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DEFAULT RISK ASSESSMENT IN MEXICAN CORPORATIONS: EMPIRICAL VALIDATION OF THE MERTON-LÖFFLER MODEL

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Abstract

This work evaluates the applicability of the Merton–Löffler structural credit risk model to Mexican publicly traded companies, examining its potential to complement traditional credit assessments in emerging markets. By conceptualizing corporate equity as a call option on firm assets and estimating unobservable parameters through Löffler's iterative methodology, the study finds that the model captures relevant patterns of relative credit risk, albeit with only moderate alignment to rating agency classifications. While short-term predictive accuracy remains limited, the model exhibits some capacity to reflect longer-term credit dynamics consistent with theoretical expectations. Overall, the Merton–Löffler framework offers a rigorous and transparent approach for market-based credit risk evaluation in contexts characterized by informational constraints, though its implementation should be viewed as a complementary analytical tool rather than a substitute for comprehensive credit analysis.[1]

[1] ChatGPT (OpenAI) was used throughout the writing of this thesis to assist with grammar, syntax, and clarity. All ideas, arguments, and conclusions are the author's own.

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1 Introduction

Credit risk assessment remains one of the most fundamental challenges in modern finance, particularly in emerging market economies where traditional credit analysis faces unique constraints including limited historical data, information asymmetries, and frequent delays in external credit rating updates. In Mexico's financial markets, these challenges are compounded by the nature of publicly traded companies, creating need for transparent, market-based credit risk measurement tools that can complement or substitute traditional ratings when they are unavailable, costly, or significantly delayed.

The structural approach to credit risk modeling, pioneered by Merton,[2] offers a theoretical solution by treating corporate securities as options on firm assets. This framework's appeal lies in its economic intuition and its ability to link credit risk directly to observable market variables, providing real-time assessments based on market prices rather than backward-looking accounting measures. However, the practical implementation of structural models presents significant challenges, particularly estimating unobservable asset values and volatilities from observable market data. The Löffler iterative methodology [3] represents a significant advancement in addressing these implementation challenges, providing a robust framework for parameter estimation that has become widely adopted in both academic and practical applications.

Previous research on structural credit risk models has primarily focused on developed markets application,[4] leaving important questions about their effectiveness in an emerging market context with different institutional frameworks. This gap is relevant given the growing importance of them on corporate credit and the increasing need for sophisticated risk management tools in these dynamic environments.[5] The Mexican context presents both opportunities and challenges for structural model implementation, offering a diverse cross-

[2] Robert C. Merton, “On the Pricing of Corporate Debt: The Risk Structure of Interest Rates,” *Journal of Finance 29*, no. 2 (1974): 449, https://doi.org/10.2307/2978814.
[3] Gunter Löffler and Peter N. Posch, *Credit Risk Modeling Using Excel and VBA* (Hoboken, NJ: Wiley, 2007), 27–44.
[4] Jorge Antonio Chan-Lau and Amadou Nicolas Racine Sy, “Distance-to-Default in Banking: A Bridge Too Far?,” IMF Working Paper No. 06/215 (September 2006): 12, SSRN, https://ssrn.com/abstract=941277.
[5] International Monetary Fund, *Global Financial Stability Report: The Last Mile: Financial Vulnerabilities and Risks* (Washington, DC: IMF, April 2024), 56.

section of companies across multiple sectors while also presenting market issues that may affect model performance.

This study directly addresses the question: To what extent does the distance-to-default calculated through the Merton model with Löffler's dynamic estimation generate effective credit risk rankings for companies listed on the Mexican Stock Exchange? The objective is to evaluate the predictive capacity of the Merton-Löffler model to generate default risk rankings for Mexican listed companies, encompassing both the technical feasibility of implementing the model in an emerging market and the practical utility of its outputs for credit risk assessment.

This research aims to make several contributions to both academic literature and practical risk management applications. It provides the first comprehensive implementation of the Merton-Löffler model specifically for Mexican corporate credit risk, demonstrating that a quantitative framework for developed markets can provide value in emerging market. The latter by developing a comprehensive validation framework that addresses practical implementation questions facing credit risk practitioners, evaluating the distance-to-default metric through both descriptive concordance analysis and forward-looking performance analysis. The findings provide guidance on appropriate applications and limitations of this structural credit risk model, revealing that while the model shows no predictive capacity over short horizons, it results effective at the twelve-month horizon.

The study is organized into seven sections. Following this introduction, the literature review situates the work within the broader context of structural credit risk modeling and emerging market applications. The theoretical model section develops the Black-Scholes-Merton foundation and explains the distance-to-default framework as a credit risk metric. The data section describes the comprehensive dataset assembly from multiple sources, including equity prices, financial statements, and credit ratings for twenty-two Mexican companies over 2002-2020. The model implementation section details the Löffler iterative methodology and its adaptation to the Mexican market, addressing the challenge of estimating unobservable asset parameters from market data. The validation section presents empirical results through both concordance analysis with Fitch ratings and forward-looking predictive content assessment, revealing a 55% agreement rate with the Fitch risk ratings and predictive power for the returns of a company. The conclusion synthesizes findings and provides practical guidance for emerging market applications of structural credit risk models.

2 Literature Review

2.1 Backbone of Credit Risk Analysis

Building on Black and Scholes's[6] insight, Merton formalized the structural approach by modeling a firm's equity as a European call option on its assets. This means that shareholders possess the right—but not the obligation—to purchase the firm's assets at debt maturity by paying an amount equal to the face value of outstanding liabilities, which functions as the option's strike price. Under the risk-neutral measure, both equity and risky debt have a valid valuation via the Black–Scholes formula, and default arises endogenously at maturity whenever the firm's asset value falls below the debt threshold. An implication of this framework is the distance-to-default metric, measuring the number of asset volatility standard deviations by which current asset value exceeds the default point, where debt exceeds assets, directly linking a firm's leverage and asset volatility to its probability of default.

Black and Cox observed that in practice firms rarely wait until bond maturity to default,[7] creditors can force bankruptcy the moment a firm violates debt covenants, restriction established by the lender. To capture this, they introduced a first-passage barrier model: if a firm's asset value ever falls to a pre-specified covenant threshold, default is triggered immediately.

Crosbie and Bohn's Moody's KMV[8] framework mapped distance-to-default to an empirical Expected Default Frequency through extensive calibration to historical defaults. Their implementation relies on an iterative inversion algorithm: one repeatedly adjusts unobservable asset values and volatilities so that the Black-Scholes equity valuation equation holds each day. Christoffersen, Lando, and Nielsen demonstrated that, while the KMV iterative solution closely approximates the MLE under typical conditions, small discrepancies emerge in edge cases,[9] meaning very low equity values or highly volatile markets, highlighting the need for robust estimation and validation, especially in emerging-market settings.

