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Introducing Soil Nutrient Dynamics in the Evaluation of Soil Remediation Programs. Evidence from Chile

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Abstract

In this article I use data from Chile to analyze the targeting policy of a cost sharing program aiming to replenish the phosphorus fertility of agricultural soils. I begin by solving an optimal-control problem to determine the farmer's optimal soil fertilization strategy. I show that two optimal paths exist. Depending on farmer characteristics and initial level of phosphorus in the soil, the farmer may choose either to follow a subsistence fertilization strategy (a low yield path) or remediate soil fertility until a higher level of phosphorus is achieved (the high yield path). In order to evaluate the impact of the program on each of the two possible regimes, I formulate an endogenous-switching regression framework with unobserved switch ing points. A switching equation determines the probability of being in each regime conditional on farm characteristics; then specific fertilization-path equations are used to determine the effect of cost sharing on each regime. Estimation results indicate that program effects are greater on financially and/or technologically constrained farms being on the low yield path, while cost sharing might have only short run effects or be equivalent to a pure transfer on farms already in the high yield path.

Resumen

Este artículo usa datos de Chile para analizar la política de asignación en programas que subsidian la recuperación de la fertilidad química de suelos agrícolas. La elección por parte de un agricultor de la estrategia óptima de fertilización es modelada mediante un problema de control óptimo. Diferentes simulaciones para las condiciones chilenas indican que existen dos posibles estrategias óptimas. Dependiendo de las características del agricultor y de las condiciones iniciales de fertilidad del suelo, el agricultor puede elegir entre aplicar modestas cantidades de fertilizante y seguir una estrategia de subsistencia (baja producción) o corregir el déficit inicial del nutriente mediante el empleo de altas dosis de fertilización hasta alcanzar un régimen permanente de alta producción. Un modelo de selección con umbrales no observados y ecuaciones independientes para cada posible estrategia es empleado para evaluar el impacto del subsidio sobre los agricultores. Una ecuación de selección determina la probabilidad de que el agricultor siga una u otra estrategia y ecuaciones específicas para cada estrategia determinan el efecto del subsidio sobre el nivel de fósforo en el suelo. Los resultados indican que los efectos del programa son mayores en granjas con limitaciones financieras y/o tecnológicas que siguen la estrategia de bajas dosis de fertilización. Por otra parte, el subsidio parece

tener sólo efectos de corto plazo o ser equivalente a una transferencia monetaria en granjas que ya estaban siguiendo una estrategia de altas dosis de fertilización.

Introduction

Most of the literature dealing with the economics of soil degradation focuses on the problem of soil depth reduction. Soil depth is modeled usually as a capital stock whose dynamics are determined by farmers' actions and the natural regeneration rate of the soil (McConnell, 1983). Few studies analyze other aspects of soil degradation, such as the reduction of fertility, which may happen even when soil depth reduction is negligible. Additionally, plant nutrients are often included in economic analyses as a strictly variable input in the production function without consideration of nutrient pools. This approach is probably correct when soil nutrient dynamics have only marginal effects on plant nutrition or the prices of fertilizers are low relative to output prices, but these cases do not cover all the possible situations. At least two cases can be mentioned where the inclusion of the dynamics of the soil nutrient pools in the analysis is justified. The first one corresponds to the presence of significant carry-over effects (Kennedy, 1986; Schnitkey *et al.*, 1996). The second case happens when soils act like sinks by fixing a significant proportion of the fertilizer applied by the farmer and thus preventing plants from using it.

This article discusses both theoretically and empirically some policy implications when soil nutrient dynamics are mistakenly ignored in the second case mentioned above. Specifically, the phosphorus fixation problem is used to illustrate how a program that subsidizes fertility replenishment can become inefficient if farmer characteristics and dynamics of the soil nutrient reservoirs are not considered when defining the targeting strategy.

Phosphorus (P) is not a limitation for crop production in most of the developed world. For instance, Sharpley *et al.* (1999) indicate that many states in the US show P-levels in their soils over the agronomic threshold[1] implying that little or no additional P fertilization is required for commercial production of most crops. On the contrary, P-deficit has become a major problem in developing countries as result of either natural low levels of phosphorus or depletion of once well stocked soil reserves (Buresh *et al.*, 1997). Phosphorus fertility depletion occurs when no phosphate fertilization is provided to compensate for the phosphorus exported with products harvested in the farm (including beef or milk from permanently grazed pastures). The main reasons for soil fertility depletion are lack of financial resources to afford an adequate fertilization program, ineffectual or absent land property rights. In other cases the soil itself contributes to the problem. Depending on parent materials and weathering factors, the capacity of the soil to fix

[1] The agronomic threshold is the amount of a plant nutrient in the soil at which this nutrient is no longer a limitation to achieve the potential yield of the crop.

phosphorus can dramatically increase the cost of fertilization. Phosphorus fixation occurs when phosphate molecules react with soil particles leading to the formation of insoluble complexes. Only a small group of plants (see for example Gilbert *et al.*, 1999; Hisinger and Gilkes, 1996) is capable of breaking apart some of those complexes and releasing P back to soil solution. For most crops, however, phosphorus becomes unavailable for plant uptake once it is tied up in these complexes. The capacity to fix phosphorus varies widely among soils. In many sandy soils fixation is minimal, while the fixation capacity of soils derived from volcanic ashes (Andisols) and rich in allophanic clays can present a serious challenge for farming (Vander Zaag and Kagenzi, 1986; Espinosa, 1992). The P-fixation ability of allophanic soils are normally strong enough to reduce carryover effects to negligible levels and crops are able to utilize no more than 10 to 30% of the phosphorus applied as fertilizer (Ludwick, 2002).

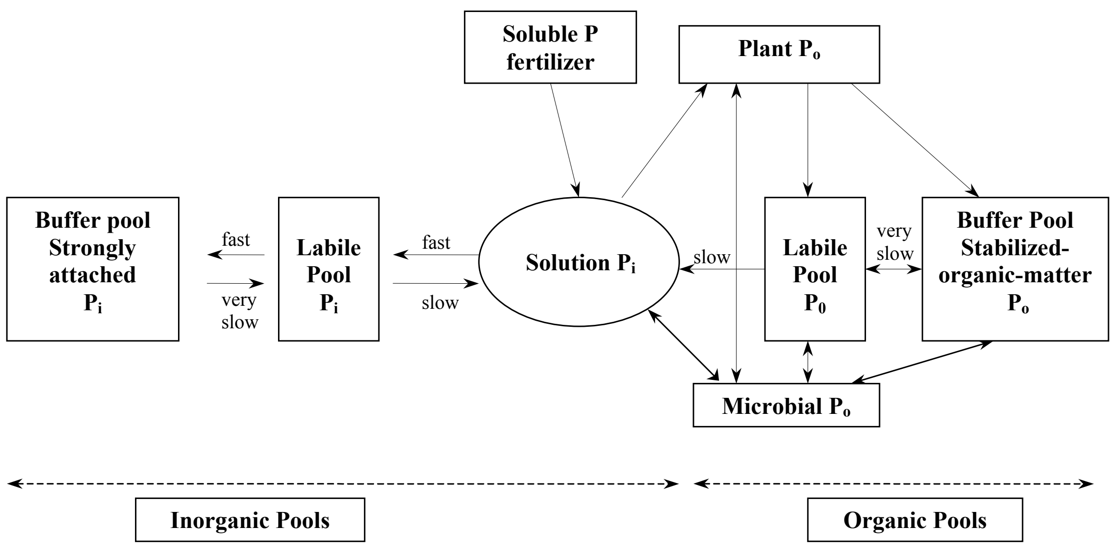
There are not many studies that analyze the economic impact of phosphorus depletion (some of the few cases include Abelson and Rowe, 1987; Buresh *et al.*, 1997), and none, to the best of author knowledge, examine the impact of programs that subsidize soil fertility replenishment. Thus, a main contribution of this work is to fill this gap in the literature. In this article I use data from Chile to analyze the implications of including soil phosphorus dynamics when selecting the targeting strategy for a soil remediation program.

The article is organized in the following way. Section 1 presents a brief description of the dynamics of soil phosphorus. Section 2 discusses the existence of a steady state in the farmer's problem of determining the optimal rate of fertilization. The discussion includes the solution of a continuous optimal control problem where the level of soil phosphorus is treated as a stock variable. The third section presents data from a fertility replenishment program from Chile and the econometric framework used to evaluate the theoretical model presented in Section 2. Then I use a Monte Carlo EM algorithm to solve an endogenous switching regression model with unobserved switching points, which permits evaluating the impact of the cost sharing program conditional on alternative fertilization strategies the farmer can follow. Section 4 discusses the results of the econometric estimation. Section 5 concludes.

1.- Soil phosphorus stocks and supply of plant-available phosphorus

Phosphorus is one of the three necessary macro elements for plant nutrition (the others are nitrogen and potassium). Plant roots are able to absorb this nutrient only from the soil solution and only in inorganic forms. When the fixation capacity of the soil is high, phosphate fertilizer applied is attached to soil particles almost immediately (Potash and Phosphate Institute, 2001). As indicated by Ludwick (2002), the strongest attachment occurs with oxides of iron (Fe) primarily in soils with pH below 4.0. As soil pH increases, P is fixed preferentially in aluminum (Al) compounds. This binding is not that strong as with iron, but the availability of phosphorus to plants is still reduced dramatically. Finally, binding to relatively weak alkaline (Ca) compounds can occur in soils with pH higher than 7.5. In many soils, binding of P happens at the surface of soil particles, which permits the process be reverted by mass action processes. However, natural release of P back to solution happens at a lower rate than binding does (Barrow, 1983a and 1983b). On the other hand, in volcanic soils with allophanic clays and humus-Al complexes, the fixation process tends to be irreversible (Espinosa, 1992; Nanzyo *et al.*, 1997).

Figure 1
Soil-phosphorus reservoirs and phosphorus dynamics.



(Modified from Steward and Sharpley, 1987) P_i and P_o indicate sources of inorganic or organic phosphorus, respectively.

Traditionally, phosphorus in the soil is treated as occurring in three reservoirs or pools (Figure 1): a buffer pool, a labile pool, and phosphorus in the soil solution or solution-P. All of them receive contributions from both organic and inorganic sources. The buffer pool includes inorganic phosphorus contained in soil minerals and strongly attached to Al and Fe compounds, and organic phosphorus occurring in stabilized organic matter. The labile pool includes inorganic phosphorus loosely attached to the surface of clay particles, and a limited amount of organic phosphorus that can be rapidly mineralized and thus made available for plant uptake. Solution-P, on the other hand, is non-attached inorganic phosphorus that can either be immediately taken up by plants, used by soil biota and converted in organic phosphorus, or, alternatively, attached and become part of either the labile pool or the buffer pool. Significant movement of phosphorus between the buffer and labile pools only happens in the long run, while the interaction between the labile pool and the soil solution is much faster. Large contributions to solution-P can be provided through fertilization with a highly soluble source of inorganic phosphorus.

