

NÚMERO 315

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**Conditional Heteroskedasticity and Cross-Sectional
Dependence in Panel Data: An Empirical Study
of Inflation Uncertainty in the G7 Countries**

JULIO 2005



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Acknowledgments

The authors would like to thank Paul Evans, Tim Bollerslev, Badi Baltagi and two anonymous referees for valuable comments. Earlier versions of this paper were presented at the 2001 North American Summer Meetings and 2003 Latin American Meetings of the Econometric Society, the 6th and 8th Annual Meeting of LACEA, and the 9th and 11th International Conference on Panel Data. We thank all comments from participants at these conferences. Any remaining mistakes are only the authors' responsibility.

Abstract

Despite the significant growth of macroeconomic and financial empirical panel studies the modeling of time dependent variance-covariance processes has not yet been addressed in the panel data literature. In this paper we specify a model that accounts for conditional heteroskedasticity and cross-sectional dependence within a typical panel data framework. We apply the model to a panel of monthly inflation rates of the G7 countries over the period 1978.2-2003.9 and find significant and quite persistent patterns of volatility and cross-sectional dependence. We then use the model to test two hypotheses about the interrelationship between inflation and inflation uncertainty, finding no support for the hypothesis that higher inflation uncertainty produces higher average inflation rates and strong support for the hypothesis that higher inflation is less predictable.

Resumen

No obstante el enorme crecimiento de estudios empíricos sobre temas macroeconómicos y financieros con datos panel, la modelación de procesos de varianza-covarianza cambiantes en el tiempo aún no ha sido tratada en la literatura de modelos panel. En este artículo modelamos heterocedasticidad condicional y correlación de corte transversal dentro de un modelo panel típico. El modelo es aplicado a un panel de inflación mensual de los países del G7 durante el periodo 1978.2-2003.9 encontrándose patrones de volatilidad y correlación de corte transversal significativos y altamente persistentes. El modelo es también utilizado para evaluar dos hipótesis sobre la interrelación entre inflación e incertidumbre inflacionaria, encontrándose fuerte evidencia a favor de la hipótesis de que inflaciones altas son menos predecibles, pero ninguna evidencia que respalde la hipótesis de que mayores niveles de incertidumbre inflacionaria conlleven a tasas de inflación mas altas.

Introduction

The empirical panel data literature on financial and macroeconomic issues has grown considerably in the few past years. A recent search of ECONLIT using the keyword phrases "financial panel data" and "macroeconomic panel data" produced 687 and 309 hits respectively.¹ While it is well known that most financial and macroeconomic time series data are conditionally heteroskedastic, rendering traditional estimators consistent, but inefficient, this rapidly growing literature has not yet addressed the issue. On the other hand, sophisticated multivariate GARCH models already are in wide use but they are confined to a time series context.²

In this paper we specify a panel model that accounts for conditional heteroskedasticity and cross-sectional correlation. The model is used to characterize the patterns of volatility and cross-sectional dependence of inflation in the G7 countries and to evaluate the hypotheses that (i) higher inflation uncertainty produces higher average inflation rates and (ii) higher inflation rates become less predictable. The main contribution of the paper is to account for a time dependent error covariance processes in panel models with fixed effects (dynamic or static), thus opening an avenue for empirical panel research of financial or macroeconomic volatility.

Although the volatility processes can be studied on an individual basis (i.e. country by country) using existing GARCH models (e.g. Engle, 1982, Bollerslev et al., 1988, and Bollerslev, 1990), panel modeling is still worth pursuing since taking into account the cross-sectional dependence will increase efficiency and provide potentially important information about patterns of cross-sectional dependence.

It is important to remark, though, that identification of time dependent variance-covariance processes in panel data is feasible as long as the cross-sectional dimension N is relatively small since the number of covariance parameters will increase rapidly otherwise, which limits the applicability of the model to relatively small N and large T panels.³

The rest of the paper is organized as follows. In section 2 we formulate the basic panel model with conditional heteroskedastic and cross-sectionally correlated disturbances and briefly discuss some special cases and generalizations. Section 3 discusses the strategy that will be followed in order to determine the presence of time dependent variance-covariance processes and to specify a preliminary panel model with such effects. Section 4 provides the empirical results, characterizing volatility and cross-sectional dependence

¹ Search conducted November 8, 2004.

² See Bollerslev et al., 1992 for a survey on ARCH models. For a comprehensive survey on multivariate GARCH models see Bauwens et al., 2003.

³ Phillips and Sul (2003) point out this limitation in the context of heterogeneous panels with (unconditional) cross-sectional dependence.

in the G7 countries, as well as testing two hypotheses about the interrelationship between inflation and its predictability. Finally, section 5 concludes.