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[6] Martin Haugh, "The Black-Scholes Model," IEOR E4706: Foundations of Financial Engineering lecture notes (2016): 1.

[7] Yunkun Shi et al., "Stock Price Default Boundary: A Black-Cox Model Approach," *International Review of Financial Analysis* 83 (2022): 8, https://doi.org/10.1016/j.irfa.2022.102284.

[8] Cathrine Jessen and David Lando, "Robustness of Distance-to-Default," 26th Australasian Finance and Banking Conference 2013 (2013): 7, https://ssrn.com/abstract=2311228.

[9] Bruce Christoffersen, David Lando, and S. F. Nielsen, "Estimating Volatility in the Merton Model: The KMV Estimate Is Not Maximum Likelihood," *Mathematical Finance* 32, no. 4 (2022): 1225, https://doi.org/10.1111/mafi.12362.

2.2 Recent Approach to Credit Risk Analysis

More recently, Suárez Torres extends Merton's insight that equity behaves like a call option on firm assets by calibrating the model to Colombian market data.[10] Drawing on Löffler and Posch's two-equation iterative procedure, she recovers unobservable asset values and volatilities from observed equity prices and book debt, thereby overcoming the strong assumption of constant debt structure. To capture real-world interdependencies, the thesis further augments the classical distance-to-default framework with Monte Carlo simulations under geometric Brownian motion. By integrating these methodological enhancements, the study provides a comprehensive toolkit for assessing default risk in emerging markets.

Löffler and Posch introduce a two-equation procedure that simultaneously exploits the Black–Scholes equity valuation equation and the linkage between equity volatility and asset volatility to infer the unobservable asset value and its volatility.[11] The algorithm begins with initial guesses for asset parameters, then alternates between solving for asset values using the rearranged Black–Scholes formula and updating volatility estimates until a convergence criterion is satisfied, all within a rolling-window framework that smooths market noise and accommodates changing liability values and interest rates.

Building on Suárez Torres and Löffler, this thesis implements the Merton–Löffler two-equation, rolling-window algorithm on a comprehensive sample of Mexican listed firms. Using daily equity prices and quarter-end balance-sheet data, the approach consists of iteratively infer each firm's unobservable asset value and volatility by alternating between the rearranged Black–Scholes equity valuation equation and the volatility linkage condition until convergence. This approach not only assesses the structural model's performance in an emerging-market environment but also identifies necessary adaptations to enhance its robustness and practical applicability in the Mexican context.

[10] Nilia Yizel Suárez Torres, "El modelo de Merton para la estimación del riesgo de incumplimiento en Colombia," tesis de grado, Facultad de Economía, Universidad Colegio Mayor de Nuestra Señora del Rosario (Bogotá D.C., semestre I, 2012), 5.
[11] Löffler and Posch, *Credit Risk Modeling Using Excel and VBA*, 39.

3 Model

3.1 Theoretical Foundation

As seen before, structural credit risk models are built upon the Black-Scholes-Merton option pricing framework, which revolutionized finance by demonstrating how derivatives can be priced without requiring knowledge of investors' risk preferences or expected returns. The key insight is that corporate securities can be viewed as options on the firm's underlying assets, where equity's limited liability feature creates option-like payoffs. In corporate finance, a firm's total value represents the underlying asset, while equity and debt represent different claims with distinct characteristics. Limited liability protection means equity holders cannot lose more than their initial investment, creating an asymmetric payoff structure—unlimited upside with limited downside—mathematically equivalent to a call option on firm assets.

Under the Black-Scholes framework, asset prices follow a geometric Brownian motion that captures both the random nature of asset price movements and ensures prices remain positive:

$$dV_t = \mu V_t dt + \sigma_V V_t dW_t \tag{1}$$

where V_t represents firm asset value, μ is the expected return, σ_V is asset volatility, and W_t is a standard Brownian motion.

The framework's crucial insight is risk-neutral valuation: any derivative can be valued as if investors were risk-neutral by adjusting the probability measure rather than the discount rate. Under the risk-neutral measure Q:

$$dV_t = rV_t dt + \sigma_V V_t dW_t^Q \tag{2}$$

where r is the risk-free rate. This eliminates the need to estimate risk premiums or investor preferences, requiring only observable market data: current asset value, risk-free rate, and asset volatility. The risk-neutral valuation principle is fundamental because it encapsulates all risk preferences and premiums in the measure change, allowing simplified valuation using the risk-free rate as the discount factor.

The transition from physical to risk-neutral measures reflects that while expected asset growth rates matter for investment decisions, they do not affect relative values of derivative claims on that asset. What matters for derivative valuation is the volatility of the underlying asset and the risk-free rate, both more directly observable from market data. At debt maturity, payoff structures reveal the option-like nature:

$$E_T = \max(V_T - L, 0) \tag{3}$$

$$D_T = \min(V_T, L) = L - \max(L - V_T, 0) \tag{4}$$

where equity receives the maximum of zero or the difference between asset value and debt face value (call option), while debt receives the minimum of asset value and face value, equivalent to a risk-free bond minus a put option representing the credit risk component.

3.2 The Merton Model

Merton's key advancement was recognizing that the limited liability feature of equity creates an option-like payoff structure, where shareholders have the right but not the obligation to repay debt holders and claim remaining firm assets. The model makes several simplifying assumptions to enable closed-form solutions while capturing essential credit risk features. The assumptions include frictionless markets characterized by continuous trading, no transaction costs or taxes, perfect asset divisibility, unlimited borrowing and lending at the risk-free rate, and no short-selling restrictions.

To account for the complexity of stochastic rates and time-varying volatility with the use of constant risk-free interest rates and firm asset volatility throughout the analysis period. The firm's capital structure is simplified to consist of only equity and zero-coupon debt. This captures the essential conflict between debt and equity holders while maintaining mathematical convenience. Default can only occur at debt maturity when asset value is insufficient to meet debt obligations, ruling out strategic default or early default due to liquidity constraints.

