3 A Uniform Example

Consider the case when the prior distribution F is uniform over the interval $[0, 1]$. Moreover, the transfer rule establishes that any shareholder must reach at least the 50% of the shares (votes) in order to become the controller. We are going to illustrate the implications of Propositions 2 and 3.

In the uniform case we have that, imposing ex post efficiency given the transfer rule, (8) becomes:

$$
\begin{aligned}
W[\alpha, U[0,1], n, \phi_1, \phi|\tau] &= \frac{n-1}{n}\left(1 - v_1^n\right)\left(\frac{3+\phi}{2}\right) \\
&- \left(1 - v_1^{n-1}\right)\left(v_1 + \frac{1+\phi}{2}\right) \\
&-(1-\alpha_1)\left[(1-\phi_0)v_0 - (1-\phi_1)v_1\right] \qquad (10)
\end{aligned}
$$

where:

$$
(1-\phi_0)v_0 = (1-\phi)(n-2)\left[\frac{1}{n-1} - \frac{v_1^{n-1}}{n-1}\right] + (1-\phi_1)v_1^{n-1}
$$

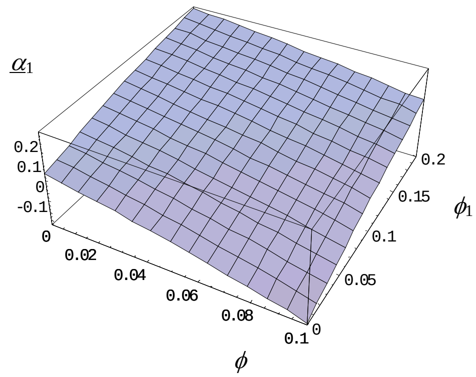

Figure 1: Dependence of $\underline{\alpha}_1$ on private benefits.

Figure 1 illustrates the dependence of $\underline{\alpha}_1$ on ϕ and ϕ_1 when the other parameters in the problem are fixed at: $v_1 = 0.5$ and $n = 10$. Above the surface represented by $\max\{\underline{\alpha}_1, 0\}$, it is possible to find an incentive feasible mechanism to transfer control ex post efficiently. Figure 1 shows that, for a given n and some values of ϕ and ϕ_1 there exists a mechanism that transfers control ex post efficiently for *any* initial endowment. This happens when $\underline{\alpha}_1 \leq 0$ (and thus $\mathcal{F}_n$ becomes $[0,1]$) as is the case for ϕ sufficiently large and ϕ_1 sufficiently small. The figure also shows that in the absence of private benefits (that is $\phi = 0$ and $\phi_1 = 0$) $\underline{\alpha}_1 > 0$, and thus ex post efficiency is not achievable for any initial endowment, which is consistent with Nagarajan's results. The totally symmetric case is illustrated in Figure 1 by the diagonal $\phi = \phi_1$. Proposition 2 says that $\underline{\alpha}_1$ is decreasing over this diagonal.

5 Solving in the Presence of Deadweight Loss

In this section we are going to study the role of private benefits on the efficient transfer of control in the presence of deadweight loss. Intuitively, the process of extraction of private benefits could naturally generate deadweight losses. For example, consider the case when the controller decides to cancel contracts of some efficient providers and give them to other less efficient ones who are related to him. In such a case, in order to generate private benefits it is necessary to incur in some level of deadweight loss.

From a theoretical standpoint, however, the presence of a deadweight loss can only be justified, in the absence of other constraints, when there is no agent i who can be the sole owner of the firm, because in such a case the central planner can avoid the extraction of private benefits, and the subsequent deadweight loss generated, by assigning all the shares to the agent with maximal valuation. Consequently, in this section we make the following assumptions.

Assumption 6 *(No Concentrated Ownership (NCO)). The final ownership structure under the mechanism can not be concentrated, that is, there exist* $\overline{x} \in (k, 1)$ so that*:* $x_i(v) \leq \overline{x} < 1 \ \ \forall i \in N, \ \ \forall v \in D.$

Assumption 7: *(Properties of the Private Benefit Functions (PPBF)) The functions* $d(\cdot)$ *and* $d_1(\cdot)$ *are strictly increasing and strictly concave on* $[0, 1]$*, with* $d_i(0) = 0$, $d_i'(0) = 1$ *and* $d_i'(1) = 0, \ \ \forall i \in N.$

Proposition 1 implies that the central planner has an incentive to minimize the shares assigned to the new controller under the mechanism in order to minimize the informational rents. Given this, Assumption 6 might seem innocuous. Unfortunately this is not true. Throughout the paper we have assumed that the controller extracts a constant proportion of private benefits, which does not depend on the shares assigned to him. Certainly, in the absence of any deadweight loss, the central planner does not care about how the total value is divided between stock value and private benefits because the total remains the same. However, in the presence of deadweight losses, the total value of the firm increases when private benefits decrease (because deadweight losses are increasing in private benefits). Consequently, the central planner might want to concentrate the ownership in the hands of the agent with maximal potential value in order to avoid deadweight losses.[20] In such an scenario, the assumption that the controller is going to extract a constant proportion of private benefits, at the cost of a deadweight loss, would not be appropriate. Assumption 6 is then necessary for the model in the presence of deadweight losses.[21]

How can we justify the assumption that the final ownership structure can not be completely concentrated? One possibility could be some legal constraints to the concentration in the ownership structure of the firms. These legal constraints will not be binding at the social optimum because the total welfare decreases in the share assigned to the controller.

We have argued that Assumption 6 is necessary to make our model theoretically consistent in the presence of deadweight loss, but it is not sufficient. If $\overline{x}$ is sufficiently high, say 0.95, it could be the case that for some agent, say i, it is more profitable to not extract private benefits or to extract less than what his idiosyncratic parameter ϕ_i permits him to do.[22] If this agent becomes the controller with a share of 0.95 he is going to choose ϕ_i' to maximize $0.95(1-\phi_i')v_i + d_i(\phi_i')v_i$ subject to $0 \leq \phi_i' \leq \phi_i$. The resulting ϕ_i' does not necessarily coincides with ϕ_i. Assumption 7 permits us to deal

[20]This incentive has to be weighted with the effect that a more concentrated final ownership structure implies higher informational rents (see the expression for W in Proposition 1) which reduce the feasibility to achieve ex post efficiency.