2. The Model

Consider the following dynamic panel data (DPD) model with fixed effects:⁴

$$y_{it} = \mu_i + \phi y_{it-1} + \mathbf{x}_{it} \boldsymbol{\beta} + u_{it} \quad i = 1, \dots, N, \quad t = 1, \dots, T \quad (1)$$

Where N and T are the number of cross sections and time periods respectively; y_{it} is the dependent variable, μ_i is an individual specific effect, which is assumed fixed, \mathbf{x}_{it} is a row vector of exogenous explanatory variables of dimension k, and $\boldsymbol{\beta}$ is a k by 1 vector of coefficients. We assume that the AR parameter satisfies the condition $|\phi| < 1$ and that T is relatively large so that we can invoke consistency of the Least Squares estimators⁵. In the case $\phi = 0$, the process given by equation (1) becomes static.⁶ The disturbance term u_{it} is assumed to have a zero mean normal distribution with the following conditional moments:

$$\begin{aligned} \text{(i)} \quad & E[u_{it} u_{js} / u_{it-1}, u_{js-1}] = \sigma_{it}^2 \quad \text{for } i = j \text{ and } t = s \\ \text{(ii)} \quad & E[u_{it} u_{js} / u_{it-1}, u_{js-1}] = \sigma_{ijt} \quad \text{for } i \neq j \text{ and } t = s \\ \text{(iii)} \quad & E[u_{it} u_{js} / u_{it-1}, u_{js-1}] = 0 \quad \text{for } i = j \text{ and } t \neq s \\ \text{(iv)} \quad & E[u_{it} u_{js} / u_{it-1}, u_{js-1}] = 0 \quad \text{for } i \neq j \text{ and } t \neq s \end{aligned} \quad (2)$$

Assumption (iii) states that there is no autocorrelation while assumption (iv) disallows non-contemporaneous cross-sectional correlation.⁷ Assumptions (i) and (ii) define a very general conditional variance-covariance process; some structure needs to be imposed in order to make this process tractable. We propose the following specification which is an adaptation of the model in Bollerslev et al., 1988.

$$\sigma_{it}^2 = \alpha_i + \delta \sigma_{i,t-1}^2 + \gamma u_{i,t-1}^2 \quad i = 1 \dots N \quad (3)$$

⁴ This class of models is widely known in the panel data literature. See Baltagi (2001) and Hsiao (2003) for details.

⁵ For dynamic models with fixed effects and i.i.d. errors, it is well known that the LSDV estimator is downward biased in small T samples. See, for example, Kiviet (1995).

⁶ It is worth emphasizing that we are only considering the case of stationary panels. In practice, we will have to assure that all variables are indeed stationary or I (0).

⁷ Ruling out autocorrelation might be a restrictive assumption but it is convenient because of its simplicity. In practice, we will need to make sure that this assumption is not violated.

$$\sigma_{ijt} = \eta_{ij} + \lambda \sigma_{ij,t-1} + \rho u_{i,t-1} u_{j,t-1} \quad i \neq j \quad (4)$$

The model defined by equations (1) (conditional mean), (3) (conditional variance) and (4) (conditional covariance) is simply a DPD model with conditional covariance. Thus, we can use the acronym DPDCCV.⁸ Modeling the conditional variance and covariance processes in this way is quite convenient in a panel data context since by imposing a common dynamics to each of them, the number of parameters is considerably reduced. In this case there are $(\frac{1}{2}N(N+1)+4)$ parameters in the covariance matrix. It is important to emphasize that (3) and (4) imply that the conditional variance and covariance processes follow, respectively, a common dynamics but their actual values, however, are not identical for each unit or pair of units (conditionally or unconditionally).

It can be shown that the conditions $\alpha_i > 0$, $(\delta + \gamma) < 1$, and $(\lambda + \rho) < 1$ are sufficient for the conditional variance and covariance processes to converge to some fixed (positive in the case of the variance) values. However, in general there is no guarantee that the covariance matrix of disturbances be positive definite (at each point in time) and that it converges to some fixed positive definite matrix. Thus, assuming positive definiteness of the covariance matrix, the error structure of the model will reduce, unconditionally, to the well-known case of groupwise heteroskedasticity and cross-sectional correlation.

In matrix notation and assuming given initial values y_{i0} , equation (1) becomes

$$\mathbf{y}_t = \boldsymbol{\mu} + \mathbf{Z}_t \boldsymbol{\theta} + \mathbf{u}_t \quad t = 1, \dots, T \quad (5)$$

Where $\mathbf{y}_t, \mathbf{u}_t$, are vectors of dimension $N \times 1$. The matrix $\mathbf{Z}_t = [\mathbf{y}_{t-1} : \mathbf{X}_t]$ has dimension $N \times (K+1)$, $\boldsymbol{\mu}$ is a $N \times 1$ vector of individual specific effects, and $\boldsymbol{\theta} = [\phi : \beta']$ is a conformable column vector of coefficients. Given our previous assumptions the N -dimensional vector of disturbances \mathbf{u}_t will follow a zero-mean multivariate normal distribution, denoted as $\mathbf{u}_t \sim N(\mathbf{0}, \boldsymbol{\Omega}_t)$. The covariance matrix $\boldsymbol{\Omega}_t$ is time dependent and its diagonal and off-diagonal elements are given by equations (3) and (4) respectively. The vector of observations \mathbf{y}_t is therefore conditionally normally distributed with mean $(\boldsymbol{\mu} + \mathbf{Z}_t \boldsymbol{\theta})$ and variance-covariance matrix $\boldsymbol{\Omega}_t$. That is, $\mathbf{y}_t \sim N(\boldsymbol{\mu} + \mathbf{Z}_t \boldsymbol{\theta}, \boldsymbol{\Omega}_t)$ and its conditional density is

⁸ We should remark that equations (3) and (4) could have a more general GARCH (p,q) formulation.