The mathematical development leverages option pricing theory where asset value evolves under the risk-neutral measure as a geometric Brownian motion with drift equal to the risk-free rate. The limited liability feature creates a payoff function identical to a European call option, where equity holders receive the maximum of zero or the difference between asset value and debt face value at maturity. Using the Black-Scholes formula, current equity value can be expressed as:

$$E_0 = V_0 N(d_1) - Le^{-rT} N(d_2) \tag{5}$$

where:

$$d_1 = \frac{\ln(V_0/L) + (r + \sigma_V^2/2)T}{\sigma_{V\sqrt{T}}} \tag{6}$$

$$d_2 = d_1 - \sigma_{V\sqrt{T}} \tag{7}$$

3.3 Distance to Default as Credit Risk Metric

The distance-to-default (DD) represents a key metric from the Merton model, providing an intuitive metric for measuring credit risk that uses market assessment of firms' default risk instead of traditional accounting measures. The DD measures how many standard deviations separate a firm's expected asset value from the default threshold, answering how much the firm's asset value needs to decline, measured in terms of its typical volatility, for the firm to reach default:

$$DD = \frac{\text{Expected log asset value} - \text{Default threshold}}{\text{Standard deviation of log asset value}} \tag{8}$$

Higher DD values indicate that the firm's expected asset value is many standard deviations away from the default point, suggesting low default risk, while low DD indicates proximity to or below the default threshold, implying high default risk.

The DD provides several advantages over traditional credit metrics such as debt-to-equity ratios, interest coverage ratios, or current ratios because traditional metrics are primarily backward-looking, based on historical accounting data that may not reflect current market conditions or prospects. In contrast, Merton’s metric incorporates forward-looking market information through current market value of equity and implied asset volatility. This market approach captures real-time investor sentiment and expectations about the firm's prospects, making it more responsive to changing conditions than accounting-based measures. The DD metric also provides a standardized measure that allows comparison across firms of different sizes and industries, as it expresses default risk in terms of standard deviations rather than absolute dollar amounts.

3.4 The Unobservable Asset Problem

The fundamental challenge in the Merton model is the fact that a firm's total asset value and asset volatility are not directly observable in the market. We can observe market values of individual securities, book values of liabilities from financial statements, and historical volatility of equity returns, but the model requires total market value of firm assets and asset volatility. Market value of assets typically differs significantly from book values due to intangible assets, market expectations, and accounting conventions that may not reflect current market conditions.

The estimation problem is complicated by the nonlinear relationship between equity value and asset value inherent in the option pricing framework, where small changes in asset value or volatility can lead to disproportionately large changes in equity value, particularly for highly leveraged firms where equity resembles a deep out-of-the-money option. The Löffler approach is based on the insight that at each point in time, we have two equations linking observable and unobservable variables: the Black-Scholes equity valuation equation and the volatility relationship connecting observable equity volatility to unobservable asset volatility:

$$E_0 = V_0 N(d_1) - Le^{-rT} N(d_2) \tag{9}$$

$$\sigma_E = \sigma_V \frac{V_0}{E_0} N(d_1) \tag{10}$$

In this system, equity value and equity volatility are observable from market data, while asset value and asset volatility are the unknowns to be determined.

Instead of solving the system for a single date, the method uses historical data over an estimation window, typically a year of trading days (252 days), to create a system with more equations than unknowns. While asset values change daily, asset volatility can be assumed to be constant over the estimation period, providing the necessary constraint to solve the over-identified system. For each day in the estimation period, we can write:

$$E_t = f(V_t, \sigma_V, L_t, r_t, T - t) \tag{11}$$

where the function represents the Black-Scholes formula.

The iterative solution procedure begins with initialization by setting starting values for asset values at each date, using a reasonable initial guess:

$$V_t^{(0)} = E_t + L_t \tag{12}$$

The algorithm alternates between updating asset values using the rearranged Black-Scholes formula and recalculating asset volatility using the updated asset value series, continuing iterations until convergence is achieved. The method allows for time-varying liability values and interest rates while maintaining the assumption of constant asset volatility, providing a realistic balance between model flexibility and parameter stability, and automatically ensures that estimated asset values and volatility are mutually consistent with observed equity values throughout the estimation period.

4 Data

4.1 Data Sources and Collection

It was necessary to assemble a comprehensive dataset from a wide range of sources. The historical equity prices were drawn from the web page Investing[12] from where there can be obtained the historical daily prices. As the proxy for the risk-free rate for Mexico, it was used the one-year CETES (Certificados de la Tesoreria de la Federación), obtained from the Banco de Mexico's Sistema de Información Económica;[13] these government-issued securities are the standard benchmark for risk-free yields in Mexico. It was also employed the S&P/BMV IPC index as the Mexican market reference, which is available daily.[14] For firm-level accounting data, monthly reported debt and outstanding shares were obtained from each company and subsequently verified against actual quarterly balance sheets. The monthly data was sourced from the capital structure breakdown provided by S&P Bloomberg, covering all listed companies on the BMV from 2002 to 2020,[15] supplemented by quarterly reports obtained directly from companies' websites where they publish this information for investors.[16] Using this approach, it was possible to construct historical debt and share series for all companies in the sample.

[12] Investing.com, "S&P/BMV IPC (MXX)," *Investing.com*, accessed March 15, 2025, https://www.investing.com/indices/ipc.

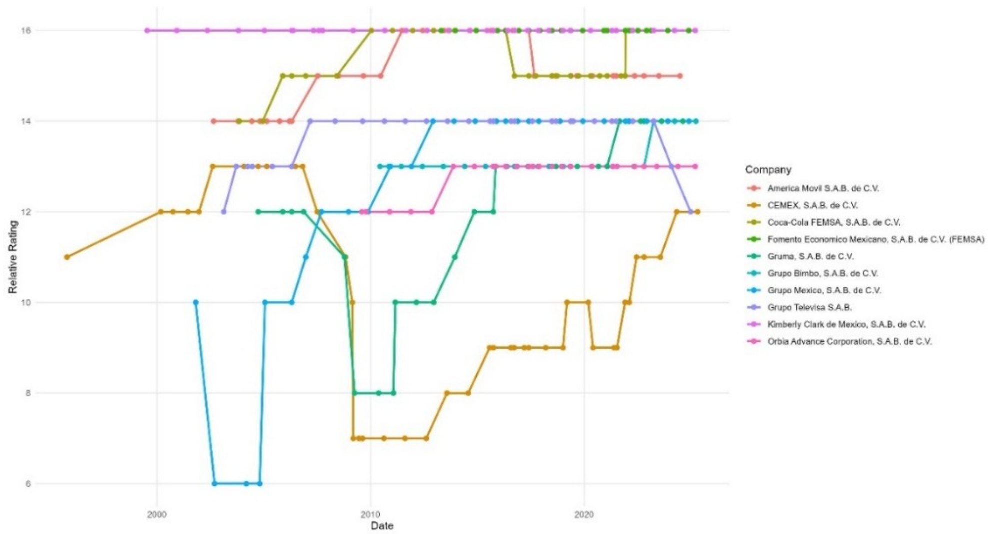
[13] Banco de México, "Government Securities – (CF107)," SIE-Banxico, accessed March 15, 2025, https://www.banxico.org.mx/SieInternet/consultarDirectorioInternetAction.do?sector=22&accion=consultarCuadro&idCuadro=CF107&locale=es.