[21]In a companion paper we study a model with endogenous selection of private benefits and deadweight loss in the spirit of Burkart, Gromb and Panunzi (2000). In that model, if the ownership structure becomes concentrated then no deadweight loss is incurred.

[22]So far we have assumed that agent i actually extracts $d_i(\phi_i)v_i$ as private benefits when he becomes the controller and no less than that. In the absence of deadweight loss it is clear that agent i is going to extract as much as possible as private benefits, because he receives a one dollar return for each dollar extracted as private benefits, whereas his shares give him $x_i(v)$ dollars of return for each dollar the stock value of the firm increases.

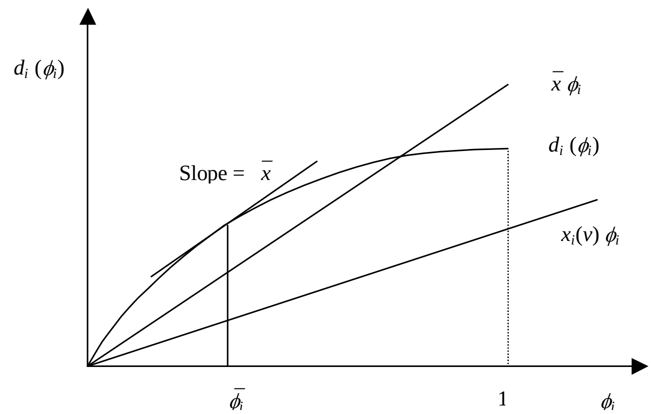

Figure 2: Zone for a rational adoption of ϕ_i.

with this problem, establishing sufficient conditions so that agent i always select ϕ_i.[23]

The reason why Assumption 7 is required to generate sufficient conditions for agent i to select ϕ_i instead of other $\phi_i' < \phi_i$ is illustrated in figure 2. Consider any amount of shares $x_i(v)$ assigned to the controller in the mechanism and satisfying Assumption 6. The controller should solve:

$$\underset{0 \leq \phi_i' \leq \phi_i}{Max} \{x_i(v)(1 - \phi_i')v_i + d_i(\phi_i')v_i\}$$

Assumption 7 permit us to guarantee that there exists $\overline{\phi_i} = d_i'^{\ -1}(\overline{x})$ such that for any value of the idiosyncratic parameter $\phi_i \leq \overline{\phi_i}$ the optimal selection for agent i is just ϕ_i and not lower.

[23]Assumption 7 coincides with assumption 1 in Burkart, Gromb and Panunzi (2000). However, the role of the assumption is completely different.

5.1 The Totally Symmetric Case

This case is defined by the conditions $\phi = \phi_1$ and $d(\cdot) = d_1(\cdot)$. The effect of an increase in private benefits over $\underline{\alpha}_1$ is given by the following proposition.

Proposition 4. *Under the (VP), (SOC), (PIV), (NCO) and (PPBF) assumptions and the induction hypothesis (ESSG), if* $\phi = \phi_1$ and $d(\cdot) = d_1(\cdot)$ *then:*

$$\frac{\partial \underline{\alpha}_1(\phi, \phi, F, \tau, n)}{\partial \phi} \leq 0 \qquad\qquad \forall n > 2\ ,\ \ 0 < \phi < \overline{\phi} = (d')^{-1}(\overline{x})$$

where $\underline{\alpha}_1(\phi, \phi, F, \tau, n)$ *is defined by the condition* $W[(\underline{\alpha}_1, \alpha_{-1}), F, n, \phi, \phi|\tau] = 0$ *and* $\overline{x}$ *satisfies (NCO).*

Proposition 4 is the natural extension of Proposition 2 in the presence of deadweight loss. It says that when private benefits increase, the set of firms $\mathcal{F}_n[\phi, \phi, F|\tau]$, where the takeover mechanism τ is ex post efficient and incentive feasible, is expanded. That is $\mathcal{F}_n[\phi, \phi, F|\tau] \subseteq \mathcal{F}_n[\phi + d\phi, \phi + d\phi, F|\tau]$ However, in contrast to the situation in the absense of deadweight loss, in this case the feasible values for ϕ are constrained to the interval $(0, \overline{\phi})$.

5.2 The Non-Symmetric Case

In the non-symmetric case in the presence of deadweight loss the equations become very difficult to work with, and we were not able to find an analytical analogue of Proposition 3. Instead we took a numerical approach.

In order to simplify the notation, let us define $l(\phi, \phi_1)$ as follows:

$$l(\phi, \phi_1) = \frac{1 - \phi_1 + d_1(\phi_1)}{1 - \phi + d(\phi)} \tag{11}$$

Using this definition, we can rewrite (6) as follows:

$$
\begin{aligned}
W[\alpha, F, n, \phi_1, \phi|\tau] \;\; = \;\; & \sum_{i=2}^{n}(1-\phi+d(\phi)) \int_{D_{-1}} \Big\{ v_i - l(\phi,\phi_1)v_1 \\
& -\frac{(1-F(v_i))}{f(v_i)} \left[\frac{(1-\phi)x_i(v)+d(\phi)}{1-\phi+d(\phi)}\right]\Big\} 1_{T_i}(v)dF_{-1}(v) \\
& -(1-\alpha_1)\left[(1-\phi_0)v_0-(1-\phi_1)v_1\right]
\end{aligned} \tag{12}
$$

Under Assumptions 6 and 7, in addition to 1 to 4, and imposing ex post efficiency, equation (12) becomes:

$$
\begin{aligned}
W[\alpha, F, n, \phi_1, \phi|\tau] \;\; = \;\; & (1-\phi+d(\phi)) \int_{l(\phi,\phi_1)v_1}^{\overline{v}} v_i \; dF^{n-1}(v_i) \\
& -(1-\phi_1+d_1(\phi_1))v_1 \int_{l(\phi,\phi_1)v_1}^{\overline{v}} dF^{n-1}(v_i) \\
& -\left[(1-\phi)k+d(\phi)\right] \int_{l(\phi,\phi_1)v_1}^{\overline{v}} \frac{1-F(v_i)}{f(v_i)} \; dF^{n-1}(v_i) \\
& -(1-\alpha_1)\left[(1-\phi_0)v_0-(1-\phi_1)v_1\right]
\end{aligned} \tag{13}
$$

with:

$$
(1-\phi_0)v_0 = (1-\phi)\int_{l(\phi,\phi_1)v_1}^{\overline{v}} v_i \; dF^{n-2}(v_i) + (1-\phi_1)v_1F^{n-2}(l(\phi,\phi_1)v_1)
$$

The complex dependence of W on ϕ and ϕ_1 makes it very difficult to find an analytical expression for the effect of an increase in private benefits over $\underline{\alpha}_1$.[24] However, we performed a simulation in the uniform case.