$$f(\mathbf{y}_t / \mathbf{Z}_t, \boldsymbol{\mu}, \boldsymbol{\theta}, \varphi) = (2\pi)^{-\frac{N}{2}} |\boldsymbol{\Omega}_t|^{-\frac{1}{2}} \exp\left(-\frac{1}{2}(\mathbf{y}_t - \boldsymbol{\mu} - \mathbf{Z}_t \boldsymbol{\theta})' \boldsymbol{\Omega}_t^{-1} (\mathbf{y}_t - \boldsymbol{\mu} - \mathbf{Z}_t \boldsymbol{\theta})\right) \quad (6)$$

Where φ includes the parameters in equations (3) and (4). For the complete panel we have the following log-likelihood function:⁹

$$l = -\left(\frac{NT}{2}\right) \ln(2\pi) - \left(\frac{1}{2}\right) \sum_{t=1}^T \ln |\boldsymbol{\Omega}_t| - \left(\frac{1}{2}\right) \sum_{t=1}^T (\mathbf{y}_t - \boldsymbol{\mu} - \mathbf{Z}_t \boldsymbol{\theta})' \boldsymbol{\Omega}_t^{-1} (\mathbf{y}_t - \boldsymbol{\mu} - \mathbf{Z}_t \boldsymbol{\theta}) \quad (7)$$

This function is similar to those derived in the context of multivariate GARCH models (e.g., Bollerslev et al., 1988).¹⁰ It can be shown straightforwardly that if the disturbances are cross-sectionally independent the $N \times N$ matrix $\boldsymbol{\Omega}_t$ becomes diagonal and the log-likelihood function takes the simpler form:

$$l = -\frac{NT}{2} \ln(2\pi) - \frac{1}{2} \sum_{i=1}^N \sum_{t=1}^T \ln(\sigma_{it}^2(\varphi)) - \frac{1}{2} \sum_{i=1}^N \sum_{t=1}^T \frac{(y_{it} - \mu_i - \phi y_{it-1} - \mathbf{x}_{it} \boldsymbol{\beta})^2}{\sigma_{it}^2(\varphi)}, \quad (8)$$

Further, in the absence of conditional heteroskedasticity and cross-sectional correlation the model simply reduces to a typical DPD model.

Even though the LSDV estimator in equation (1) is still consistent it will no longer be efficient in the presence of conditional heteroskedastic and cross-sectionally correlated errors, either conditionally or unconditionally. In this case, the proposed non-linear MLE estimator based upon (7) or (8) (depending on whether we have cross-sectionally correlated disturbances or not) will be appropriate. Note that, by using the MLE estimator we are able to obtain both the parameters of the conditional mean and conditional variance-covariance equations while the LSDV estimator will only be able to compute the coefficients in the mean equation.

It is well known that under regularity conditions the MLE estimator is consistent, asymptotically efficient and asymptotically normally distributed. Also it is known that these properties carry through when the observations are time dependent as is the case of multivariate GARCH processes. Therefore, the MLE estimator in (7) or (8) is asymptotically normally distributed with mean equal to the true parameter vector and a covariance matrix equal to the inverse of the corresponding information matrix. It is important to note that these asymptotic properties would hold for N fixed and T approaching to infinity since we are modeling the N -dimensional vector of disturbances of the panel as a multivariate time series process.

⁹ It should be remarked that the normality assumption may not hold in practice leading to Quasi-MLE estimation. See Davidson and McKinnon (1993) for a general discussion. Although this issue needs further investigation it is worth pointing out that Bollerslev and Wooldrige (1992) find that the finite sample biases in the QMLE appear to be relatively small in time series GARCH models.

¹⁰ Also it is similar to the log likelihood function derived in the context of prediction error decomposition models for multivariate time series. See for example Brockwell and Davis (1991) and Harvey (1990).

Estimation of the DPDCCV model will be made by direct maximization of the log likelihood function given by (7), using numerical methods.¹¹ The asymptotic covariance matrix of the MLE estimator of this type will be approximated by the negative inverse of the Hessian of l evaluated at MLE parameter estimates. It is important to remark that the total number of coefficients to be estimated depends on the squared cross-sectional dimension of the panel, N^2 , which in practice suggests applying the model to relatively small N panels in order to make the estimation feasible and to retain the asymptotic properties, namely consistency and efficiency, of this estimator.¹²

In practice, the individual effects in the mean equation may not be significantly different from each other giving rise to a mean equation with a single intercept (often called “pooled regression model”). Also, it is possible that the conditional variance or covariance processes do not exhibit individual effects. A combination of these possibilities could occur as well. A completely heterogeneous panel with individual specific coefficients for all the parameters in the mean and variance-covariance equations can also be considered, although in this last case we can run into estimation problems given the considerably large number of parameters that will arise even if the number of cross sections is relatively small.

Finally, it is worth mentioning some alternative specifications for the variance and covariance processes along the lines of those developed in the multivariate GARCH literature. For example, a variation of equation (4) that specifies the analogous of the constant correlation model as in Bollerslev (1990) or its generalized version, the dynamic conditional correlation model, given in Engle (2002). Also, depending on the particular subject of study, exogenous regressors can be included in the variance equations as well as the variance itself can be included as a regressor in the conditional mean equation, as in multivariate M-GARCH-M models.

3. Empirical Strategy

Since the proposed DPDCCV models are non-linear and estimation by direct maximization of the log-likelihood can be tedious work, it may be helpful to make some preliminary identification of the most appropriate model. In what follows we outline an empirical methodology for this purpose although it should be remarked that it is only done in an informal way.