[14] Investing.com, "S&P/BMV IPC (MXX)."

[15] S&P Bloomberg, "Capital Structure Breakdown for All Listed Companies on the Bolsa Mexicana de Valores (BMV), January 2002–December 2020," database, accessed March 30, 2025.

[16] Quarterly financial reports, company investor-relations websites, accessed March 2025.

Figure 1 *Fitch Local Currency Long Term Issuer Default Rating History*



Note: The fitch ratings don't have a fixed time to be updated so the time between updates varies between the same company and across all of them. **Source:** Own elaboration based on data from Fitch Ratings.

The Fitch Ratings are available for public access but not for public download at the level this model requires, so the approach involved systematic web scraping with Python to obtain the ratings from the Fitch website.[17] Fitch generally provides four different types of ratings per company, but for the validation of this model, only the Long-Term Issuer Default Rating and Local Currency Long Term Issuer Default Rating were used. These ratings are "forward-looking opinions on the relative ability of an entity or obligation to meet financial commitments";[18] the difference between the two types of ratings is that one ignores the transfer and convertibility risk associated with currency by being measured in local currency.

4.2 Data Construction and Processing

The creation of a comprehensive daily data frame integrating all these metrics required matching data across different timeframes, particularly the Fitch ratings. The entire dataset was constructed using daily stock prices as foundation, because they constitute the data set with the highest frequency. The matching process with other metrics involved taking each company's stock price date and identifying the most recent available data for other metrics prior to that date, then matching them accordingly. This process was repeated for all companies' stock prices until the complete dataset was assembled. Since stock prices had a higher frequency than other

[17] Fitch Ratings, accessed April 30, 2025, https://www.fitchratings.com/.
[18] Fitch Ratings, "Rating Definitions."

metrics, some data was necessarily repeated; however, given that the model relies on creating default probabilities using daily prices, there remains daily variation even though debt data only has quarterly frequency.

For the model implementation, total debt was defined as all liabilities reported in the companies' balance sheets, encompassing both short-term and long-term obligations. The market value of equity was calculated as the product of outstanding shares and daily closing stock prices. This resulted in some repetition of quarterly debt data across daily observations but maintained the essential daily variation in equity values required for volatility estimation while preserving the temporal integrity of the underlying financial relationships.

4.3 Sample Selection and Composition

The sample was selected using S&P/BMV IPC companies, which had already been chosen as a diverse and representative index for the BMV, considering the liquidity, size, and overall performance of the selected companies. This approach had the main advantage of maintaining the model's representativeness for the BMV without requiring the use of all listed companies. The companies in the sample represent a diverse cross-section of the Mexican economy. This sectoral diversity ensures that the model's performance can be evaluated across different business risk profiles and capital structure patterns typical of the Mexican market.

Table 1 *Sample of Companies Across Different Sectors*

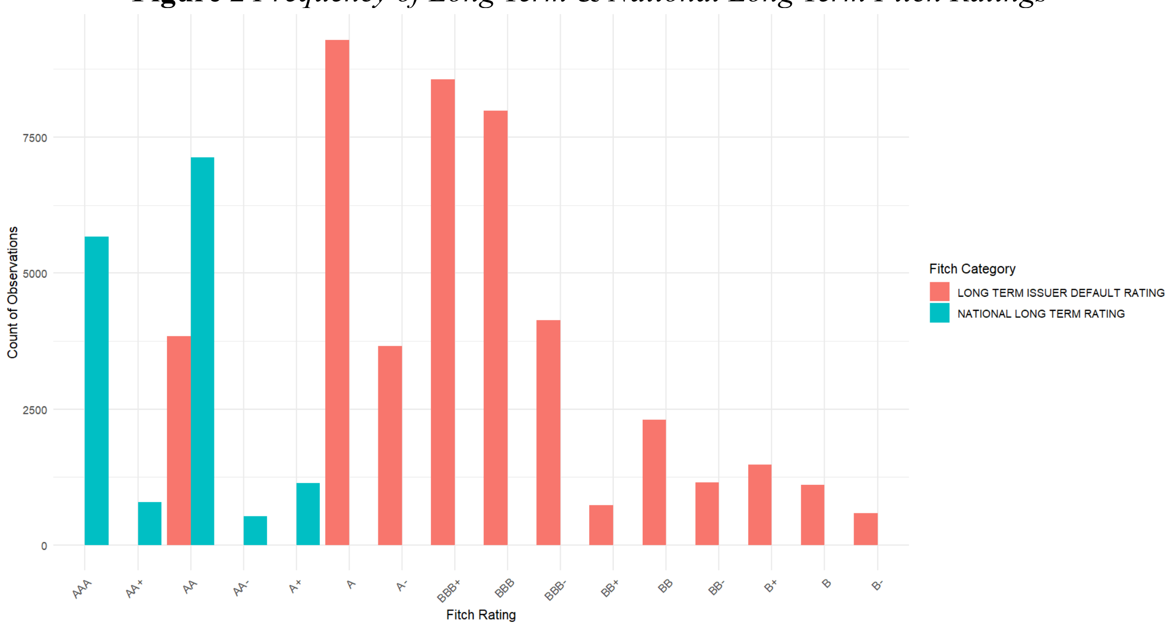
Sector	Companies
Telecommunications	América Móvil, Megacable, Grupo Televisa
Consumer Goods & Retail	Alsea, Walmart de México, Liverpool, Chedraui, FEMSA
Materials & Mining	CEMEX, GCC, Grupo México, Industrias Peñoles
Food & Beverages	Arca Continental, Becle, Coca-Cola FEMSA, Gruma, Grupo Bimbo
Real Estate	Corporación Inmobiliaria Vesta
Pharmaceuticals	Genomma Lab

Infrastructure	Grupo Aeroportuario del Centro Norte
Consumer Products	Kimberly-Clark
Diversified Conglomerate	Grupo Carso
Chemicals	Orbia Advance Corporation

Source: Own elaboration based on sectors classification from IPC S&P index.

All data was limited to the period from 2002 to 2020 due to the availability of companies' debt and outstanding shares as historical series; nonetheless, the exclusion of the COVID-19 period helps assess the model's performance outside of a global crisis such as the pandemic. Due to the limitations in the sample, the final number of companies available for the application resulted in 22 companies, from which it was possible to obtain approximately 68,000 observations for the estimation of the model parameters. In addition to 1,470 individual ratings for model validation; after matching these ratings to the rest of the daily data frame through the previously described process, a total of 60,080 ratings were successfully assigned to corresponding dates, considering that ratings were not assigned if they had been issued more than five years prior to the assigned date. The matching process resulted in an average of 279 days after the last available rating for each assigned observation.