Consider the following functional forms for $d(\phi)$ and $d_1(\phi_1)$:

$$
d(\phi) = \phi\left(1-\frac{\phi^{\beta}}{1+\beta}\right) \quad \text{and} \quad d_1(\phi_1) = \phi_1\left(1-\frac{\phi_1^{\gamma}}{1+\gamma}\right) \tag{14}
$$

[24] Note that in the symmetric case $l(\phi,\phi_1)=1$ and then equation (13) is greatly simplified. That is the reason why we could get an analytical result in that case.

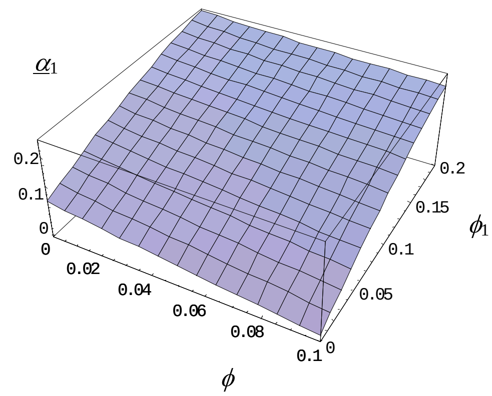

Figure 3: Dependence of $\underline{\alpha}_1$on private benefits in the presence of deadweight loss.

It is easy to check that these functions satisfy Assumption 7 (PPBF).

Figure 3 illustrates the dependence of $\underline{\alpha}_1$ on ϕ and ϕ_1 when F is a $U[0,1]$, the private benefit functions have been modeled as in (14) and the other parameters have been fixed in: $v_1 = 0.5$, $n = 10$, $\beta = 0.15$, $\gamma = 0.95$. Again, we see that $\underline{\alpha}_1$ depends negatively on ϕ and positively on ϕ_1.

We can conclude that the results obtained in the absence of deadweight loss can be extended, at least partially, to environments with a deadweight loss. However, although in the asymmetric case the qualitative effect under both cases is the same (see figures 1 and 3), the quantitative effect is very different. Figure 1 showed that, for some values of ϕ and ϕ_1, the set of firms where an ex post efficient transfer of control is achievable becomes $\mathcal{F}_n = [0,1]$ (this is the case when $\underline{\alpha}_1 \leq 0$). On the contrary, in the particular case analyzed in this section in the presence of deadweight loss, for any combination of values for ϕ and ϕ_1 in the range of the figure, it does not exist a mechanism to transfer control ex post efficiently for any initial endowment ($\underline{\alpha}_1$ is never below zero).

6 Rational Beliefs and Generic ex post Efficiency

Finally, we need to show that the beliefs sustained by the induction hypothesis are indeed rational. Specifically, we need to find a set of ownership structure where the beliefs are self sustained. Given that the agents are assuming that all the subgames will be solved ex post efficiently, it seems natural to claim that the set of firms where these beliefs are rational is the intersection of all $\mathcal{F}_j$ with $2 \leq j \leq n$. Let us call this set:

$$\mathcal{H}[\phi_1, \phi, F|\tau] = \bigcap_{j=2}^{n} \mathcal{F}_j[\phi_1, \phi, F|\tau] \qquad n \geq 2$$

Note that if $\alpha \in \mathcal{H}$ then, by definition of $\mathcal{F}_j$, it is possible to transfer control ex post efficiently in any subgame where the number of players is between 2 and n. Moreover, note that each set $\mathcal{F}_j$ is affected in the same direction when ϕ or ϕ_1 change and so does $\mathcal{H}$. For example, if ϕ increases and ϕ_1 remains constant, then $\mathcal{F}_j$ expands $\forall j = 2 \ldots n$ and, consequently, $\mathcal{H}$ expands.

It is important to note, however, that $\mathcal{H}$ is only a lower bound for the set of firms where in each subgame an incentive feasible mechanism exists to transfer control ex post efficiently. This is because in real subgames, if one agent, say k, decides to stay out of the game then his share α_k has the same qualitative effect on the mechanism as a symmetric increase in private benefits, which expands $\mathcal{F}_j$. On the other hand, the relevant initial share for the incumbent in the subgames is bigger than the original α_1, because some players are not playing the game, which also expands $\mathcal{F}_j$.

Figure 4 illustrates how $\mathcal{H}[\phi_1, \phi, F|\tau]$ is obtained in the no deadweight loss, non-symmetric uniform case. The figure shows $\underline{\alpha}_1$ as a function of ϕ and j when the other parameters in the problem have been fixed in $n = 10$, $v_1 = 0.5$ and $\phi_1 = 0.2$. For any fixed ϕ, $\mathcal{H}$ is obtained as $\bigcap_{j=2}^{n} \mathcal{F}_j[\phi_1, \phi, F|\tau] = \bigcap_{j=2}^{n} [\underline{\alpha}_1(\phi, j), 1]$. When ϕ increases, figure 4 depicts that each set $\mathcal{F}_j[\phi_1, \phi, F|\tau] = [\underline{\alpha}_1(\phi, j), 1]$ increases and then so does