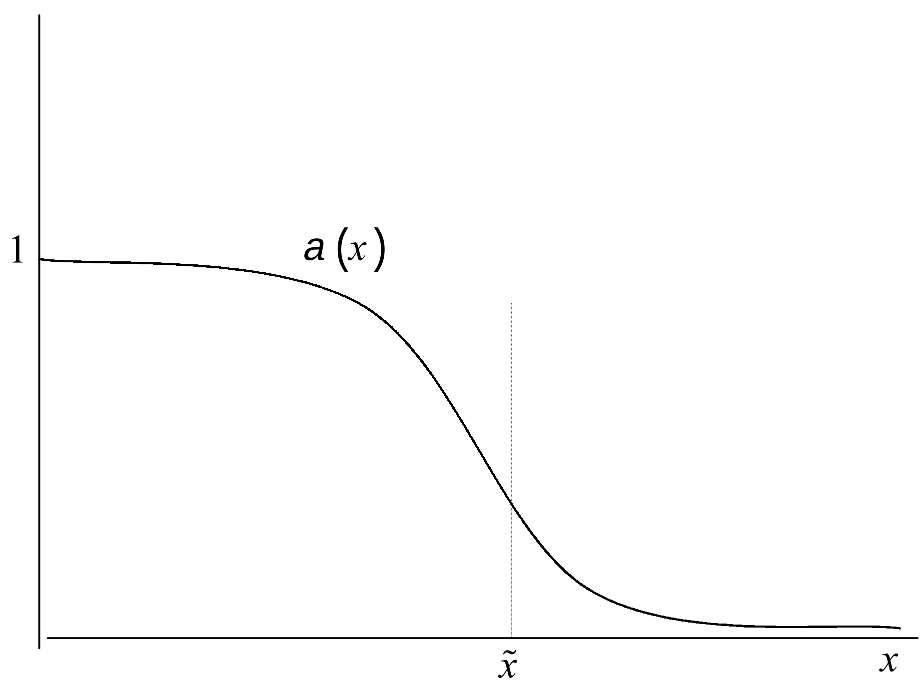
In absence of fertilization and phosphorus exportation, there exists a chemical equilibrium between the three reservoirs mentioned above. If equilibrium is altered, e.g. when concentration of solution-P is reduced by plant uptake, the labile pool releases phosphorus to solution to restore the equilibrium between the labile-pool and the soil solution. This, in turn, reduces the phosphorus level in the labile pool, which alters the equilibrium between labile and buffer pools. In this case, however, given the low rate at which phosphorus can be released from the buffer pool, the speed at which the labile pool recovers is slower than the speed at which phosphorus is released to solution. Pastures used to raise cattle, for instance, feature a vegetative cover during most of the season. Periodic harvest or grazing of this cover permanently alters the equilibrium among phosphorus reservoirs. The labile pool is progressively depleted unless phosphate fertilization is provided. If phosphorus fertilizer is not applied, less and less phosphorus is available from soil solution, which, in turn, reduces forage and crop production.

If farmers wait to fertilize until signals of depletion become apparent, chemical linkages among phosphorus pools often make it difficult to recover the initial fertility levels. Fertilization causes an abrupt increase in the concentrations of phosphorus in the solution, which triggers a change in the kinetics of soil phosphorus. Since the equilibrium among the pools must be restored, most of phosphorus provided by fertilization will go to enrich the labile and buffer pools. The lower the level in the buffer pool the less phosphorus will remain available in solution to plant uptake. This is one of the reasons explaining why, once a certain level of depletion is achieved, soils with high phosphorus-fixation capacity are not fertilized in developing countries. Unless cheap sources of phosphate are available, the gains from fertilization do not compensate (at least in the short run) the cost of the massive applications necessary for replenishment.

1.1.- State equation of the phosphorus pools

Modeling of phosphorus dynamics in this article is simplified by assuming the existence of only two reservoirs in the soil: the buffer pool and the plant-available pool. The buffer pool is responsible for both P-fixation and long run release of phosphorus to plant-available pool. The plant-available pool includes phosphorus loosely attached in the labile pool and phosphorus free in the soil solution. Considering two stocks instead of three is not a restrictive assumption. Given the fast kinetics of phosphorus between solution and the labile pool, it seems reasonable considering them as a single reservoir for purposes of policy analysis.

Figure 2
Phosphorus fixation function.



Let x be the non-negative level of phosphorus in the buffer pool. I assume that soil fixation power follows a function $\alpha(x)$, which depends only on the level of the buffer pool and represents the proportion of phosphorus fertilizer that is fixed during the season (and, consequently, contributes to the level of phosphorus in the buffer pool x). The properties of $\alpha(x)$ are: $0 < \alpha(x) \leq 1$, $\alpha^{'}(x) \leq 0$, $\alpha^{'}(0) = 0$, and $\alpha^{''}(x)$ is negative for $x < \tilde{x}$ and positive for $x > \tilde{x}$ (Figure 2). The phosphorus binding power of the soil attains its maximum at $x = 0$, i.e. when all possible binding sites in the buffer pool are unoccupied, and approaches zero as *x* becomes large. The state equation giving the seasonal change in the buffer pool is.

$$\dot{x} = \alpha(x)z - \gamma(x) + \kappa \qquad (1)$$

According to equation (1), the level of the buffer pool increases by capturing phosphorus from fertilization z in amount of $\alpha(x)z$ and via natural weathering of parent materials and organic matter at constant rate κ. On the other hand, it is reduced by phosphorus desorption according to a function $\gamma(x)$, which is quasi-convex in x and satisfies $\gamma'(x) > 0$ for $x > 0$, $\gamma'(0) = 0$ and $\gamma(0) = 0$. Desorpted phosphorus goes to the plant-available pool in order to sustain the chemical equilibrium between the two pools.

Accordingly, there are two sources of phosphorus for the plant-available pool: a proportion $1-\alpha(x)$ of the fertilizer applications z, and contributions from the buffer pool in amount $\gamma(x)$.

2.- The optimal fertilization strategy

2.1.- The farmer's dilemma as an optimal control problem

I represent the problem in a continuous-time infinite-horizon optimal-control framework with one control and one state variable. The control variable is the rate of fertilization z, while the state variable is the phosphorus level in the buffer pool. Thus, the farmer solves,

$$\begin{aligned} &\max \int_0^{\infty} \left\{ f\left(z \cdot \left(1-\alpha(x)\right) + \gamma(x)\right) - w \cdot z \right\} e^{-rt} dt \\ &s.t. \quad \dot{x} = z \cdot \alpha(x) - \gamma(x) + \kappa \qquad (2) \\ &\qquad z(t) \geq 0, \quad x(0) = x_0 \end{aligned}$$

where $z(t)$ is the phosphorous fertilization rate at time t, $x(t)$ is the corresponding phosphorus level in the buffer pool, x_0 is the initial buffer pool level, w is the phosphorous price, r is the farmer's discount rate, and $f(\cdot)$ is a twice-differentiable production function. Prices have been normalized such that output price equals one.

The current-value Hamiltonian is,

$$H = f\left(z\left(1-\alpha\right)+\gamma\left(x\right)\right)-wz+\lambda\left(z\alpha-\gamma\left(x\right)+\kappa\right) \quad (3)$$

where λ is the shadow value of a marginal increase in the level of the buffer pool. The first order necessary conditions for interior solutions of (3) are,

$$\frac{\partial H}{\partial z} \leq 0 \quad \Rightarrow \quad f^{'}\left(1-\alpha\right)-w+\lambda\alpha = 0 \quad (4)$$

$$\frac{\partial H}{\partial x} = \dot{\lambda}-r\lambda \quad \Rightarrow \quad \dot{\lambda} = \left(z\alpha^{'}-\gamma^{'}\right)\left(f^{'}-\lambda\right)+r\lambda \quad (5)$$

$$\frac{\partial H}{\partial \lambda} = \dot{x} \quad \Rightarrow \quad \dot{x} = z\alpha-\gamma+\kappa \quad (6)$$

The interpretation of (4) follows from the well-known rule of profit maximization: it will be optimal to fertilize until marginal benefits equalize marginal costs. In this case we have two kinds of benefits: on one side there are the revenues from output sale, and on the other side we have an enriched buffer pool, which ensures less phosphorus fixation and fertilization savings in future crop seasons.

Non-convexities make it difficult to find a closed form for the internal solution(s), if any, of problem (2). Therefore, I combine graphical and numerical tools in the next section to analyze the existence of and nature of potential equilibria.

2.2.- Phase-diagram analysis

I begin with the phase diagram in the $z-x$ plane. Differential equations giving the rates of change of x and z are needed in order to characterize the shapes of the curves $\dot{x}=0$ and $\dot{z}=0$. The equation for $\dot{x}$ is simply the state equation (6), while a differential equation involving the first derivative of z with respect to time can be obtained by taking the time derivative of equation (4). We get,

$$\left(1-\alpha\right)\left[\left(\gamma^{'}-z\alpha^{'}\right)\dot{x}+\dot{z}\left(1-\alpha\right)\right]f^{''}-\alpha^{'}\left(f'-\lambda\right)\dot{x}+\dot{\lambda}\alpha = 0 \quad (7)$$

Equation (7) can be combined with equations (4) through (6) to obtain,

$$\dot{z} = -\frac{1}{(1-\alpha)^2 f''}\left\{(1-\alpha)\left[\left(\gamma' - z\alpha'\right)\dot{x}\right]f'' - \alpha'\left(f' - \lambda\right)\dot{x} + \dot{\lambda}\alpha\right\}$$

$$= -\frac{1}{(1-\alpha)^2 f''}\left\{(1-\alpha)\left(\gamma' - z\alpha'\right)\left(z\alpha - \gamma + \kappa\right)f'' - \frac{\alpha'}{\alpha}\left(f' - w\right)\left(z\alpha - \gamma + \kappa\right)\right. \qquad (8)$$

$$\left. + r\left(w - (1-\alpha)f'\right) + \left(f' - w\right)\left(z\alpha' - \gamma'\right)\right\}$$

Equations involving $\dot{x}=0$ and $\dot{\lambda}=0$ needed to construct the phase diagram in the $\lambda - x$ plane are provided by the state equation (6) and the adjoint equation (5).

I draw the phase diagrams following four steps: 1) choosing functional forms for $f(\cdot)$, $\alpha(\cdot)$, and $\gamma(\cdot)$, 2) solving numerically the equations $\dot{x}=0$ and $\dot{z}=0$ as functions of x and z, and equations $\dot{x}=0$ and $\dot{\lambda}=0$ as functions of x and λ, 3) drawing the curves $\dot{x}=0$ and $\dot{z}=0$ in the $z-x$ plane and curves $\dot{x}=0$ and $\dot{\lambda}=0$ in the $\lambda - x$, and 4) determining the sign of $\dot{x}$, $\dot{z}$, and $\dot{\lambda}$ in each isosector of the phase diagrams.