Figure 2 *Frequency of Long Term & National Long Term Fitch Ratings*



Note: The National Rating are skewed towards higher ratings, as the issuer ones are more balanced, but still tend to middle ratings. **Source:** Own elaboration based on data from Fitch Ratings.

5 Model Implementation

5.1 Overview of the Löffler Iterative Approach

The estimation of asset values and volatilities in the Merton framework presents a computational challenge. Both asset value and asset volatility are unobservable parameters, yet they are critical inputs for calculating DD measures and subsequent default probabilities. This creates what is essentially an inverse problem: the outcomes (equity values and their volatility) can be observed, but one must infer the underlying drivers (asset values and their volatilities) that generate these observable variables. The challenge is further complicated by the nonlinear relationship between equity value and asset value in the option pricing framework, where small changes in asset value or volatility can lead to disproportionately large changes in equity value, particularly for highly leveraged firms.

The iterative methodology offers several advantages over alternative estimation approaches. First, it provides enhanced stability using time-series data rather than relying on single-point estimates that may be heavily influenced by temporary market conditions or data anomalies. Second, it reduces sensitivity to market noise by incorporating multiple observations into the parameter estimation process, allowing temporary fluctuations to be smoothed. Third, it keeps estimates coherent over time by enforcing the same theoretical relationship at each step, so the resulting asset values and volatilities don't fluctuate a lot between periods.

5.2 The Iterative Estimation Algorithm

A rolling window framework forms the temporal foundation of the analysis, utilizing one-year (252 trading day) windows for parameter estimation. This window length represents a balance between competing statistical and economic considerations. The one-year horizon aligns with common practice in credit risk modeling and provides sufficient observations for meaningful statistical inference while maintaining relevance to current market conditions. Each rolling window requires validation for data sufficiency and quality before proceeding with parameter estimation, requiring at least 30 valid return observations for meaningful volatility calculations.

The iterative solution process begins with parameter initialization based on observable information and their theoretical relationships. Asset values start with the accounting identity;

the firm value equals the sum of equity and debt values. Initial volatility estimates utilize historical return calculations based on these preliminary asset value estimates. The algorithm then alternates between updating asset values using the rearranged Black-Scholes formula and recalculating asset volatility using the updated asset value series, continuing iterations until successive iterations produce minimal changes in the estimated parameters.

Convergence criteria must balance computational efficiency with estimation accuracy, typically employing thresholds based on the sum of squared differences between successive parameter estimates. The implementation includes comprehensive safeguards against numerical instability and computational failures, with boundary condition checks identifying situations where calculated values fall outside reasonable ranges. The iterative nature of the algorithm allows for self-correction, enabling the system to recover from temporary computational difficulties and find stable solutions even when initial estimates are imprecise or when data quality varies across the estimation window.

Once stable estimates of asset values and volatilities are obtained through the process the DD can be calculated, this being the central risk metric for the analysis. The DD calculation represents the central risk measure, quantifying how many standard deviations the firm's current asset value stands above the level that would trigger default, providing an intuitive assessment of financial stability that naturally accounts for both the firm's leverage position and the volatility of its underlying business operations.

The DD incorporates expected returns estimated through the Capital Asset Pricing Model, reflecting the actual economic probability of default as perceived by market participants. This approach requires the estimation of systematic risk factors through beta calculations that measure the co-movement between individual firm asset returns and broad market returns, with expected return calculations following the standard CAPM relationship that incorporates the risk-free rate, market risk premium, and firm-specific beta to produce expected returns reflecting both systematic and firm-specific risk factors. The risk-neutral measure uses the risk-free rate in place of expected returns, providing default probabilities that align with market pricing principles rather than economic fundamentals. The risk-neutral framework ensures that calculated default probabilities are consistent with the option pricing theory that underlies the entire Merton model framework, facilitating direct comparison with other derivatives-based risk measures and market pricing models.

Default probability calculations convert the DD measures into probabilities through the cumulative standard normal distribution function, if the standardized DD follows a standard normal distribution under the respective probability measures. This transformation provides intuitive probability estimates that can be directly interpreted as the likelihood of default over the specified time horizon, enabling straightforward comparison with other probability-based risk measures and facilitating integration with broader risk management frameworks. After converting the DD's into probabilities, they didn't result intuitive because of the low values generated through this process, almost none resulting in a probability above 1%, so for better clarity the results are presented in DD which are values strictly positive and more intuitive to read.

5.3 Implementation Robustness and Quality Control

Memory management becomes critical when processing multiple companies over extended time periods, requiring careful optimization of data structures and processing sequences. The solution involves processing companies individually, with strategic use of garbage collection and incremental result aggregation to manage computational constraints effectively. Data quality control operates at multiple levels, from individual observation validation to company-level assessments and dataset-wide consistency checks. Window-level validation assesses each rolling window's data quality individually, ensuring adequate observations and reasonable data ranges before proceeding with parameter estimation. Company-level assessments track success rates and identify systematic issues that might indicate fundamental problems with data quality or model applicability. Robust measures address the inherent challenges of working with emerging market data, including missing observations, extreme market conditions, and varying data quality across companies and time periods.

The modular architecture facilitates validation and sensitivity analysis by enabling individual components to be modified or tested independently. This design approach supports systematic evaluation of different modeling choices and parameter specifications, enabling comprehensive assessment of model sensitivity to various assumptions and implementation decisions. The comprehensive output structure captures primary risk measures alongside diagnostic information and quality indicators, providing the metadata necessary to support subsequent analysis and interpretation while maintaining full traceability of the estimation process and parameter choices.