¹¹ We use the GAUSS Optimization module.

¹² In this paper we only consider small values of N . Further work will focus on using existing multivariate GARCH two-step methods which allow consistent, although inefficient, estimation of a considerably large number of parameters as would be the case of larger N panels. See Engle (2002), and Ledoit, Santa-Clara and Wolf (2003).

Two issues are fundamental in our empirical strategy: (i) Specifying the best model for the mean equation and (ii) Identifying conditional variance-covariance processes in the panel. We consider that, provided there are a large enough number of time series observations so that we can rely on consistency of LS estimators, these issues can be addressed using conventional panel data estimation results as we discuss next.

3.1 *Specifying the mean equation*

An important issue in empirical panel work is the poolability of the data. In the context of equation (1) we need to determine whether there are individual specific effects or a single intercept.¹³ For this purpose we can test for individual effects in the mean equation using the LSDV estimator with a heteroskedasticity and autocorrelation consistent covariance matrix, along the lines of White (1980) and Newey and West (1987) estimators applied to panel.¹⁴

Under the assumption of cross-sectional independence, and for models where the variance process is identical across units, the LSDV and OLS estimators respectively are still best linear estimators. However if the variances are not equal across units the unconditional variance process will differ across units and the previous estimators will no longer be efficient. Given that we do not know a priori the appropriate model and that we may have auto correlation problems in practice, it seems convenient to use a covariance matrix robust to heteroskedasticity and autocorrelation. Specifically, we can test the null hypothesis $H_0 : \mu_1 = \mu_2 = \dots = \mu_N$ by means of a Wald-test, which will follow a $\chi^2_{(N-1)}$ distribution asymptotically.

3.2 *Identifying conditional variance-covariance processes*

Once we have determined a preliminary model for the mean equation, we can explore the possibility of a time dependent pattern in the variance process by examining whether the squared LSDV or LS residuals (depending on whether individual specific effects are included or not in the mean equation) exhibit a significant autocorrelation pattern.¹⁵ Depending upon the number of significant partial autocorrelations obtained we can choose a preliminary order for the variance process. As a practical rule, we can consider an ARCH

¹³ From a much broader perspective, however, we need to determine if full heterogeneity or some form of pooling is more appropriate for the conditional mean equation.

¹⁴ Arellano (1987) has extended White's heteroskedasticity consistent covariance estimator to panel data but this estimator is not appropriate here since it has been formulated for small T and large N panels which is not our case.

¹⁵ This argument is along the lines of Bollerslev (1986) who suggests examining the squared least squares residuals in order to determine the presence of ARCH effects in a time series context.

(1) process if only the first lag is significant, or a GARCH (1, 1) if more lags are significant.

A related important issue is to determine if there are individual effects in the variance process. This can be done by testing for individual effects in the AR regression of squared residuals. Complementarily, a test for unconditional groupwise heteroskedasticity (which can be done in a conventional way) can lead us to decide for individual effects in the variance process if the null hypothesis is rejected.

Next, we can carry out a conventional test for the null hypothesis of no cross-sectional correlation (unconditionally) which if not rejected will allow us to consider the simpler model under cross-sectional independence as a viable specification. Rejection of the previous hypothesis will indicate that the (unconditional) covariance matrix of the N vector of disturbances is not diagonal making it worth to explore a possible time dependent pattern of the covariance among each pair of units. This can be done in a similar way as outlined previously for the case of the variance. Specifically, we can examine if the cross products of LSDV or LS residuals show a significant autocorrelation pattern. The inclusion of pair specific effects in the covariance process can be decided after testing for individual effects in the AR regression of cross products of residuals.

We need to remark that the previous guidelines can be quite helpful to determine a preliminary specification of the model. However, in order to determine the most appropriate model we need to estimate a few alternative specifications via maximum likelihood and compare the results. At this point, it is important to make sure that all conditional heteroskedasticity has been captured in the estimation. We can accomplish this in two ways. First, we can add additional terms in the conditional variance equation and check for their significance. Second, we can test the squared normalized residuals for any autocorrelation pattern. If significant patterns remain, alternative specifications should be estimated and checked.

4. Inflation and Inflation Uncertainty in the G7 countries

Several studies have found using time series GARCH models that inflation uncertainty, measured by the estimated conditional variance, is a significant phenomenon in the G7 and other countries, and that it interacts in various ways with nominal or real variables.¹⁶ In this paper we attempt to characterize the conditional variance-covariance process of inflation in the G7

¹⁶ See, for example, Caporale and Caporale (2002), Apergis (1999) and Grier and Perry (1996, 1998, 2000) among others. It is also worth mentioning the seminal paper by Robert Engle (1982).

countries taken as a panel. We also evaluate the hypotheses that (i) higher inflation uncertainty increases average inflation and (ii) higher inflation rates become less predictable. We use monthly observations on inflation rates (π) during the period 1978.2 to 2003.9.¹⁷

Before proceeding, we evaluate the stationarity of the inflation process. In Table 1 we present time series as well as panel unit root tests for inflation. In all cases the regression model for the test includes an intercept. For each individual country, we use the Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) tests.¹⁸ The results reject the null hypothesis of unit root except in the cases of France and Italy when using the ADF test. At the panel level, both Levin, Lin and Chu's (2002) t -star and Im, Pesaran and Shin's (2003) t -bar and W (t -bar) tests reject the null of unit root, which enables us to treat this panel as stationary.¹⁹

4.1 *Conditional heteroskedasticity and cross-sectional dependence in G7 inflation*

In this section we present and briefly discuss the estimation results of various DPDCCV models after performing some preliminary testing following the empirical strategy outlined in Section 3. In all cases, we consider an AR (12) specification for the mean equation since we are using seasonally unadjusted monthly data.