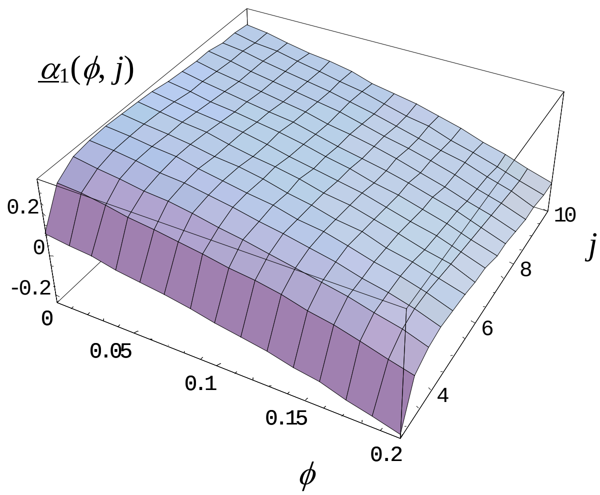

Figure 4: $\underline{\alpha}_1$ in the no deadweight loss, non symmetric uniform case as a function of ϕ and j.

their intersection.

It is also convenient to define a mechanism τ as *n-generic* ex post efficient if τ is able to transfer control ex post efficiently for *any* initial ownership structure in an n takeover game. In other words, when $\mathcal{H}[\phi_1, \phi, F|\tau] = [0, 1]$. This definition does not coincide with *generic* ex post efficiency, because in order to be generic, the mechanism has to achieve ex post efficiency independently of the number of stockholders. However, as the example in Figure 4 shows, n-generic and generic ex post efficiency could coincide for n sufficiently large when $\bigcap_{j=2}^{n} \mathcal{F}_j = \bigcap_{j=2}^{\infty} \mathcal{F}_j$.

7 Conclusion

This paper studies the existence and properties of incentive feasible, ex post efficient mechanisms to transfer control of a firm in the presence of private benefits and incomplete information.

We have shown that the role of private benefits in the efficient transfer of control could be positive, as opposed to what seems to be implied by the existing literature. This means that the existence of private benefits could make it feasible to find an ex post efficient mechanism, even when, for the same set of parameters for the problem, such a mechanism does not exist in the absence of private benefits. Moreover, in some cases it is possible to find a generic efficient mechanism due to the presence of private benefits.

Summarizing the insights of the paper, we identified two key elements in the analysis of how private benefits affect the set of firms $\mathcal{F}_n$ where an efficient transfer of control is feasible. First, the beliefs about how the central planner will solve the subgames if one player stays out. Along the paper, we worked with the induction hypothesis that each shareholder thinks that takeover subgames will be solved ex post efficiently if he remains out of the game with his shares. Under our assumptions and (ESSG) inductive hypothesis, an increase in private benefits of rivals (i.e. an increase in ϕ) expands $\mathcal{F}_n$, while an increase in private benefits of the incumbent (i.e. an increase in ϕ_1) shrinks it. Second, even under the (ESSG) inductive hypothesis, a change in the level of private benefits could expand or shrink $\mathcal{F}_n$ depending on the comovements of ϕ and ϕ_1. The simplest case is the symmetric one, where ϕ and ϕ_1 coincide and then we show that an increase in private benefits will always expand $\mathcal{F}_n$. However, if they do not coincide we need to analyze the feasibility that ϕ and ϕ_1 move together or in opposite directions, and the magnitude of such movements. For example, if an increase in ϕ is associated with a bigger increase in ϕ_1, then the net effect over $\mathcal{F}_n$ is ambiguous. On the other hand, if ϕ and ϕ_1 move in opposite directions, then an increase in ϕ will always expand $\mathcal{F}_n$ and a decrease in ϕ will always shrink it.

Consider, for example, the case of a second stage in a privatization process, i.e., when the government has to define the process to select the new controller of a firm where private shareholders have already been admitted, but the firm is initially under public administration. In such a case, it is more reasonable to assume that ϕ and ϕ_1 move in opposite directions because if the public administrators of a firm increase their capacity to extract private benefits when they are in control, then they are usually more able to keep a close surveillance on a private controller in case of losing control. Consequently, two policy recomendations arise. First, in the first stage when the government retains control and veto power, it should also keep a minimum amount of shares consistent with propositions 2 or 3 in this paper, otherwise it is not possible to guarantee an efficient transfer of control in the next stage. Second, in the design of the second stage, the authority should limit the bargaining power of the original public administrators (that means it should bound ϕ_1), because such a policy increases the feasibility to reach an efficient transfer of control.

Finally it is important to mention some extensions that appear desirable. First, we can study the case when private benefits are endogenously selected by the controller. Second, we can study the effect of departing from the one share-one vote's implicit assumption. Third, we could analyze the impact of introducing to the model a group of minority shareholders with no strategic role. All of these issues are part of our agenda.

8 Appendix

Proof of Lemma 1. Consider first the only if part. The takeover mechanism $\tau = \langle x, p, 1_{\{T\}} \rangle$ is Bayesian incentive compatible iff $\forall v_i, \hat{v}_i \in [\underline{v}, \overline{v}]$:

$$U_i[v_i|\tau] \geq (1-\phi_i)v_i X[\widehat{v}_i|\tau] + \sum_{k \neq i} Z_k[\widehat{v}_i|\tau] + d_i(\phi_i)v_i I[\widehat{v}_i|\tau] + P_i[\widehat{v}_i|\tau] \qquad (15)$$

Replacing the value of $U_i[\hat{v}_i|\tau]$ we have that (15) is equivalent to:

$$U_i[v_i|\tau] \geq U_i[\hat{v}_i|\tau] + (v_i - \hat{v}_i)\left[(1-\phi_i)X[\widehat{v}_i|\tau] + d_i(\phi_i)I[\widehat{v}_i|\tau]\right] \tag{16}$$

But this condition holds $\forall v_i, \hat{v}_i \in [\underline{v}, \overline{v}]$ so the same expression holds interchanging v_i and $\hat{v}_i$, that is:

$$U_i[\hat{v}_i|\tau] \geq U_i[v_i|\tau] + (\hat{v}_i - v_i)\left[(1-\phi_i)X[v_i|\tau] + d_i(\phi_i)I[v_i|\tau]\right] \tag{17}$$