5.4 Model Results

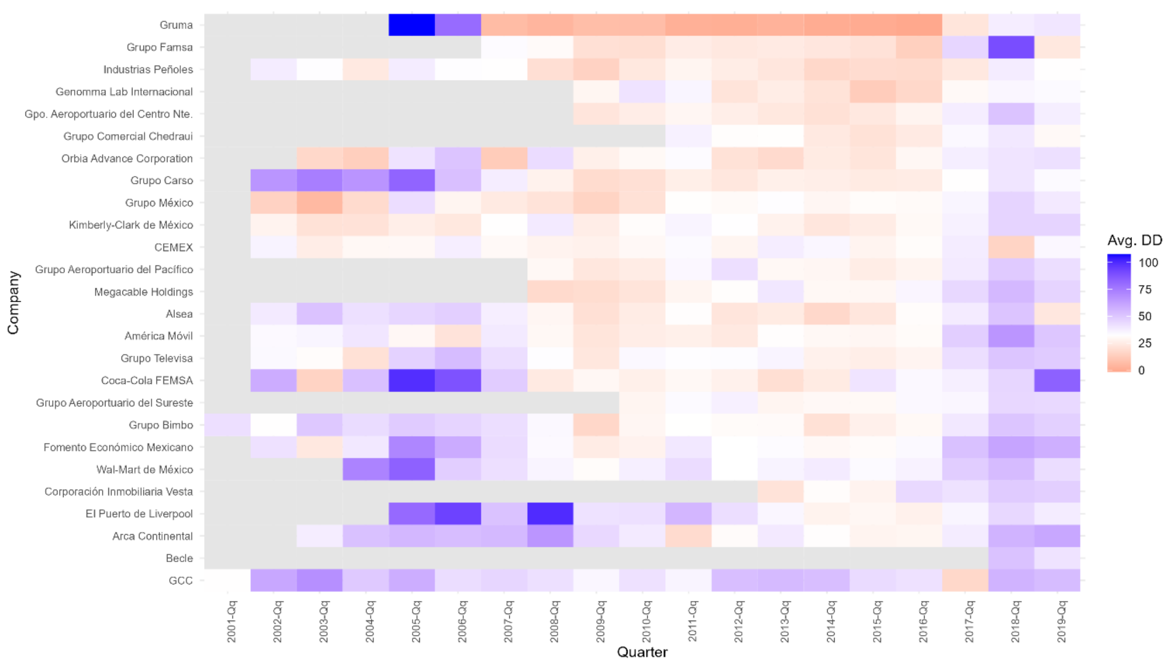
Figure 3's daily, rolling-window series offer a high-resolution view of how distance-to-default responded to key macro-financial events and idiosyncratic firm shocks. During the 2008–09 crisis, almost all firms experienced dramatic compressions in their DD's before embarking on staggered recoveries in the subsequent years. Capital intensive issuers such as CEMEX and Grupo México display the deepest troughs and most pronounced volatility, reflecting their greater leverage sensitivity, whereas large conglomerates like América Móvil and Wal-Mart de México retain relatively better stability. Commodity companies as Industrias Peñoles exhibit persistent cyclical swings tied to price cycles, and consumer-oriented firms show smoother, more stable trajectories.

Figure 3 *Distance-to-Default by Company*

Note: Not all companies in the sample have the same start, therefore the variation in the beginning of each company sample. **Source:** Own elaboration based on the distance-to-default from own calculations.

Figure 4 aggregates daily distance-to-default values into quarterly averages across all firms, arranged by sector to facilitate direct comparisons over the credit cycle. The intense red hues during 2007–2009 confirm the systemic nature of the crisis, while the shift back to cooler blues after 2010 reflects widespread balance-sheet repair. For example, consumer-focused firms rapidly regain high distance-to-default levels, whereas capital-heavy groups linger in mid-range territory. The consistent patterns between the two visualizations validate the Löffler iterative estimates: the daily dynamics captured in Figure 3 aggregate naturally into the broader credit-cycle rhythms and sectoral resilience profiles revealed by the quarterly averages in Figure 4. Together, they underscore both the sensitivity of structural risk metrics to market shocks and their utility for benchmarking credit health across firms and through time.

Figure 4 *Distance-to-Default by Company and Quarter*



Source: Own elaboration based on quarterly averages of the distance-to-default from own calculations.

6 Model Validation

The validation framework evaluates the metric through two complementary approaches. The first approach examines descriptive concordance, measuring how frequently DD's firm rankings align with established external benchmarks, with particular emphasis on relative ordering rather than absolute default probabilities. This focus deliberately sidesteps the well-documented challenges that structural models face in generating precise default probabilities, instead emphasizing DD's capacity to reproduce the hierarchy of credit risk.

The second validation strand employs forward-looking performance analysis to examine whether firms deemed safer by DD today actually demonstrate superior equity performance in subsequent periods. This approach tests whether DD captures forward-looking information that markets value, providing evidence of genuine predictive content rather than merely reflecting historical relationships or statistical artifacts. By anchoring concordance analysis in pairwise comparisons and predictive power assessment in return differentials, this dual validation framework clearly delineates when DD can serve as an effective screening tool and when it provides meaningful signals for portfolio construction and risk management decisions.

This validation addresses the distinct investor questions that any structural credit risk measure must ultimately satisfy. How capable is this model to order a company relative to others? And is this model useful to generate profits? Together, these validation components establish the specific contexts where DD adds genuine analytical insight and identify situations where its inherent limitations warrant caution or supplementation with additional risk measures.

6.1 Swap Matrix Methodology and Fitch Ratings Benchmark

The Swap Matrix methodology operationalizes descriptive concordance systematically enumerating every possible pair of firms in the sample and recording whether DD and the external benchmark agree on which firm exhibits lower credit risk. For each pair of companies (i, j), a "match" occurs when both methods rank the same firm as safer, while disagreement constitutes a mismatch. This pairwise comparison approach provides an intuitive interpretation: When three firms produce two matches out of three possible comparisons, the resulting concordance of 66,7% directly quantifies ranking accuracy without requiring calibration of probability thresholds or subjective interpretation of correlation statistics. As in the following example:

Table 2 *Visual Example Swap Matrix*

Firm	Fitch Rating	DD Merton	Match?
A	AAA	15,2	✓
B	BBB	8,7	✓
C	BB	10,1	✗

- A vs. B: Fitch says A > B, Merton says A > B → Match
- A vs. C: Fitch says A > C, Merton says A > C → Match
- B vs. C: Fitch says B > C, Merton says C > B → Mismatch

Fitch Ratings serve as the primary validation benchmark because they provide comprehensive long-term issuer default ratings for the complete universe of companies listed on the Mexican Stock Exchange during the 2002-2020 analysis period. This complete coverage ensures that each pairwise comparison draws upon a consistent, market-recognized ordinal scale that aggregates extensive fundamental and market-based analysis into a unified hierarchy. Fitch's Long-Term Issuer Default Ratings represent forward-looking opinions on entities' ability to meet financial commitments, making them ideally suited for rigorous cross-firm comparison with market-based DD measures that similarly attempt to capture default risk differentials.

Application of the Swap Matrix methodology to the twenty-two BMV firms yields an average concordance score of 0,55, indicating that DD and Fitch agree on rankings in 55% of pairwise comparisons across monthly, quarterly, and annual rebalancing frequencies. Supporting rank correlation measures including Spearman's rho of 0,27 and Kendall's tau of 0,22 reinforce this moderate alignment, confirming that DD successfully captures more than half of the relative credit quality judgments embedded in professional agency ratings. The stability of these concordance metrics across different rebalancing frequencies demonstrates that DD reflects persistent structural features of credit risk, including leverage positions, asset volatility patterns, and embedded market expectations, rather than transient market noise or temporary valuation dislocations.