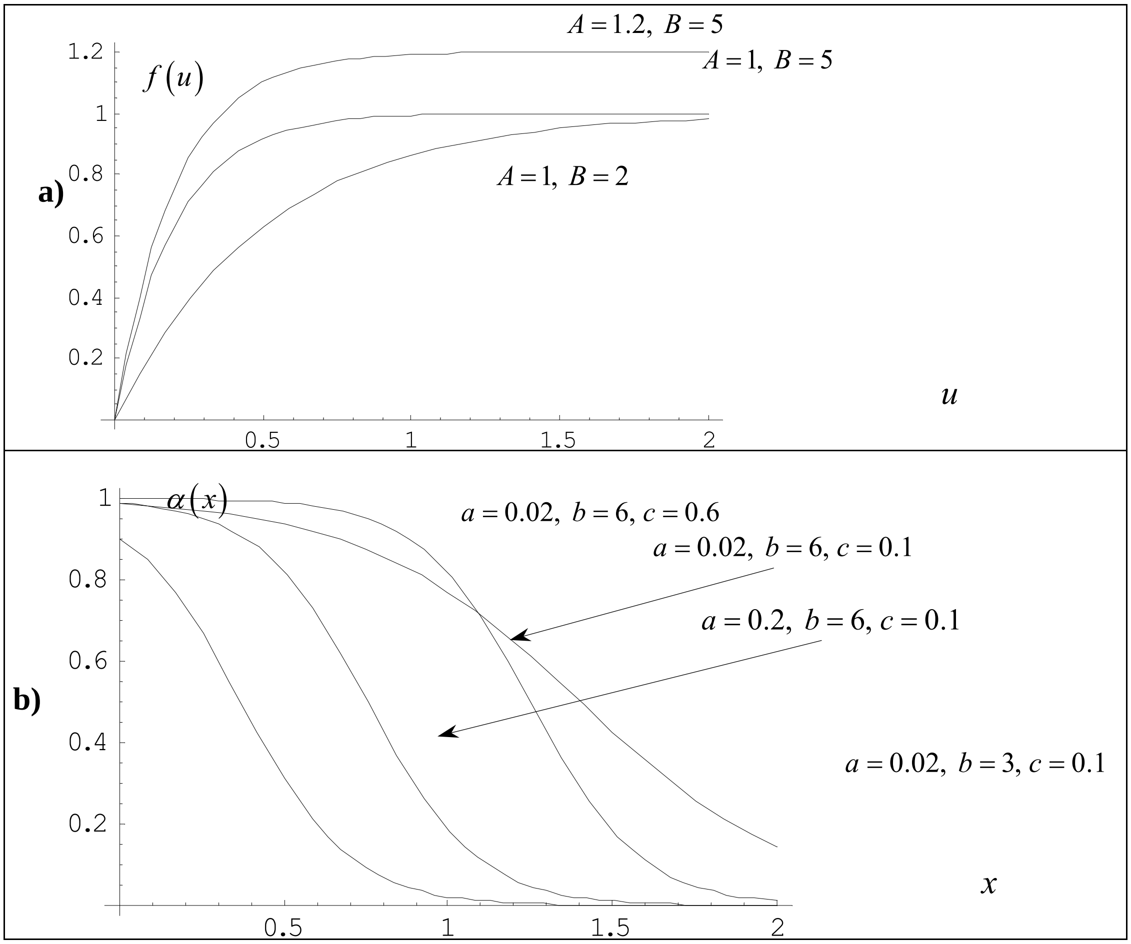
Functional forms and baseline parameter values

I assume the following functional forms for the functions involved.

Production function: $f(u) = A\left(1 - e^{-Bu}\right)$, where A represents the maximum yield the farmer can achieve using the technology available and B is a measure of the "speed" at which that potential is attained (Figure 3a). In this case a larger B corresponds to more efficient use of soil phosphorus.

To choose the values of w, A and B for the baseline simulation, I use the work of Smith-Ramírez *et al.* (2002), which classifies Chilean dairy farms according to technical, productive and human capital characteristics. I chose to work with dairy farms because dairy and beef farms are the main target of fertility remediation programs in Chile. To simplify the notation I normalize the maximum yield to the unity, i.e. $A=1$, which transforms w in the price of phosphorus relative to the maximum revenue for acre achievable by the farmer conditional on the technology available.

Figure 3
Production and fixation functions.



In what follow, I use information from Region X in Chile, which produces 70% of the Chilean milk and includes about 6,700 dairy farms. Smith-Ramírez *et al*. (2002) classify the farms in four groups according to their production levels and technologies used in their production process. Table 1 gives some characteristics of the groups.

The Chilean dairy pays different prices to farmers according annual production. Currently, farmers in the S1 group receive an average of 13.6 cents per kilogram of fluid milk, those in the S2 group receive 15.3 cents, those in the S3 group receive 17.0 cents, and those in the S4 group receive an average of 18.7 cents per kilogram. The price of the ton of phosphorus (in its form of P_2O_5) is 645 dollars in Chile. In order to obtain values of w for each group in Table 1, I need to know the potential yield per hectare for each of them. Since these data are not available, I used the 95th percentile for the production per hectare for each group. After combining all this information, the values for w are 2.39, 1.30, 0.79, and 0.57 for groups S1, S2, S3, and S4, respectively. The selection of a value for parameter B is discussed in conjunction with the selection of the parameters for the phosphorus fixation function.

Table 1
Production indicators of Region X dairy farms (Chile)

Group	Indicator	Quartile 1[2]	Median	Quartile 3
S1	production per ha[1] (Kg ha^{-1} year^{-1})	459	701	959
	farm size (ha)	6	10	15
	production per cow (Kg year^{-1})	741	1131	1547
S2	production per ha (Kg ha^{-1} year^{-1})	1345	1906	2588
	farm size (ha)	65	91	154
	production per cow (Kg year^{-1})	1868	2647	3594
S3	production per ha (Kg ha^{-1} year^{-1})	2575	3542	4685
	farm size (ha)	50	83	144
	production per cow (Kg year^{-1})	2220	3053	4038
S4	production per ha (Kg ha^{-1} year^{-1})	3768	4408	5134
	farm size (ha)	120	197	274
	production per cow (Kg year^{-1})	3925	4592	5348

[1] 1 hectare (ha) is equivalent to 2.471 acres.
[2] 25% of the population have a value below quartile 1; 25% of the population have a value above quartile 3.

Phosphorus fixation function: $\alpha(x)=\dfrac{1}{1+ae^{b(x-c)}}$, where parameters $a, b, \text{and } c$ determine together the location of the inflection point of the curve in Figure 3b. The most influential parameter in this function is c, which controls the curvature of the function before it reaches the inflection point. According to Escudey *et al*. (2001), fertile Chilean Ultisols and Andisols (both volcanic ash soils) contain from 733 mg P kg $^{-1}$ to 3470 mg P kg $^{-1}$. For purposes of the simulation, I take the middle value, 2200 mg P kg $^{-1}$ as a measure of the aggregated capacity of the buffer and plant-available pools. The plant-available pool, however, contains only a small fraction of the phosphorus in the soil, which means that 2100 mg P kg $^{-1}$ is also a good measure of the buffer pool only. To reproduce the high fixation power of Region X Andisols, I choose parameters $a, b, \text{and } c$ such that the soil fixes 90% of the fertilizer applied when the buffer pool is at half of its capacity. Soil fixation power declines then rapidly to fix only 20% of the fertilizer when the buffer pool is at two thirds of its capacity. Both conditions, along with the requirement the $\alpha(x)$ goes to zero as x approaches 2200, are satisfied if $a=0.018,\ b=6$ and $c=0.1$ (Figure 3 b).

To choose parameter B in the production function, I use two pieces of information: (1) the average dairy farm in Region X produces no more than a third of his potential yield (Smith-Ramírez, 1999) and (2) the phosphorus content of the plant-available pool never exceeds 5% of the phosphorus content of the buffer pool (Rowell, 1994). Therefore, I choose B such as the yield is one third of the potential when the level of the plant-available pool is 0.1 mg P kg $^{-1}$, which corresponds to $B=5$ (Figure 3a, u is the level of the plant-available pool).

Desorption function: $\gamma(x)=sx$. A linear form is assumed, where the parameter s denotes the proportion of the buffer pool being released to the plant-available pool during the crop season and ranges from zero to one. There is little useful information to guide the selection of a value for s to use in the simulation. However, since the plant available pool rarely contains more than 5% of the phosphorus in the buffer pool, I chose s equals to 0.05. For the contribution from parental material to the buffer pool (the parameter κ in the state equation), I chose a value $\kappa=0$ to better represent volcanic soil conditions.

Determination of the curves $\dot{x}=0$, $\dot{z}=0$ and $\dot{\lambda}=0$

The $\dot{x}=0$ and $\dot{z}=0$ curves were drawn in the $x-z$ plane by writing the right-hand sides of expressions (6) and (8) as functions of z and x. Both expressions were then set equal to zero and solved as simultaneous equations by numerical methods[2] for various combinations of parameter values. The $\dot{x}=0$ and $\dot{\lambda}=0$ curves were drawn in an analogous manner using equations (5) and (6).

According to the previous discussion, the parameter values used for the baseline simulation were $\kappa=0,\ r=0.15,\ w=1.8,\ A=1,\ B=5,\ a=0.018,\ b=6,\ c=0.1 \text{ and } s=0.05$. A value $w=1.8$ was chosen in order to represent farmers in the groups S1 and S2 (Table 1), which accounts for about the 80% of the farmers in Region X and have similar characteristics as three quarters of Chilean dairy farmers (Smith-Ramírez *et al*. 2002). Common values for the farmer's discount rate in the literature for natural resource economics range between 10% to 20% (Kremen *et al*. 2000; Lu and Stocking 2000). Here, I have chosen $r=0.15$. Graphical outcomes are presented in Figure 4.

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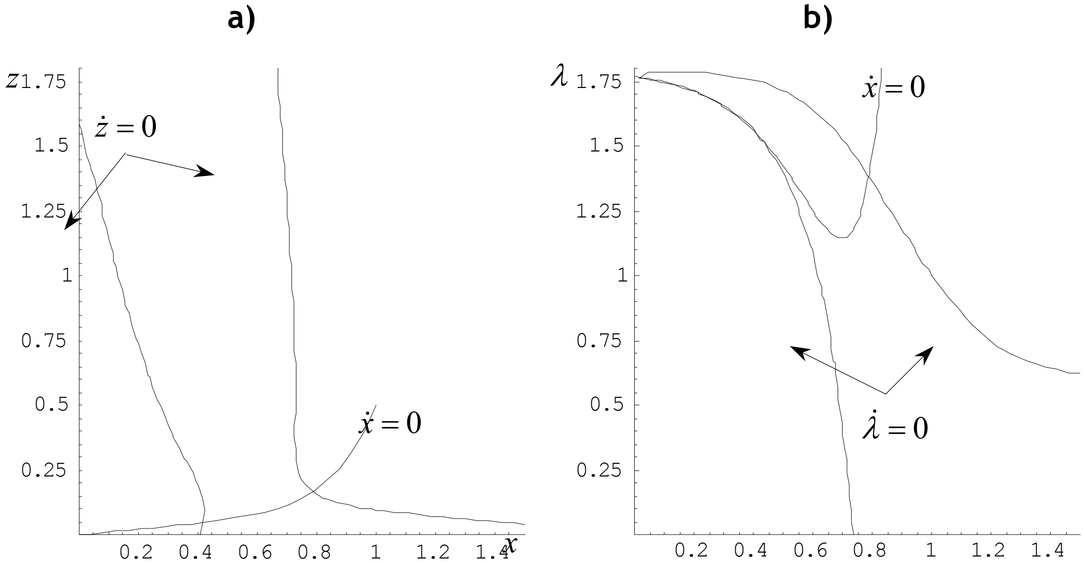
[2] I used Mathematica 4.1 from Wolfram research Inc.

Figure 4



a) Curves $\dot{x}=0$ and $\dot{z}=0$ in plane $x-z$, b) curves $\dot{x}=0$ and $\dot{\lambda}=0$ in plane $x-\lambda$.