So combining (16) and (17) we get:

$$(v_i - \hat{v}_i)\left[(1-\phi_i)X[v_i|\tau] + d_i(\phi_i)I[v_i|\tau]\right] \geq U_i[v_i|\tau] - U_i[\hat{v}_i|\tau] \geq$$

$$(v_i - \hat{v}_i)\left[(1-\phi_i)X[\widehat{v}_i|\tau] + d_i(\phi_i)I[\widehat{v}_i|\tau]\right] \tag{18}$$

From (18) we directly have that $XI[\cdot|\tau] \equiv (1-\phi_i)X[\cdot|\tau] + d_i(\phi_i)I[\cdot|\tau]$ is non decreasing. Moreover, taking the limit when $\hat{v}_i \to v_i$ we also get:

$$U_i'[v_i|\tau] = XI[v_i|\tau] \equiv (1-\phi_i)X[v_i|\tau] + d_i(\phi_i)I[v_i|\tau] \qquad \forall v_i \in [\underline{v}, \overline{v}]$$

So, in particular, $U_i[\cdot|\tau]$ is convex. To prove the if part, it is enough to use the reverse argument. ■

Proof of Theorem 1. To prove the first part of the Theorem, note that the ex ante expected utility of stockholder i is:

$$\begin{aligned}
\int_{\underline{v}}^{\overline{v}} U_i[v_i]dF(v_i) &= (1-\phi_i)\int_D v_i x_i(v) 1_{\{T_i\}}(v)dF_n(v) \\
&\quad + \sum_{k\neq i}(1-\phi_k)\int_D v_k x_i(v) 1_{\{T_k\}}(v)dF_n(v) \\
&\quad + d_i(\phi_i)\int_D v_i 1_{\{T_i\}}(v)dF_n(v) + \int_D p_i(v)dF_n(v)
\end{aligned}$$

Adding over i we get the cumulative ex ante expected welfare:

$$\begin{aligned}\sum_{i=1}^{n}\int_{\underline{v}}^{\overline{v}} U_i[v_i]dF(v_i) &= \sum_{i=1}^{n}\sum_{k=1}^{n}(1-\phi_k)\int_D v_k x_i(v)1_{\{T_k\}}(v)dF_n(v) \\ &+\sum_{i=1}^{n} d_i(\phi_i)\int_D v_i 1_{\{T_i\}}(v)dF_n(v)\end{aligned}$$

which imply:

$$\sum_{i=1}^{n}\int_{\underline{v}}^{\overline{v}} U_i[v_i]dF(v_i) = \sum_{i=1}^{n}(1-\phi_i+d_i(\phi_i))\int_D v_i 1_{\{T_i\}}(v)dF_n(v) \tag{19}$$

On the other hand, if $\tau = \langle x, p, 1_{\{T\}}\rangle$ is Bayesian incentive compatible then:

$$U_1[v_1|\tau] = U_1[v_m|\tau] + \int_{v_m}^{v_1} XI[u_1|\tau]du_1$$

$$U_i[v_i|\tau] = U_i[\underline{v}|\tau] + \int_{\underline{v}}^{v_i} XI[u_i|\tau]du_i \qquad i = 2\ldots n$$

So the respective ex ante expected utilities are:

$$\int_{\underline{v}}^{\overline{v}} U_1[v_1|\tau]dF(v_1) = U_1[v_m|\tau] + \int_{\underline{v}}^{\overline{v}}\int_{v_m}^{v_1} XI[u_1|\tau]du_1 dF(v_1)$$

$$\int_{\underline{v}}^{\overline{v}} U_i[v_i|\tau]dF(v_i) = U_i[\underline{v}|\tau] + \int_{\underline{v}}^{\overline{v}}\int_{\underline{v}}^{v_i} XI[u_i|\tau]du_i dF(v_i) \qquad i = 2\ldots n$$

So adding over i we got a second expression for the cumulative ex ante expected welfare:

$$\sum_{i=1}^{n}\int_{\underline{v}}^{\overline{v}} U_i[v_i|\tau]dF(v_i) = U_1[v_m|\tau] + \sum_{i=2}^{n} U_i[\underline{v}|\tau] + \int_{\underline{v}}^{\overline{v}}\int_{v_m}^{v_1} XI[u_1|\tau]du_1 dF(v_1) + \sum_{i=2}^{n}\int_{\underline{v}}^{\overline{v}}\int_{\underline{v}}^{v_i} XI[u_i|\tau]du_i dF(v_i)$$

$$= U_1[v_m|\tau] + \sum_{i=2}^{n} U_i[\underline{v}|\tau] + \int_{v_m}^{\overline{v}}\int_{v_m}^{v_1} XI[u_1|\tau]du_1 dF(v_1) - \int_{\underline{v}}^{v_m}\int_{v_1}^{v_m} XI[u_1|\tau]du_1 dF(v_1) + \sum_{i=2}^{n}\int_{\underline{v}}^{\overline{v}}\int_{\underline{v}}^{v_i} XI[u_i|\tau]du_i dF(v_i)$$

and then:

$$\sum_{i=1}^{n}\int_{\underline{v}}^{\overline{v}} U_i[v_i|\tau]dF(v_i) = U_1[v_m|\tau] + \sum_{i=2}^{n} U_i[\underline{v}|\tau] + \sum_{i=1}^{n}\int_{\underline{v}}^{\overline{v}} [1 - F(u_i)]\, XI[u_i|\tau]du_i - \int_{\underline{v}}^{v_m} XI[u_1|\tau]du_1$$

replacing the expressions for $XI[u_i|\tau]$ we have:

$$\sum_{i=1}^{n}\int_{\underline{v}}^{\overline{v}} U_i[v_i|\tau]dF(v_i) = U_1[v_m|\tau] + \sum_{i=2}^{n} U_i[\underline{v}|\tau] + \sum_{i=1}^{n}\int_{D} \frac{[1 - F(u_i)]}{f(u_i)} [(1-\phi_i)x_i(u) + d_i(\phi_i)]\, 1_{\{T_i\}}(u)dF_n(u) - \int_{D_{-1}}\int_{\underline{v}}^{v_m} \frac{[(1-\phi_1)x_1(u) + d_1(\phi_1)]}{f(u_1)} 1_{\{T_1\}}(u)dF_n(u) \qquad (20)$$