First, we test for individual effects in the mean equation. The Wald test statistic (using White/Newey-West's HAC covariance matrix) is $\chi^2_{(6)} = 2.82$, which is not significant at any conventional level and lead us to consider a common intercept in the mean equation.

Secondly, we perform likelihood ratio tests for (unconditional) groupwise heteroskedasticity and cross-sectional correlation obtaining the values of $\chi^2_{(6)} = 255.08$ and $\chi^2_{(21)} = 214.28$ respectively. These tests statistics are highly significant and indicate that the unconditional variance-covariance matrix of disturbances is neither scalar identity nor diagonal.

More explicitly, these results show that there is significant unconditional groupwise heteroskedasticity and cross-sectional correlation. Clearly, the

¹⁷ These data are compiled from the International Monetary Fund's (IMF) International Financial Statistics.

¹⁸ For the ADF and PP tests the number of lags was determined by the floor $\{4(T/100)^{5/9}\}$ which gives a value of 5 lags in all cases.

¹⁹ It is important to remark that the alternative hypothesis is not the same. In Levin-Lin-Chu test all cross sections are stationary with the same AR parameter while in the case of Im-Pesaran-Shin the AR parameter is allowed to differ across units and not all individual processes need to be stationary.

second test suggests that the assumption of cross-sectional independence does not hold in these data.

Next, in order to explore if a significant conditional variance-covariance process exists, we estimate *AR (12)* regressions using the squared as well as the cross products of the residuals taken from the pooled *AR (12)* mean inflation regression. For the squared residuals, lag 1 is significant at the 5% while lags 3, 9 and 12 are significant at the 1% level. In the case of the cross products of residuals, lags 1, 3, 6, 7, 9 and 12 are significant at the 1% level.²⁰

We also perform simple tests for individual effects in the previous *AR (12)* regressions. We find that the null of no individual effects in the regression using squared residuals is rejected at the 5% significance level. This result, together with the previous evidence on unconditional groupwise heteroskedasticity, leads us to include individual effects in the conditional variance equation. For the case of cross products of LS residuals, the joint null of no pair specific effects is not rejected pointing to a covariance process with a single intercept.²¹

To summarize, the preliminary testing suggests a dynamic panel model without individual effects in the mean equation for inflation rates (π). For both the conditional variance and conditional covariance processes, a GARCH (1, 1) specification seems to be appropriate given the persistence exhibited by the squares and cross products of the LS residuals respectively. The variance and covariance equations may include individual specific and a single intercept respectively. This DPDCCV model will be estimated and referred to as Model 2. We will also consider a few relevant alternative specifications based on the following benchmark model:

$$\pi_{it} = \mu + \sum_{j=1}^{12} \beta_j \pi_{it-j} + u_{it}, \quad i = 1, \dots, 7; t = 1, \dots, 296 \quad (9)$$

$$\sigma_{it}^2 = \alpha_i + \delta \sigma_{i,t-1}^2 + \gamma u_{i,t-1}^2 \quad (10)$$

$$\sigma_{ijt} = \eta_{ij} + \lambda \sigma_{ij,t-1} + \rho u_{i,t-1} u_{j,t-1} \quad (11)$$

This model will be referred to as Model 3. Model 2 is a special case of Model 3 in that $\eta_{ij} = \eta$ in equation (11). We also consider a model with cross-sectional independence, which is defined by equations (9) and (10) only. This will be referred to as Model 1. For comparison, two versions of the simple dynamic panel data (DPD) model without GARCH effects are also considered. The first one, which includes country specific effects, is estimated using the

²⁰ The results are available upon request.

²¹ It is important to note, though, that 9 out of the 21 pair specific coefficients resulted positive and significant at the 10% or less, indicating that a model with pair specific effects in the covariance process may not be discarded.

SDV as well as Arellano and Bond's (1991) GMM estimators.²² The pooled regression model (common intercept) is estimated by OLS.²³

In Table 2 we report some conventional DPD estimation results. Two issues are worth noting. First, the estimated coefficients for the mean equation are numerically quite close, although the GMM1 estimator is the most efficient (as it would have been expected) and gives a higher number of significant coefficients than the other estimators. Second, when comparing OLS and LSDV results we find that the (implied) values of the log likelihood function are also quite close, which is congruent with the non-rejection results from the Wald test for no individual specific effects reported before.

Given the previous results, we consider that a specification without country specific effects in the mean equation is justified and therefore we use it for the DPDCCV models. The estimation results of these models are shown in Table 3. All of them were obtained by MLE. It should be remarked that we estimated 22, 25 and 45 parameters in Models 1, 2 and 3 respectively.

Clearly, the last DPDCCV model (Model 3) outperforms all the other models based on the value of the log-likelihood function. Notice that our specification strategy picked Model 2 rather than Model 3, so that actually estimating several reasonable models is probably important to do in practice. In what follows we use the results of Model 3 to characterize the G7's mean inflation process as well as its associated conditional variance and covariance processes.