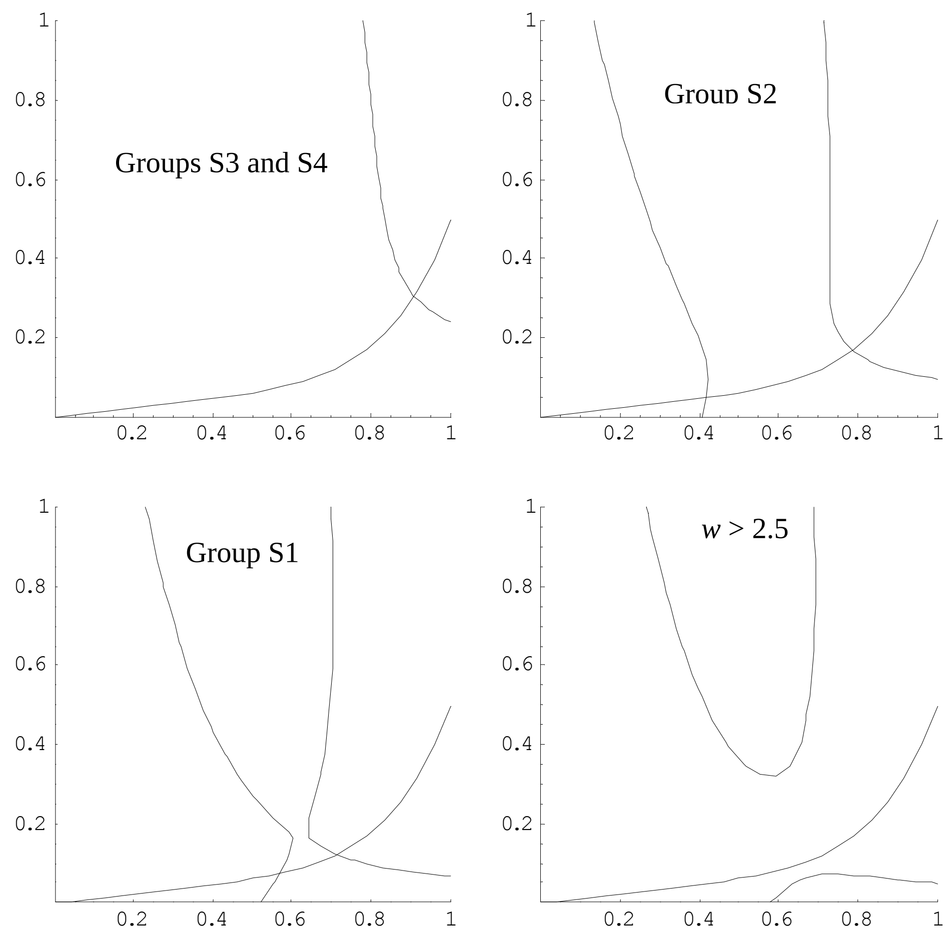
A noteworthy characteristic of the solutions depicted in Figure 4 is the existence of two candidates for interior solutions, which originate in double branched curves for the loci $\dot{z}=0$ and $\dot{\lambda}=0$. The two solutions $\left(x^*, z^*, \lambda^*\right)$ for the parameter values used in the simulation are $\left(0.467,\ 0.027,\ 1.383\right)$ and $\left(0.947,\ 0.185,\ 1.261\right)$. The only observable amounts in these two sets of values are the fertilization rates: 0.027 and 0.185 tons of phosphorus (in its P_2O_5) form) per hectare. They correspond to 60 and 400 kilograms of Triple Superphosphate (the most used phosphate fertilizer in Chile). Fertilization rates of 60 Kg of Triple Superphosphate are typical in subsistence farming in Chile and characterize most of the farmers in the S1 group. Fertilizer application rates between 300 and 400 Kg of Triple Superphosphate are usual among the most productive farms in groups S3 and S4.

The existence of two branches in the loci $\dot{z}=0$ and $\dot{\lambda}=0$, however, is conditional on the values of function parameters, phosphorus price, and discount rate. Figure 5 sketches the phase diagram for different combinations of values on the $x-z$ plane. In order to give some structure to the next discussion, I label the branches of $\dot{z}=0$ as the “low-yield branch” (the left-hand side branch) and “high-yield branch” (the right-hand side branch). The labels follow the crop yields at each solution. For instance, for the parameter values used to construct Figure 4, the yield at the low-yield (LY) solution is 0.127, while it is 0.603 at the high-yield (HY) solution. This outcome reproduces closely Chilean conditions: farmers in groups S1 and S3 produce milk on grassland, however, farmers in S3 quintuplicate the production per hectare of those farmers in group S1 (Table 1).

According to Figure 5a, under low fixation power and/or high production potential, only one internal solution exists: the high-yield equilibrium. Thus, under high production levels and milk prices (groups S3 and S4), soil remediation is always optimal and every farmer attains the steady state.

Figure 5

a) **b)**



c) **d)**

Phase diagrams on plane $x-z$ for different parameter values

Figures 5b and 5c, on the other hand, show two alternative fertilization paths with two corresponding interior solution. Conditions for the existence of low-yield and high-yield solutions include: medium to high phosphorus prices (relative to revenue per hectare, groups S1 and S2 in Table 1), medium to high phosphorus fixation power, and/or limited technology for achieving high crop yields. I discuss this case extensively below since it characterizes the conditions facing Chilean dairy farmers better.

Finally, Figure 5d presents the case in which low milk prices and/or poor technological levels prevent the existence of an interior solution. This is the case in which, after the resource is exhausted, farm operations are abandoned or the farm is sold. This case represents to many small farmers that in the 1990s migrated to cities and sold their farms either to bigger farmers or to people who currently use the land for forestry[3].

Analysis of the phase diagram and optimal paths

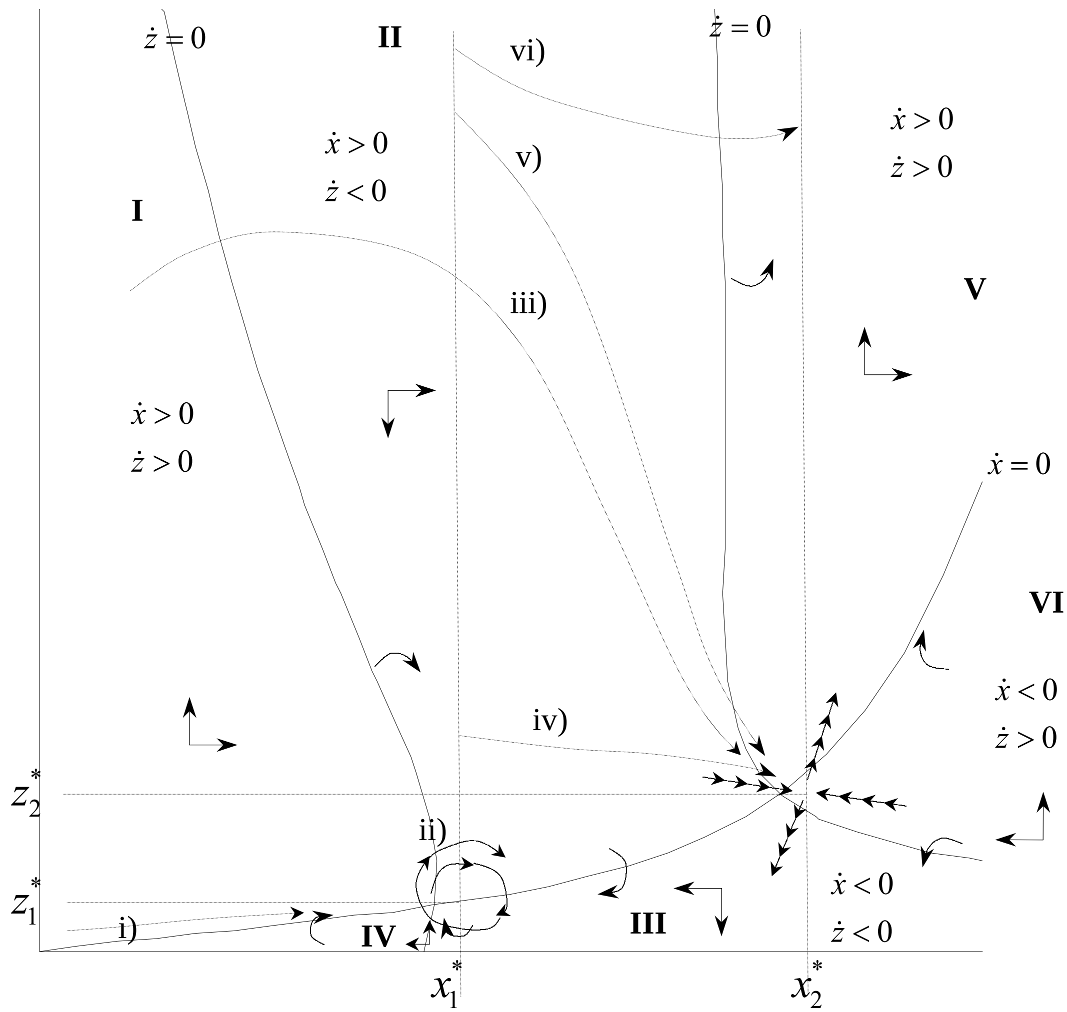
Since we are interested in determining optimal fertilization strategies, I analyze here the case depicted in Figure 5b on the $x-z$ plane. As mentioned previously, cases depicted in figures 5b and 5c are the ones that better represent the conditions of Chilean farms. The full diagram is presented in Figure 6.

First, I determine the nature of the internal solutions. I follow the procedure in Léonard and Van long (1992: 96), which includes taking a first order approximation of the system composed by the differential equations[4] for $\dot{x}$ and $\dot{z}$ (i.e. equations (6) and (8)) in the neighborhood of each solution and finding the characteristic roots (eigenvalues) of the linearized system. The nature of the solutions is determined then according to the magnitudes and signs of the roots and whether they are real or complex.

[3] Chilean government supports actively the planting of fast-growing trees on eroded soil or on soil whose fertility has been mined.

[4] Alternatively, we can use a different pair of equations such as those for $\dot{x}$ and $\dot{\lambda}$ or those for $\dot{z}$ and $\dot{\lambda}$.

Figure 6



Phase diagram on plane $x-z$

For the baseline simulation parameters, the roots at the LY node are $0.075+1.168i$ and $0.075-1.168i$, i.e. one is the complex conjugate of the other. Since the real part of the roots is positive, the LY node is an unstable focus spiraling away from the node. The analysis of the eigenvalues, however, characterizes only the behavior of the system in the closest neighborhood of the LY node. Therefore, the existence of a limit cycle around the LY solution cannot be ruled out (Clark, 1990:185-192). If a limit cycle exists, both the LY fertilization rate and the LY buffer stock level oscillate around the node $\left(x_1^*, z_1^*\right)$. Regarding to the HY node, its roots are 0.476389 and -0.326389. Since they are real, distinct, and have different signs, the HY solution is a saddle point.

In summary, two fertilization regimes can be postulated: a cyclic regime represented by the LY solution $\left(x_1^*, z_1^*\right)$ and the HY stable regime at $\left(x_2^*, z_2^*\right)$. Intuitively, these two solutions arise from the behavior of the production and fixation power functions, $f(\ )$ and $\alpha(\ )$ respectively. In the LY regime, although the marginal productivity of phosphorus is high, a larger share of phosphorus applied goes into the buffer stock so it is optimal to limit phosphorus application. In the HY regime, the opposite occurs. How high x_1^* can be depends on the relative behaviors of the functions $f(\ )$ and $\alpha(\ )$. In soils with high fixation power (high value of c), since the marginal productivity of phosphorus decreases faster than the marginal reduction in fixation power does, x_1^* cannot correspond to a significant enrichment of the buffer pool. Therefore, the solution $\left(x_1^*, z_1^*\right)$ represents a regime associated with low crop yields, a "subsistence" fertilization path.