Equating (19) and (20) and reordering we have:

$$\begin{aligned}
& U_1[v_m|\tau] + \sum_{i=2}^{n} U_i[\underline{v}|\tau] = \\
& \sum_{i=1}^{n} (1-\phi_i) \int_D \left[v_i - \frac{[1-F(v_i)]}{f(v_i)} x_i(v) \right] 1_{\{T_i\}}(v) dF_n(v) \\
& + \sum_{i=1}^{n} d_i(\phi_i) \int_D \left[v_i - \frac{[1-F(v_i)]}{f(v_i)} \right] 1_{\{T_i\}}(v) dF_n(v) \\
& + \int_{D_{-1}} \int_{\underline{v}}^{v_m} \frac{[(1-\phi_1) x_1(v) + d_1(\phi_1)]}{f(v_1)} 1_{\{T_1\}}(v) dF_n(v)
\end{aligned}$$

Which proves the first part of Theorem 1.

For the second part of the Theorem, consider first the *only if* part. Consider a feasible mechanism $\tau = \langle x, p, 1_{\{T\}} \rangle$. Combining (2) and (3) we directly get (4).

To prove the if part of the second part of Theorem 1, for any given $x(v)$ and $1_{\{T\}}(v)$ and we must construct a payment mechanism $p(v)$ such that $\tau = \langle x, p, 1_{\{T\}} \rangle$ is incentive feasible. The proof is similar to Nagarajan (1995), where in our case:

$$\begin{aligned}
p_i(v) &= c_i - \int_{\underline{v}}^{v_i} u_i dXI[u_i|\tau] + \frac{1}{n-1} \sum_{j \neq i} \int_{\underline{v}}^{v_j} u_j dXI[u_j|\tau] \\
&\quad - \sum_{k \neq i} \int_{\underline{v}}^{v_i} dZ_k[u_i|\tau] + \frac{1}{n-1} \sum_{j \neq i} \sum_{k \neq j} \int_{\underline{v}}^{v_j} dZ_k[u_j|\tau]
\end{aligned}$$

$$\begin{aligned}
c_1 &= C + [\alpha_1(1-\phi_1) + d_1(\phi_1)]\, v_m - v_m XI[v_m|\tau] - \sum_{k \neq 1} Z_k[v_m|\tau] \\
&\quad + \int_{\underline{v}}^{v_m} u_1 dXI[u_1|\tau] + \sum_{k \neq 1} \int_{\underline{v}}^{v_m} dZ_k[u_1|\tau] \\
&\quad - \frac{1}{n-1} \sum_{j \neq 1} \int_{\underline{v}}^{\overline{v}} [1-F(u_j)]\, u_j dXI[u_j|\tau] \\
&\quad - \frac{1}{n-1} \sum_{j \neq 1} \sum_{k \neq j} \int_{\underline{v}}^{\overline{v}} [1-F(u_j)]\, dZ_k[u_j|\tau]
\end{aligned}$$

$$\begin{aligned}
c_i &= C + \alpha_i(1-\phi_0)v_0 - \underline{v}XI[v_i = \underline{v}|\tau] - \sum_{k\neq i} Z_k[v_i = \underline{v}|\tau] \\
&\quad -\frac{1}{n-1}\sum_{j\neq i}\int_{\underline{v}}^{\overline{v}} [1-F(u_j)]\, u_j dXI[u_j|\tau] \\
&\quad -\frac{1}{n-1}\sum_{j\neq i}\sum_{k\neq j}\int_{\underline{v}}^{\overline{v}} [1-F(u_j)]\, dZ_k[u_j|\tau]
\end{aligned}$$

and

$$C = \frac{1}{n}\left[U_1[v_m|\tau] + \sum_{i=2}^{n} U_i[\underline{v}|\tau] - [\alpha_1(1-\phi_1) + d_1(\phi_1)]\, v_m - (1-\alpha_1)(1-\phi_0)v_0\right]$$

the rest of the proof is omitted. ■

Proof of Proposition 1. We just need to look for an expression for W in the case when the incumbent's valuation is common knowledge.

The ex ante expected utility of stockholder i is given by:

$$\begin{aligned}
\int_{\underline{v}}^{\overline{v}} U_i[v_i]dF(v_i) &= (1-\phi_i)\int_{D_{-1}} v_i x_i(v) 1_{\{T_i\}}(v) dF_{-1}(v) \\
&\quad + \sum_{k\neq i}(1-\phi_k)\int_{D_{-1}} v_k x_i(v) 1_{\{T_k\}}(v) dF_{-1}(v) \\
&\quad + d_i(\phi_i)\int_{D_{-1}} v_i 1_{\{T_i\}}(v) dF_{-1}(v) + \int_{D_{-1}} p_i(v) dF_{-1}(v)
\end{aligned}$$

Following the same steps as in the proof of Theorem 1 we get:

$$\begin{aligned}
\sum_{i=1}^{n}\int_{\underline{v}}^{\overline{v}} U_i[v_i]dF(v_i) &= (1-\phi_1+d_1(\phi_1))v_1\int_{D_{-1}} 1_{\{T_1\}}(v)dF_{-1}(v) \\
&\quad + \sum_{i=2}^{n}(1-\phi_i+d_i(\phi_i))\int_{D_{-1}} v_i 1_{\{T_i\}}(v) dF_{-1}(v)
\end{aligned} \tag{21}$$