Table 3 *Concordance Between Merton-Löffler and Fitch Rankings*

Horizon	Swap Matrix Score	Spearman's	Kendall's	Pearson's
Annual	0,55	0,25	0,21	0,30
Quarterly	0,55	0,26	0,21	0,28
Monthly	0,55	10,1	0,22	0,30
Average	0,55	0,28	0,22	0,29

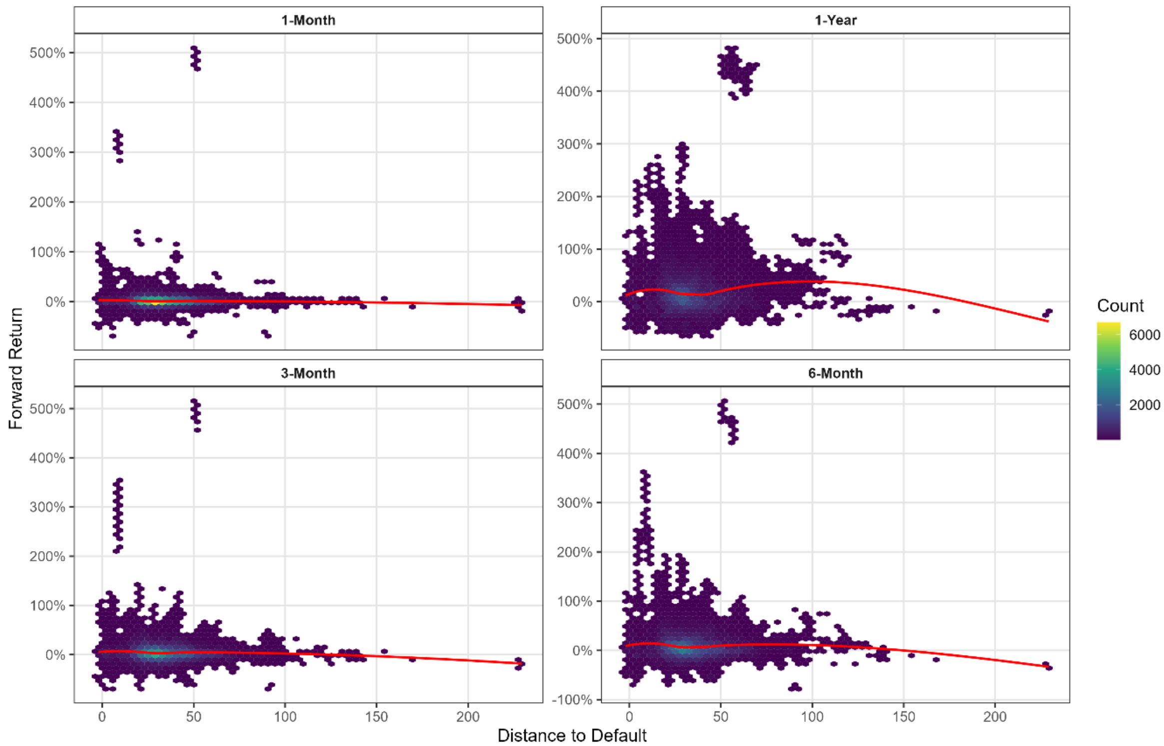
Source: Own elaboration with the model outputs.

6.2 Forward Returns Analysis and Predictive Content

The forward returns validation tests whether a meaningful risk premium can be inferred by examining whether firms with higher distance-to-default values subsequently earn superior equity returns. The economic logic underlying this test recognizes that credit-risky firms should compensate investors through higher expected returns, while safer companies should deliver more stable but potentially lower returns reflecting their reduced risk profiles. The testing methodology sorts firms each period into low, medium, and high credit risk terciles based on their DD measures, then tracks equally weighted portfolio returns for each tercile over one, three, six, and twelve-month forward-looking horizons.

Results reveal a temporal pattern in DD's predictive capacity that has important implications for practical application. Returns measured over shorter horizons of one to six months exhibit no meaningful differential between risk terciles indicating that credit risk signals require a more time to manifest equity price performance. Significant predictive capacity emerges at the twelve-month horizon, where the lowest-risk tercile meaningfully outperforms the highest-risk tercile with statistical significance. This temporal evolution suggests that while credit conditions may not immediately impact stock performance, they influence longer-term returns in concordance with the model's theoretical foundations.

Figure 5 *Distance-to-Default vs Forward Returns*



Source: Own elaboration with the model outputs.

The gradual emergence of DD-based return premiums carries several important practical implications for implementation and application. First, it validates that DD contains genuine forward-looking content that markets recognize and eventually price, but through processes that require extended periods to fully materialize in observable price movements. Second, it provides clear guidance against deploying DD for short-term trading strategies or high-frequency tactical adjustments, where its signal remains obscured by market volatility and other short-term price drivers. Instead, DD's predictive power proves most valuable for annual portfolio reviews, medium-term risk budgeting exercises, and identification of individual companies warranting deeper fundamental credit analysis within longer-term investment frameworks.

6.3 Model Limitations and Methodological Constraints

The validation exercise operates under several important limitations that constrain both the generalization of findings and conclusions about the Merton-Löffler model's practical utility. The sample composition presents a fundamental constraint, encompassing only twenty-two companies from the BMV's most liquid and established firms over the 2002-2020 period. This selection, while ensuring data quality and computational feasibility, creates potential biases that may limit applicability to broader corporate populations. Large, established companies with sufficient trading history may exhibit different credit risk characteristics than smaller, less liquid firms that constitute the majority of emerging market landscapes. These index constituents typically possess sophisticated financial reporting, regular analyst coverage, and institutional investor attention, potentially overstating the model's effectiveness when applied to companies with less transparent operations or irregular reporting practices.

The temporal scope limitation excludes the COVID-19 pandemic period and subsequent market disruptions, representing both a methodological choice and a constraint on the model's demonstrated robustness. While this exclusion enables assessment of model performance under normal market conditions, it prevents evaluation of DD's behavior during extreme stress periods when credit risk measures are most critically needed.

The validation framework relies exclusively on Fitch ratings as the external benchmark, creating several methodological constraints that may affect the interpretation of concordance results. This singular dependence assumes that Fitch's approach represents the optimal standard for credit risk evaluation, when different rating agencies may reach varying conclusions about relative risk rankings based on their distinct methodological frameworks and analytical emphasis. The approach cannot distinguish between cases where DD provides superior insight versus situations where rating agencies possess information advantages through private company interactions, regulatory filings, or industry expertise. Additionally, the reliance on specific distributional assumptions, particularly that asset returns follow geometric Brownian motion with constant volatility or may not hold during periods of structural change. These assumptions become particularly problematic when applying the model to companies experiencing significant business model evolution, major acquisitions, or fundamental operational restructuring.