The existence of an oscillatory "equilibrium" for the system has its counterpart on the field. A common practice among farmers in groups S1 and S2 is to carry out periodical "low-scale" fertility remediations. The cycle begins with the planting of grass on a degraded pasture, which includes the application of "high" rates of phosphorus (most commonly 200-250 Kg of Triple Superphosphate per hectare). Unable to sustain this level of fertilization for more than one season, the farmer will apply only a reduced fraction (some farmers apply nothing) of the initial rate during the next seasons. Thus, the initial "remediation" corresponds to investing in improving the content of phosphorus in the buffer pool. During the next seasons, the farmer gets his investment back by harvesting from the enhanced soil and reducing the phosphorus applications. After a certain number of seasons (three years usually), the soil is back to its original condition and it is time to repeat the process.

The relevant issue from a policy standpoint at this stage of the analysis is the identification of the determinants that make a farmer to choose one regime over the other. The phase diagram in Figure 6 provides several clues pointing toward which those determinants are. No optimal fertilization path combining rates below z_2^* can reach the HY steady state. Shifting from the LY to the HY regime involves a large short run increase in phosphorus applications since attaining the HY steady state is feasible only by applying $z > z_2^*$. But that short run increase in the fertilization rate may not be affordable for farmers who face a high effective price of phosphorus or interest rate because of credit constraints or who utilize less efficient technology. In the field, this is the case of many small Chilean farmers, who lack of adequate technological and managerial skills (low values for parameters A and B). Many of them have little schooling; many are also old and have short planning horizons (hence high discount rates). Farms with highly depleted soils in region I can reach the HY equilibrium following an optimal trajectory such as iii). It begins with fertilization rates well above z_2^* and then gradually reduces phosphorus applications as it approaches the HY equilibrium. Farmers already in the LY steady state can move to the HY equilibrium by applying fertilization rates greater than z_2^*. The higher are these fertilization rates the faster is the approaching to the equilibrium. Trajectory iii), for instance, is faster than trajectory iv) at early stages of the fertility remediation, while v) is faster than iii). A more aggressive approach is depicted by trajectory vi), which considers the use of very high fertilization rates to move from the LY regime to x_2^*. Only once the equilibrium buffer pool level is attained, phosphorus applications can be reduced until reaching the HY solution $\left(x_2^*, z_2^*\right)$.

From the analysis, there are two issues relevant for policy analysis. First, we have the existence of two alternative long-term regimes, or, in other words, two optimal long-term fertilization strategies. Second, the endogenous nature of the regimes raises questions as to the appropriateness of using exogenous indicators as the core components of the targeting policy of a soil fertility remediation program. As the preceding analysis has shown, the optimal levels of remediation of the buffer pool depend on individual characteristics such as soil properties, discount rate, efficiency in the production process, and technology used in the farm.

According to the analysis of the phase diagram, it seems that the capacity to afford high fertilization rates is crucial in determining the fertlization a farmer will follow. In order to find worthwhile to remediate the soil, the present value of the HY steady state must exceed the present value of the cost of attaining it. For farmers with very low initial buffer pools, i.e. $x(0) < x_1^*$, this may not be the case, particularly if they face managerial or financial constraints. These farmers may not have a choice except to follow a LY fertilization path. On the other hand, we have farmers that are neither technologically nor financially constrained and thus they will consider achieving the HY steady state $\left(x_2^*, z_2^*\right)$ as the optimal option.

In order to allocate a limited budget efficiently, a program aiming soil fertility remediation should provide financial support preferentially to those farms considering the LY solution optimal if left to their own, and support should be provided only up to the point at which soil fertility makes the HY equilibrium affordable. It can be argued that providing funding beyond that point will bring benefits from accelerating the transition to the HY steady state. However, unless enough money is available to accomplish the two tasks, program budget will bring more benefits if aimed to move farmers from a LY regime to a path ending at the HY steady state. Making it affordable for farmers who would otherwise choose the LY regime to attain the HY equilibrium brings long run benefits, while accelerating the transition brings short run gains only.

Adequate targeting is expected to be difficult in one-time subsidies for remediation, like the one to be discussed in the next sections, which do not consider follow-up subsidies for fertility maintenance. If such a program fails in collecting adequate information from farmers, then it is possible that part of the budget goes to people that, after cost sharing is stopped, find optimal to move back to the LY regime. Consider, for instance, old farmers with short time-horizons or farmers with technological constraints that make unaffordable to sustain a fertilization rate z_2^*. Graphically, this outcome corresponds to a trajectory that begins at some point on the vertical line representing x_2^*, moves through region III in Figure 6, and returns to the cyclical path around $\left(x_1^*, z_1^*\right)$.

The second important issue is the endogenous nature of the steady state levels $\left(x_1^*, z_1^*\right)$ and $\left(x_2^*, z_2^*\right)$. According to the simulations in Figure 5, the steady state level of the buffer pool depends on soil properties (κ, γ, and α), phosphorus price, and farmer's attributes (such as age, farming experience, and financial condition). Thus, steady state levels are not exogenously determined but rather depend on individual characteristics, a fact that has implications for targeting cost-sharing.

Fertility remediation programs like the one to be discussed in the next section currently use an exogenous target level to allocate the program budget. Thus, only farms having a phosphorus level below the target are eligible to receive funding. These programs provide remediation cost sharing (or, in other words, a subsidized phosphorus price) to enrolled farmers until the target level of phosphorus in the soil is achieved. This could take more than one crop season, but, once the target level is reached, cost sharing is stopped.

The consequences of such a policy on program efficiency depend on where the target level is located with respect to the alternative equilibria in Figure 6. Consider, for instance that the exogenous target level x_t is greater than x_2^*. In this case, the farmer is eligible for receiving cost sharing; however, if the buffer pool level was already at x_2^*, then cost sharing will have only short run effects. After x_t is attained and cost sharing ceased, the farmer will return to the HY equilibrium and cost sharing effects will vanish. On the other hand, if the farmer condition was on some cyclical path around $\left(x_1^*, z_1^*\right)$, then, after attaining the target, the farmer will follow some optimal trajectory back to $\left(x_2^*, z_2^*\right)$. In this case, cost sharing has the desired effect, although some cost sharing money may be spent inefficiently when taking the phosphorus level beyond x_2^*.

Consider now a target level between the two equilibriums, i.e. $x_1^* < x_t < x_2^*$. In this case farmers already in the HY equilibrium are not eligible. Farmers at the LY equilibrium or on the way to the HY equilibrium, but still below x_t, are eligible for cost sharing. Awarded farmer will move from region I to region II until reaching $x = x_t$, then cost sharing will be stopped. What happen afterwards will depend on farmer attributes. Some farmers may follow a stable path, such as trajectories iii) and iv), and reach the HY steady state. Others may move from region II to region III and finally finish again in LY regime. In the first case, cost sharing has the desired effect of altering the long run fertility regime. In the second case, there are short term effects only. The effect of cost sharing on farmers on the way to the HY equilibrium will be to accelerate the transition, but the effect will be short run as well. Thus, if $x_1^* < x_t < x_2^*$, cost sharing will have long run effects only on farmers who would otherwise have been in the LY regime, but not on all of them.

Evaluating the performance of a fertility remediation program under the theoretical framework developed above requires dealing with two econometric issues. First, it is necessary to handle the problem of determining which long-term fertilization strategy the farmer would choose in the absence of cost sharing. Note that even we may assume the farmer has attained some equilibrium, we, in general, do not know in which of the two possible regimes he is. Second, both short and long run effects of the program must be estimated in order to identify clearly the two fertilization regimes predicted by the theoretical model. Actually, if farmers that are not financially constrained receive cost sharing, then long run effects should be negligible for them. I discuss these two issues in the following sections, where I describe the program from which data was collected and develop the econometric model.

2.3.- A description of the Chilean soil-fertility replenishing program

Beginning in 1996 after successive measures to open the local economy to foreign markets, the Chilean government implemented a cost-sharing program aiming to replenish soil fertility in those agricultural operations whose existence could be jeopardized in a free-trade environment. The program provides support to farms with phosphorus pools depleted either naturally or by human activities, which are widespread in the country due to the volcanic origin of most of its soils and the fact that managerial and financial constraints prevent many farmers from replenishing phosphorus stocks. So far, farms awarded by the program have been mainly beef and dairy operations on grassland in central and southern Chile.

Participation in the program is voluntary and farmers must submit an application to be considered for funding. Applications can be prepared by an authorized agronomist, who can be a private consultant or on the staff in a public agency. Applications are ranked according to a number of factors, the principal one being whether the soil phosphorus level falls below a target level previously established by the program. Applicants who are awarded funding receive enough to ensure that the target level is attained, a process that might take more than one season.

The current phosphorus target level has been kept constant in the last years at 15 mg Kg^{-1} Olsen in the plant-available pool. This level is motivated by agronomic considerations: 15 mg Kg^{-1} Olsen in the plant-available pool is about the minimal level necessary to guarantee survival of most grass species and hence commercial production on most type of grasslands in Chile. The premise of the program is that, once the target is achieved, farmers will maintain a phosphorus level of at least 15 mg Kg^{-1} Olsen without additional funding.

The Chilean program gives us a good opportunity to test the theoretical model presented in Section 2. At least three testable hypotheses emerge from the preceding analysis. First, there exist two farmer sub-populations. One is constrained financially and/or technologically and thus unable to achieve the HY equilibrium level of soil phosphorus. The other sub-population faces no significant constraints. Thus, the level of phosphorous observed in the second group in absence of the program is the HY steady state level (or the farm is on the way to that equilibrium level).

Second, the short run effect of cost sharing on the LY subpopulation exceeds the short run effect on the HY subpopulation. Although program administrators want to grant cost-share funding to the constrained group preferentially and to those farms on the way to the HY equilibrium alternatively, they cannot avoid awarding farmers already at the HY steady state because they do not know neither which sub-population farmers belong to nor the individual HY phosphorous levels. Nonetheless, from the discussion in section above the short run effect of cost sharing should be greater on farms in the LY regime because those farms are the ones receiving the largest phosphorus applications.

Finally, there is a third hypothesis: cost sharing has no long run impact on the HY subpopulation but does on the LY subpopulation. However, panel data, which is not available in this study, are needed to test this hypothesis.

The following sections present an econometric framework that models the existence of two phosphorus fertilization paths in a farmer population, where the adoption of one or the other depends on whether the farmer faces or not constraints that prevent him from attaining the HY equilibrium.

Simultaneously, the impact of a cost-sharing program is evaluated conditional on the fertilization strategy adopted by the farmer. By using an endogenous-switching regression with unobserved switching points, I analyze the suitability of using an exogenous target level as the core of the targeting policy in a cost-sharing program aiming the recovery of a natural-resource stock.

3.- Data, Econometric model and Estimation

3.1.- Data

The Chilean Agricultural Policy and Statistics Office provided the data used to evaluate the theoretical model. Data were collected in 2001 by surveying a total of 856 farms from the population targeted by the cost-sharing program. The survey sample was stratified according to geographic location (4 north-to-south strata) and total acreage (2 strata). The sample was distributed proportionally within each stratum; the Agricultural Policy and Statistics Office provided the corresponding expansion factors. After cleaning the data set from observations with contradictory or missing information the sample contained observations on 505 farms, 177 of which received cost sharing for at least one year and 328 of which were never awarded cost share funding. A short description of the variables used in this study follows. Descriptive statistics of the full set of variables is presented in Table 2.