According to Model 3, the G7's inflation volatility can be characterized as a significant and quite persistent although stationary GARCH (1, 1) process. Similarly, the results for the covariance equation indicate that this process is also a quite persistent GARCH (1, 1).

We find that all individual specific coefficients in the variance equation are statistically significant at the 1% level. Also, all but two of the pair specific coefficients in the covariance equation are positive and about half of them are statistically significant at the 10% level or less.²⁴

Some interesting patterns of individual volatility and cross-sectional dependence among the G7's inflation shocks are worth mentioning. First, our results suggest that Italy, France and USA have the lowest levels of unconditional volatility. Second, the USA has relatively high and significant positive cross-sectional dependence with Canada and to a lesser extent with France, Germany and Italy. Third, Japan's inflation shocks do not seem to be correlated with any of the other G7 countries. Fourth, the three biggest European economies, namely France, Germany and UK show a relatively significant pattern of positive cross-sectional dependence.

²² Given that we are dealing with a large T and small N panel we only use the GMM1 estimator after restricting the number of lagged values of the dependent variable to be used as instruments to a maximum of 7. Specifically, we use lags 13th through 19th as instruments. See also Baltagi (2001), pp. 131-136 for details on these estimators.

²³ For both OLS and LSDV we computed standard errors using White/Newey-West's HAC covariance matrix.

²⁴ These results as well as the ones we referred to in the rest of this section are available upon request.

We also find some interesting patterns for the conditional volatility processes. For example while in most G7's the volatility levels appear to be lower at the end of the sample compared with those experienced in the eighties, this does not appear to be the case for Canada and Germany. Also, the volatility levels appear to have been rising in the last two years of the sample in the cases of Canada, France, Germany, and the USA.

We have also calculated the implied conditional cross correlations between the USA and the other G7 countries and between France, Germany and Italy. The dependence of USA with Canada, France and Italy seems to have increased over time. On the other hand, the process does not seem to exhibit a clear pattern over time in the case of the three biggest European economies.

4.2 *The interrelationship between average inflation and inflation uncertainty*

One advantage of our DPDCCV model over conventional DPD models and their associated estimation methods, including GMM, is that it allows us to directly test some interesting hypotheses about the interrelationship between average inflation and inflation uncertainty. The most famous of these, that higher average inflation is less predictable, is due to Friedman (1977) and was formalized by Ball (1992). We can test this hypothesis for the G-7 countries by including lagged inflation as a regressor in our conditional variance equation.

It has also been argued that increased inflation uncertainty can affect the average inflation rate. The theoretical justification for this hypothesis is given in Cukierman and Meltzer (1986) and Cukierman (1992) where it is shown that increases in inflation uncertainty increase the policy maker's incentive to create inflation surprises, thus producing a higher average inflation rate. In order to evaluate the previous hypothesis we simply include the conditional variance as an additional regressor in the mean equation. To conduct these tests, we alter equations 9 and 10 as shown below and call the resulting system Model 4 (we continue to use equation 11 for the covariance process).

$$\pi_{it} = \mu + \sum_{j=1}^{12} \beta_j \pi_{it-j} + \kappa \sigma_{it} + u_{it}, \quad i = 1, \dots, 7; t = 1, \dots, 296 \quad (9a)$$

$$\sigma_{it}^2 = \alpha_i + \delta \sigma_{i,t-1}^2 + \gamma u_{i,t-1}^2 + \psi \pi_{i,t-1} \quad (10a)$$

A positive and significant value for the parameter κ supports the Cukierman and Meltzer hypothesis that inflation volatility raises average inflation, while a positive and significant value for the parameter ψ supports the Friedman-Ball hypothesis that higher inflation is more volatile.

The results are shown in the Table 4. As it can be seen, the parameter ψ is positive and highly statistically significant, indicating that higher inflation

rates do become less predictable as argued by Friedman. On the other hand, we find that the parameter κ is significant at the 5% level although its sign is negative, which clearly rejects the hypothesis that higher inflation uncertainty produces higher average inflation rates.

This negative sign actually supports previous findings by Holland (1995) for the USA and by Grier and Perry (1998) for the USA and Germany. These authors argue that if inflation uncertainty has deleterious real effects that central banks dislike and if higher average inflation raises uncertainty (as we have found here) then the Central Bank has a stabilization motive to reduce uncertainty by reducing average inflation. In our G-7 panel we find the stabilization motive dominates any potentially opportunistic Central Bank behavior.

Overall, when comparing Model 3 in Table 3 with Model 4 in Table 4 by means of a likelihood ratio test we find that the later outperforms to the former and lead us to conclude that (i) higher inflation rates are less predictable and (ii) higher inflation uncertainty has been associated with lower average inflation rates.

Conclusion

In this paper we have specified a model, (DPDCCV), which accounts for conditional heteroskedasticity and cross-sectional correlation within a panel data framework, an issue that has not yet been addressed in the panel data literature. We have also outlined a methodology to identify these phenomena, which could be useful for empirical research.