On the other hand:

$$\begin{aligned}\sum_{i=1}^{n}\int_{\underline{v}}^{\overline{v}} U_i[v_i|\tau]dF(v_i) &= U_1[v_1|\tau]+\sum_{i=2}^{n}U_i[\underline{v}|\tau] \\ &+\sum_{i=2}^{n}\int_{\underline{v}}^{\overline{v}}\int_{\underline{v}}^{v_i} XI[u_i|\tau]du_i dF(v_i)\end{aligned}$$

and following the usual steps:

$$\begin{gathered}\sum_{i=1}^{n}\int_{\underline{v}}^{\overline{v}} U_i[v_i|\tau]dF(v_i) = U_1[v_1|\tau]+\sum_{i=2}^{n}U_i[\underline{v}|\tau] \\ +\sum_{i=2}^{n}\int_{D_{-1}}\frac{[1-F(u_i)]}{f(u_i)}\left[(1-\phi_i)x_i(u)+d_i(\phi_i)\right]1_{\{T_i\}}(u)dF_{-1}(u)\end{gathered} \tag{22}$$

Equating (21) and (22) and imposing the individual rationality constraints we have:

$$\begin{aligned}W &= \sum_{i=2}^{n}\int_{D_{-1}}\{(1-\phi_i+d_i(\phi_i))v_i-(1-\phi_1+d_1(\phi_1))v_1 \\ &\quad -\frac{[1-F(v_i)]}{f(v_i)}\left[(1-\phi_i)x_i(v)+d_i(\phi_i)\right]\Big\}1_{\{T_i\}}(v)dF_{-1}(v) \\ &\quad -(1-\alpha_1)((1-\phi_0)v_0-(1-\phi_1)v_1)\end{aligned}$$

Finally, noting that under the SOC Assumption: $\phi_i=\phi$ and $d_i(\phi_i)=d(\phi)$, for $i=2...n$ we have the result. ■

Proof of Proposition 2: In the symmetric case we have:

$$\underline{\alpha}_1=\frac{1}{a_3}(a_2+a_3-a_1)\equiv\frac{A}{a_3}$$

where:

$$
\begin{aligned}
a_1 &= \int_{v_1}^{\overline{v}} (v_i - v_1) dF^{n-1}(v_i) \\
a_2 &= [\phi + (1-\phi)k] \int_{v_1}^{\overline{v}} \frac{1-F(v_i)}{f(v_i)} dF^{n-1}(v_i) \\
a_3 &= (1-\phi)\left[\int_{v_1}^{\overline{v}} v_i \; dF^{n-2}(v_i) - v_1\left(1 - F^{n-2}(v_1)\right)\right] \equiv (1-\phi)\beta(v_1, n)
\end{aligned}
$$

Using this notation:

$$
\frac{\partial \underline{\alpha}_1}{\partial \phi} = \frac{a_3 \partial A/\partial \phi - A \partial a_3/\partial \phi}{a_3^2}
$$

then we have:

$$
sgn\left\{\frac{\partial \underline{\alpha}_1}{\partial \phi}\right\} = sgn\left\{a_3 \frac{\partial a_2}{\partial \phi} - (a_2 - a_1)\frac{\partial a_3}{\partial \phi}\right\}
$$

so

$$
sgn\left\{\frac{\partial \underline{\alpha}_1}{\partial \phi}\right\} = sgn\left\{\beta(v_1, n)\left[(1-\phi)\frac{\partial a_2}{\partial \phi} + a_2 - a_1\right]\right\} \tag{23}
$$

After some calculations and noting that $\beta(v_1, n) \geq 0$ we have:

$$
(1-\phi)\frac{\partial a_2}{\partial \phi} + a_2 - a_1 = \int_{v_1}^{\overline{v}} \left[\frac{1-F(v_i)}{f(v_i)} - v_i + v_1\right] dF^{n-1}(v_i)
$$

But:

$$
\int_{v_1}^{\overline{v}} \frac{1-F(v_i)}{f(v_i)} dF^{n-1}(v_i) = (n-1)\int_{v_1}^{\overline{v}} \left[F^{n-2}(v_i) - F^{n-1}(v_i)\right] dv_i
$$

and:

$$
\int_{v_1}^{\overline{v}} (v_i - v_1)\, dF^{n-1}(v_i) = \overline{v} - v_1 - \int_{v_1}^{\overline{v}} F^{n-1}(v_i) dv_i
$$

so

$$(1-\phi)\frac{\partial a_2}{\partial \phi}+a_2-a_1=\int_{v_1}^{\overline{v}}\left[(n-1)F^{n-2}(v_i)-(n-2)F^{n-1}(v_i)-1\right]dv_i$$

Calling:

$$g(v_i)=(n-1)F^{n-2}(v_i)-(n-2)F^{n-1}(v_i)$$

it is easy to check that $g(v_i)\geq 0$ and non-decreasing in $[\underline{v},\overline{v}]$, $\forall n>2$. Moreover, $g(\overline{v})=1$ so:

$$(1-\phi)\frac{\partial a_2}{\partial \phi}+a_2-a_1=\int_{v_1}^{\overline{v}}\left[g(v_i)-1\right]dv_i\leq 0$$

which implies, by (23), that $\partial\underline{\alpha}_1/\partial\phi\leq 0$. ■

Proof of Proposition 3: In the non symmetric case, we have:

$$\underline{\alpha}_1=\frac{1}{a_3}(a_2+a_3-a_1)\equiv\frac{A}{a_3}$$

where in this case:

$$\begin{aligned}
a_1 &= \int_{v_1}^{\overline{v}}(v_i-v_1)dF^{n-1}(v_i)\\
a_2 &= [\phi+(1-\phi)k]\int_{v_1}^{\overline{v}}\frac{1-F(v_i)}{f(v_i)}dF^{n-1}(v_i)\\
a_3 &= (1-\phi_0)v_0-(1-\phi_1)v_1
\end{aligned}$$

with

$$(1-\phi_0)v_0=(1-\phi)\int_{v_1}^{\overline{v}}v_i\ dF^{n-2}(v_i)+(1-\phi_1)v_1F^{n-2}(v_1)$$

We must analyze the sign of: a) $\partial\underline{\alpha}_1/\partial\phi_1$ and b) $\partial\underline{\alpha}_1/\partial\phi$.

a) By differentiation we get:

$$\frac{\partial \underline{\alpha}_1}{\partial \phi_1} = \frac{(a_1 - a_2)\partial a_3/\partial \phi_1}{a_3^2}$$