The variable SHARE indicates the share of land enrolled in the program at some point between years 1996 and 2000 inclusively and is censored from below at zero. The variable POLSEN gives the current (year 2001) level of phosphorus in the plant available phosphorus. Twelve soil samples were used to build a composite sample for each farm, which was analyzed to obtain an estimate of the average level of soil phosphorus content. Phosphorus fixation power is proxied by the variable ALSAT, which gives the aluminum saturation of the soil or level of aluminum in the soil solution. In chilean soils, most of the phosphorus is fixed to aluminum compounds. Thus, following the discussion from the introduction, the higher the aluminum concentration in a soil, the higher is its phosphorus fixation power (e.g. the higher the parameter c in $\alpha(x)$).

Variables related to credit accessibility and cash flow included in the econometric estimation are AACRE, and REV. AACRE gives farm acreage suitable for agricultural operations, and REV provides an estimation of average annual revenue per hectare. It is expected that farms with more utilizable acreage have more access to the credit market, while farms with higher average revenue face lower financial constraints.

An operation-type dummy, CATTLE, was included to control for cost sharing targeting preferences. Program regulations prohibit awarding cropping operations unless crops are being used at the initial stage of a rotation previous to establish pasture land. The variable CATTLE takes the value one if the farm held more than one animal unit of dairy or beef cattle.

Two sets of geographic dummies were included as explanatory variables. Excluding the Metropolitan Area (which includes the capital city, Santiago), Chile is divided administratively in twelve regions that are numbered in ascending order from north to south. The phosphorus cost sharing program has concentrated on Regions Seven through Ten. It is well known among Chilean agronomists that soil parental materials change north to south from alluvial and granite materials to volcanic ash. Most of the volcanic soils are located in Regions Nine and Ten. To control for soil properties other than aluminum saturation, a set of four location dummies for Regions Seven, Eight, Nine, and Ten were included in the empirical models. It is expected that probability of being awarded cost-sharing is higher in more southern regions since volcanic soils show a particularly strong phosphorus fixation power. Consequently, the current level of phosphorus is expected to decrease north to south (other things equal).

A second set of three location dummies (ANDES, VALLEY and COAST was included. The purpose of these three dummies was to characterize soil limitations to agricultural activities. In Chile the more productive soils are located in the valley between the coastal range next to the Pacific and the Andes Mountains. Soils in the piedmont of the Andes are young volcanic soils with poor chemical properties that limit crop and grass production. Soils on the coastal range, on the other hand, are highly erodible and sensitive to droughts during the dry season (summer in Chile). Thus, it is expected a higher level of cost-share awarding among farmers close to the Andes or located on the coastal range, because they are likely to be more financially constrained than farmers on the Valley and thus a preferred target for program administrators. Finally, a higher level of phosphorus is expected on farms on the Valley given the less restrictive qualities of their soil.

Table 2
Dependent and Explanatory variables

Variable	Description	Mean	St. dev.
SHARE	Binary variable indicating whether or not the farm has received cost sharing between 1996 and 2000 inclusively.	0.3626	0.4812
POLSEN	Logarithm of the phosphorus level (measured by the Olsen method) in the plant-available pool.	2.0123	0.7915
AACRE	Acreage usable for agricultural production (10^3 ha)	0.0321	0.0528
REV	Average annual revenue per hectare (10^6 Ch$ ha^{-1})	0.0223	0.0345
AlSat	Percentage of Al saturation in the soil solution	0.8456	0.8180
CATTLE	Farm holds more than 1 animal unit of beef or dairy cattle (yes=1)	0.8474	0.3600
SEVEN	Farm is located in Region Seven (yes=1)	0.0746	0.2630
EIGHT	Farm is located in Region Eight (yes=1)	0.1215	0.3271
NINE	Farm is located in Region Nine (yes=1)	0.1899	0.3926
TEN	Farm is located in Region Ten (yes=1)	0.6140	0.4873
ANDES	Farm is located on the hills at the feet of Andes range (yes=1)	0.4097	0.4923
VALLEY	Farm is located in the valley between the Coast and Andes ranges (yes=1)	0.3160	0.4654
COAST	Farm is located on the Coastal range or in the hills at the eastern side of that range (yes=1)	0.2743	0.4466

The available data include farms that have received funding during one or more years between 1996 and 2000 inclusively. A positive short-run effect of the program is expected on every farm because the program monitors the application of the fertilizer. Hence, the 2001 survey data used in this study is likely to detect some positive effect even on those farms facing no constraint to achieve the HY equilibrium. These data are cross-sectional and thus do not permit investigation of the long run effects. The theoretical model does, however, indicate that the effect of cost-sharing on fertilizer use during the transition period on farms in the LY regime should exceed that on farms in the HY regime, permitting a test of this hypothesis.

In what follows I develop a framework that allows testing the existence of two farm subpopulations with different fertilization strategies as indicated by the theoretical analysis. Simultaneously, I determine whether or not the effect of the program is conditional on the subpopulation a farmer belongs to. This framework also allows for an examination of how well cost sharing funds have been targeted. If two separate groups of farmers do exist and those groups can be distinguished by observable characteristics, then those characteristics can and should be used to determine how cost share funds are allocated. If those groups cannot be distinguished by observable characteristics, then the current allocation strategy of the program may be adequate and an exogenous target level may be a reasonable criterion for determining funding awards.

3.2.- The econometric model

Let y_{2i}^* denote the amount of cost-share money allocated to farm i and y_{3i}^* be the phosphorus content in the corresponding plant-available pool. A model that allows evaluating the impact of cost sharing on soil phosphorus level is the following,

$$\begin{aligned} y_{2i}^* &= X_{2i}\beta_2 + \varepsilon_{2i} \\ y_{3i}^* &= \alpha y_{2i} + X_{3i}\beta_3 + \varepsilon_{3i} \end{aligned} \qquad (9)$$

where the X_{ji} $(j = 2,3)$ are vectors of exogenous explanatory variables, and β_j are parameter vectors to estimate. Cost sharing funding is not an event that is exogenous to farmer decisions since farmers self-select by deciding whether to apply for funding. Consequently, the equations in (9) cannot be estimated independently and the correlation between equation disturbances must be allowed to adjust freely during the estimation.

The theoretical model suggests that farmers' fertilization strategies, hence their responses to receiving cost sharing, depend on farm characteristics. To introduce the process of selecting a fertilization strategy (in simpler words: whether to be in the LY regime or in the HY regime), I include the following equation to the equation system (9)

$$y_{1i}^* = X_{1i}\beta_1 + \varepsilon_{1i} \qquad (10)$$

The unobserved variable y_{1i}^* gives "farmer's propensity to attain the HY equilibrium". Thus, y_{1i}^* is negative or positive depending on whether farmer i faces constraints that reduce his chances to attain the HY equilibrium. Farmers facing no constraint should be close to the HY phosphorus level so that the receipt of cost share funds should have only a small short-run effect on the level of their phosphorus stocks. By contrast, cost share funds should allow constrained farmers ($y_{1i}^* \leq 0$) to switch from an LY to a HY regime and should thus have a larger long-run effect on the plant-available buffer phosphorus stocks.

The econometric model is now,

$$\begin{aligned} y_{1i}^* &= X_{1i}\beta_1 + \varepsilon_{1i} \\ y_{2i}^* &= X_{2i}\beta_2 + \varepsilon_{2i} \\ y_{3i}^* &= \alpha_3 y_{2i} + X_{3i}\beta_3 + \varepsilon_{3i} \qquad y_{1i}^* < 0 \\ y_{3i}^* &= \alpha_4 y_{2i} + X_{4i}\beta_4 + \varepsilon_{4i} \qquad y_{1i}^* \geq 0 \end{aligned} \tag{11}$$

Model (11) is a switching regression model with endogenous switching and fertilization strategy conditional on cost-share funding. Since farmer "propensities", y_{1i}^*, are unobserved, the model has unobserved switching points.

Note that if fertilization decisions are not conditional on farmer constraints, the switching should generate no differential effects on the parameters of the remaining equations. In other words: if a single equilibrium exists, then elements in the parameter vectors β_3 and β_4 should be equal to each other. However, if alternative regimes do exist, then these parameters should be different. I use aWald test to check equality between the two sets of estimated parameters.

3.3.- Implementing a MCEM for a switching regression with unobserved switching points

If distribution assumptions are made for the disturbances then the parameters in the equation system (11) can be estimated by maximum likelihood. Before proceeding to the estimation method it is necessary to establish the relation between the dependent variables and its observed counterparts. Two out of the three dependent variables are latent. As discussed previously, the variable y_{1i}^* is fully unobserved and y_{2i}^* is binary. The variable y_{3i}, on the other hand, is observed fully. Thus,

y_{1i}^* is unobserved fully $\qquad y_{2i} = \begin{cases} 1 & \text{if } y_{2i}^* > 0 \\ 0 & \text{if } y_{2i}^* \leq 0 \end{cases} \qquad y_{3i} = y_{3i}^*$

The model in (11) is a system of structural equations combining latent and observed variables and, in consequence, a MCEM algorithm can be used for its estimation. For estimation purposes the observed counterpart y_{2i} is estimated by the dichotomous variable SHARE and y_{3i} by the logarithm of the continuous variable POLSEN.

Let proceed now with the implementation of the MCEM algorithm for this problem. First, let assume that the disturbance terms in (11) are distributed

$$\text{according to } \begin{pmatrix} \varepsilon_{1i} \\ \varepsilon_{2i} \\ \varepsilon_{3i} \\ \varepsilon_{4i} \end{pmatrix} \sim N(0,\Sigma), \text{ where } \Sigma = \begin{bmatrix} 1 & \sigma_{12} & \sigma_{13} & \sigma_{14} \\ \sigma_{12} & 1 & \sigma_{23} & \sigma_{24} \\ \sigma_{13} & \sigma_{23} & \sigma_{33} & \sigma_{34} \\ \sigma_{14} & \sigma_{24} & \sigma_{34} & \sigma_{44} \end{bmatrix} \qquad (12)$$

where σ_{33} and σ_{44} have been set equal to one according to the usual normalization required for identification. Then the complete information likelihood function can be written as,

$$L^c(\boldsymbol{a},\boldsymbol{b},\Sigma \mid \boldsymbol{y}) = \prod_{i,y_{1i}^*<0}\left[\frac{1}{(2\pi)^{3/2}|\Gamma_1||\Sigma_1|^{1/2}}\exp\left(-\frac{1}{2}\boldsymbol{\varepsilon}_i^{'}\Sigma_1^{-1}\boldsymbol{\varepsilon}_i\right)\right]\times\prod_{i,y_{1i}^*\geq 0}\left[\frac{1}{(2\pi)^{3/2}|\Gamma_2||\Sigma_2|^{1/2}}\exp\left(-\frac{1}{2}\boldsymbol{\varepsilon}_i^{'}\Sigma_2^{-1}\boldsymbol{\varepsilon}_i\right)\right]$$

$$\text{where } \boldsymbol{\beta} = (\beta_1,\beta_2,\beta_3,\beta_4),\ \boldsymbol{\alpha} = (\alpha_3,\alpha_4),\ \Sigma_1 = \begin{bmatrix} 1 & \sigma_{12} & \sigma_{13} \\ \sigma_{12} & 1 & \sigma_{23} \\ \sigma_{13} & \sigma_{23} & \sigma_{33} \end{bmatrix}, \text{ and,}$$

$$\Sigma_2 = \begin{bmatrix} 1 & \sigma_{12} & \sigma_{14} \\ \sigma_{12} & 1 & \sigma_{24} \\ \sigma_{14} & \sigma_{24} & \sigma_{44} \end{bmatrix}$$