The DPDCCV model has been applied to a panel of monthly inflation rates for the G7, over the period 1978.2 -2003.9, showing that there exist highly persistent patterns of volatility as well as cross-sectional dependence. Further, we have found that higher inflation rates become less predictable. Also, we have found that the hypothesis that higher inflation uncertainty produces higher average inflation rates is not supported in these data. On the contrary, we find that this relationship is negative indicating that Central Banks dislike inflation uncertainty.

Although the model formulated here is practical for small N and large T panels, it is especially relevant due to the following 4 factors: (1) The rapid growth of empirical panel research on macroeconomic and financial issues, (2) The ubiquity of conditional heteroskedasticity in macroeconomic and financial data, (3) The potential extreme inefficiency of estimators that do not account for these phenomena, and (4) The rapid growth of multivariate GARCH models outside the panel data literature. Further work, particularly theoretical, to account for these phenomena in a more general panel setting is certainly necessary.

T A B L E 1

TIME SERIES AND PANEL DATA UNIT ROOT TESTS FOR INFLATION IN THE G7 COUNTRIES

TIME SERIES UNIT ROOT TESTS			
	AUGMENTED DICKEY-FULLER	PHILLIPS-PERRON $Z(\rho)$	PHILLIPS-PERRON $Z(t)$
CANADA	-3.905	-260.545	-13.174
FRANCE	-2.131	-71.268	-6.525
GERMANY	-5.464	-222.100	-12.638
ITALY	-2.247	-68.437	-6.282
JAPAN	-5.521	-219.774	-14.866
U.K.	-3.525	-215.237	-12.291
U.S.	-3.636	-96.258	-7.626
PANEL DATA UNIT ROOT TESTS			
POOLED T -STAR TEST: (LEVIN, LIN AND CHU, 2002)		-4.79577	(0.0000)
T -BAR TEST: (IM, PESARAN AND SHIN, 2003)		-6.227	(0.0000)
$W(T$ -BAR) TEST: (IM, PESARAN AND SHIN, 2003)		-14.258	(0.0000)

The time series unit root tests correspond to the model with intercept only. For the Augmented Dickey-Fuller (ADF) and the Phillips-Perron (PP) tests, the lag truncation was determined by $\text{floor } 4(T/100)^{2/9}$. For the ADF and PP $Z(t)$ tests, the approximate 1, 5 and 10 percent critical values are -3.456, -2.878 and -2.570 respectively. For the PP $Z(\rho)$ test the approximate 1 percent critical value is -20.346. For the panel unit root tests, the number of lags for each individual country was also set to $\text{floor } 4(T/100)^{2/9}$. Numbers in parenthesis are p -values.

T A B L E 2
CONVENTIONAL DPD ESTIMATION RESULTS

CONDITIONAL EQUATION	ESTIMATION RESULTS
DPD MODEL (INDIVIDUAL SPECIFIC EFFECTS): LSDV ESTIMATOR	
LOG LIKELIHOOD = -5691.96	
MEAN:	$\pi_{it} = \mu_i + 0.176\pi_{it-1} - 0.008\pi_{it-2} + 0.025\pi_{it-3} + 0.052\pi_{it-4} + 0.033\pi_{it-5}$ $+ 0.086\pi_{it-6} + 0.046\pi_{it-7} + 0.009\pi_{it-8} + 0.012\pi_{it-9} - 0.013\pi_{it-10}$ $+ 0.046\pi_{it-11} + 0.446\pi_{it-12} + \hat{u}_{it}$
VARIANCE:	$\sigma_{it}^2 = 14.38$
COVARIANCE:	$\sigma_{ijt} = 0$
DPD MODEL (INDIVIDUAL SPECIFIC EFFECTS): ARELLANO-BOND GMM1 ESTIMATOR	
MEAN:	$\pi_{it} = \mu_i + 0.173\pi_{it-1} - 0.004\pi_{it-2} + 0.019\pi_{it-3} + 0.054\pi_{it-4} + 0.033\pi_{it-5}$ $+ 0.086\pi_{it-6} + 0.046\pi_{it-7} + 0.004\pi_{it-8} + 0.015\pi_{it-9} - 0.008\pi_{it-10}$ $+ 0.048\pi_{it-11} + 0.446\pi_{it-12} + \hat{u}_{it}$
VARIANCE:	$\sigma_{it}^2 = 13.72$
COVARIANCE:	$\sigma_{ijt} = 0$
DPD MODEL (COMMON INTERCEPT): OLS ESTIMATOR	
LOG LIKELIHOOD = -5692.45	
MEAN:	$\pi_{it} = 0.172 + 0.177\pi_{it-1} - 0.007\pi_{it-2} + 0.026\pi_{it-3} + 0.052\pi_{it-4}$ $+ 0.034\pi_{it-5} + 0.087\pi_{it-6} + 0.047\pi_{it-7} + 0.010\pi_{it-8} + 0.012\pi_{it-9}$ $- 0.012\pi_{it-10} + 0.047\pi_{it-11} + 0.447\pi_{it-12} + \hat{u}_{it}$
VARIANCE:	$\sigma_{it}^2 = 14.25$
COVARIANCE:	$\sigma_{ijt} = 0$

For each model we show the estimated mean equation followed by the estimated (or implied) equations for the conditional variance and covariance processes. Values into parenthesis are *t*-ratios and the symbols ***, **, * , indicate significance levels of 1%, 5% and 10% respectively. The *t*-ratios for the OLS and LSDV estimators are based on White / Newey-West's HAC standard errors. For the GMM1 estimator the number of lagged values of the dependent variable to be used as instruments is restricted to a maximum of 7. Specifically, we use lags 13th through 19th as instruments.