It is easy to see that:

$$\frac{\partial a_3}{\partial \phi_1} = v_1(1 - F^{n-2}(v_1)) \geq 0 \quad \forall n > 2$$

so:

$$sgn\left\{\frac{\partial \underline{\alpha}_1}{\partial \phi_1}\right\} = sgn\left\{a_1 - a_2\right\} \tag{24}$$

But we know that by definition of $\underline{\alpha}_1$:

$$a_1 - a_2 = (1 - \underline{\alpha}_1)a_3 \geq 0$$

and then, using (24), we conclude that $\partial \underline{\alpha}_1/\partial \phi_1 \geq 0, \ \forall n > 2$.

b) The direct derivative is given by:

$$\frac{\partial \underline{\alpha}_1}{\partial \phi} = \frac{a_3 \partial A/\partial \phi - A \partial a_3/\partial \phi}{a_3^2}$$

so:

$$sgn\left\{\frac{\partial \underline{\alpha}_1}{\partial \phi_1}\right\} = sgn\left\{a_3 \partial A/\partial \phi - A \partial a_3/\partial \phi\right\} \tag{25}$$

But:

$$
\begin{aligned}
& a_3\frac{\partial A}{\partial \phi}-A\frac{\partial a_3}{\partial \phi}=a_3\frac{\partial a_2}{\partial \phi}-(a_2-a_1)\frac{\partial a_3}{\partial \phi}= \\
& \left[(1-\phi_0)v_0-(1-\phi_1)v_1\right](1-k)\int_{v_1}^{\overline{v}}\frac{1-F(v_i)}{f(v_i)}dF^{n-1}(v_i)+ \\
& \left[[\phi+(1-\phi)k]\int_{v_1}^{\overline{v}}\frac{1-F(v_i)}{f(v_i)}dF^{n-1}(v_i)-\int_{v_1}^{\overline{v}}(v_i-v_1)dF^{n-1}(v_i)\right] \\
& \cdot\int_{v_1}^{\overline{v}}v_i dF^{n-2}(v_i)
\end{aligned}
$$

Replacing the value of $(1-\phi_0)v_0$ we have:

$$
\begin{aligned}
& a_3\frac{\partial A}{\partial \phi}-A\frac{\partial a_3}{\partial \phi}= \\
& \left[(1-\phi)\int_{v_1}^{\overline{v}}v_i\; dF^{n-2}(v_i)+(1-\phi_1)v_1F^{n-2}(v_1)-(1-\phi_1)v_1\right] \\
& \cdot(1-k)\int_{v_1}^{\overline{v}}\frac{1-F(v_i)}{f(v_i)}dF^{n-1}(v_i)\quad + \\
& \left[[\phi+(1-\phi)k]\int_{v_1}^{\overline{v}}\frac{1-F(v_i)}{f(v_i)}dF^{n-1}(v_i)-\int_{v_1}^{\overline{v}}(v_i-v_1)dF^{n-1}(v_i)\right] \\
& \cdot\int_{v_1}^{\overline{v}}v_i dF^{n-2}(v_i)
\end{aligned}
$$

Reordering and simplifying

$$
\begin{aligned}
& a_3\frac{\partial A}{\partial \phi}-A\frac{\partial a_3}{\partial \phi}= \\
& \left[F^{n-2}(v_1)-1\right](1-\phi_1)(1-k)\,v_1\int_{v_1}^{\overline{v}}\frac{1-F(v_i)}{f(v_i)}dF^{n-1}(v_i)+ \\
& \left[\int_{v_1}^{\overline{v}}\frac{1-F(v_i)}{f(v_i)}dF^{n-1}(v_i)-\int_{v_1}^{\overline{v}}(v_i-v_1)dF^{n-1}(v_i)\right]\int_{v_1}^{\overline{v}}v_i dF^{n-2}(v_i)\leq 0
\end{aligned}
$$

The inequality follows because the square bracket in the second term is negative, which was proved in Proposition 2, and because the first term is also negative. From (25) we get that $\partial\underline{\alpha}_1/\partial\phi\leq 0\;\;\forall n>2.$ ■

Proof of Proposition 4: Assumptions (NCO) and (PPBF) permit us to work with a constant ϕ in our proof provided that $\phi < \overline{\phi} = d_i' \ ^{-1}(\overline{x})$, where $\overline{x}$ has been arbitrarily fixed according to (NCO).

In the totally symmetric case we have:

$$\underline{\alpha}_1 = \frac{1}{a_3}(a_2 + a_3 - a_1) \equiv \frac{A}{a_3}$$

where in this case:

$$\begin{aligned}
a_1 &= (1-\phi+d(\phi))\int_{v_1}^{\overline{v}}(v_i - v_1)dF^{n-1}(v_i) \\
a_2 &= [d(\phi)+(1-\phi)k]\int_{v_1}^{\overline{v}}\frac{1-F(v_i)}{f(v_i)}dF^{n-1}(v_i) \\
a_3 &= (1-\phi)\left[\int_{v_1}^{\overline{v}} v_i \; dF^{n-2}(v_i) - v_1\left(1-F^{n-2}(v_1)\right)\right] \equiv (1-\phi)\beta(v_1,n)
\end{aligned}$$

by the usual steps:

$$sgn\left\{\frac{\partial \underline{\alpha}_1}{\partial \phi}\right\} = sgn\left\{(1-\phi)\left(\frac{\partial a_2}{\partial \phi} - \frac{\partial a_1}{\partial \phi}\right) + a_2 - a_1\right\}$$

and after some calculations:

$$\begin{aligned}
&(1-\phi)\left(\frac{\partial a_2}{\partial \phi} - \frac{\partial a_1}{\partial \phi}\right) + a_2 - a_1 = \\
&[d'(\phi)(1-\phi)+d(\phi)]\int_{v_1}^{\overline{v}}\left[\frac{1-F(v_i)}{f(v_i)} - v_i + v_1\right]dF^{n-1}(v_i)
\end{aligned}$$

but the first term is non-negative by the properties of $d(\phi)$ and the second is negative according to the proof of Proposition 2. Consequently $\partial\underline{\alpha}_1/\partial\phi \leq 0.$■

9 References

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