Since Γ_1 and Γ_2 are identity matrices, the complete information log-likelihood function reduces to,

$$\begin{aligned} \ell^c(\boldsymbol{a},\boldsymbol{b},\Sigma \mid \boldsymbol{y}) &= -\frac{3N}{2}\ln(2\pi) - \frac{1}{2}\sum_{y_{1i}^*<0}\ln|\Sigma_1| - -\frac{1}{2}\sum_{y_{1i}^*\geq 0}\ln|\Sigma_2| - \frac{1}{2}\mathrm{tr}\left(\Sigma_1^{-1}\sum_{y_{1i}^*<0}\boldsymbol{\varepsilon}_i\boldsymbol{\varepsilon}_i^{'}\right) - \frac{1}{2}\mathrm{tr}\left(\Sigma_2^{-1}\sum_{y_{1i}^*\geq 0}\boldsymbol{\varepsilon}_i\boldsymbol{\varepsilon}_i^{'}\right) \\ &= -\frac{3N}{2}\ln(2\pi) - \frac{1}{2}\sum_{y_{1i}^*<0}\ln|\Sigma_1| - -\frac{1}{2}\sum_{y_{1i}^*\geq 0}\ln|\Sigma_2| - \frac{1}{2}\mathrm{tr}\left(\Sigma_1^{-1}\sum_{y_{1i}^*<0}\boldsymbol{\varepsilon}_i\boldsymbol{\varepsilon}_i^{'} + \Sigma_2^{-1}\sum_{y_{1i}^*\geq 0}\boldsymbol{\varepsilon}_i\boldsymbol{\varepsilon}_i^{'}\right) \end{aligned} \qquad (13)$$

where N is the total number of observations. Note that the parameter σ_{34} in (12) cannot be estimated since there are no observations in both regimes simultaneously. Also note that the last two terms between brackets in (13) cannot be written in practice because, as y_{1i}^* is not observed, we do not know the regime in which each observation must be included. This is standard in switching models with unobserved switching points (Dickens and Lang, 1985). In the classical maximum likelihood approach the log-likelihood function for each individual is the weighted sum of the likelihoods of being in each regime, where the weights are the conditional probabilities of being in the respective regimes. In (13) we still have a sum of two terms; however, instead of weighting the sum, the idea is to simulate y_{1i}^{*} as if the individual were in one regime in order to calculate the first sum and then simulate y_{1i}^{*} as if the individual were in the other regime in order to calculate the second sum. Details are given below when describing the implementation of the Gibbs sampler.

The Expectation step is straightforward from (13) and it requires the calculation of,

$$\underset{y_{1i}^{*}\le 0}{Q_i}\left(\boldsymbol{a},\boldsymbol{b}\mid\boldsymbol{a}^{(m)},\boldsymbol{b}^{(m)},\Sigma^{(m)},\boldsymbol{y}\right)=E\left[\varepsilon_i\varepsilon_i^{'}\mid\boldsymbol{a}^{(m)},\boldsymbol{b}^{(m)},\Sigma^{(m)},\boldsymbol{y},y_{1i}^{*}\le 0\right]=E\left[\begin{pmatrix}y_{1i}^{*}-X_{1i}\beta_1\\ y_{2i}^{*}-X_{2i}\beta_2\\ y_{3i}^{*}-\alpha_3 y_{2i}-X_{3i}\beta_3\end{pmatrix}\begin{pmatrix}y_{1i}^{*}-X_{1i}\beta_1\\ y_{2i}^{*}-X_{2i}\beta_2\\ y_{3i}^{*}-\alpha_3 y_{2i}-X_{3i}\beta_3\end{pmatrix}^{'}\right] \quad (14)$$

$$=\sigma_i^{2(m)-}+\begin{pmatrix}\mu_{y_{1i}^{*}}^{(m)-}-X_{1i}\beta_1\\ \mu_{y_{2i}^{*}}^{(m)-}-X_{2i}\beta_2\\ y_{3i}-\alpha_3 y_{2i}-X_{3i}\beta_3\end{pmatrix}\begin{pmatrix}\mu_{y_{1i}^{*}}^{(m)-}-X_{1i}\beta_1\\ \mu_{y_{2i}^{*}}^{(m)-}-X_{2i}\beta_2\\ y_{3i}-\alpha_3 y_{2i}-X_{3i}\beta_3\end{pmatrix}^{'}$$

Analogously,

$$\underset{y_{1i}^{*}>0}{Q_i}\left(\boldsymbol{a},\boldsymbol{b}\mid\boldsymbol{a}^{(m)},\boldsymbol{b}^{(m)},\Sigma^{(m)},\boldsymbol{y}\right)=E\left[\varepsilon_i\varepsilon_i^{'}\mid\boldsymbol{a}^{(m)},\boldsymbol{b}^{(m)},\Sigma^{(m)},\boldsymbol{y},y_{1i}^{*}>0\right]=E\left[\begin{pmatrix}y_{1i}^{*}-X_{1i}\beta_1\\ y_{2i}^{*}-X_{2i}\beta_2\\ y_{3i}^{*}-\alpha_3 y_{2i}-X_{3i}\beta_3\end{pmatrix}\begin{pmatrix}y_{1i}^{*}-X_{1i}\beta_1\\ y_{2i}^{*}-X_{2i}\beta_2\\ y_{3i}^{*}-\alpha_3 y_{2i}-X_{3i}\beta_3\end{pmatrix}^{'}\right] \quad (15)$$

$$=\sigma_i^{2(m)+}+\begin{pmatrix}\mu_{y_{1i}^{*}}^{(m)+}-X_{1i}\beta_1\\ \mu_{y_{2i}^{*}}^{(m)+}-X_{2i}\beta_2\\ y_{3i}-\alpha_4 y_{2i}-X_{4i}\beta_4\end{pmatrix}\begin{pmatrix}\mu_{y_{1i}^{*}}^{(m)+}-X_{1i}\beta_1\\ \mu_{y_{2i}^{*}}^{(m)+}-X_{2i}\beta_2\\ y_{3i}-\alpha_4 y_{2i}-X_{4i}\beta_4\end{pmatrix}^{'}$$

where,

$$\sigma_i^{2(m)-}=\mathrm{Cov}\left(y_{1i}^{*},y_{2i}^{*},y_{3i}\mid\boldsymbol{a}^{(m)},\boldsymbol{b}^{(m)},\Sigma^{(m)},\boldsymbol{y},y_{1i}^{*}\le 0\right)$$

$$\sigma_i^{2(m)+}=\mathrm{Cov}\left(y_{1i}^{*},y_{2i}^{*},y_{3i}\mid\boldsymbol{a}^{(m)},\boldsymbol{b}^{(m)},\Sigma^{(m)},\boldsymbol{y},y_{1i}^{*}>0\right)$$

and

$$\mu_{y_{ji}^{*}}^{(m)-}=E\left[y_{ji}^{*}\mid\boldsymbol{a}^{(m)},\boldsymbol{b}^{(m)},\Sigma^{(m)},\boldsymbol{y},y_{1i}^{*}\le 0\right]\qquad j=1,\dots,3$$

$$\mu_{y_{ji}^{*}}^{(m)+}=E\left[y_{ji}^{*}\mid\boldsymbol{a}^{(m)},\boldsymbol{b}^{(m)},\Sigma^{(m)},\boldsymbol{y},y_{1i}^{*}>0\right]\qquad j=1,\dots,3$$

Recall that $E\left[y_{ji}^{*}\mid\boldsymbol{a}^{(m)},\boldsymbol{b}^{(m)},\Sigma^{(m)},\boldsymbol{y}\right]$ equals y_{ji} if y_{ji} is observed and must be estimated by Gibbs sampling otherwise.

I replace the M-step by two conditional M-steps. Given the simplicity of expressions (14) and (15), it is useful to define the vector $\theta=\left(\beta_1,\beta_2,\alpha_3,\beta_3,\alpha_4,\beta_4\right)$. Thus, the first conditional M-step maximizes,

$$-\frac{3N}{2}\ln(2\pi)-\frac{1}{2}\sum_{y_{1i}^*\le 0}\ln|\Sigma_1|--\frac{1}{2}\sum_{y_{1i}^*>0}\ln|\Sigma_2|-\frac{1}{2}\operatorname{tr}\left(\Sigma_1^{-1}\sum_{y_{1i}^*\le 0}Q_i\left(\theta\mid\theta^{(m)},\Sigma^{(m)},\boldsymbol{y}\right)\right)-\frac{1}{2}\operatorname{tr}\left(\Sigma_2^{-1}\sum_{y_{1i}^*>0}Q_i\left(\theta\mid\theta^{(m)},\Sigma^{(m)},\boldsymbol{y}\right)\right)$$

with respect to θ conditional on the elements of $\Sigma_1^{(m)}$ and $\Sigma_2^{(m)}$ to produce,

$$\theta^{(m+1)}=\left[\sum_{y_{1i}^*\le 0}X_i'\tilde{\Sigma}_1^{(m)}X_i+\sum_{y_{1i}^*>0}X_i'\tilde{\Sigma}_2^{(m)}X_i\right]^{-1}\left[\sum_{y_{1i}^*\le 0}X_i'\tilde{\Sigma}_1^{(m)}\mu_{y_i^*}^{(m)}+\sum_{y_{1i}^*>0}X_i'\tilde{\Sigma}_2^{(m)}\mu_{y_i^*}^{(m)}\right] \quad (16)$$

where the matrices in (16) are define as,

$$X_i=\begin{cases}\begin{bmatrix}X_{1i}&0&0&0\\0&X_{2i}&0&0\\0&0&X_{3i}&0\\0&0&0&0\end{bmatrix} & \text{if } y_{1i}^*\le 0\\ \begin{bmatrix}X_{1i}&0&0&0\\0&X_{2i}&0&0\\0&0&0&0\\0&0&0&X_{4i}\end{bmatrix} & \text{if } y_{1i}^*>0\end{cases},\qquad \mu_{y_i^*}^{(m)}=\begin{cases}\begin{pmatrix}\mu_{y_{1i}^*}^{(m)}\\ \mu_{y_{2i}^*}^{(m)}\\ y_{3i}^*\\ 0\end{pmatrix} & \text{if } y_{1i}^*\le 0\\ \begin{pmatrix}\mu_{y_{1i}^*}^{(m)}\\ \mu_{y_{2i}^*}^{(m)}\\ 0\\ y_{3i}^*\end{pmatrix} & \text{if } y_{1i}^*>0\end{cases}$$

$$\tilde{\Sigma}_1^{(m)}=\begin{bmatrix}\tilde{\sigma}_{11}^{(1,m)}&\tilde{\sigma}_{12}^{(1,m)}&\tilde{\sigma}_{13}^{(1,m)}&0\\ \tilde{\sigma}_{12}^{(1,m)}&\tilde{\sigma}_{22}^{(1)}&\tilde{\sigma}_{23}^{(1,m)}&0\\ \tilde{\sigma}_{13}^{(1,m)}&\tilde{\sigma}_{23}^{(1,m)}&\tilde{\sigma}_{33}^{(1,m)}&0\\0&0&0&0\end{bmatrix},\qquad \tilde{\Sigma}_2^{(m)}=\begin{bmatrix}\tilde{\sigma}_{11}^{(2,m)}&\tilde{\sigma}_{12}^{(2,m)}&0&\tilde{\sigma}_{14}^{(2,m)}\\ \tilde{\sigma}_{12}^{(2,m)}&\tilde{\sigma}_{22}^{(2)}&0&\tilde{\sigma}_{24}^{(2,m)}\\0&0&0&0\\ \tilde{\sigma}_{14}^{(2,m)}&\tilde{\sigma}_{24}^{(2,m)}&0&\tilde{\sigma}_{34}^{(2,m)}\end{bmatrix}$$

and $\tilde{\sigma}_{ij}^{(1,m)}$ and $\tilde{\sigma}_{ij}^{(2,m)}$ are the elements in the ***i***-th row and ***j***-th column of $\Sigma_1^{(m)-1}$ and $\Sigma_2^{(m)-1}$ respectively. The second conditional M-step then maximizes

$$-\frac{3N}{2}\ln(2\pi)-\frac{1}{2}\sum_{y_{1i}^*\le 0}\ln|\Sigma_1|--\frac{1}{2}\sum_{y_{1i}^*>0}\ln|\Sigma_2|-\frac{1}{2}\operatorname{tr}\left(\Sigma_1^{-1}\sum_{y_{1i}^*\le 0}Q_i\left(\theta^{(m+1)}\mid\theta^{(m)},\Sigma^{(m)},\boldsymbol{y}\right)\right)-\frac{1}{2}\operatorname{tr}\left(\Sigma_2^{-1}\sum_{y_{1i}^*>0}Q_i\left(\theta^{(m+1)}\mid\theta^{(m)},\Sigma^{(m)},\boldsymbol{y}\right)\right)$$

with respect to the elements in Σ to obtain an estimate for $\Sigma^{(m+1)}$.