T A B L E 3
ESTIMATION RESULTS FOR THE DPDCCV MODEL

CONDITIONAL EQUATION	ESTIMATION RESULTS
DPDCCV MODEL 1 (CONDITIONAL VARIANCE ONLY): MLE ESTIMATOR	
LOG LIKELIHOOD = -5466.22	
MEAN:	$\pi_{it} = 0.331 + 0.193\pi_{it-1} - 0.036\pi_{it-2} + 0.025\pi_{it-3} + 0.036\pi_{it-4}$ $+ 0.004\pi_{it-5} + 0.071\pi_{it-6} + 0.060\pi_{it-7} + 0.027\pi_{it-8} - 0.027\pi_{it-9}$ $+ 0.024\pi_{it-10} + 0.056\pi_{it-11} + 0.434\pi_{it-12} + \hat{u}_{it}$
VARIANCE:	$\sigma_{it}^2 = \alpha_i + 0.769\sigma_{i,t-1}^2 + 0.148u_{i,t-1}^2$
COVARIANCE:	$\sigma_{ijt} = 0$
DPDCCV MODEL 2 (CONDITIONAL VARIANCE AND COVARIANCE): MLE ESTIMATOR	
LOG LIKELIHOOD = -5355.61	
MEAN:	$\pi_{it} = 0.367 + 0.153\pi_{it-1} - 0.039\pi_{it-2} + 0.012\pi_{it-3} + 0.021\pi_{it-4}$ $+ 0.018\pi_{it-5} + 0.078\pi_{it-6} + 0.062\pi_{it-7} + 0.022\pi_{it-8} - 0.026\pi_{it-9}$ $+ 0.034\pi_{it-10} + 0.052\pi_{it-11} + 0.443\pi_{it-12} + \hat{u}_{it}$
VARIANCE:	$\sigma_{it}^2 = \alpha_i + 0.884\sigma_{i,t-1}^2 + 0.072u_{i,t-1}^2$
COVARIANCE:	$\sigma_{ijt} = 0.072 + 0.877\sigma_{ijt-1} + 0.037u_{i,t-1}u_{j,t-1}$
DPDCCV MODEL 3 (CONDITIONAL VARIANCE AND COVARIANCE): MLE ESTIMATOR	
LOG LIKELIHOOD = -5328.08	
MEAN:	$\pi_{it} = 0.407 + 0.154\pi_{it-1} - 0.033\pi_{it-2} + 0.020\pi_{it-3} + 0.020\pi_{it-4}$ $+ 0.022\pi_{it-5} + 0.075\pi_{it-6} + 0.061\pi_{it-7} + 0.016\pi_{it-8} - 0.029\pi_{it-9}$ $+ 0.033\pi_{it-10} + 0.052\pi_{it-11} + 0.433\pi_{it-12} + \hat{u}_{it}$
VARIANCE:	$\sigma_{it}^2 = \alpha_i + 0.882\sigma_{i,t-1}^2 + 0.069u_{i,t-1}^2$
COVARIANCE:	$\sigma_{ijt} = \eta_{ij} + 0.806\sigma_{ijt-1} + 0.034u_{i,t-1}u_{j,t-1}$

For each model we show the estimated mean equation followed by the estimated (or implied) equations for the conditional variance and covariance processes. Values in parenthesis are t -ratios and the symbols ***, **, * indicate significance levels of 1%, 5% and 10% respectively. All DPDCCV models were estimated by direct maximization of the log-likelihood function using numerical methods.

T A B L E 4

**ESTIMATION RESULTS FOR THE DPDCCV MODEL WITH VARIANCE EFFECTS IN THE
CONDITIONAL MEAN AND LAGGED INFLATION IN THE CONDITIONAL VARIANCE**

CONDITIONAL EQUATION	ESTIMATION RESULTS
DPDCCV MODEL 4 (CONDITIONAL VARIANCE AND COVARIANCE): MLE ESTIMATOR	
LOG LIKELIHOOD = -5306.03	
MEAN:	$\pi_{it} = 0.643 + 0.156\pi_{it-1} - 0.025\pi_{it-2} + 0.021\pi_{it-3} + 0.026\pi_{it-4}$ $+ 0.031\pi_{it-5} + 0.086\pi_{it-6} + 0.061\pi_{it-7} + 0.018\pi_{it-8} - 0.022\pi_{it-9}$ $+ 0.026\pi_{it-10} + 0.057\pi_{it-11} + 0.438\pi_{it-12} - 0.117\sigma_{it} + \hat{u}_{it}$
VARIANCE:	$\sigma_{it}^2 = \alpha_i + 0.867\sigma_{i,t-1}^2 + 0.050u_{i,t-1}^2 + 0.092\pi_{i,t-1}$
COVARIANCE:	$\sigma_{ijt} = \eta_{ij} + 0.855\sigma_{ijt-1} + 0.031u_{i,t-1}u_{j,t-1}$

This model has been estimated by direct maximization of the log-likelihood function by numerical methods. We show the estimated equations for the conditional mean, variance and covariance processes. Values in parenthesis are *t*-ratios and the symbols ***, **, * indicate significance levels of 1%, 5% and 10% respectively.

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