The forward returns analysis suffers from limited statistical power due to the relatively small sample size and concentration of observations within specific time periods and market conditions. The identification of significant effects only at the twelve-month horizon relies on a relatively small number of independent observations across the full sample period, limiting the robustness of statistical inference. Data quality and availability constraints further limit both scope and reliability, as the requirement for complete time series naturally selects for companies with consistent reporting practices, potentially creating survivorship bias.

6.4 Integrated Validation Results and Practical Applications

The combined evidence from Swap Matrix concordance analysis and forward returns testing presents a coherent picture of DD’s capabilities and appropriate applications within the Mexican market context. The 55% concordance rate with Fitch ratings confirms DD’s suitability for screening and relative ranking applications when external ratings are unavailable, prohibitively expensive, or significantly delayed. This level of agreement substantially exceeds random chance while acknowledging that DD should augment rather than replace comprehensive credit analysis in critical decision-making contexts. The consistency of this concordance across monthly, quarterly, and annual rebalancing frequencies demonstrates that the model captures persistent structural features of credit risk.

The emergence of significant predictability only at the twelve-month horizon provides clear temporal guidance for DD deployment, validating its use in medium-term asset allocation frameworks, annual risk committee assessments, and strategic portfolio construction processes. The absence of short-term predictive power reinforces that DD captures fundamental credit characteristics that influence long-term firm performance rather than market-timing signals suitable for tactical trading strategies. This temporal specificity helps practitioners align their expectations with DD's demonstrated capabilities while avoiding inappropriate applications that could lead to disappointing results. The gradual emergence of risk premiums over extended horizons aligns with theoretical expectations about how credit risk information gets incorporated into market prices through slower-moving adjustments in investor perceptions and risk assessments.

For practical implementation, the principal lesson emerging from this comprehensive validation exercise is that DD functions most effectively as a complementary tool within broader risk management frameworks rather than as a standalone solution. In contexts demanding rapid signal identification or tactical portfolio adjustments, integrating DD with higher-frequency credit spread indicators, traditional fundamental ratios, or more responsive factor models will enhance both timing precision and overall analytical robustness. The model's transparency and market-based foundation make it particularly valuable for explaining investment decisions to stakeholders and committees who require clear, data-driven rationales for credit risk assessments. Conversely, for strategic allocation decisions, credit committee screening processes, and long-term risk assessment frameworks, DD offers a transparent, market-based analytical lens that effectively complements established benchmarks while providing additional insight into relative creditworthiness across firms and time periods, particularly in emerging market contexts where traditional credit analysis may be complicated by information asymmetries or institutional development constraints.

7. Conclusion

This study demonstrates the practical feasibility of implementing the Merton structural credit risk model in emerging market contexts through the Löffler iterative methodology. Applied to twenty-two Mexican corporations over 2002-2020, the analysis yields several findings that define the model's capabilities and appropriate applications.

The distance-to-default measure achieves a 55% concordance rate with Fitch ratings, substantially exceeding random chance while establishing clear boundaries for screening effectiveness. This moderate alignment confirms that the model captures meaningful credit risk differentials but should complement rather than replace comprehensive credit analysis in critical decisions. The temporal analysis reveals a fundamental characteristic that shapes practical deployment: while the model shows no predictive capacity over short horizons, significant differentiation emerges at the twelve-month horizon where safer firms meaningfully outperform riskier counterparts. This temporal pattern validates the model's use in strategic investment decisions while contraindicating deployment for short-term trading strategies.

The successful implementation demonstrates that sophisticated quantitative frameworks developed for mature markets can provide value in emerging market applications when properly adapted and validated. The model's transparency and market-based foundation make it particularly valuable for situations where external ratings are unavailable or delayed. The model functions most effectively as a complementary screening tool within broader risk management frameworks, particularly suited for medium-term asset allocation, annual risk assessments, and strategic portfolio construction. For emerging market practitioners, it represents a valuable addition to the credit risk toolkit when applied within its demonstrated temporal characteristics and screening capabilities. This research contributes evidence supporting the measured application of structural credit risk models in emerging markets while emphasizing the continued importance of comprehensive validation frameworks that clearly delineate appropriate model applications.

References

Banco de México. “Government Securities – (CF107).” SIE-Banxico. Accessed March 15, 2025. https://www.banxico.org.mx/SieInternet/consultarDirectorioInternetAction.do?sector=22&accion=consultarCuadro&idCuadro=CF107&locale=es.

Chan-Lau, Jorge Antonio, and Amadou Nicolas Racine Sy. “Distance-to-Default in Banking: A Bridge Too Far?” IMF Working Paper No. 06/215 (September 2006). https://ssrn.com/abstract=941277.

Christoffersen, Bruce, David Lando, and S. F. Nielsen. “Estimating Volatility in the Merton Model: The KMV Estimate Is Not Maximum Likelihood.” *Mathematical Finance* 32, no. 4 (2022): 1214–30. https://doi.org/10.1111/mafi.12362.

Fitch Ratings. Accessed April 30, 2025. https://www.fitchratings.com/.

Haugh, Martin. “The Black-Scholes Model.” IEOR E4706: Foundations of Financial Engineering lecture notes. 2016.

International Monetary Fund. *Global Financial Stability Report: The Last Mile: Financial Vulnerabilities and Risks*. Washington, DC: IMF, April 2024.

Investing.com. “S&P/BMV IPC (MXX).” *Investing.com*. Accessed March 15, 2025. https://www.investing.com/indices/ipc.

Jessen, Cathrine, and David Lando. “Robustness of Distance-to-Default.” 26th Australasian Finance and Banking Conference 2013, 2013. https://ssrn.com/abstract=2311228.

Löffler, Gunter, and Peter N. Posch. *Credit Risk Modeling Using Excel and VBA*. Hoboken, NJ: Wiley, 2007.

Merton, Robert C. “On the Pricing of Corporate Debt: The Risk Structure of Interest Rates.” *Journal of Finance 29*, no. 2 (1974): 449–70. https://doi.org/10.2307/2978814.

S&P Bloomberg. “Capital Structure Breakdown for All Listed Companies on the Bolsa Mexicana de Valores (BMV), January 2002–December 2020.” Database. Accessed March 30, 2025.

Shi, Yunkun, Charalampos Stasinakis, Yaofei Xu, Cheng Yan, and Xuan Zhang. "Stock Price Default Boundary: A Black-Cox Model Approach." *International Review of Financial Analysis* 83 (2022): Article 102284. https://doi.org/10.1016/j.irfa.2022.102284.

Various issuers. Quarterly financial reports. Company investor-relations websites. Accessed March 2025.