There remains the implementation of the Gibbs sampler necessary to estimate the matrices Q_i in the objective function. To simulate the unobserved "observations" of the dependent variables proceed as follows.

The variable y_{1i}^* is fully unobserved. As we do not know what regime y_{1i}^* belongs to, we have to consider the possibility that y_{1i}^* may belong to either. Consequently, in order to estimate $\sigma_i^{2(m)-}$ and $\mu_{y_{1i}^*}^{(m)-}$ in (14) we must sample from a normal distribution with mean $\mu_{1i|i(-1)}^{(m)}$ and variance $\sigma_{1|-1}^{2(m)}$ truncated from above at zero. Analogously, the simulation must be performed from a normal distribution with mean $\mu_{1i|i(-1)}^{(m)}$ and variance $\sigma_{1|-1}^{2(m)}$ truncated from below at zero when estimating $\sigma_i^{2(m)+}$ and $\mu_{y_{1i}^*}^{(m)+}$ in (15).

The variable y_{2i}^* is binary. Accordingly, we simulate y_{2i}^* from a normal distribution with mean $\mu_{2i|i(-2)}^{(m)}$ and variance $\sigma_{2|-2}^{2(m)}$ truncated below at zero if y_{2i}^* equals one and truncated above at zero if y_{2i}^* equals zero.

The Gibbs sampler was started with 300 simulations and increased by 15 simulations at every iteration of the MCEM algorithm. The number of dismissed simulations, k_{burn}, was kept constant at 150. The routine converged after 420 iterations. A simulation of size $R = 3400$ and $r_{burn} = 400$ was used for the estimation of the information matrix and asymptotic standard errors. Results are presented in Table 3 (location dummies COAST and SEVEN were excluded from the estimation as required for identification). Estimation outcomes are presented in Table 3.

Table 3
Estimates of the switching regression model

Equation	Variable	Estimate	As. st. error	As. t-stat.	P-value
Switching	Constant	-0.0197	0.0395	-0.4986	0.6183
	AACRE	0.7208	0.3809	1.8922	0.0594
	ALSAT	-0.2014	0.0992	-2.0293	0.0430
	REV	0.9573	0.2036	4.7012	0.0000
	VALLEY	0.2639	0.2618	1.0081	0.3139
	ANDES	0.1249	11.6548	0.0107	0.9915
Cost share	Constant	-0.1395	0.2367	-0.5893	0.5559
	REV	-0.9582	0.8294	-1.1553	0.2485
	ALSAT	0.8262	0.3517	2.3495	0.0192
	CATTLE	-0.0463	0.0359	-1.2875	0.1986
	VALLEY	-0.9630	0.4981	-1.9332	0.0542
	ANDES	0.8390	1.5384	0.5454	0.5858
Phosphorus pool level HY regime	Constant	0.7499	0.3258	2.3018	0.0218
	SHARE	0.1082	0.0061	17.8456	0.0000
	REV	1.4109	0.1926	7.3238	0.0000
	EIGHT	-0.2377	0.0601	-3.9531	0.0001
	NINE	-0.3188	0.1427	-2.2343	0.0259
	TEN	-0.7226	0.0472	-15.3075	0.0000
	VALLEY	0.5283	0.1188	4.4462	0.0000
	ANDES	-0.2634	0.0626	-4.2078	0.0000
Phosphorus pool level LY regime	Constant	1.8991	0.3977	4.7752	0.0000
	SHARE	0.8270	0.0546	15.1443	0.0000
	REV	0.3741	0.2935	1.2749	0.2030
	EIGHT	-0.1993	0.0402	-4.9600	0.0000
	NINE	-0.5512	0.2098	-2.6270	0.0089
	TEN	-0.9331	0.1937	-4.8177	0.0000
	VALLEY	0.1430	0.1937	0.7384	0.4606
	ANDES	0.5217	0.6220	0.8387	0.4021
	σ_{12}	-0.0116	0.0125	-0.9255	0.3552
	σ_{13}	0.2633	0.0553	4.7629	0.0000
	σ_{23}	-0.4355	0.0486	-8.9554	0.0000
	σ_{33}	1.0256	0.0260	39.4721	0.0000
	σ_{14}	0.6556	0.0721	9.0881	0.0000
	σ_{24}	-0.2472	0.0149	-16.5854	0.0000
	σ_{44}	1.1550	0.0985	11.7285	0.0000

4.- Results

Overall, the signs of the coefficients in Table 3 correspond closely to what was expected from the theoretical model. In what follows I identify regime $y_{1i}^{*} \leq 0$ as the "low yield" regime and regime $y_{1i}^{*} > 0$ as the "high yield" regime.

4.1.- Existence of two subpopulations and two fertilization regimes.

To confirm whether there are or not two fertilization regimes, I use a Wald test to compare the beta coefficients of the two Phosphorus-pool-level equations in Table 3. The test provides a value $W = 307.6$ (7 df), which permits rejecting the hypothesis of equality between the two sets of coefficients. The test result confirms that there exist two farm sub-populations following different fertilization regimes.

Results from the switching equation suggest that the larger the share of land usable for agricultural purposes and the higher the revenue per hectare, the higher is the likelihood that a farm belongs to the high yield regime. The evidence is particularly strong for the revenue variable, which confirms the importance of financial condition on fertilization strategy. From the same equation, we have that the more intense is the power of the soil to fix phosphorus (measured by variable ALSAT), the higher the probability of being in the low yield regime. This outcome was expected since a higher level of free aluminum in the soil is a signal of greater depletion and thus of a more costly soil remediation. The coefficients of the regional dummies in the phosphorus equations are negative and increasing in magnitude north to south. This indicates that phosphorus level decreases as we move to south, which was expected since the presence of volcanic ashes in soil parental materials increases north to south in Chile. Soils in the central valley show more phosphorus in the plant-available pool than those close to the Andes or on the Coastal range, which is an indication of the greater productivity of valley soils. This is evidence of farmers' economic rationality: they fertilize more on soils having greater production potential. However, this is true only if farmers are not financially or technologically constrained. From the results for the equation of the low yield regime, we can see that constrained farmers show no significant differences between the phosphorus levels in valley farms and those in farms located on the Andes piedmont or on the Coastal range.

4.2.- Determinants of cost share allocation

The only statistically significant coefficient in the cost share equation is the coefficient of the ALSAT variable. Since the key funding requirement of the program is being below the target level in the plant-available pool, a positive and significant coefficient of our indicator of phosphorus fixation power was expected. None of the remaining variables in the equation is significantly different from zero, indicating that agronomic considerations alone were used to determine cost share awards, with economic considerations playing no role. This result confirms the stated policy of the cost sharing program. Results from the switching equation suggest that financial condition is an important determinant of fertilization strategy, so that it is feasible to improve targeting using observable characteristics, e.g. using farm revenue to determine awards.

4.3.- Effects of cost sharing on fertilization

The final evidence supporting the theoretical model comes from the comparison of program effects between the two regimes, i.e.

$$\begin{aligned}&\left(E\left[y_{3i} \mid y_{1i} \le 0, y_{2i}=1\right]-E\left[y_{3i} \mid y_{1i} \le 0, y_{2i}=0\right]\right)-\left(E\left[y_{3i} \mid y_{1i} > 0, y_{2i}=1\right]-E\left[y_{3i} \mid y_{1i} > 0, y_{2i}=0\right]\right)\\&=\alpha_4-\alpha_3+E\left[\varepsilon_{3i} \mid \varepsilon_{1i} \le -X_{1i}\beta_1, \varepsilon_{2i} > -X_{2i}\beta_2\right]-E\left[\varepsilon_{3i} \mid \varepsilon_{1i} \le -X_{1i}\beta_1, \varepsilon_{2i} \le -X_{2i}\beta_2\right]-\\&\quad E\left[\varepsilon_{3i} \mid \varepsilon_{1i} > -X_{1i}\beta_1, \varepsilon_{2i} > -X_{2i}\beta_2\right]+E\left[\varepsilon_{3i} \mid \varepsilon_{1i} > -X_{1i}\beta_1, \varepsilon_{2i} \le -X_{2i}\beta_2\right] \qquad (17)\end{aligned}$$

where the last expectations can be written as function of the standard normal pdf and the conditional bivariate normal cdf. After calculating (17) for every individual in the sample and taking the average, we obtain an estimate of 0.415 with a standard error calculated using the delta method of 0.133. This estimate is positive and different from zero at a 1% significance level. This result supports the hypothesis that cost sharing has a greater effect on the phosphorus level of constrained farmer who would be in the LY regime in the absence of cost sharing.

Overall, these results support the idea that the threshold separating the two sub-populations is actually determined endogenously and that the use of an exogenously determined target phosphorus level should not be used as the sole cost share allocation criterion.

Program administrators should care more about determinants of farmer behavior at the moment to allocate the program budget. By identifying the characteristics that makes a farmer more likely to be in one regime or in the other it would be possible to increase program efficiency by a more careful targeting. The way targeting should be improved, however, is not an easy problem. Determining the actual financial situation of applicants requires collecting sensitive information, which would make the process more complicated and require extra paperwork. As a result we may find that the additional transaction costs of application may discourage participation on the part of farmers for whom cost sharing would do the most good, i.e. financially constrained farmers with low managerial and technical skills.

5.- Final remarks

This article analyzed the targeting policy of a soil remediation program aiming to replenish phosphorus fertility of Chilean soils. By using an optimal-control approach it was shown that, depending on soil and farmer characteristics, a farmer may choose between two long-run phosphorus fertilization paths: 1) apply low rates of fertilization and sustain a low yield level of phosphorus in the soil, or 2) apply high fertilization rates initially and then attain and maintain a high yield level of phosphorus in the soil pools.

The existence of two possible equilibria, which are endogenously determined, questions the suitability of using an exogenous target level as the primary determinant in the allocation policy of a fertility remediation program. Thus, if the target is set too high with respect to the individual HY phosphorus level, at least part of cost sharing money becomes a net transfer. On the other hand, if the target is set too low, cost sharing may cause only short run effects and the farmer will move back to the LY equilibrium level in the long run.

An empirical cross-sectional evaluation of the model confirmed that two fertilization regimes do exist. As predicted by the theory, financial conditions are important determinants of the fertilization path followed by the farmer and they must be considered in the targeting policy of the program.

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